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PARTICIPANT DEMAND FUNCTIONS FOR
IN-KIND TRANSFER PAYMENTS

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ABSTRACT

This essay analyzes the market demand by eligible participants for in-kind transfer payments of the type that reduce the free market price participants would otherwise have to pay for the product being subsidized. Examples are the Food Stamp Program, public housing programs, and such proposed programs as rent subsidy and health co-insurance programs. Two common forms for such programs are analyzed: a variable purchase form whereby those eligible may purchase any amount of the subsidized product at the subsidized price, perhaps up to some maximum; a required purchase form whereby those eligible may purchase only a given amount of the product at the subsidized price. Both cases are examined.

The demand for a variable purchase in-kind transfer payment has many properties similar to that of the demand for the subsidized product from which it is derived. For instance, the price elasticity of demand for the in-kind transfer payment is equal to the price elasticity of demand for the subsidized product times the fraction to which the subsidized product price is reduced by the program. Further, the income and cross elasticities of demand for the in-kind transfer payment are identical to the income and cross elasticities of demand for the in-kind transfer payment. These elasticities are muted if a maximum purchase is established.

The demand for a required purchase in-kind transfer payment is likewise derived from the demand for the subsidized product. But since those eligible choose only whether to participate rather than the amount they wish to purchase, a demand to participate results. The market demand for the in-kind transfer payment is derived from the demand to participate in the program.

The antecedents of the present paper are the works of Brehm and Saving [3], Albin and Stein [1, 2] and David and Miller [4]. Brehm and Saving estimated a demand relation for general assistance payments while Albin and Stein first took issue with the Brehm and Saving work [1] and subsequently provided a complex geometrical analysis of general assistance payments [2]. The David and Miller work is a general household model with money transfer payments included, focusing upon the work-leisure choice. This essay is concerned with in-kind transfer payments.

The paper is in two parts each containing the analysis of a different demand function. These functions differ because the regulations surrounding the subsidy differ in each case. The two demand relations analyzed here stem from two currently politically popular ways of constructing subsidy programs. Undoubtedly other ways exist for constructing subsidy programs which imply other functions. The demand function analyzed in Part 2 emanates from the way the Food Stamp Program has operated from its inception to date. Such a function also flows from the regulations typical of present public housing projects. The demand relation discussed in Part 1 emanates from the way the Food Stamp Program will operate when the "variable purchase requirement" specified in the 1970 amendments to the Food Stamp Act is put into practice. However, both demand functions are general and can be used to represent the demand for any in-kind transfer with similar features.

Part 1: Demand under a Variable Purchase Option

Imagine a program that subsidizes the cost of a particular commodity for a small subgroup of the population by reducing the market price of the good by some fraction. Let the subsidized good be "f", the quantity of "f" traded be q_f, and its free market price be p_f. Let the fraction to which the market price is reduced be α, where $0 < \alpha < 1$. Suppose further that an eligible household may purchase any quantity of "f" it chooses at the subsidized price, αp_f. There could instead be a regulation permitting the purchase of any quantity of "f" below some maximum. This type of regulation is a slight simplification of the "variable purchase requirement" of the Food Stamp Program.[2] However, the former and simpler regulation will be analyzed in detail and differences between its results and those of the slightly more complicated regulation will be noted. Suppose finally that regulations are effectively enforced so that eligible households do not re-sell the subsidized good to ineligible households.

We wish to analyze the aggregate demand function for the "f"-specific money created by the supposed program. For simplicity we will call the "f"-specific money "f" stamps, denote it as "s", the quantity of "s" traded as q_s, and the price of "s" as p_s. These concepts are made operational by defining a unit of "s" as a dollar that can only be spent on "f" in the period in which it is purchased. The quantity of "s" traded is, then, participant households' total expenditures on "f" valued at the free market price of "f"; that is, (1.0) $q_s = p_f q_f$. The expenditures of participants on q_s are equal to their out-of-pocket cash outlay on q_s. Their out-of-pocket outlay is α times $p_f q_f$. Thus, (1.1) $p_s q_s = \alpha p_f q_f$. It follows from the definitions of q_s and of $p_s q_s$ that the price of "s", p_s, is equal to α; that is, (1.2) $p_s = p_s q_s / q_s = \alpha p_f p_f / p_f q_f = \alpha$.

Two examples can be given to illustrate the definitions. For the Food Stamp Program, q_s is the number of stamps (each valued at \$1.00) purchased by participant households per month. Clearly, q_s is equal to the expenditures on food the households make under the program for they redeem the stamps for food at grocery stores; thus, $q_s = p_f q_f$. And p_s, of course, is the average cost to a participant of a stamp redeemable for \$1.00 worth of food. In reality, participants are charged prices of food stamps according to their net income so that p_s in our example is a weighted average of the prices charged households of different income classes.

As another example, take a housing subsidy program under which eligible households may rent housing at say 50% of the market rent; i.e., $\alpha = .50$. Suppose the rent for a specific household is \$200 per month; i.e., $p_f q_f =$ \$200, where q_f is the quantity of housing the household purchases. The landlord collects \$100 from the participating household and subsequently collects the remainder from the housing authority (or in the established manner exchanges housing stamps for cash). In this case the participating household purchases $p_f q_f = q_s =$ \$200 of housing stamps for which it pays \$100. The price of a \$1.00 "housing stamp" is, therefore, \$.50.

Clearly, the demand for "f"-stamps is intimately related to the demand for "f". The relationship is made clear in equation (1.0). If q_s can be regarded as the quantity of "s" demanded per eligible household and q_f as the quantity of "f" demanded per eligible household at the subsidized price, then the demand for "s" per eligible household is simply the demand for "f" times the market price of "f". Further, if the demand for "f" is a function of the subsidized price, $p_s p_f$; the price of other goods, p_o, (p_o may be regarded as a vector of the prices); and average eligible household income,

1^e; then so is the demand for "s".[3] Thus, (1.3) $q_s = p_f\,[g(p_s p_f,\ p_o,\ 1^e)]$ represents the demand for "s" per eligible household where g is the analagous demand for "f". We now investigate the function.

<u>Responses to changes in p_s</u>

Assume a small reduction p_s; i.e., an increase in the subsidy. So long as "f" is a normal good, q_f will rise and with it q_s. Hence, (1.4) $\partial q_s/\partial p_s = g_f {p_f}^2 < 0$, where $g_f = \partial q_f/\partial p_f$ and p_f is independent of p_s.[4]

If we denote the price elasticity of demand for "s" as η_{ss} and price elasticity of demand for "f" as η_{ff}, then (1.5) $\eta_{ss} = p_s \eta_{ff}$.[5]

The price elasticity of "s" also equals the subsidized price elasticity of "f", η^*_{ff}, where the subsidized price is $p_s p_f$.[6] The interpretation of η_{ss} is clearest through its relationship to η^*_{ff}. Elasticity is, of course, a shorthand method of discussing changes in total expenditures given changes in exogenous variables. Total expenditures on "f" by recipients are $p_s p_f q_f$ and are by definition equal to total expenditures on "s", $p_s q_s$. Obviously then, a 1% change in p_s will affect total expenditures on "f" and "s" identically. Thus, $\eta_{ss} = \eta_{ff} = p_s \eta_{ff}$. Now consider the source of the difference between η_{ff} and η_{ff}. If p_s changes by 1% then the subsidized price, $p_s p_f$ changes by 1%. But since $p_s p_f$ is 100 percent of p_f, a one percent change is $p_s p_f$ is only 100 percent of an analagous one percent change in p_f. Consequently, the subsidized price elasticity of "f" and thus the price elasticity of demand for "s" are 100 percent of the price elasticity of demand for "f". Thus, $\eta_{ss} = p_s \eta_{ff}$.

<u>Responses to changes in p_f</u>

Since $q_s = p_f q_f$, the response of q_s to a change in p_f is similar to the response of total revenue to a change in the price. Consequently, whether

q_s rises or falls as p_f rises depends on the price elasticity. Thus, (1.6) $\partial q_s/\partial p_f = q_f\ (1 + p_s\eta_{ff})$ assuming again that p_s and p_f are independent.[7]

If the demand for "f" is price inelastic, a small drop in the market price of the subsidized good will bring about a reduction in the quantity of "s" demanded. But if the demand for "f" is price elastic, then the response of q_s to a change in the market price of "f" depends on the size of the subsidy, $(1 - \alpha)$; the larger the subsidy, the more likely a fall in the price of "f" will result in a decline in the quantity of "s" demanded.

To express the relationship in relative terms, denote the elasticity of demand for "s" with respect to p_f as η_{sf}. Then, (1.7) $\eta_{sf} = 1 + p_s\eta_{ff}$, assuming p_s and p_f are independent.

<u>Responses to changes in p_o</u>

The effect of a change in the price of another good, p_o, on the demand for "s" depends on whether "f" and "o" are complements or substitutes. If "f" and "o" are substitutes, then a rise in p_o will lead to a rise in the demand for "f" and also for "s". If "f" and "o" are complements the opposite will occur. Thus, (1.8) $\partial q_s/\partial p_o = p_f g_o \gtreqless 0$ according as "f" and "o" are substitutes, neutral, or complements respectively. Denote the cross elasticity of demand for "f" and "s" as η_{fo} and η_{so} respectively. In this case the two elasticities happen to be identical; thus (1.9) $\eta_{so} = \eta_{fo}$.[8]

<u>Responses to changes in 1^e</u>

A rise in average eligible household income will increase the demand for "f" and with it the demand for "s". Hence, (1.10) $\partial q_s/\partial 1^e = p_f g_1 > 0$. As with the cross elasticities, the income elasticities of demand for "s" and "f" happen to be equal; i.e., (1.11) $\eta_{s1} = \eta_{f1}$.[9]

Responses to other variables

The demand for "s" is more than a function of the demand for "f", however. There are two other classes of variables that are influential in determining the demand for "s". First are those variables on which eligibility is defined while the second class emanate from the certification and subsidy receipt procedures and locations.

Net household income, is perhaps, the most common of the eligibility criteria variables. Assets is another common variable as is employment status. Changes in these eligibility variables or in the ranges of the variables which define eligibility will swell or shrink the number of eligible households and so swell or shrink the total demand for "s". Only if the households becoming eligible or losing eligibility given such changes have different demands for "f", will the relationships between q_s and the variables discussed above change from what has been postulated.

The relationship between changes in average household income and the total demand for "s" if income is an eligibility criterion is not at all clear. Average household income may rise, for instance, removing some once-eligible households from the ranks of the eligible. The decline in the number of eligible households will reduce the total demand for "s" at the same time the rise in the incomes of those remaining eligible will increase the demand for "f" and so for "s" also. Whether the net change in the total demand for "s" is positive or negative depends on the magnitude of the increase in income and the income distribution of households. Changes in other eligibility variables have similar effects.

The second class of variables affecting the demand for "s" but not for "f" has to do with the certification and subsidy receipt procedures and locations as well as the stigma or prestige attached to receipt of the subsidy.[10] Participation in a subsidy program requires the eligible household to depart from its accustomed behavior and to go places and to do things it might not otherwise. Thus, participation requires expenditures of time and cash as well as perhaps the use depreciation on durables such as automobiles in order to become certified as eligible, to maintain certification, and to take receipt of the subsidy when, where, and in the fashion officials, agencies, and/or the law require.

Clearly, the expenditures of time, money, and status required to participate can be so high as to make participation not worthwhile and thus reduce the total demand for "s". Just as clearly, certification procedures and locations and the locations and the manner in which the subsidy is received can be arranged so that these costs are minimized so that participation rises and with it the total demand for "s". Whether the per eligible household demand for "s" is altered by these variables (i.e., the responses with respect to changes in p_s, p_f, p_o, and 1^e) is a moot question. Certification and subsidy receipt procedures and locations appear to be largely a matter of local rather than national determination and hence quite variable in a national program. Since the paper concentrates on the aggregate demand relation, these are not discussed further.

Finally, one should note the effects of variations in the effectiveness of enforcing regulations preventing re-sale of the "f"-stamps or of the "f" purchased with "f"-stamps. To the extent that regulations are not enforced, then it becomes a profitable business to exchange "f"-stamps for money or to sell the "f" purchased with "f"-stamps. In consequence, the price elasticity of demand for "s" becomes quite elastic; a reduction in p_s will call forth a greatly increased demand for "s".[11]

A variable purchase option with a maximum

Before concluding this part, let us briefly analyze the option allowing households to purchase up to some maximum quantity of "f" at the subsidized price. Such a regulation alters the results stated above only when the maximum constrains the behavior of at least some of the participating households. So long as there are households which do not purchase the maximum quantity of "f", the directions of the responses to changes in p_s, p_f, p_o, and l^e are the same as were discussed. But the magnitudes of the changes will be muted as will the elasticities of demand of "s". If the maximum constrains all participants, then no positive changes in q_s will be observed with changes in the exogenous variables. Changes in eligibility variables or criteria, of course, might evoke positive changes in the total quantity of q_s demanded as would changes in certification and subsidy receipt procedures and locations.

Part 2: Demand under a Purchase Requirement Option

Imagine now a program identical to that described above with one important exception. Rather than being allowed to purchase any quantity of the subsidized good at the subsidized price, the household is required to purchase a specified quantity of "f", q_{fr}, or not participate at all. In essence, the eligible household has a choice of: (a) buying q_{fr} at the subsidized price, $p_s p_f$; or (b) not buying any "f" at $p_s p_f$ and purchasing the quantity of "f" it chooses at the market price, p_f. Its problem is to decide whether to participate rather than to decide how much "s" to consume.

Since the individual household decides only whether to participate, the concept of the demand to participate in the program is relevant at the household level rather than the concept of the demand for "s". At the aggregate level, however, both concepts are relevant. The demand to participate is directly reflected in aggregate participation and the number of participants can be regarded as a measure of the aggregate quantity of program participation demanded. The aggregate quantity of "s" demanded at a given price of "s" is simply the quantity of "s" each eligible household must purchase to participate, $p_f q_{fr}$ multiplied by the number of participants.[12]

The household's participation decision must be analyzed, therefore, and the linkages traced between it and the aggregate demands for participation and for "s". We begin by assuming that eligible households maximize satisfaction. We imagine that an eligible household calculates the satisfaction to be derived from participating in the program and also that to be obtained by not participating. It then chooses to participate if participation yields the greater satisfaction.

To formalize this idea denote the gain from participation to the i^{th} eligible household as G_i, the total utility obtainable by participating in the program as V_{ri}, and the total utility obtainable by not participating as V_{wi}. Then, (2.0) $G_i = V_{ri} - V_{wi}$ $(i = 1, 2, \ldots, N)$ where N is the number of households eligible to participate in the program. If $NG_i > 0$, household i participates whereas if $NG_i \leq 0$, then it will not.

Variables important in determining G_i are the subsidy and purchase requirement. The subsidy increases satisfaction and disposes the household toward participation. It does so through the "income effect" of the reduction in the price in "f" to participants. The subsidy lowers p_f to

$p_s p_f$ and so allows the participant to purchase more of both "f" and other things, increasing real income and thereby total satisfaction.

The purchase requirement either reduces the added satisfaction brought about by the subsidy or does not affect satisfaction at all. The former occurs when the household is required to purchase more "f" and, therefore, fewer other things than it prefers. The purchase requirement thus reduces the household's freedom to spend its income and consequently reduces satisfaction from what it would be if there were no purchase requirement. The latter occurs when the eligible household is required to purchase no more "f" and thus no fewer things than it prefers. Consequently, the purchase requirement does not infringe on the household's freedom to spend its income and therefore does not diminish the satisfaction added by the subsidy.

Important also are the prices of other things, the price of "f" and eligible household income. The gain from participating in the program varies directly with the extent to which changes in them raise real income and thereby total satisfaction; i.e., their "income effects". The gain is diminished to the extent changes in them evoke changes in the mix of "f" and other things the household prefers to purchase which are frustrated by the purchase requirement. Variables on which eligibility criteria are based are influential too for they swell or shrink the number of eligible participants. And changes in variables emanating from certification and subsidy receipt procedures and locations change the net gain from participating also. In short, the variables entering the demand function for "f" specified in Part 1 all continue to be relevant to the present analysis.

The gain, therefore, can be written as a function of p_s, p_f, p_o, q_{fr}, and l_i. The variables implied by certification and subsidy receipt procedures and locations can be collected into a random disturbance term, e_i. Hence, (2.1) $G_i = h\ (p_s, p_f, p_o, q_{fr}, l_i) + e_i$ $(i = 1, 2, \ldots, N)$ where e_i has a $N\ (0, \sigma^2)$ distribution.[13] Equation (2.1) implies that eligible households differ only in their reactions to the variables collectively represented by e_i.[14]

Equation (2.1) reveals the variables postulated to determine the participation behavior of the i^{th} household. Ceteris paribus changes in the arguments of h increase or decrease G_i. If G_i turns positive the i^{th} household will begin to participate. If G_i turns negative or zero, it will cease participating. If G_i remains positive (negative or zero) given the change, the household will not change its participation status.

The link between the participation behavior of the i^{th} household and aggregate participation can be specified by defining an index of participation, P_i, such that

$$(2.2) \quad P_i = \begin{cases} 1, & \text{if } G_i > 0; \\ 0, & \text{if } G_i \leq 0. \end{cases} \qquad (i = 1, 2, \ldots, N)$$

As G_i turns positive (negative or zero), P_i becomes one (zero). P_i can be aggregated over all eligible households to give the number of participants; i.e., (2.3) $P_{\cdot} = \Sigma_i P_1$. $(i = 1, 2, \ldots, N)$ Changes in $P_{\cdot}$ according to equation (2.2) reflect sign changes in the gains obtainable by individual eligible households and these, as postulated in equation (2.1) are a function of p_s, p_f, p_o, q_{fr}, and l_i or random shocks introduced by changes in certification and subsidy receipt procedures and locations.

Equation (2.3) is transformed into the aggregate quantity of "f"-stamps demanded, Q_s, by multiplying $P_{\cdot}$ by the quantity of "s" each household must purchase if it participates. Thus, (2.4) $Q_s = p_f \Sigma_i q_{fri} P_i$ $(i = 1, 2, \ldots, N)$

We turn now to investigate responses in G_i, $P_.$, and Q_s to _ceteris paribus_ changes in the exogenous variables.

Responses to changes in p_s

Consider the effect on G_i of a small reduction in p_s. A reduction in p_s causes G_i to rise for at least two of three reasons. First, a reduction in p_s raises real income via the income effect and increasing the total utility obtainable from participation; thus, G_i increases. Second, if the income effect on "f" is positive, the income effect also increases the household's demand for "f", reducing the difference between q_{fr} and q_{fpi} (the subscripts r and p refer to required and preferred respectively), reducing the burden of the purchase requirement. Third, since "f" has become relatively cheaper than other things, the household will want to change the mix of "f" and "o" in favor of more "f" and so reduce the burden of the purchase requirement further.[15]

In short, both the income and substitution effects of an increase in the subsidy, $(1 - \alpha)$, act to increase NG_1. If the income effect on "f" is positive, the effect on G_i of the income effect is even stronger. By way of summary, (2.5) $\partial G_i/\partial p_s = h_s < 0$, where $h_s = \partial h/\partial p_s$.

If h_s is sufficient to turn G_i positive (negative or zero), then the i^{th} household will begin (cease) participating, bringing about an increase (decrease) in $P_.$ of one. Given that N is large there will be a number of households whose participation status will change in like fashion to a change in p_s. Thus, the effect of a change in p_s on $P_.$ has the same sign as its effect on G_i. Further, since the aggregate quantity of "s" demanded is proportional to $P_.$ in the case where $q_{frj} = q_{fr}$, and a weighted sum of P_i with the q_{frj} as weights when, $q_{frj} \neq q_{frj+m}$ $(j = 1, 2, \ldots, J)$, $(m = 0, 1, \ldots, J-1)$ then the effect on Q_s of a change in p_s also has the same sign as its effect on G_i. In short, (2.6) $\Delta P_. / \Delta p_s < 0$; and (2.7) $\Delta Q_s / \Delta p_s < 0$.

Responses to changes in p_o

A small reduction in p_o will have two effects on the gain of the i^{th} household, their directions depending on whether "f" and "o" are complements or substitutes. The income effect of a reduction in p_o will increase G_i and G_i will be increased even more if the income effect on "f" is positive.

If "f" and "o" are complements, then not only will the i^{th} household increase its demand for "o" given a reduction in p_o, but it also will increase its demand for "f". In consequence, the difference between q_{fr} and q_{fpi} will decline and with it the utility diminishing effect of the purchase requirement. If, however, "f" and "o" are substitutes, then the relative cheapening of "o" will cause a decrease in the demand for "f", increasing the difference between q_{fr} and q_{fpi}, consequently increasing the participation dampening effect of the purchase requirement.

Thus, the effect of a change in p_o on G_i will be positive (negative) as "f" and "o" are substitutes (complements). In brief, (2.8) $\partial G_i / \partial p_o \gtreqless 0$ as "f" and "o" are substitutes, neutral, or complements respectively.

Logic identical to that use in the discussion of responses to changes in p_s lead to the conclusion that changes in the aggregate demand for participation, $P_.$, and for "s", Q_s, have the same signs as $\partial G_i / \partial p_o$. Hence, (2.9) $\Delta P_{.}/\Delta p_o \lesseqgtr 0$; and (2.10) $\Delta Q_s/\Delta p_o \gtreqless 0$ as "f" and "o" are substitutes, neutral, or complements.

Responses to changes in l_i

Suppose the income of the i^{th} household increases. If the income effect on "f" is positive, then the household will demand more of it. The difference between q_{fr} and q_{fpi} consequently diminishes and G_i rises. A change in the income of the i^{th} eligible household, therefore, has a positive effect on NG_i, $P_{.}$, and on Q_s. In symbols, (2.11) $\partial G_i/\partial l_i > 0$; (2.12) $\Delta P_{.}/\Delta l_i > 0$; and (2.13) $\Delta Q_s/\Delta l_i > 0$, if income effect on "f" is positive.

The effect of a change in average eligible household income, l^e, (or, average household income, l, for that matter) on $P_{.}$ and Q_s is the same as that above if low income is not among the eligibility criteria or closely correlated with one of the eligibility criteria (unemployment, for instance). If low income is among the eligibility criteria, then the response of $P_{.}$ and Q_s to an increase in l^e (or l) depends on the number of participants that become ineligible versus the number of eligible non-participants who become participants. In short, (2.14) $\Delta P_{.}/\Delta l \lesseqgtr 0$; (2.15) $\Delta Q_s/\Delta l \lesseqgtr 0$.

Responses to changes in p_f

A small reduction in p_f induces an increase in real income and with it total utility. If the income effect on "f" is positive, its demand will increase. It will be further increased by the substitution effect as the relatively cheaper "f" is substituted for relatively more expensive other things. Thus, the difference between q_{fr} and q_{fpi} diminishes with a reduction in p_f, reducing the burden of the purchase requirement, and therefore, increasing G_i. The direction of the resulting change in $P_{.}$ is the same. Thus, (2.16) $\partial G_i/\partial p_f < 0$; and, (2.17) $\Delta P_{.}/\Delta p_f < 0$.

The effect of a change in p_f on Q_s is ambiguous because Q_s is equal to $p_f q_{fr} P_.$. It turns out that the effect of a change in p_f on Q_s depends on the elasticity of demand for participation with respect to p_f; i.e., $\eta_{P_{p_f}}$. Hence, (2.18) $\Delta Q_s/\Delta p_f = q_{fr} P_. \ (1 + \eta_{P p_f}) \gtreqless 0$ as $/\eta_{P_{pf}}/ \gtreqless 1$. In other words, the more responsive is participation with respect to changes p_f, the more likely the effect of p_f on Q_s is negative.[16]

Responses to changes in q_{fr}

Suppose the purchase requirement is reduced by a small amount. The difference between q_{fr} and q_{fpi} is reduced and with it the burden of the requirement for those households whose behavior was constrained by the requirement. Thus, the gain increases for households who are constrained by the requirement. For those households for which $q_{fr} \leq q_{fpi}$ before the change, the reduction in the constraint does not affect them. Hence, (2.19) $\partial G_i/\partial p_{fr} \leq 0$ as $q_{fr} \geq q_{fpi}$ before the change. The effect on participation of q_{fr} is negative assuming that the behavior of at least some of all participating households is constrained by the purchase requirement. Hence, (2.20) $\Delta P_. /\Delta q_{fr} < 0$.

The effect of changes in the purchase requirement on Q_s is ambiguous, again because of the definition of Q_s. In this instance, the direction of the effect depends on the elasticity of demand for participation with respect to the purchase requirement, $\eta_{Pq_{fr}}$. If it is elastic, then the effect of changes in q_{fr} on Q_s is negative whereas if it is inelastic, then the effect of changes in q_{fr} on Q_s is positive; i.e., (2.21) $\Delta Q_s/\Delta q_{fr} = p_f P_.$ $(1 + \eta_{Pq_{fr}}) \gtreqless 0$ as $/\eta_{Pq_{fr}}/ \gtreqless 1$.[17]

Concluding Remarks

It has been argued that an income in-kind transfer program that subsidizes the price at which a good can be purchased by eligible households creates a market for a quasi-money spendable only on the subsidized good. The quasi-money was called "f"-stamps and its market demand was analyzed under two alternative program operating procedures: that eligible households could purchase as much "f" at the subsidized price as it chose; and, that an eligible household was required to purchase a specified quantity of "f" at the subsidized price or not participate at all.

In both cases the market demand for "f"-stamps turned out to be a function of the demand for "f", a set of variables on which eligibility criteria are based, and variables emanating from the certification and subsidy receipt procedures and locations. Under a required purchase option there exists an additional demand relation: that of the demand by eligible households to participate in the program. The paper concentrated its analysis on the variables that enter the demand for "s" and the demand for participation via the demand for "f". This was done because the demands at the federal level were of concern and these variables along with the eligibility variables were postulated to be most important at that level.

There do not appear to be any grave problems of estimating the market demand functions for "s" and for participation. Under the variable purchase option the parameters and elasticities of the demand for "s" can all be derived from a knowledge of the demand for "f" on the part of participating households. The reverse also holds. This is not the case under the purchase requirement option. As constructed both the demand for "s" and the demand for participation under the required purchase option have stochastic elements. Further, the central limit theorem suggests that both $P_{\cdot}$ and Q_s have normal distributions despite the fact that they are built up from a variable with a binomial distribution. [See: 5; chap. vii]. Finally, while estimated functions for the demand for "s" under a variable purchase option can be homogeneous, the demands for "s" and for participation under the required purchase option cannot be. This is so because the random disturbance term of $P_{\cdot}$ and of Q_s have nonzero means which are included in the intercept terms of the estimated functions.

It should be noted that the participant demand functions that have been analyzed do not represent the total demands for "s" and for participation. Since "s" is a transfer payment, it is a public good and, therefore, there is a public component of the total demand presumably reflected by legislative appropriations for the subsidy program. No analysis of the public components of the demand functions for "s" has been attempted.

FOOTNOTES

[1]For an example of a formal model of a "public market" see Niskanen [6].

[2]In reality familes will be able to purchase 25%, 50%, 75%, or 100% of the maximum number of stamps they may purchase. They will be free to families with monthly net income of less than \$30.

[3]Let the demand function for "f" per eligible household be: (1.3') $q_f = \sigma(p_f, p_o, 1^e)$ where p_o may be regarded as a vector. If we assume that the introduction of a price subsidy only alters the price of the subsidized good and not the demand relation itself, then (1.3") $q_f = g(p_s p_f, p_o, 1^e)$ has all the properties possessed by (1.3'). Specifically, we assume: (1.3‴a) $\partial\sigma/\partial p_f = \partial g/\partial(p_s p_f) = g_f < 0$; i.e., "f" is a normal good; (1.3‴b) $\partial\sigma/\partial p_o = \partial g/\partial p_o = g_o \lesseqgtr 0$ as "f" and "o" are substitutes, neutral or complements; (1.3‴c) $\partial\sigma/\partial 1^e = \partial g/\partial 1^e = g_{1^e} > 0$.

[4]If p_s and p_f are not independent, then (1.4') $\partial q_s/\partial p_s = g_f p_f^2 + (\partial p_f/\partial p_s)(p_f)(q_f + p_s g_f)$. Note: $0 < p_s < 1$; $g_f < 0$ if "f" is a normal good; and $q_f > 0$. Hence it is quite unlikely that $(q_f + p_s g_f) \lessgtr 0$. Also note that $p_f > 0$. It would be strange if $\partial p_f/\partial p_s > 0$ for this would mean an increase in the subsidy, $(1 - \alpha)$, forces the market price of "f" down; the reverse is far more likely. Hence, in all probability $\partial p_f/\partial p_s \leq 0$. If the equality holds then (1.4') collapses to (1.4). If the inequality holds, then it is likely that $\partial q_s/\partial p_s < 0$. Hence, even if p_f varies with p_s, the demand for "s" is most likely to be downward sloping so long as "f" is a normal good.

[5]$\eta_{ss} = (\partial q_s/\partial p_s)(p_s/q_s) = g_f p_f^2 p_s/q_s = p_s g_f p_f/q_f$. Since $\eta_{ff} = g_f p_f/g_f$, then, $\eta_{ss} = p_s \eta_{ff}$.

[6]$\eta_{ff}^* = (\partial q_f/\partial(p_s p_f))(p_s p_f/q_f) = g_f p_s p_f/q_f = p_s \eta_{ff}$ where η_{ff}^* denotes the subsidized price elasticity of demand for "f".

[7]If p_s and p_f are not independent, then (1.6) becomes (1.6') $\partial q_s/\partial p_f = q_f(1 + p_s\eta_{ff}) + p_f q_f(\partial p_s/\partial p_f)\eta_{ff}$. In footnote 3 it was argued that $\partial p_f/\partial p_s \leq 0$. If the equality holds, then (1.6') collapses to (1.6). If the inequality holds and since $\eta_{ff} < 0$ and $p_f q_f > 0$, then the last term on the right-hand side of (1.6') is positive. The first term on the right-hand side of (1.6') is discussed in the text.

[8] $\eta_{so} = (\partial q_s/\partial p_o)\ (p_o/q_s) = p_f g_o p_o/p_f q_f = g_o p_o/g_f = \eta_{fo}$

[9] $\eta s_{1e} = (\partial q_s/\partial 1^e)\ (1^e/q_s) = p_f g_{1^e} 1^e/p_f q_f = g_{1e} 1^e/q_f = \eta_{f1e}$.

[10] See Weisbrod [7] for an economic analysis of the stigma cost of welfare payments.

[11] I am indebted to B. Weisbrod for this point.

[12] Most programs require different classes of households to purchase different quantities of "f". Family size is a typical variable by which q_{fr} varies. Incorporating this complication into the analysis poses no problems.

[13] NG_i can be shown to be a function of h ( ) as follows. Assume a two good world, "f" and "o", with notation as defined elsewhere. Let the utility function of the typical eligible household be defined on "f" and "o"; i.e., (2.1'a) $U = u(q_f, q_o)$ Assuming purchases of "f" and "o" exhaust household income a Lagrangean expression can be formed as follows: (2.1'b) $V_w = u(q_f,\ q_o) + \lambda_1^w (p_f q_f + p_o q_o - 1)$. The maximized value of (2.1'b) is the household's total utility from remaining without the program. If it were to participate in the program then the household faces an additional constraint composed of the purchase requirement; i.e., $p_f q_f \ p_f q_{fr}$. Furthermore, the subsidized price, $p_s p_f$, replaces the market price, p_f, in the income constraint. Thus, (2.1'c) $V_r = u(q_f,\ q_o) + \lambda_1^r (p_s p_f q_f + p_o q_o - 1) + \lambda_2^r\ (p_f q_f - p_f q_{fr})$. It is assumed that the eligible household maximizes each and compares V_r and V_w. If $V_r > V_w$ _ceteris paribus_, then it participates. If $V_r \leq V_w$ _ceteris paribus_, it remains without the program. It is clear from equations (2.1'b) and (2.1'c) that $V_r - V_w$ and, therefore, NG are functions of p_s, p_f, p_o, q_{fr}, and 1.

[14] This is a simplification. Indeed most in-kind transfer programs with which the author is familiar are based on the assumption that h in equation (2.1) varies systematically with at least household size and income. Thus, consider J household sizes (j = 1, 2, ..., J) and K household income classes (k = 1, 2, ..., K), for instance. Then equation (2.1) is written as: (2.1) $G_{ijk} = h_{jk}(p_{sk}, p_f, p_o, q_{frj}, 1_i) + e_i$; where $h_{jk} \neq h_{j+m,\ k+n}$ (m = 0, 1, ..., J-1) and (n = 0, 1, ..., K-1). Equation (2.1') incorporates the assumption that the price of "s" varies with household income classes and the purchase requirement varies with household size classes. This is in fact the case with respect to the Food Stamp Program in which the price of stamps rises with eligible household income and the purchase requirement rises with household size.

[15]The analysis here as elsewhere is conducted on the assumption that the purchase requirement is effective ($q_{fr} > q_{fpi}$). If it is not, then it does not affect behavior at all. The results are no different under this alternative assumption.

[16]This is shown as follows. Assume $P_{.}$ is continuous and therefore the calculus applies. (If the Δ notation is retained, the notation is laborious but the result is the same.) Then, $\partial Q_s/\partial p_f = \partial(p_f q_{fr} P_{.})/\partial p_f = q_{fr}\,(P_{.} + p_f \partial P_{.}/\partial p_f)$. Define $\eta_{Pp_f} = (\partial P_{.}/\partial p_f)\,(p_f/P_{.})$ and then $p_f \partial P_{.}/\partial p_f = P_{.}\eta_{Pp_f}$. Substituting $P_{.}\eta_{Pp_f}$ for $p_f \partial P_{.}/\partial p_f$ above yields equation (2.18).

[17]Given that $\eta_{Pq_{fr}} = (\partial P_{.}/\partial q_{fr})\,(q_{fr}/P_{.})$, logic similar to that in footnote 16 yields equation (2.21).

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