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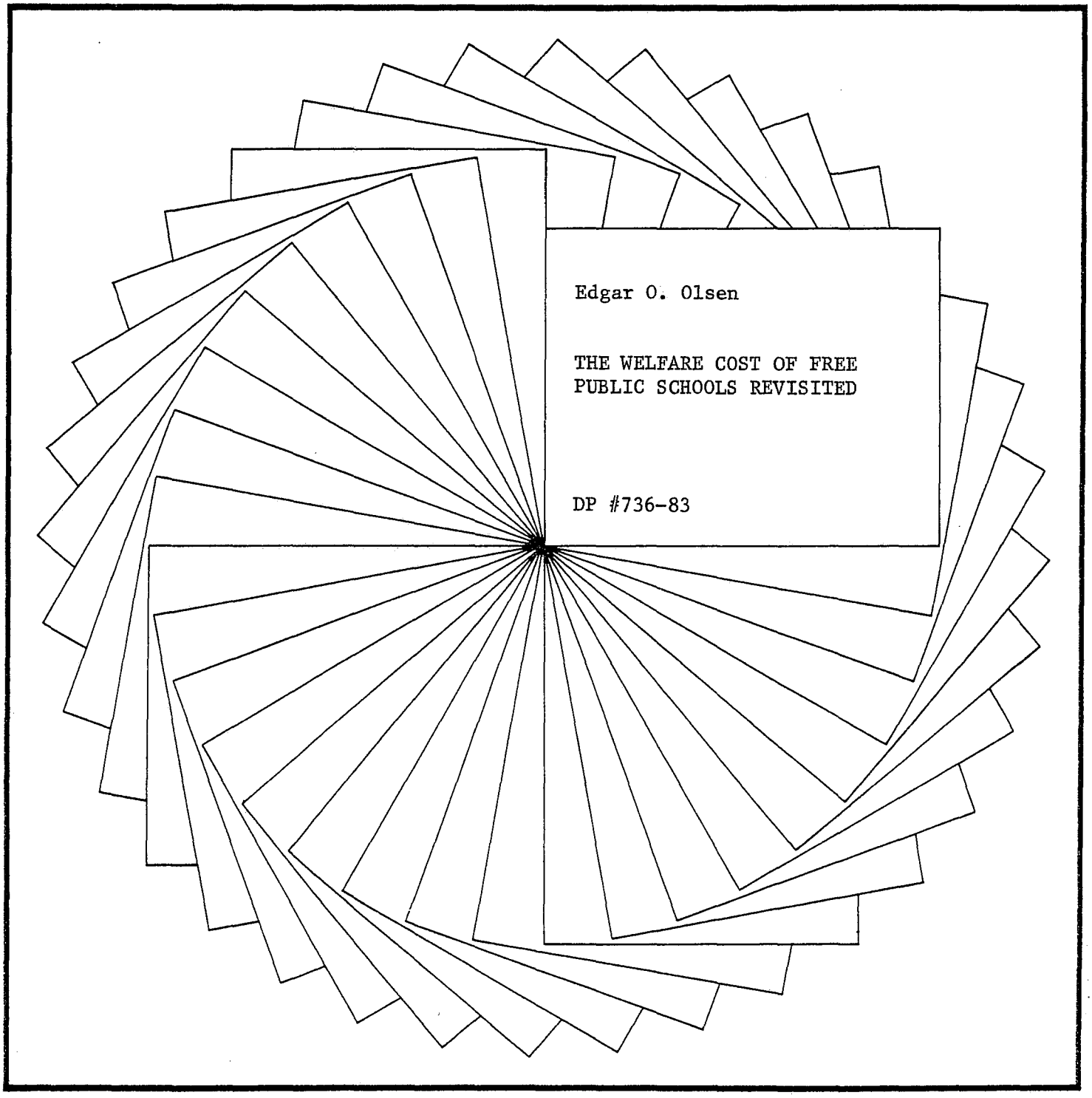

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The Welfare Cost of Free Public Schools Revisited

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Abstract

Sonstelie (1982) has produced a promising approach to estimating the effects of the public school system on the allocation of resources and the distribution of well-being. This paper points out some potentially serious problems with his methodology and suggests how they can be eliminated.

THE WELFARE COST OF FREE PUBLIC SCHOOLS REVISITED

In a pathbreaking article, Sonstelie (1982) presents a method for estimating the efficiency with which services are produced in public schools and the excess of the cost of the system over its direct benefit to the parents of public school students. His approach could be extended to answer many other important questions about the public school system, such as the effect on consumption patterns of replacing it with educational vouchers for private school attendance. Although more than $100 billion is spent on public schools each year, little is known about the effects of the system or the effects of replacing it with alternative government programs (Blaug, 1978).

While Sonstelie offers a promising approach to answering a broad range of questions about public schools, his methodology can be improved in a number of respects. These include the measure of welfare cost, its decomposition into technical inefficiency and consumption distortion, and the recognition of a relationship between the functional forms of his two estimated equations that will eliminate a major problem with his method for estimating the productive efficiency of public schools. This paper will explain the problems with Sonstelie's treatment and suggest how these problems can be eliminated.

SONSTELIE'S MODEL

To understand the problems with these aspects of Sonstelie's methodology, it is desirable to review the assumptions and conclusions of the underlying model. In the process, I will make explicit many of the assumptions that seem to be implicit in his analysis. This will suggest other ways in which the model can be refined.

Assume that the quality of education is the same in all public schools in a school district and that this quality is decided by a vote. Each household has one child in school, though not necessarily in a public school, and the same number of voters. Each voter is required to state his or her preference concerning the quality of public education. The median of the most preferred qualities will be the outcome of the voting process.

Assume that each household cares about the quality of education received by its child and its consumption of other private goods. Except for elementary and secondary education, the goods and services provided by governments and the taxes necessary to finance them are assumed to be fixed. Assume that households have identical tastes.

Assume that income is not subject to individual choice and that each household pays a fixed share of the total expenditure on the public schools in its district. This fixed share is larger for richer families within a district. For a given number of public school students, total government expenditure on education is assumed to be proportional to public school quality. For a given public school quality, it is assumed to be proportional to the number of public school students. Assume that there are no intergovernmental grants for education and that the vector

of market prices for education and other goods is the same everywhere. Assume that there is no charge to send a child to a public school, but it is impossible to supplement the quality of education received therein.

In equilibrium, there is a quality of public schools for each household such that the household will send its child to a public school if and only if actual quality is greater than this quality. Sonstelie assumes that this magnitude, called the reservation quality, increases with after-tax income, and his estimates support this belief. This implies that the richest households will send their children to private schools and oppose expenditure on public schools. Among households that send their children to public schools, Sonstelie's estimates support the belief that the richest will vote for the highest quality public schools. Therefore, in equilibrium, the richest and poorest households will prefer less expenditure on public schools, while middle income households will favor more.

Figure 1 depicts this equilibrium. The quality of education received by a child is measured along the horizontal axis; expenditure on other goods along the vertical axis. The quality Q_e is the equilibrium quality of public schools. Household A contains a voter whose most preferred quality of public schools is the equilibrium quality. This household has an after-tax income W_a. Units of school quality are measured so that the cost of providing an additional unit of quality in public schools is \$1 per student. The total cost of providing an additional unit is equal to the number of public school students, and the tax price to this household, S_a, is its tax share multiplied by the number of students. Richer households have budget lines relevant for voting that are further from

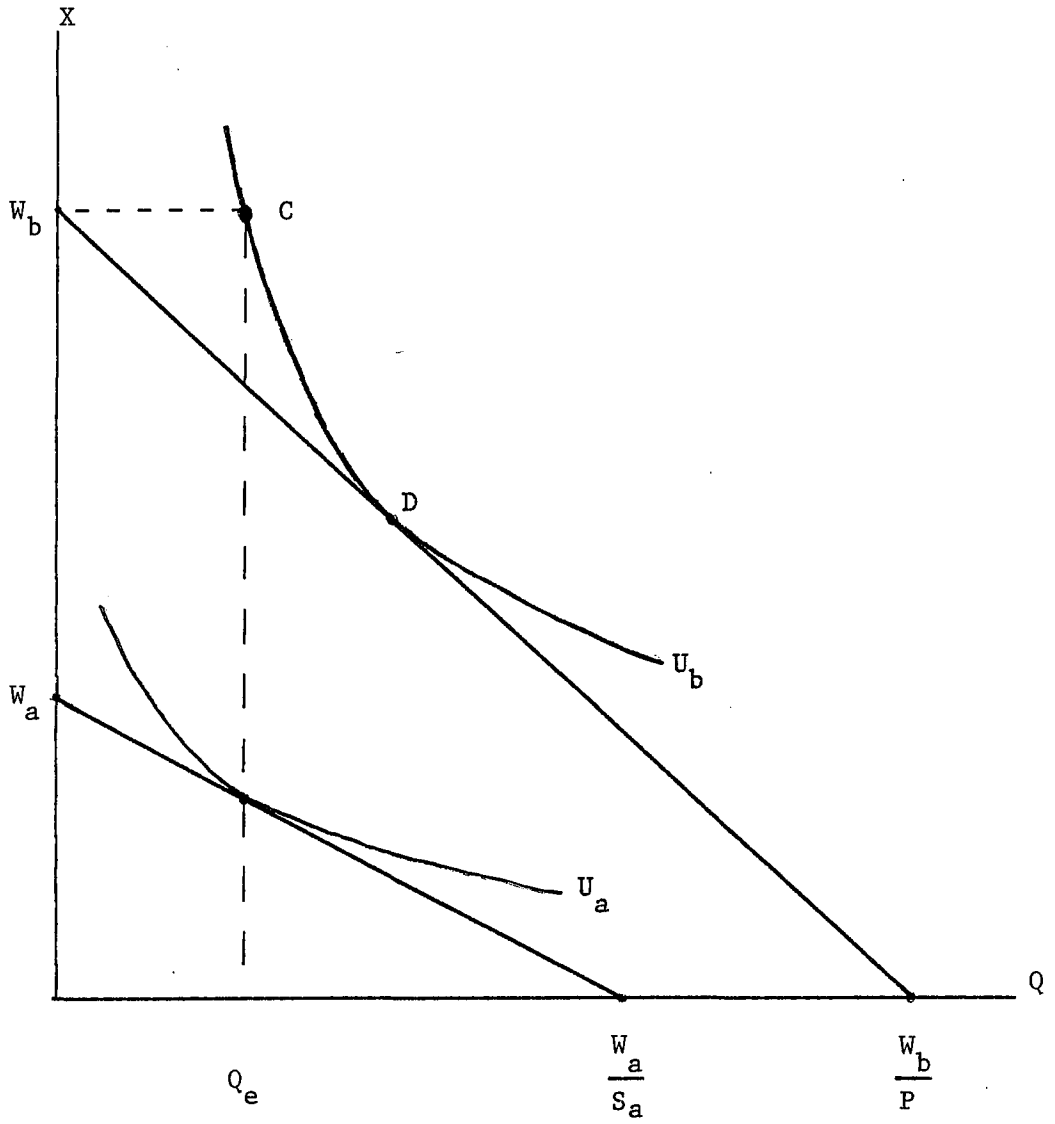


Figure 1. Equilibrium in Sonstelie's Model

the origin and steeper. Household B is a household that is indifferent between sending its child to a public or a private school. Its after-tax income is W_b. The price of school quality in the private sector is P. If the household were to send its child to a private school, its consumption bundle would be D. In the public school, it would be C. Richer households have reservation qualities greater than Q_e and so send their children to private schools.

To calculate the welfare cost and technical inefficiency of public schools, Sonstelie estimates simultaneously two equations describing the behavior of households on these two margins. With this background, it is possible to understand several problems with his methods for making these calculations.

TECHNICAL INEFFICIENCY

A major purpose of Sonstelie's paper is to compare the per-unit cost of quality in the public and private schools. Each is assumed to be the same everywhere. Units of quality are defined so that the per-unit cost in the public schools is $1 per student, and the price of quality in the private sector is treated as an unobserved parameter. An acceptable estimation procedure would yield a unique estimate of this parameter. Sonstelie uses a roundabout method for estimating it that yields a different estimate for each household income. This section shows that if his two estimated equations were correctly specified, this would not be the case.

In order to see the problem with Sonstelie's method, it is necessary to explain it in more detail than in his paper. Sonstelie estimates two

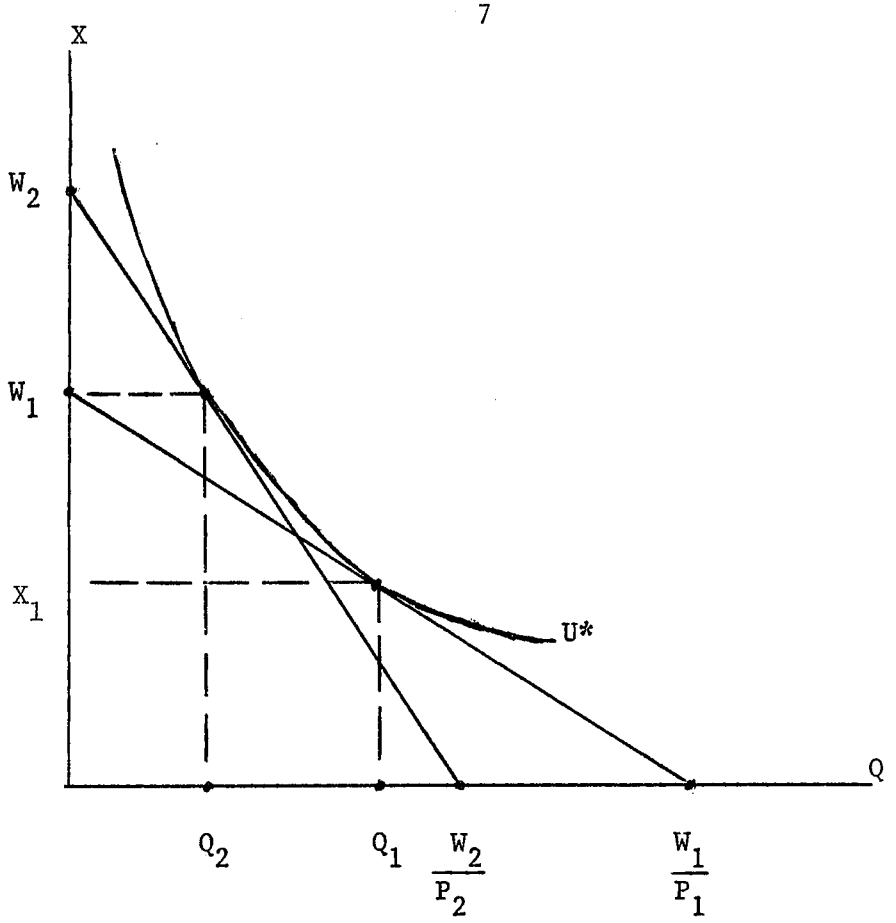
equations. One explains reservation quality as a function of the household's after-tax income. The other is the household's Marshallian demand function D_m, from which he derives a compensated demand function D_c.

Sonstelie's method for using these equations to calculate the price per unit of quality in the private sector can be understood most easily by reference to Figure 2. Figure 2(a) is an indifference curve diagram for a household; Figure 2(b) contains a compensated demand curve for this household. Sonstelie first selects an after-tax income, W_1 in Figure 2(a). The income used in his calculations was some sort of average for his data, but a satisfactory method will produce the same answer for any choice. There is some unobserved price P_1 in the private sector that he wants to estimate. The indifference curve U* in Figure 2(a) represents the highest attainable level of well-being for the household if it sends its child to a private school. The second step in the analysis is to substitute W_1 into the reservation quality equation to predict Q_2. There is some after-tax income W_2 and some price of school quality P_2 such that, in the absence of the public school system, the household would choose the bundle (Q_2, W_1). Clearly, W_2 is equal to $W_1 + P_2Q_2$. Therefore,

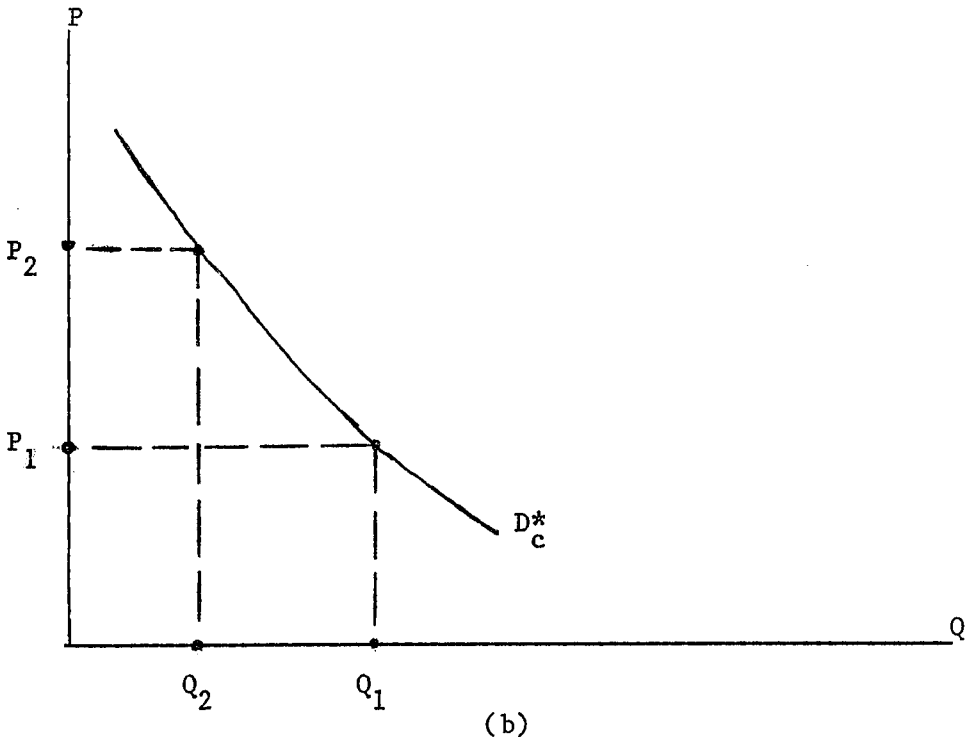
$$Q_2 = D_m(W_1 + P_2Q_2, P_2).$$

The third step is to solve this equation for P_2. In figure 2(b), D_c^* is the compensated demand curve corresponding to the level of well-being U*. Therefore,

$$Q_2 = D_c(P_2; P_1, W_1).$$



(a)



(b)

Figure 2. Sonstelie's Method for Calculating Technical Inefficiency

Sonstelie's estimate of the price of quality in the price sector is obtained by solving this equation for P_1.

This estimate should not, but does, depend upon the income selected in the first step. Sonstelie's best estimate, .37, is based on an after-tax income of $11,393. For incomes $9,393 and $13,393, his method produces estimates of .29 and .50.

Sonstelie's procedure does not yield a unique estimate of the price of quality in the private sector because his demand and reservation quality functions are inconsistent. If they had been derived from the same indifference map, his method would produce the same answer for all incomes. Furthermore, it would not be necessary to use such a roundabout procedure to make the estimate.

To illustrate these points, suppose that all households have an identical Cobb-Douglas indifference map:

$$U = Q^{\alpha}X^{1-\alpha}.$$

For this indifference map, the Marshallian demand function is

$$Q = \alpha W/P. \tag{1}$$

The equilibrium relationship between reservation quality and after-tax income is obtained by solving

$$Q^{\alpha}W^{1-\alpha} = (\alpha W/P_1)^{\alpha}((1-\alpha)W)^{1-\alpha}$$

for Q, where P_1 is the price of quality in private schools. (In Figure 2(a), if W_1 is the household's after-tax income, Q_2 is its reservation quality.) Therefore, the reservation quality function is

$$Q = [\alpha(1-\alpha)^{(1-\alpha)/\alpha}/P_1]W = \beta W. \qquad (2)$$

In estimating equations (1) and (2), the actual quality of the public schools in a locality would be used in both. In the reservation quality function (2), the relevant income is the income of households in the Π percentile of the income distribution, where Π is the percentage of households that send their children to public schools. In the demand function, the relevant income is the income of households in the $\Pi-(1/2)$ percentile of the income distribution, and the relevant price is that facing voters at this income level.

Now suppose that α and β are consistent estimators of the parameters α and β in equations (1) and (2). A consistent estimator of P_1 is

$$\hat{P}_1 = \hat{\alpha}(1-\hat{\alpha})^{(1-\hat{\alpha})/\hat{\alpha}}/\hat{\beta}.$$

This is the expression that results from Sonstelie's method based on the new Marshallian demand and reservation quality functions no matter what income is selected at the outset.

WELFARE COST

Measure

In general, the welfare cost of a government program is the difference between the level of well-being that a household attains under the program and the level that would be attained if it were replaced with an equally costly lump-sum grant. A satisfactory measure of the welfare cost would yield a value of zero if and only if the two alternatives

resulted in the same level of well-being. Sonstelie's measure does not have this property.

His intuitively appealing measure is the excess of the cost of educating a child in the public school over the maximum amount that the household is willing to pay for its child to go to this school. The defect of this measure is easiest to see in the case where the per-student cost of quality is the same in public and private schools. For simplicity, assume that both are $1. The household depicted in Figure 3 has an after-tax income W. Suppose that this household lives in a locality with public schools of quality Q_c. It will send its child to a public school because consumption bundle C is preferred to bundle A. The cost of educating this household's child in the public school is Q_c, and the maximum amount that the household is willing to pay to send its child to the public school is $W - X_m$. Therefore, Sonstelie's measure of welfare cost is $Q_c - W + X_m$. Since X_n in Figure 3 is equal to $W - Q_c$, his measure is $X_m - X_n$.

This measure will be zero if the quality of the public schools is equal to the quality that the household would select if it were not allowed to send its child to a public school, Q_a in Figure 3. Unfortunately, unless the income elasticity of demand for school quality is zero, there is a welfare cost in this case. Assuming that the two goods are normal, the income consumption curve IC is upward sloping. If the household were given a lump-sum grant equal to the cost of educating its child in a public school, Q_a in this case, it would choose consumption bundle K and be better off than under the public school system where its best alternative is bundle H.

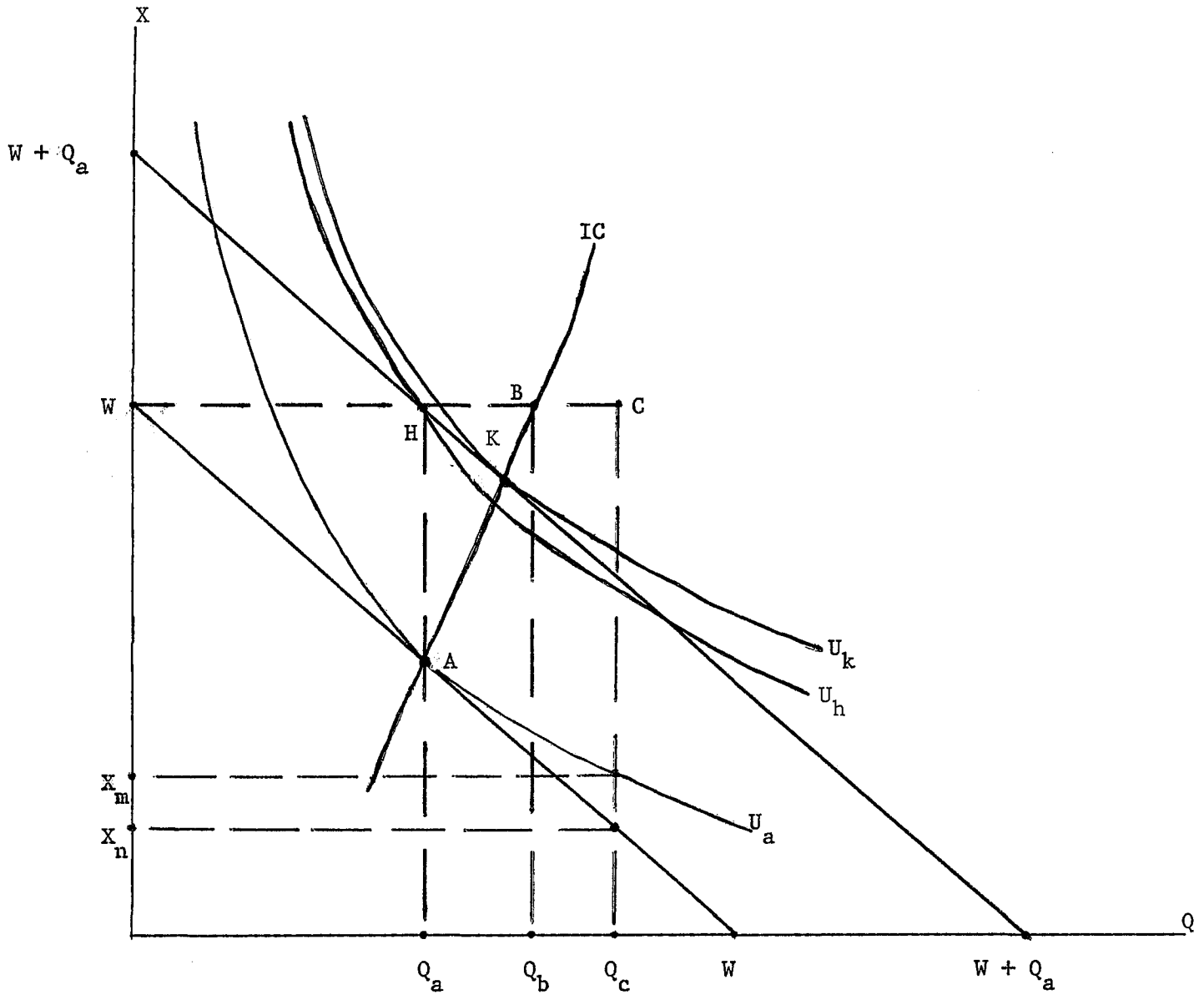


Figure 3. Defect of Sonstelie's Measure of Welfare Cost

If the household lives in a locality with public schools of quality Q_b, replacing the public school system with a program of equally costly cash grants would have no effect on this household's consumption bundle B and hence the public school system would have no welfare cost. Sonstelie's measure of the welfare cost in this case is the distance between the indifference curve U_a and the lower budget line at Q_b.

Although Sonstelie's measure of welfare cost is flawed, it is easy to devise a satisfactory measure that can be calculated from his estimated parameters. An acceptable index of a household's well-being is the income necessary at some vector of prices to attain the relevant indifference curve. Therefore, a satisfactory measure of the welfare cost of the public school system is the excess of the household's income if it were given a lump-sum grant equal to the cost of educating its child in a public school over the income necessary to attain the same level of well-being experienced under the public school system if the household were forced to buy all goods at market prices.

This measure is depicted in Figure 4. The household has after-tax income W_a, faces a price of school quality P in the private market, and sends its child to a public school of quality Q_c. Its consumption bundle is C on indifference curve U_c. The income necessary at market prices to attain this level of well-being is W_b. If the household were given a cash grant equal to the cost of educating its child in a public school in lieu of the opportunity to send its child to that school, its income would be some amount W_e and it would choose consumption bundle E on indifference curve U_e. My proposed measure of welfare cost is $W_e - W_b$, the difference between the incomes necessary at market prices to attain the relevant indifference curves. This measure is zero if the equally

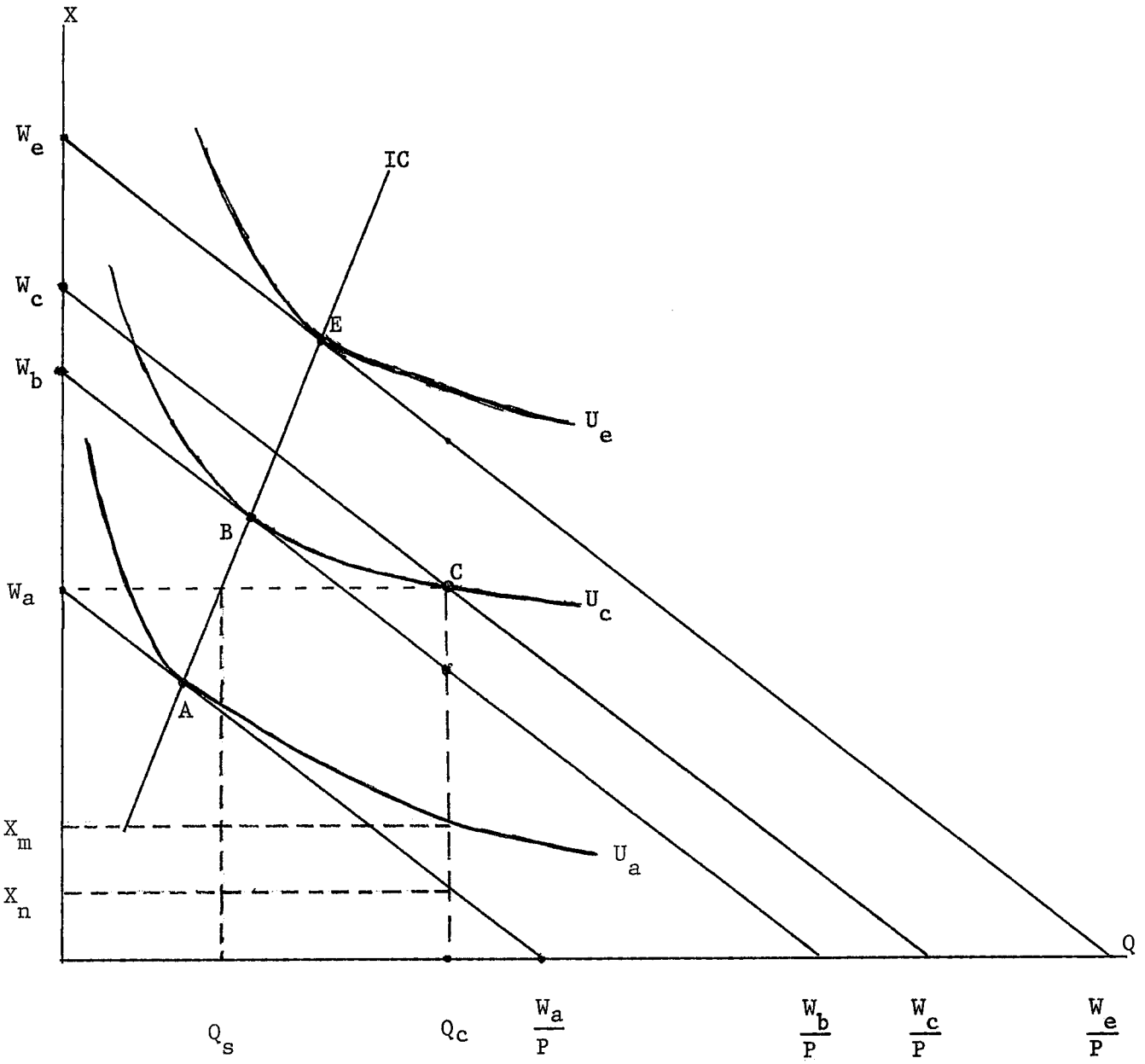


Figure 4. Satisfactory Measure and Decomposition of Welfare Cost

costly cash grant leads to the same level of well-being as the public school system. Sonstelie's measure of welfare cost in this case is $(W_e - W_c) + (X_m - X_n)$, which is identical with the proposed measure if and only if the income elasticity of demand for education is zero.

Decomposition

The welfare cost of the public school system can be decomposed into a part due to technical inefficiency and a part due to consumption distortion. In Figure 4, the cost of producing quality Q_c in the public school is $W_e - W_a$. The cost of producing this quality in private schools is $W_c - W_a$. Therefore, a plausible measure of the magnitude of the technical inefficiency is $W_e - W_c$. The remainder of the welfare cost, $W_c - W_b$, is attributable to consumption distortion.

In order to compare this decomposition with Sonstelie's, it is convenient to depict the relationship between public school quality and the components of welfare cost for a particular household. The proposed measure of technical inefficiency in monetary units for a household that sends its child to a public school of quality Q is (C - P)Q, where C is the per-student cost of a unit of quality in the public sector and P the per-student cost in the private sector. The straight line labeled TI* in Figure 5 depicts this relationship. The proposed measure of consumption distortion is a U-shaped function of public school quality with a minimum of zero at Q_s, which corresponds to Q_s in Figure 4. The curve CD* in Figure 5 depicts this relationship. Since welfare cost is the sum of technical inefficiency and consumption distortion, it is a U-shaped curve with a minimum at some quality Q_r less than Q_s.

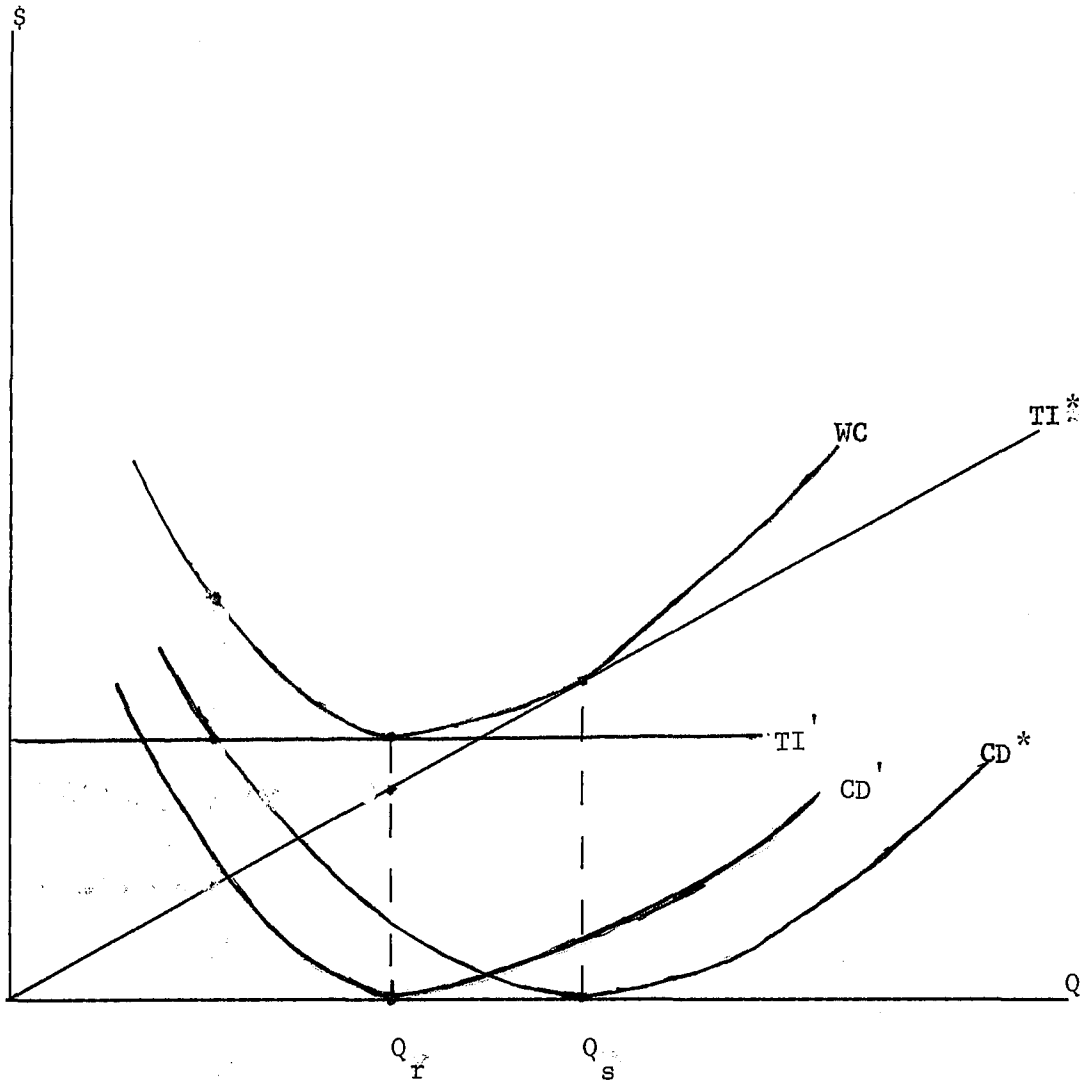


Figure 5. Comparison of Decompositions of Welfare Cost

Even if Sonstelie had used the same measure of welfare cost, his decomposition would have been quite different. Although he assumes that the difference between the per-student cost of a unit of quality in the public and private schools is a constant, his measure of technical inefficiency in monetary units is independent of the quality of education provided by the public school. Specifically, his measure (p. 803) is the minimum welfare cost. The horizontal line TI′ in Figure 5 depicts this relationship. Sonstelie measures consumption distortion as the excess of welfare cost over his measure of technical inefficiency. The curve CD′ depicts this relationship. The rationale for this decomposition is not clear, and it has undesirable properties. For example, at quality Q_s which corresponds to a point on the household's income-consumption curve (see Figure 4), it attributes a part of the welfare cost to consumption distortion even though there is none at this quality.

The flaw in Sonstelie's measure of welfare cost has little effect on the empirical results reported. Table 1 presents estimates of the welfare cost based on my measures for a subset of the cases considered in his Table 1. Like his estimates, these refer to a non-Catholic, non-Hispanic homeowner with an annual after-tax income of $11,393. The largest difference between one of Sonstelie's estimates and my corresponding estimate is about 4 percent.

The estimates of the ratio of technical inefficiency to welfare cost in Table 1 seem to support Sonstelie's claim that technical inefficiency is by far the most important source of welfare cost. In assessing this claim, it is important to recognize that his original estimates and my revised ones refer to only one income level. The credibility of the claim could be greatly increased by making calculations for all com-

Table 1

Welfare Cost Estimates

Public School Quality	Reservation Quality					
	$350		$374		$400	
	WC	TI/WC	WC	TI/WC	WC	TI/WC
$ 375	$241	.62	$355	.66	--	--
$ 446	$184	.97	$298	.94	$373	.92
$ 550	$235	.94	$350	.99	$425	1.00

Note: WC = welfare cost; TI = technical inefficiency. The cost estimates are for a family with after-tax income of $11,393 per year. For definitions of other terms, see text.

binations of income and public school quality in his sample of households with public school children and accounting for the frequencies of such combinations. Furthermore, estimates of welfare cost and its decomposition depend upon estimates of the price of quality in the private sector. The preceding section revealed a potentially serious problem with Sonstelie's method for estimating this parameter.

V. CONCLUSION

Sonstelie has produced a promising approach to estimating important effects of the public school system. This paper is devoted to improving the details of his methodology. It is shown that his measure of welfare cost and its decomposition into technical inefficiency and consumption distortion are defective, but that a satisfactory measure and decomposition are possible based on the data available and parameters estimated. These flaws do not have a marked effect on the estimates reported in Sonstelie's paper, but these estimates alone provide little support for his conclusion that technical inefficiency is by far the most important source of the welfare cost of the public school system, because they apply to an extremely restricted range of values of the variables that determine the ratio of technical inefficiency to welfare cost. A more serious problem with Sonstelie's methodology is that it does not yield a unique estimate of the difference between the cost per unit of quality in the public and private schools. It is shown that this stems from an inconsistency in the specification of the two equations estimated and that this inconsistency can be easily eliminated by deriving both equations from an explicit indifference map. Until this is done,

Sonstelie's estimate of the technical inefficiency of the public school system should not be taken too seriously.

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