University of Wisconsin-Madison

Institute for Research on Poverty

Discussion Papers

Emile Allie

LABOR SUPPLY UNDER AN INCOME MAINTENANCE PROGRAM: A MODEL OF ALLOCATION OF TIME TO HOUSEHOLD PRODUCTION, LEISURE, AND WORK

DP #685-82

Labor Supply under an Income Maintenance Program:

A Model of Allocation of Time to Household

Production, Leisure, and Work

Emile Allie

January 1982

Institute for Research on Poverty,
University of Wisconsin-Madison

and

LABREV-UQAM
(Laboratory on Income Security and Distribution,
University of Quebec at Montreal)

This research was made possible by grant no. BRS-54-580-1 from the Quebec Council of Social Research. I want to thank G. Cain, R. Fiorito, C. Flinn, and M. MacDonald for their comments on a preliminary version of this paper, and Elizabeth Evanson for her editorial help.

ABSTRACT

In most studies of labor supply, the time devoted to leisure and the time devoted to household production are aggregated, because of the difficulty in distinguishing between them in some cases--e.g., is playing with the baby leisure or household production? This paper attempts to make it possible to distinguish them and to explore the implications for willingness to participate in an income maintenance program. I first develop a "general" model with m commodities and n market goods (Becker, 1965; Lancaster, 1971; and Gronau, 1973, 1977). This model is the foundation that permits us to derive the generalized income concept, which in turn is the basis for derivation of the reservation wage. Finally, I make a distinction between two types of reservation wage--the individual reservation wage, which is related to the decision to participate in the labor market (Vickery, 1977; Cogan, 1977), and the income maintenance program reservation wage, which is related to the shape of the potential labor supply function (Hanoch and Honig, 1978; Hausman, 1980). I then derive the potential labor supply function in a general form.

I conclude that given a market wage function which depends on time, the observed time of work for the person who works full time can be a disequilibrium situation.

Labor Supply under an Income Maintenance Program: A Model of Allocation of Time to Household Production, Leisure, and Work

1. INTRODUCTION

Almost all models of allocation of time make a distinction between time devoted to paid work, to leisure, and to household production. In these models the distinction between leisure time and household production time fades out rapidly. Two hypotheses are usually offered to justify their aggregation.

1. The two elements move in the same direction when there are changes in the socioeconomic environment.
2. They satisfy the composite goods theorem (their relative price is constant).

For Gronau (1977), both hypotheses are suspect because budget-time studies have shown that leisure and household production times do not go in the same direction when there are changes in the socioeconomic environment. Yet maintaining the distinction is difficult because there are borderline cases. Is playing with a child leisure, or is it household production? "An intuitive distinction between work at home ... and leisure ... is that work at home (like work in the market) is something one would rather have somebody else do for one ... while it would be almost impossible to enjoy leisure through a surrogate" (Gronau, 1977). However, empirical studies of labor supply make references to household production when they use family composition as an explanatory variable.

The model I develop in the next section is a single-period model that does not include dynamics, uncertainty, and the family. Its goal is to permit analysis of the labor supply of people who participate in an income maintenance program. It is based on work by Becker (1965) and Lancaster (1971), where the utility level is a function of composite goods that we name commodities.

As did Johnson (1966), Gronau (1970), Cohen and Stafford (1974), and Lefebvre and Allie (1980), I define work activity as a commodity. By doing so I can say that a person will prefer to work, instead of not working, for an income producing the same utility level as the other commodities. When someone says he dislikes working, he may be referring to his marginal utility of work. There may be costs for quitting a job (job security, pension plan), or institutional constraints (trade union contracts) that prevent him from reaching his work time equilibrium.

In the Becker (1965) and Lancaster (1971) models, consumption and household production techniques were expressed by the same technical relation, without distinguishing between leisure and household production times. Gronau (1977) made the distinction when he said that market goods can be bought in the market or produced at home. But his household production function depends only on time, while in Gronau (1973) he used a production function in the manner of Leontief as a specific form for a general function, with time and goods as components. This general function is the one I will use later in the paper.

Section 2 will describe the model and examine the first-order conditions of utility maximization.

In Section 3 I will make the distinction between two types of reservation wage. The first type is the individual reservation wage, which depends on individual preferences and the program characteristics. I will discuss it in relation to other reservation wage concepts (e.g., Cogan, 1977). Using the results from 2.2, I will show with the help of the generalized income concept that there are indirect costs of work resulting from a reduction in household production. The indirect costs change the nature of the reservation wage--it is no longer a single point, but instead becomes a curve. The specific form derived in this section shows its practicability in behavior simulation of an income maintenance program (Lefebvre and Allie, 1980, 1981).

The second type of reservation wage is related to the labor supply function, and to program characteristics (Hanoch and Honig, 1978). I then derive a general form of the labor supply function.

I conclude the paper with a brief discussion of some problems related to the labor supply analysis.

2. THE MODEL

2.1. Presentation

We can say, as did Becker (1965) and Lancaster (1971), that the utility level is not a direct function of goods and services but is a function of commodities (Z) that are, at a point in time, characterized by a consumption technology. The utility function can be written as:

(1) $U = U(Z)$,

where $Z = (Z_1, \ldots, Z_m)$. The commodities (Z) are defined through the consumption technology, which is a function of goods and services ($X = (X_1, \ldots, X_m)$) and the consumption time (t^c), assuming a sufficient aggregation level for X_i.

$$(2)\ Z_j = Z_j(X,\ t_j^c),\ j = 1,\ \ldots,\ m,$$

where m identifies the work activity. We can justify the introduction of the work activity in the utility function by saying that there are people who derive satisfaction from the fact that they work, and the time devoted to work that induces a disutility is in fact related to the reduced availability of consumption (leisure) and household production times. Someone who incurs the costs of quitting a job or of an institutional constraint can find himself out of equilibrium and have a negative marginal utility of work. This may induce him to say that he dislikes working when in fact he likes to work but is in disequilibrium for the number of work hours. For the following analysis we will assume the possibility of finding an equilibrium for the number of work hours.

A related problem is job choices; I touch on that subject only briefly. With Z_m, which is a function of X and t_m^c (the time devoted to work), we can deal with fixed and variable costs of work.

Gronau (1977) distinguishes between goods according to whether they were bought on the market (X^M) or produced at home (X^H):

$$(3)\ X = X^M + X^H.$$

Equation (3) says that goods produced at home or bought on the market are perfect substitutes. Home production requires market goods and time (t^H).

$$(4)\ X_i^H = X_i^H\ (X^M,\ t_i^H),\ i = 1,\ \ldots,\ n.$$ [1]

Income (Y) comes from market work at a wage $W(t^m)$, plus non-market income (A) and net government transfers $G(\cdot)$:

$$(5)\ Y = W(t^m)\ t^m + A + G\ (W(t^m)t^m,\ A).$$

The function $G(\cdot)$ is the result of combining income taxes, social insurance, and government transfers from welfare programs. Consequently $G(\cdot)$ does not necessarily define a linear or a convex set because interactions among programs can induce discontinuities in the budget constraint.[2]

Income is allocated to purchase of market goods and to savings. "Consumers, like firms, have commitments which are fixed in the short run, so it is useful to distinguish between long-run and short-run behavior" (Pollak, 1969). In the short run there is consumption that is fixed. We can regard savings (S) as a special commodity fixed at a pre-determined level.

$$(6)\ Y = X^M P^T + S$$

where P^T is the transposed vector of prices. Total time available is divided between household production ($T^H = \sum_{i=1}^{m} t_i^H$), and leisure plus time devoted to work ($T^c = \sum_{j=1}^{m} t_j^c$).

$$(7)\ T = T^H + T^c.$$

We should note that $t_m^c < T$ because there will be at least a minimum level of household activity ($\underline{t}$), such as sleeping. There is also a maximum level of household activity ($\overline{t}$) that people can perform (if not, their behavior would be more appropriate for psychiatric analysis than economic study).

$$(8)\ \underline{t} \leq T^H \leq \overline{t}.$$

2.2. First Order Conditions of Utility Maximization

The Lagrange function to be maximized is:

$$(9)\ L = U(z) + \sum_{j=1}^{m} \lambda_j (Z_j(X, t_j^c) - Z_j)$$

$$+ \sum_{i=1}^{n} \phi_i (X_i^M + X_i^H (X^M, t_i^H) - X_i)$$

$$+ \lambda_Y [W(t^m)t^m + A + G(W(t^m)t^m, A) - P^X X^M - S]$$

$$+ \lambda_T (T - T^H - T^c)$$

$$+ \phi_{\bar{t}} (\bar{t} - T^H) + \phi_{\underline{t}} (T^H - \underline{t}).^3$$

In the short term, there are values of X^M that are predetermined, and what we are maximizing is a utility function conditional on these predetermined values (example: mortgage payments) and the income left available. When the paid work time (t^m) is given, we have the problem of utility maximization under a fixed income.

Variation of commodities.

$$(10)\ \frac{\partial L}{\partial Z_j} = \bar{U}_j - \lambda_j,\ \frac{\partial U}{\partial Z_j} = U_j,\ j = 1, \ldots, m.$$

A commodity (including work activity) is used (or pursued) until the marginal utility from its use ($\bar{U}_j$) is equal to its shadow price (λ_j). The marginal rate of substitution between two commodities (including the work activity) (MRS_Z) is equal to their shadow price ratio.

$$MRS_Z = \frac{U_a}{U_b} = \frac{\lambda_a}{\lambda_b},\quad a, b = 1, \ldots, m.$$

This can explain the choice of the work activity. People who dislike the work they do may be pushed into a corner solution.

Variation of goods.

$$(11)\ \frac{\partial L}{\partial X_{ij}} = \lambda_j Z_{j_i} - \phi_i,\ \frac{\partial Z_j}{\partial X_i} = Z_{j_i},\ j = 1, \ldots, m,\ i = 1, \ldots, n,$$

where $X_i = \sum_{j=1}^{m} X_{ij}$.

We must regard the allocation of goods and time in this model as a "general equilibrium process" where we examine the marginal effect of X_i as a specific Z_j, X_{ij}, instead of all the Z_j's. For example, transportation service, X_a, can be used for dinner at a restaurant or attending a football game.

A good X_i is used in a commodity Z_j (including the work activity) until the marginal utility resulting from its use ($\lambda_j Z_{j_i}$) is equal to the shadow price of the good (ϕ_i). This can explain why one chooses to work in a place where there is free coffee or attractive plants in one's office.

The MRS between two goods in the same commodity is equal to the marginal productivity ratio and to the goods' shadow prices ratio.

$$MRS_X = \frac{Z_{a_c}}{Z_{a_d}} = \frac{\phi_c}{\phi_d}, \qquad \begin{array}{l} a, b = 1, \ldots, m, \\ c, d = 1, \ldots, n. \end{array}$$

Between two goods in two different commodities, the MRS is equal to the goods' shadow price ratio or the product of MRS_Z with the marginal consumption productivities ratio $\frac{Z_{a_c}}{Z_{b_d}}$.

$$MRS_X^* = MRS_Z \frac{Z_{a_c}}{Z_{b_d}} = \frac{\phi_c}{\phi_d}.$$

Variation of consumption time.

$$(12)\ \frac{\partial L}{\partial t_j^c} = \lambda_j Z_{j_t} - \lambda_T, \quad j = 1, \ldots, m, \ z_{m_t} = M.$$

For $j = m$, the non-paid work time (including transportation time), has a shadow price equal to the shadow price of leisure. This means that these times have an opportunity cost equal to the shadow price of leisure (consumption time). It can explain where the worker locates himself (with the help of Eq. (11), because location is a multidimensional problem).

For $j = 1, \ldots, m - 1$, if t_j^c includes the transportation time, then the marginal utility of transportation and pure consumption times $(\lambda_j Z_{j_t})$ (or leisure) is equal to the time's shadow price (λ_T), and is the same in all $j = 1, \ldots, m - 1$ consumption activities.[4]

Variation of market goods.

For a direct use of market goods,

$$(13)\ \frac{\partial L}{\partial X_i^M} = \phi_i - \lambda_y P_i\ , \ i = 1, \ldots, n.$$

The shadow price of good $X_i(\phi_i)$ is equal to the marginal utility of using X_i directly $(\lambda_j Z_{j_i})$ and is equal to the product of the marginal utility of income (λ_y) times the market price of the good (P_i). And MRS_X or MRS_X^* is equal to the price ratio.

$$MRS_X = \frac{Z_{a_c}}{Z_{a_d}} = \frac{\phi_c}{\phi_d} = \frac{P^X_c}{P^X_d} .$$

$$MRS^*_X = MRS_Z \frac{Z_{a_c}}{Z_{b_d}} = \frac{\phi_c}{\phi_d} = \frac{P_c}{P_d} ; \qquad \begin{array}{l} a,\ b = 1,\ \ldots,\ m, \\ c,\ d = 1,\ \ldots,\ n. \end{array}$$

For market goods that are consumed through a household production function, the marginal utility of consuming X^M_i indirectly through the household production function of X^W_k $(\phi_i X^H_{k_i})$ is equal to the product of the marginal utility of income times the market price of X_i.

$$(14) \quad \frac{\partial L}{\partial X^M_{ik}} = \phi_i X^H_{k_i} - \lambda_y P_i ; \quad \begin{array}{l} i = 1,\ \ldots,\ n \\ k = 1,\ \ldots,\ n \end{array}$$

$$\frac{\partial X^H_k}{\partial X^M_{ik}} = X^H_{k_i} .$$

<u>Variation of household production time</u>.

$$(15) \quad \frac{\partial L}{\partial t^H_i} = \phi_i X^H_{i_t} - \lambda_T + \phi_{\underline{t}} - \phi_{\bar{t}} , \qquad i = 1,\ \ldots,\ n.$$

When we have an interior solution for household production ($\phi_{\underline{t}} = \phi_{\bar{t}} = 0$), the marginal utility of one time unit devoted to household production is equal to the marginal utility of time and, from Eq. (12), to the marginal utility of time consumption (leisure), and is the same in all its uses. A corner solution occurs when $\phi_{\underline{t}}$ or $\phi_{\bar{t}}$ is different from zero; then, the marginal utility of time consumption is different from that of household production time,

$$\lambda_j Z_{j_t} \neq \phi_i X^H_{i_t} \quad .$$

From Eq. (3) we have, by hypothesis, a perfect substitutability between goods produced at home or bought on the market, so there is a market price of good i below which the product is only bought on the market and over which it is only produced at home. This is when

$$\phi_i \left[\sum_{R=1}^{n} X^H_{i_R} dX^h_R + dt^H \right] = \phi_i \quad (\bullet),$$

the shadow cost of producing one marginal unit of X_i is just equal to the shadow price of buying one marginal unit of X^M_i on the market or when the cost of all the input by unit produced is equal to the market price derived in (16), below.

$$\text{(i)} \quad \phi_i \quad (\bullet) \quad = \phi_i dX^H_i = \phi_i dX^M_i = \lambda_y P_i dX^M_i \ ,$$

$$\text{(ii)} \quad \phi_i \quad (\bullet) \quad = \lambda_y \left[\sum_{R=1}^{n} P_R dX^M_R + \frac{\lambda_T}{\lambda_y} dt^H \right] \ .$$

By (i) and (ii) we have

$$\text{(16)} \quad \bar{P}_i = \sum_{R=1}^{n} P_R \quad \frac{dX^M_R}{dX_i} \quad + \pi \quad \frac{dt^H}{dX_i} \quad ,$$

where $\pi = \lambda_T/\lambda_y$ is the price of time.[5]

Variation of market time.

$$(17)\ \frac{\partial L}{\partial t_m} = \lambda_m M + \lambda_y \tilde{W} - \lambda_T,$$

where $\tilde{W} = \left[\frac{\partial W(.)}{\partial t^m} t + W(\cdot) \right] \left[g_1(\cdot) + 1 \right]$,

and $g_1(\cdot)' = \frac{\partial G(\cdot)}{\partial W(\cdot) t^m}$.

$\tilde{W}$ can be seen as the net marginal wage. Then the marginal utility of time (λ_T) is equal to the marginal utility of the working activity plus the marginal utility of income generated by work. In traditional results, we have M = 0 and the marginal utility of time is said to be proportional to market wage (not the net marginal wage).

If $\pi = \frac{\lambda_T}{\lambda_y}$ is the price of time and $\Omega = \frac{\lambda_m}{\lambda_y}$ is the price of a marginal working time unit, then

$$(18)\ \pi = \tilde{W} + \Omega M.$$

And the price of time is equal to the net marginal wage plus the increase in the productivity value in consuming more time in work activity.

For working women, for example, this would mean that a marginal increase in the amount of paid work time would raise the value of time (π) more rapidly than the marginal value of the work activity (ΩM). This could explain why women may be reluctant to increase their work time.

The first order conditions of utility maximization also include (2); (3) and (4); (5) and (6); (7); plus

$$(19)\quad \phi_{\bar{t}}(-T + \bar{t}) = 0,$$

$$(20)\quad \phi_{\underline{t}}(-\underline{t} + T^H) = 0.$$

3. THE LABOR SUPPLY

We usually assume that someone will participate in the labor market if his market wage is larger than his reservation wage. In the terms of Figure 1, this is when $W_M > W_0$.

Cogan (1977), in an analysis of the labor supply of married women, argues that there are fixed costs (monetary and in time) for participation in the labor market. His reservation wage is not W_0, but W_C on the line PL(·), which is the potential labor supply function. In this case there is a simultaneous determination of work time, t_c, and no work is observed for a wage less than W_C or for a work time less than t_c.

When we explicitly introduce household production in a model of the allocation of time, we see that there are indirect costs of work resulting from a reduction in household production that have to be compensated for by market goods, to maintain at least the same utility level. There are also direct costs of work which vary with the time spent working. The reservation wage that results is no longer a single point but is a curve. I call it the individual reservation wage, and it is related to the break-even wage developed by Vickery (1977).

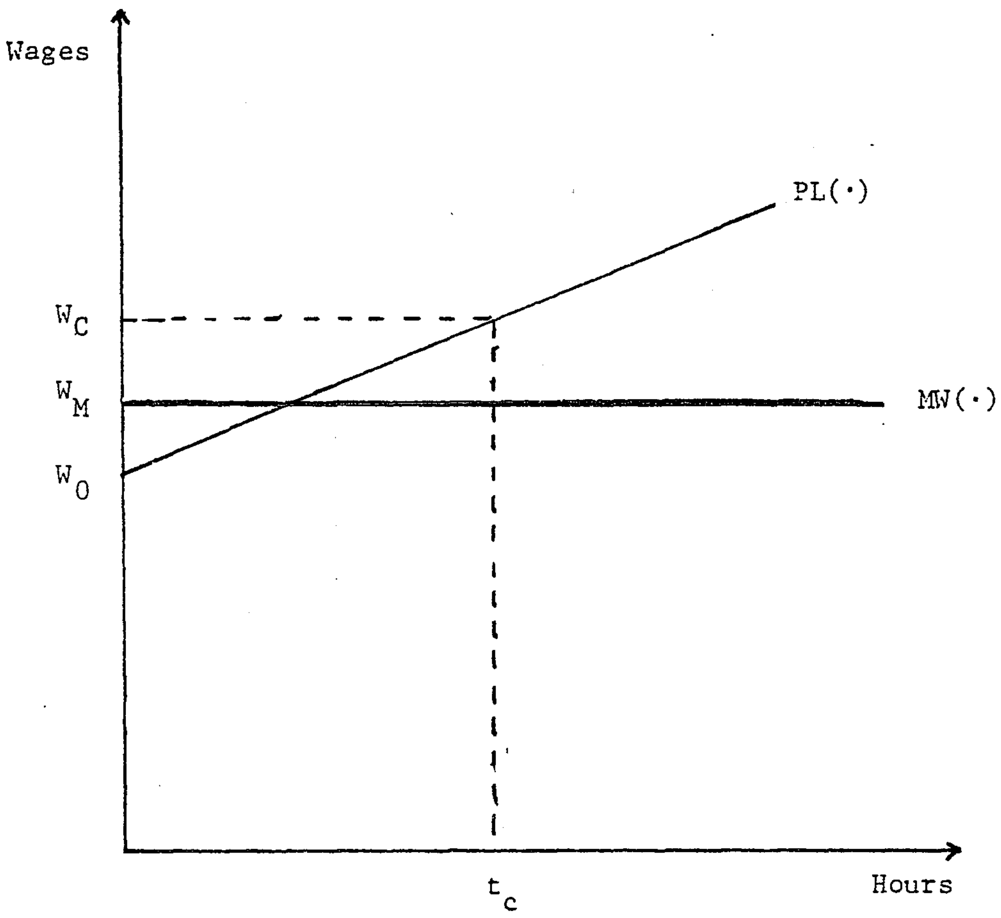

Figure 1. The "traditional" reservation wage.

On the other hand, Hanoch and Honig (1978) have shown that the non-convexity of the budget set constraint induces discontinuities in the potential labor supply function (PL). In Section 3.2 I will briefly set forth their arguments, and discuss them in relation to the individual reservation wage.

I conclude by deriving a general form of the labor supply function. Note that in this section I write Wt^m for $W(t^m)t^m$.

3.1. The Individual Reservation Wage

The reservation wage[6] results from a static analysis. At the utility level for someone who does not work, we want to know the wage which will induce him to work without changing that utility level. This analysis can also be performed by comparing the generalized income in both states.

Generalized income is defined as the sum of the values of time and of unearned (non-work monetary) income. It is related to the earnings capacity concept (Garfinkel, Haveman and Betson, 1977; Moon, 1977; Moon and Smolensky, 1977).

Because of discontinuities in the income constraint, there is more than one reservation wage.[7]

In this section, I first define generalized income; second, I define a functional form for $G(Wt^m)$, and finally I define the individual reservation wage.

The generalized income.

Let Y^* be the generalized income, T^{c*} the consumption time not devoted to market work, T^H the household production time, and π, π^* the respective values of those times. The generalized income is the sum of non-work monetary income corrected for marginal taxation $\{G(\cdot) + A - g_1(\cdot)Wt^m\}$, plus the times values $\{V = \pi T^c + \pi^* T^H + \pi t_m\}$. To have a consistent measure of Y^* we must say that $\tilde{W}$ is linearized around $W(1 + g(\cdot))$.

(21) $Y^* = V + G(\cdot) + A - g(\cdot)Wt^m.$

A functional form for $G(Wt^m)$.

Let us assume $A = 0$; when $A \neq 0$ the analysis of the reservation wage is only slightly modified. We also define $G(Wt^m)$ as the difference between an income maintenance program $G_1(Wt^m)$, and the taxation system $G_2(Wt^m)$, which includes also social insurance.

(22) $G(Wt^m) = G_1(Wt^m) - G_2(Wt^m)$

(23) $G_1(Wt^m) = \delta_1 G^* + \delta_2(D - (1 - \tau)\, Wt^m),$

$$\delta_1 = \delta_1(Wt^m) = 1 \text{ if } 0 \leq Wt^m \leq \frac{G^* + D}{1 - \tau},$$

$$0 \text{ otherwise},$$

$$\delta_2 = \delta_2(Wt^m) = 1 \text{ if } \frac{\bar{D}}{1 - \tau} \leq Wt^m \leq \frac{G^* + D}{1 - \tau},$$

$$0 \text{ otherwise},$$

$$(24)\quad G_2(Wt^m) = g_i^* Wt^m - \xi_i \quad \text{if} \quad \theta_i < Wt^m < \theta_{i+1},$$

$$\text{for } i = 1, \ldots, n,$$

$$\xi_i = \theta_i g_i^* - \sum_{p=1}^{i-1} g_p^*(\theta_{p+1} - \theta_p)$$

$$\theta_0 = \theta_1 = 0 < \theta_2 < \ldots < \theta_n < \theta_{n+1} = \infty,$$

$$g_0^* = g_1^* = 0 < g_2^* < \ldots < g_n^*,$$

where:

G^* is the maximum income available for someone who does not work,

D is a deduction from the work income that varies with the household composition,

τ is a proportional deduction from the work income,

θ_i is the limiting income for an income class,

g_i^* is the marginal taxation rate for the income in excess of θ_i, but smaller than θ_{i+1},

ξ_i is a taxation adjustment for the income class θ_i to θ_{i+1}.

We can rewrite (22) as

$$(25)\quad G(Wt^m) = \delta_1 G^* + \delta_2 D + \xi_i - \{\delta_2(1 - \tau) + g_i^*\} Wt^m$$

$$\text{for } i = 1, \ldots, n.$$

<u>Derivation of the reservation wage.</u>[8]

When a person does not work, his or her generalized income is

$$(26)\quad y_0^* = \pi_0 T_0^{C*} + \pi_0^* T^H + G^* = V_0 + G_0^*.$$

When the person works, it is

$$(27)\quad y_k^* = V_k - \pi_k t_k^m + \Omega_k M_k t_k^m$$

$$+ \delta_{1k} G^* + \delta_{2k} D + \xi_k$$

$$+ \left(1 - (\delta_{2k}(1 - \tau) + g_k^*)\right) W t_k^m .$$

The utility level is the same when the generalized income is the same, i.e., when (26) = (27). Then the reservation wage is the wage that permits such equality.

$$(28)\quad W_R = \frac{\left(V_0 - V_k\right) + \left(\pi_k - \Omega_k M_k\right) t_k^m + \left(1 - \delta_{1k}\right) G^* - \xi_k}{1 - \left(\delta_{2k}(1 - \tau) + g_k^*\right) t_k^m} .$$

From (26), (27) and the income constraint we have

$$(29)\quad \left(V_0 - V_k\right) + \left(\pi_k - \Omega_k M_k\right) t_k^m = \left(X_k^{M*} - X_0^{M*}\right) P^T + X_k^{Mm} P^T ,$$

where X^{M*} is the basket of goods and services not related to work, and X^{Mm} is the basket that is related. $\left(V_0 - V_k\right)$ is the difference in the total time value, and is expected to be negative because the price of time is larger when someone works than when he does not $\left(\pi_k > \pi_0\right)$. The term $\left(\pi_k - \Omega_k M_k\right) t_k^m$ is the net income from work. Then, as the right-hand side of (29) is positive, the total value of time out of paid work time, which is the left-hand side of (29), is smaller when someone works than when he does not work.

When a person begins to work, the time value begins to increase, and some part of the household production has to cease because it becomes too expensive relative to the market value.

The increase in expenses not directly related to work can be said to be increasing with the time devoted to work. Because one can expect that the substitution of household production for market goods is not constant with the time devoted to work, and that this time can be proportional to the paid work time, we can write

$$(30)\quad \left(X_k^{M*} - X_0^{M*}\right)P^T = \gamma t_k^m + \psi t_k^{m2}.$$

The expenses directly related to work can be said to have a fixed point, and a part that varies in proportion to income from work.

$$(31)\quad X_k^{Mm}P^T = C + \beta W t_k^m$$

Substituting (29), (30), and (31) into (28), we can rewrite the reservation wage as

$$(32)\quad W_R = \frac{\gamma t_k^m + \psi t_k^{m2} + C + \left(1 - \delta_{1k}\right)G^* - \delta_{2k}D - \xi_k}{\left(1 - \delta_{2k}(1 - \tau) - g_k^* - \beta\right)t_k^m}$$

and we must have the following condition:

$$(33)\quad \tau > g_k^* - \bar{\beta}.$$

We have to analyze three cases: (1) a person receives the maximum from the program, $G_1 = G^*$; (2) a person participates simultaneously in the program and in the labor market, but does not receive the program maximum, $G^* > G_1 > 0$; and (3) a person does not participate in the program, $G_1 = 0$. In each case, the adjusted taxation (ξ_k) reduces the reservation wage because it is not all the income that is taxed at the rate g_k^*, but only the part in excess of θ_k. It points out that the mean taxation is also an important factor in the decision to participate in the labor market.

Case 1: $G_1 = G^*$. When a person has an individual reservation wage that allows him to receive the maximum from the program, the reservation income (Wt^m) must cover the direct and the indirect costs of work minus the adjusted tax,

$$(34)\quad W_{R1} = \frac{\gamma t^m + \psi t^{m2} + C - \xi_k}{(1 - g_k^* - \beta)t^m},$$

and the individual reservation wage has a minimum at

$$(35)\quad \tilde{t}_1^m = \frac{C - \xi_k}{\psi} \approx \frac{C}{\psi}.$$

It can be shown that the individual reservation wage is equal to the minimum break-even income at time $\hat{t}_1^m$, smaller than $\tilde{t}_1^m$. Then the individual reservation wage is a declining function (see Figure 2), because the fixed costs become less important as the work time increases.

And $\hat{t}_1^m$ is small; therefore, if a person works and receives the maximum benefit from the program, this is probably because the indirect costs are small, and the direct fixed costs are smaller than the deduction allowed by the program. In this case the individual reservation wage function will go down relative to the one where the fixed costs and the deduction are equal.

Case 2: $G^* > G_1 > 0$. When a person participates in the labor market and in the program simultaneously, but does not receive the program maximum, his individual reservation wage is

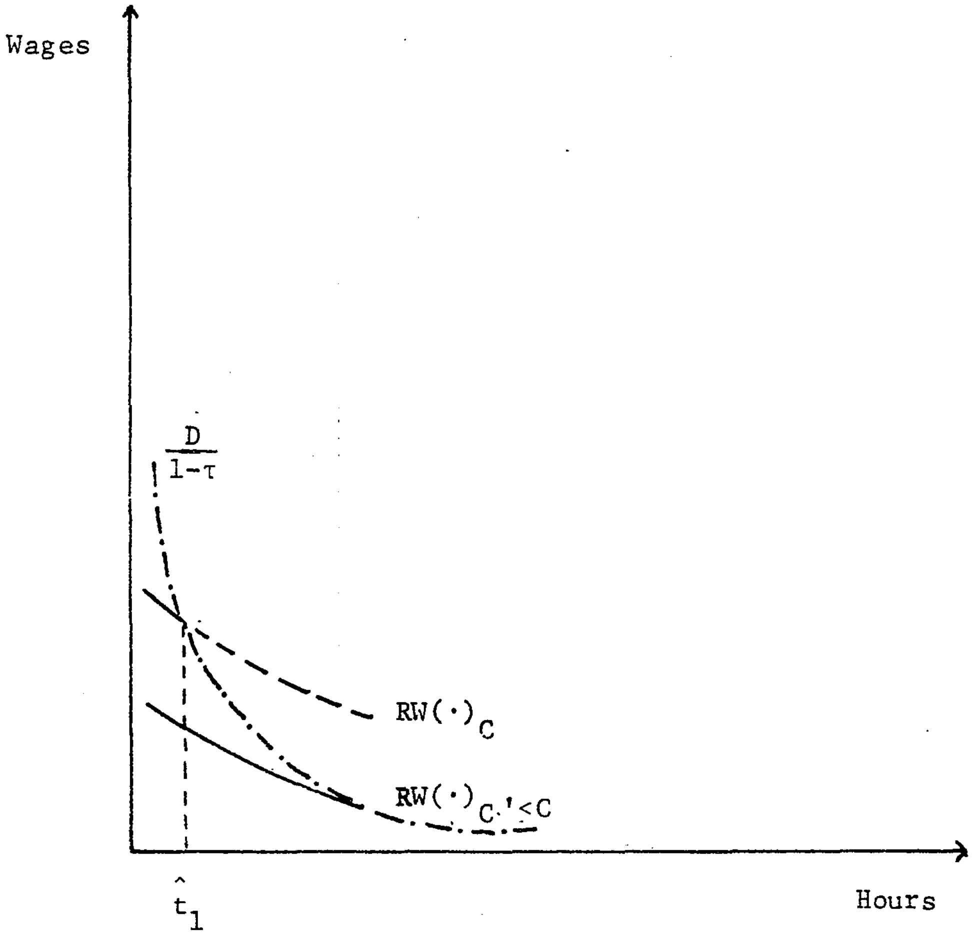

Figure 2. The individual reservation wage when a person works and receives the maximum benefit from the income maintenance program.

$$(36)\quad W_{R2} = \frac{\gamma t \;+ \psi t^{m} \;\; - \xi_k^{m2}}{(\tau - g_k^* - \beta\,)t^{m}}\;,\; C = D,$$

and has a minimum at

$$(37)\quad \tilde{t}_2^m = \frac{-\xi_k}{\psi}.$$

Then, over the relevant income area $(\frac{D}{1 - \tau}, \frac{G^* + D}{1 - \tau})$, the reservation wage is an increasing function of time, as we can see in Figure 3. The most important term here is ψt^{m2}, the increasing substitution of household production for market goods.

On the other hand, if the direct fixed costs of work are less than the allowed deduction, the reservation wage is reduced and the individual reservation wage function goes down.

<u>Case 3: $G_1 = 0$.</u> When a person participates only in the labor market, his individual reservation wage is

$$(38)\quad W_{R3} = \frac{\gamma t^{m} + \psi t^{m2} + C + G^* - \xi_k}{(1 - g_k^* - \beta)t_k^m}$$

and has a minimum at

$$(39)\quad \tilde{t}_3 = \frac{C + G^* - \xi_k}{\psi}.$$

It can be shown that the time at which the reservation income is equal to the break-even income $(\hat{t}_3)$ is smaller than $\tilde{t}_3$. Then the reservation wage is a declining function between $\hat{t}_3$ and $\tilde{t}_3$ and an increasing function for time larger than $\hat{t}_3$, as we can see in Figure 4. This result follows from the fact that the individual

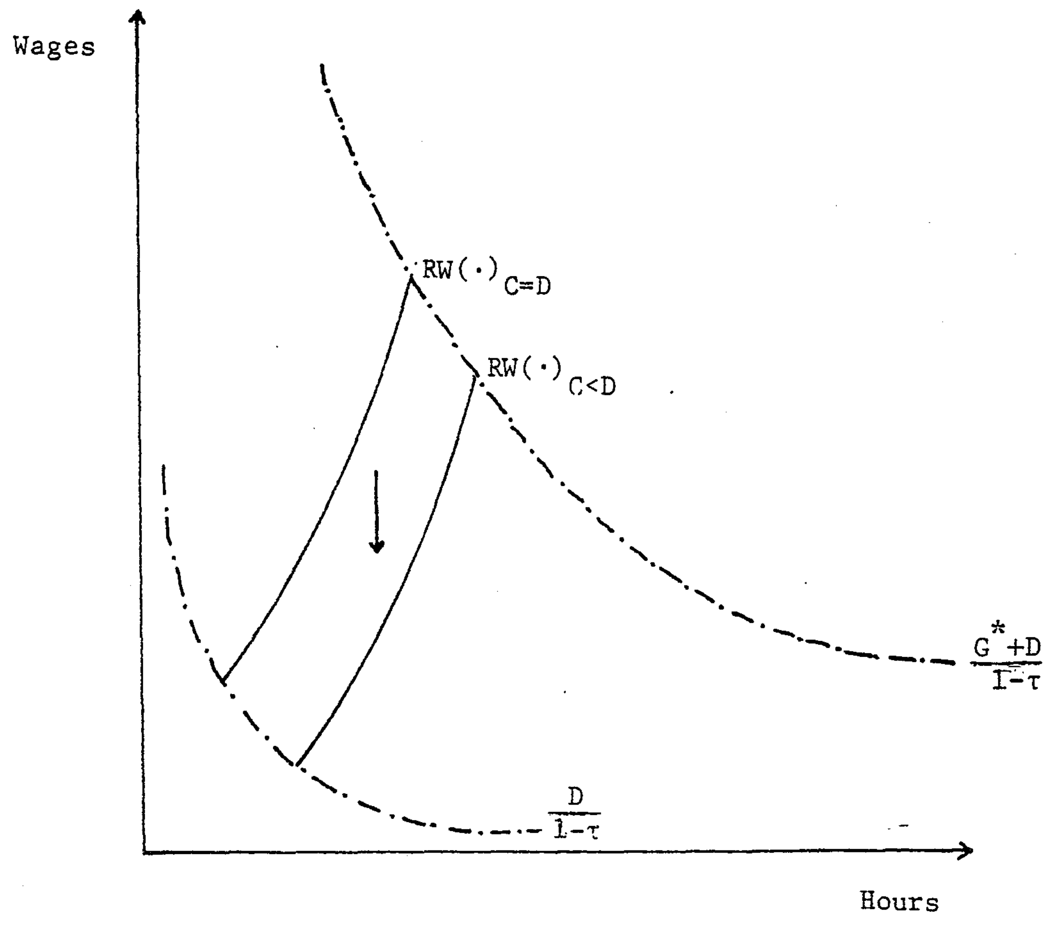

Figure 3. The individual reservation wage when a person participates in the income maintenance program but does not receive the maximum, G^*.

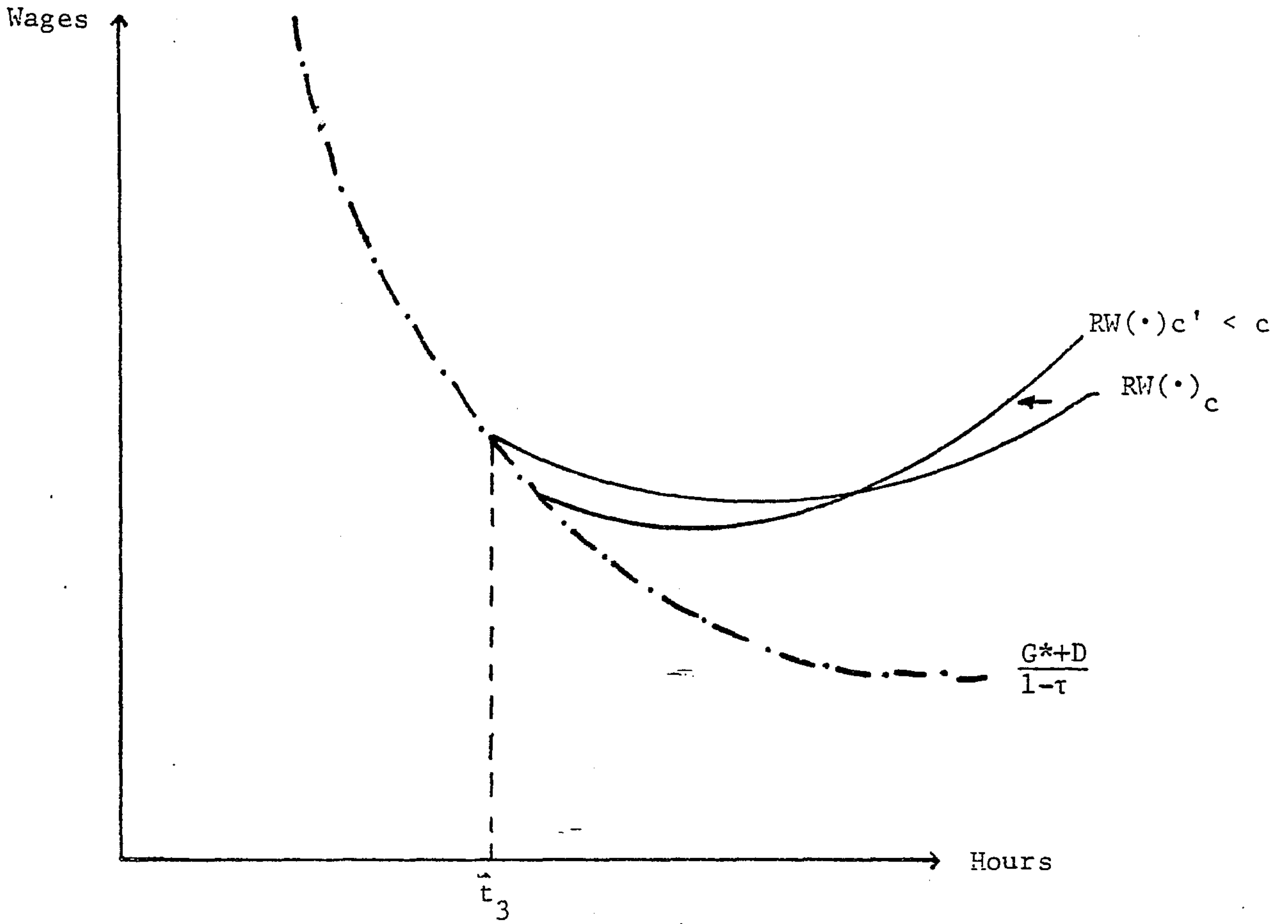

Figure 4. The individual reservation wage when a person does not participate in the income maintenance program.

reservation wage has to cover the fixed costs of work and the maximum from the program (G^*). These two terms are important elements in the individual reservation wage function.

On the other hand, if the fixed costs are small, this shifts the individual reservation wage function to the left and slightly changes the shape of the function.

3.2. The Program Reservation Wage

The individual reservation wage is the result of a static analysis that compares two points on the same utility level.

In this section, the utility level is allowed to change. Because the budget constraint set is non-convex, there may exist points where a change in the utility level accompanies steps in the potential labor supply function. I call the wages at which these steps occur the "program reservation wage."

As Hanoch and Honig (1978) (see also Hausman, 1980) have shown, there are at least two program reservation wages. One occurs at the minimum break-even income; the second occurs in the neighborhood of the break-even income. We can also say that there are as many taxation reservation wages in the potential labor supply function as there are knots in the taxation function, but that is not discussed here.

Minimum break-even income. When the consumer equilibrium is at the minimum break-even income, a change in the wage may not change the earned income. In this case we move from A along the AA' curve in Figure 5.[9] There is a wage W_B where we shift from AA' to C on CC', if the rate of exempted income (τ) is larger than a minimum rate, $\tau°$.[10] If

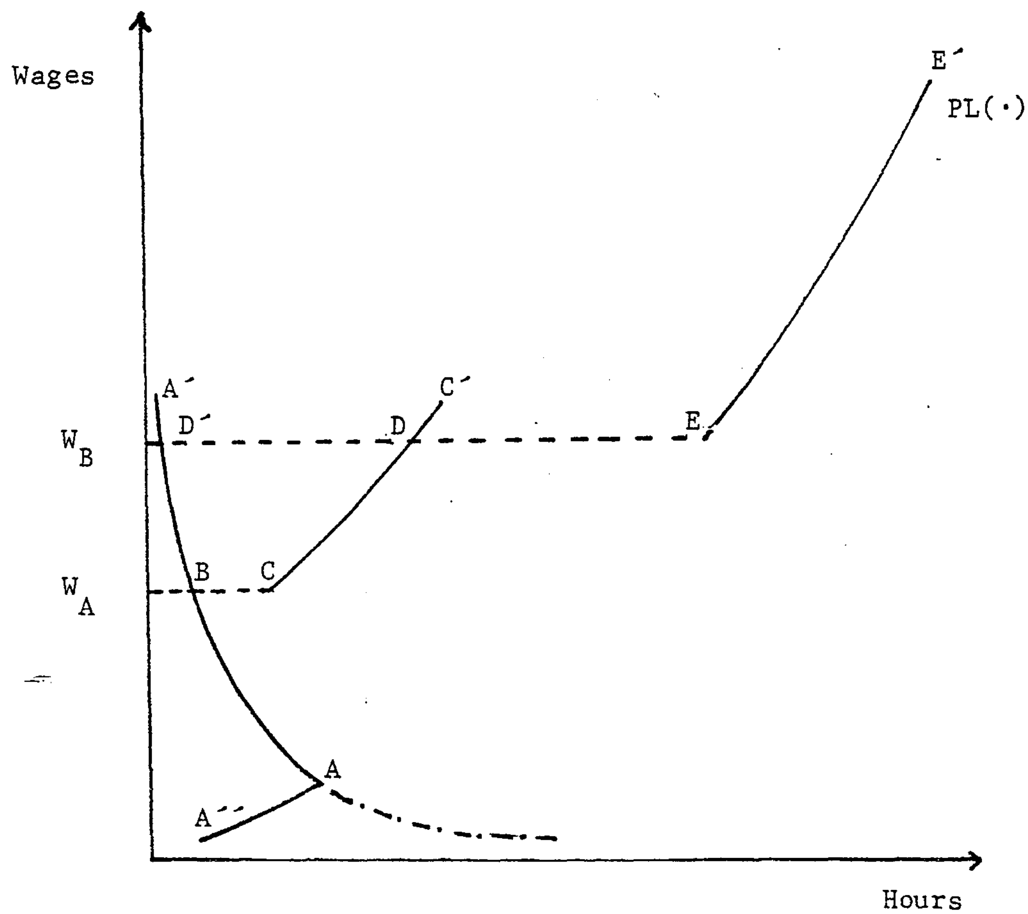

Figure 5. The potential labor supply function.

this is not the case, segment CC' of the potential labor supply function does not exist, and the shift occurs only at W_D, in the neighborhood of the break-even income.

Break-even income. As income increases from C, there is a point in the neighborhood of the break-even income where someone will be indifferent in choosing between two situations A and B, as shown in Figure 6. In this case the generalized income is the same in both situations and it induces a program reservation wage. The result is a shift in the potential labor supply function from D to E. If $\tau < \tau°$, the shift is instead from D' on AA' to E.

Synthesis. With the individual reservation wage, which is an analysis restricted to a comparison of two situations on the same utility level, I have dealt with practically all the costs of work that may be involved in the labor supply decision. I have tied in to the work of Cogan (1977) and especially of Vickery (1977).

On the other hand, Hanoch and Honig (1978) have shown that the parameters of the program induce discontinuities in the potential labor supply function. But the utility level changes, and there is nothing that assures us that the program reservation wage, W_D in Figure 5, is equal to the individual reservation wage.

The result of the preceding discussion is that the equilibrium reservation wage, W^*, is the result of a simultaneous system where WR(·) intersects PL(·), as we can see in Figure 7. We can see the reservation wage developed by Cogan as a special case of our equilibrium reservation wage.

Also, this system simultaneously determines a reservation time, t^*, and no work is observed if the equilibrium market wage, which is the

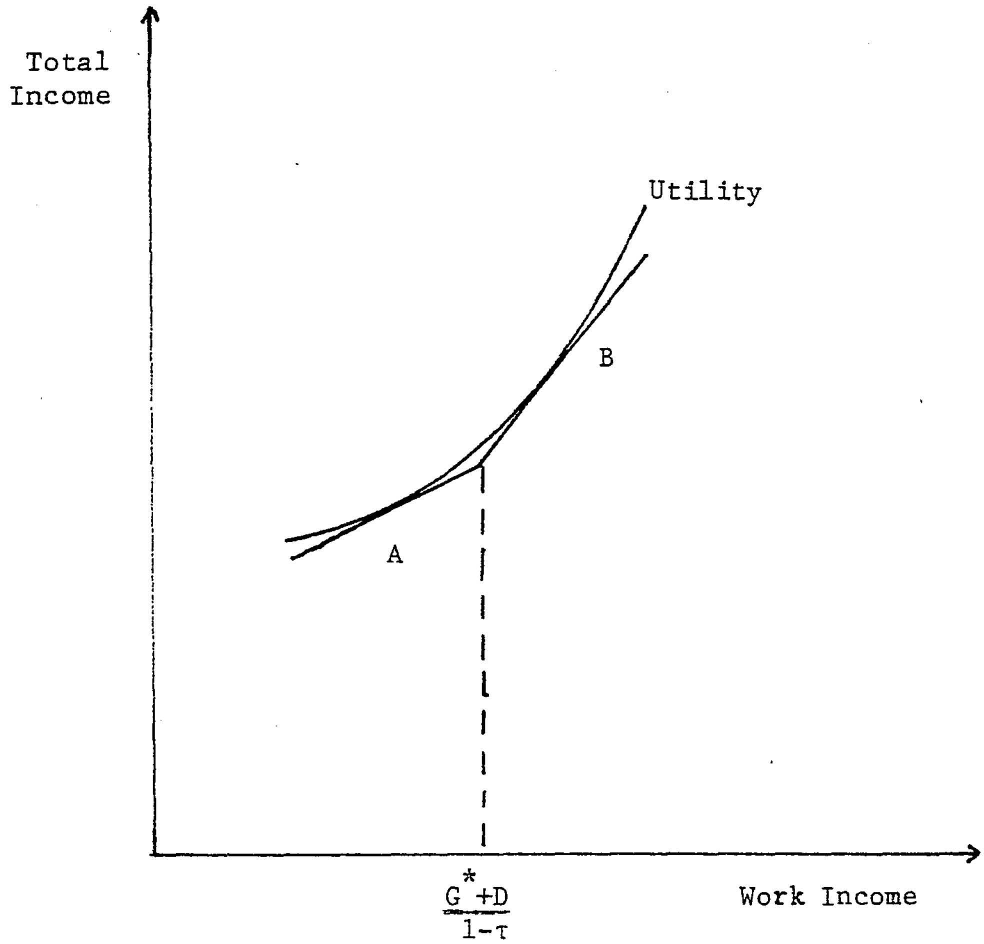

Figure 6. Indifference between A, participate in the income maintenance program <u>and</u> work; and B, work but do not participate in the program.

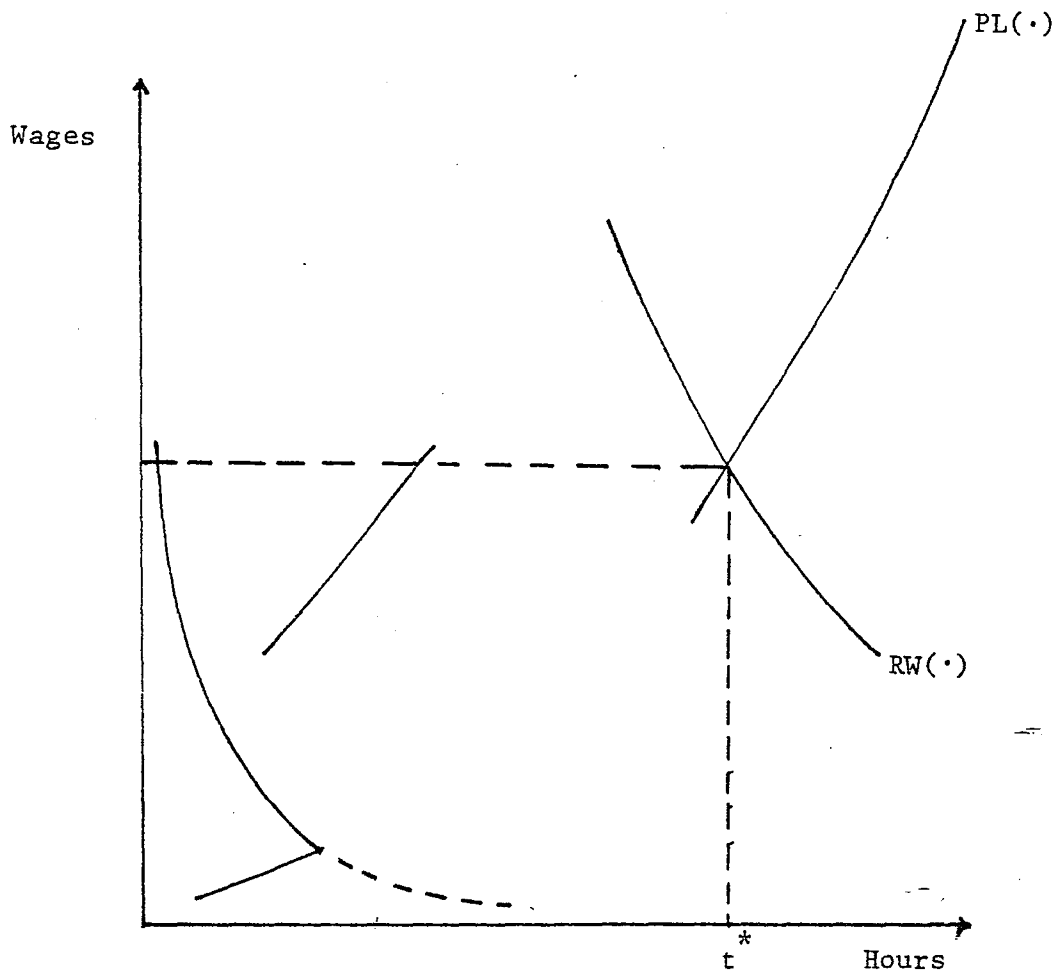

Figure 7. The ēquilibrium reservation wage.

result of the intersection of the potential labor supply function and the market wage function, is smaller than W^* or the equilibrium market time is smaller than t^*.

The next section will tell us what are, in general, the components of the potential labor supply function.

3.3. The Potential Labor Supply Function

The potential labor supply function derived here is for a family unit with only one adult. It can be easily expanded to a family unit with two adults.

From the second order conditions of utility maximization we know that the paid time of work depends on the marginal net wage, the price of the market goods, the nonearned income and savings for given tastes, consumption and household production techniques. These are given for observable (I), and nonobservable (N) individual characteristics. Tastes can also depend on the household composition (H_c: ages of the children). Then we can write the potential labor supply function as

$$(40) \quad PL = (\tilde{W}, P, A, S, I, F, X_c).$$

In a budget-time survey, household production time and leisure time can be observed, but there are only $\bar{n} - 1$ of the n possible times that need to be estimated because we have a total time constraint. Unfortunately, in most labor supply studies the only time observed is the paid work time; then the household production time and the consumption time are aggregated in a complementary term.

We must be careful in analyzing the effect of an individual characteristic on the potential labor supply function, because the characteristics that induce a person to work more can be the same that induce him to locate in the suburb and use more transportation time in most commodities. Then a change in paid work time resulting from a variation in one characteristic can be related to a change in the same direction of the transportation time.

It is also possible that if the effect is an increase of paid work time, possibly the only reverse effect would be a reduction of household production time, since the person is using more market goods with a high time content. However, the net result is in the reverse direction from the one observed in the paid work time equation.[11]

The market wage. The characteristics that make someone efficient in household production and consumption, which induce a high price for time, are probably the same, more or less, that give to this person a high potential market wage ($\tilde{W}$).

Let us assume that in the absence of any cost of hiring a person, the market wage would be equal to the potential market wage.

If we assume that there are fixed costs (C) of hiring a person and that the employer deducts them from the potential market wage, $\tilde{W}$, then the market wage is

$$(41) \quad MW = \tilde{W} - \frac{C}{t} ,$$

and is an increasing function of time up to an institutional maximum time of work $\bar{t}$.[12] We suppose that the law, or a contract, stipulates that over $\bar{t}$, the wage has to be increased by a given factor, so the employer has no incentive to hire somebody for more than $\bar{t}$ hours. If there is a minimum wage law, ($\underline{W}$), nobody will be hired for less than $\underline{t}$ hours because then the fixed costs of hiring by unit of work time are higher than the potential market wage.

We can see from Figure 8 the general shape of the market wage function. In the shaded area, no wage is observed unless someone takes a second job.

Finally, we can write the market wage as

$$(42) \quad MW = MW(L,\ PL,\ \underline{t},\ \bar{t}),$$

where L is a vector of individual characteristics related to the labor market. And to find the equilibrium market wage we have to solve a simultaneous equation system with (40), (42) and

$$(43) \quad \tilde{W} = MW\left\{1 + \frac{\partial G(MW \cdot PL)}{\partial (MW \cdot PL)}\right\}.$$

Equilibrium. A positive paid work time is observed only if the equilibrium market wage is higher than the equilibrium reservation wage.

We can say that when the observed amount of paid work time is less than the institutional maximum time of work, an equilibrium situation exists. But when this time is equal to $\bar{t}$, in most cases it is probably a disequilibrium situation, as we can see in Figure 9.

Also, a second job is taken if the potential labor supply function intersects the market wage function for this second job. In this case the first job is taken at the institutional maximum time of work in a disequilibrium situation.

We should note that this shape of market wage induces discontinuity of the budget constraint at $\bar{t}$.

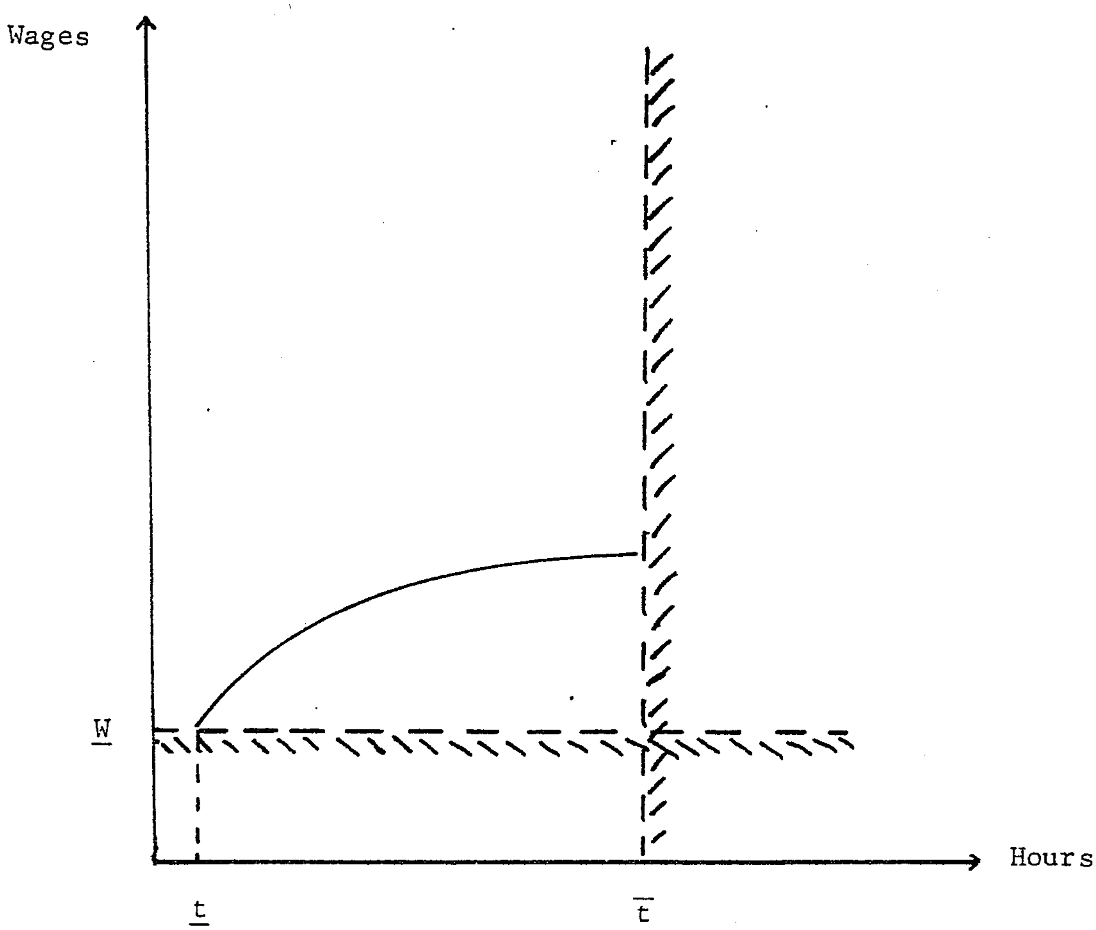

Figure 8. Market wage.

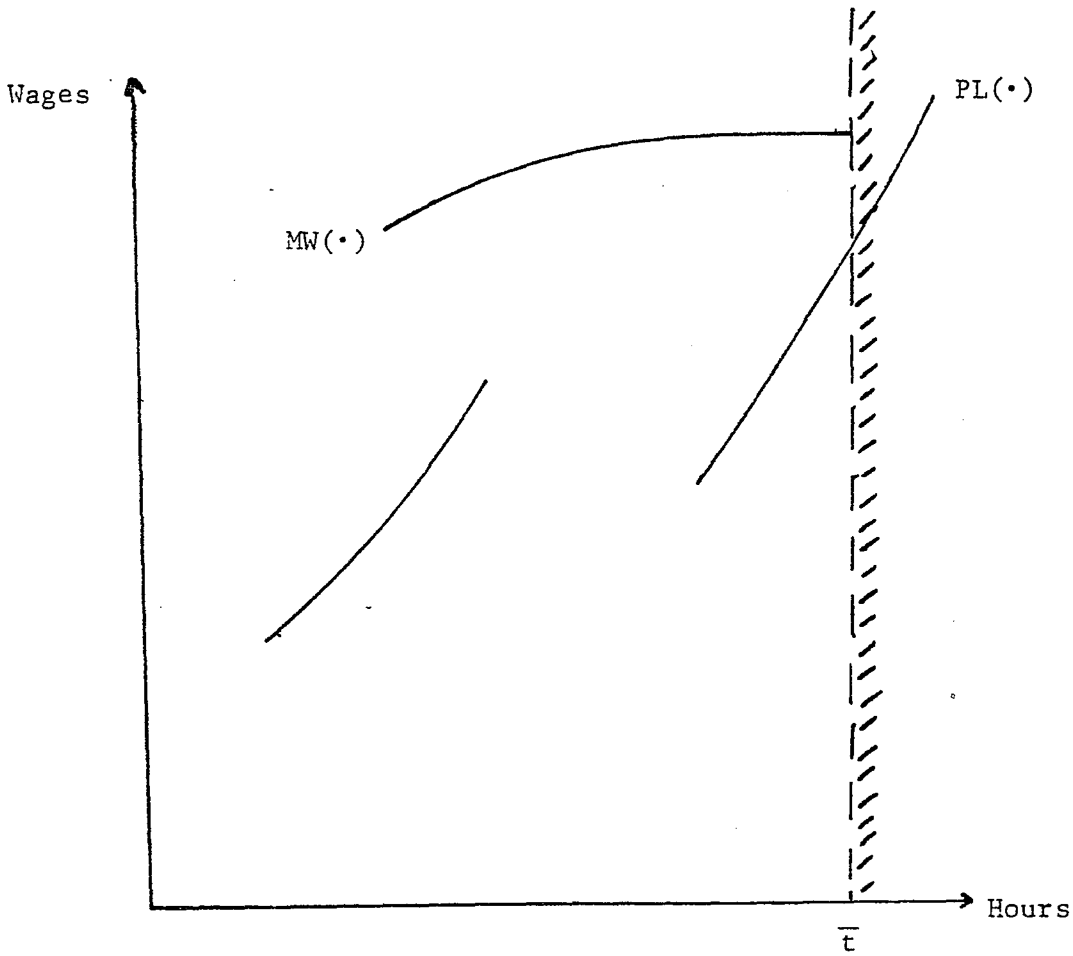

Figure 9. Disequilibrium in the number of work hours.

CONCLUSION

A model of the allocation of time to household production, leisure, and work, which includes m commodities as arguments of the utility function, with work as one of them, and n market goods, is a manageable model if we see the utility maximization process as a kind of general equilibrium process (in this case the market prices are given but the individual prices are not).

As the utility level depends on the work activity, the price of time is then equal to the marginal net wage plus the marginal value of the work activity, which is assumed equal to zero in more traditional models.

Analyzing the labor supply, we have made the distinction between the individual reservation wage and the program reservation wage. The program reservation wage is related to the shape of the potential labor supply function, while the individual reservation wage is related to the decision to participate in the labor market when there are direct and indirect costs of work. Under the hypothesis that these costs depend on time, the individual reservation wage is no longer a sole point but a function of time, and the equilibrium reservation wage, which is unique, is the result of a simultaneous equation system with the reservation wage function and the potential labor supply function.

Instead of viewing the market wage as a constant, I have made it depend on time for work, with institutional maximum time of work and minimum wage. This induces discontinuities in the budget constraint, but we can separate the analytical problem in a simultaneous equation system with the potential labor supply function and the market wage function. In such a system we see that when someone works full time it very often means that this person is in disequilibrium.

In this paper, I have left aside some problems that could be included in a less restrictive analysis:

- The program accounting period may differ from the individual planning period (Fortin, 1979).
- In the planning period, leisure may have a different value depending on whether it is taken within a work period or not (Hanoch, 1976; Fortin, 1979).
- The allocation process may be considered in the context of the life cycle (Becker and Ghez, 1974; Smith, 1973, 1976; Lillard, 1979a, 1979b).
- There may be uncertainty in the wage and in the duration of employment (Solberg, 1974).

Including one or more of these elements in the analysis will have a cost in manageability, but the benefits will reflect a more realistic behavior.

FOOTNOTES

[1]By substituting (2), (3), and (4) into (1) we can rewrite the utility function as

$$U = V(X^M, t^H, t^c).$$

In this utility function, the consumption and household production technologies are confounded with the individual utility function eccentricity. Barnett (1977), in a comment on Pollak and Wachter (1977), sees this problem as an identification problem in the econometric meaning of the word.

[2]See Hanoch and Honig (1978), Cogan (1977) and Hausman (1980). "The complete budget set must be considered in assessing labor force participation. The essentially local reservation wage theory which considers what happens in the neighborhood of zero hours is insufficient in a world of nonconvexities created by fixed cost [of participation in the labor market] and by decreases in marginal tax rate" (Hausman, 1980). And by the characteristics of welfare programs.

[3]The utility function and the consumption and household production technologies have the following properties:

(i) the utility function: $\frac{\partial U}{\partial Z_i} > 0,\ \frac{\partial^2 U}{\partial Z_i^2} < 0;$

(ii) the consumption technology:

$$\frac{\partial Z_i}{\partial X_j} > 0,\ \frac{\partial Z_i}{\partial t_i^c} > 0,\ \frac{\partial^2 Z_i}{\partial X_j^2} < 0,\ \frac{\partial^2 Z_i}{\partial t_i^{c\,2}} < 0;$$

(iii) the household production technology:

$$\frac{\partial X_i^M}{\partial X_j} > 0,\ \frac{\partial X_i^M}{\partial t_i^H} > 0,\ \frac{\partial^2 X_i^M}{\partial X_j^2} < 0,\ \frac{\partial^2 X_i^M}{\partial {t_i^H}^2} < 0.$$

We also have $\frac{\partial G(\cdot)}{\partial Wt^m} < 0$, $\frac{\partial G(\cdot)}{\partial A} < 0$, except at the knot point for the program and the taxation system.

[4] I am indebted to Riccardo Fiorito for having pointed out this generalization. The transportation time and services are not demands per se but are indirect demands for consuming a higher level of commodity Z_j.

[5] It can be argued that it takes time to buy the goods and services in the market. This does not affect the household production decision because it also takes time to buy the inputs.

[6] A part of this section is contained in Lefebvre and Allie (forthcoming in L'Actualité Economique). The presentation here is slightly different.

[7] See Rea (1971), Boskin (1973), Kurz (1974) and Masters and Garfinkel (1977).

[8] See Lefebvre and Allie (1981).

[9] AA' is an equilateral hyperbola defined as $Wt^m = \frac{D}{1 - \tau}$.

[10] Simultaneous participation in the program and in the labor market is possible if

$$(1)\quad \tau > \frac{G^*(g_b^* + \beta) + C(g_b^* - g_a^*) - (\xi_b - \xi_a)}{G^* - (\xi_b - \xi_a)}$$

when τ is constant, and

$$(2) \quad \tau > g_b^* + \beta,$$

when τ can vary.

This means there is a $\tau°$ minimum such that for every $\tau < \tau°$, the program dichotomizes participation in the program and in the labor market (Hanoch and Honig, 1978). If $\xi_{b-1} = \xi_a$, and $g_b^* - g_a^*$ is small, then (1) is equivalent to (2). If $g_b^* - g_a^*$ is not small, then (1) can be rewritten as

$$(3) \quad \tau > \frac{G^*(g_b^* + \beta) + (C - \theta_b)\,(g_b^* - g_a^*)}{G^* - \theta_b(g_b^* - g_a^*).}$$

This constraint can be very useful in designing new programs because all the parameters can be evaluated.

[11]For a good discussion of price and income component elasticities, see Becker (1965), Becker and Ghez (1974), Pollak and Wachter (1977) and Schultz (1975).

[12]This hypothesis can be criticized, but it is no more unrealistic than to suppose that the market wage function is quadratic to take care of the institutional maximum time of work.

REFERENCES

Barnett, William A. "Pollak and Wachter on the Household Production Function Approach." Journal of Political Economy, 85, 1977.

Becker, Gary S. "A Theory of the Allocation of Time." Economic Journal, 75, 1965.

Becker, Gary S. and Ghez, G. The Allocation of Goods and Time over the Life Cycle. New York: NBER, 1974.

Boskin, Michael J. "The Economics of Labor Supply." In Cain and Watts, eds., Income Maintenance and Labor Supply. Chicago: Rand McNally, 1973.

Cogan, John F. "Labor Supply with Time and Money Cost of Participation." Rand Report no. R-2044, 1977.

Cohen, Malcolm S. and Stafford, Frank P. "A Model of Work Effort and Productive Consumption." Journal of Economic Theory, 7, 1974.

Felteau, Claude. "A Theoretical Framework for Analyzing the Determinants of Participation in a Non-Experimental Guaranteed Minimum Income Program." Université Laval, Département de science économique, Working Paper 8105, 1981.

Fortin, Bernard. "A Labor Supply Analysis of Low-Income Families in the Province of Quebec," Ph.D. dissertation, University of California at Berkeley, 1979.

Garfinkel, Irwin, Haveman, Robert A., and Betson, D. Earnings Capacity, Poverty and Inequality. New York: Academic Press, 1977.

Gronau, Reuben. The Value of Time in Passenger Transportation: The Demand for Air Travel. New York: NBER, 1970.

Gronau, Reuben. "The Intrafamily Allocation of Time: The Value of Housewives' Time." American Economic Review, 63, 1973.

Gronau, Reuben. "Leisure, Home Production and Work--The Theory of the Allocation of Time Revisited." Journal of Political Economy, 85, no. 6, 1977.

Hanoch, Giora. "Hours and Weeks in the Theory of Labor Supply." Rand Report no. R-1787, 1976.

Hanoch, Giora and Honig, M. "The Labor Supply Curve under Income Maintenance Programs." Journal of Public Economics, 9, 1978.

Hausman, Jerry A. "The Effect of Wages, Taxes and Fixed Costs on Women's Labor Force Participation." Journal of Public Economics, 14, 1980.

Johnson, M. B. "Travel Time and Price of Leisure." Western Economic Journal, 1966.

Kurz, Mordecai, et al. A Cross-Sectional Estimation of Labor Supply for Families in Denver, 1970. Stanford, Calif.: Stanford Research Institute, 1974.

Lancaster, Kelvin. Consumer Demand: A New Approach. New York: Columbia University Press, 1971.

Lefebvre, Pierre and Allie, E. "L' Aide Social et le Supplément au Revenu de Travail: une simulation économique de leurs revenus (salaires) critiques et de réserve," forthcoming in L' Actualité Economique.

Lefebvre, Pierre and Allie, Emile. "Seuils généralisés de pauvreté, revenu garanti et activités de travail: analyse et simulations." LABREV (University of Quebec at Montreal, Laboratory on Income Security and Distribution), 1980.

Lefebvre, Pierre and Allie, Emile, "Une analyse êconomique de l'Aide Sociale et du Supplément au Revenu de Travail: leurs revenus (salaires) critiques et de rêserve," LABREV (University of Quebec at Montreal, Laboratory on Income Security and Distribution), Paper no. 8107, 1981.

Lillard, Lee A. "Estimation of Permanent and Transitory Response Functions in Panel Data: A Dynamic Labor Supply Model." Rand Report P-6292, 1979a.

Lillard, Lee A. "Wages and Hours in Earnings Dynamics," Rand Report P-6165, 1979b.

Masters, Stanley and Garfinkel, Irwin. Estimating the Labor Supply Effects of Income Maintenance Alternatives. New York: Academic Press, 1977.

Moon, Marilyn. The Measurement of Economic Welfare: Its Application to the Aged Poor. New York: Academic Press, 1977.

Moon, Marilyn and Smolensky, Eugene, eds. Improving Measures of Economic Well-Being. New York, Academic Press, 1977.

Pollak, Robert A. "Conditional Demand Functions and Consumption Theory." Quarterly Journal of Economics, 83, 1969.

Pollak, Robert A. and Wachter, Michael. "Reply: Pollak and Wachter on the Household Production Function Approach." Journal of Political Economy, 85, 1977.

Rea, S. A. "The Supply of Labor and the Incentive Effects of Income Maintenance Programs." Ph.D. dissertation, Harvard University, August, 1971.

Schultz, P. A. "Estimating the Labor Supply Function for Married Women." Rand Report no. R-1265, 1975.

Smith, J. P. "Family Decision Making over the Life-Cycle: Some Implications for Estimating the Effects of Income Maintenance Programs." Rand Report no. R-1121, 1973.

Smith, J. P. "Arrests, Savings and Labor Supply." Rand Report no. P-5470-1, 1976.

Solberg, Eric J. Labor Supply and Labor Force Participation Decisions of the AFDC Population at Risk. Ann Arbor, Mich.: Xerox University Microfilm, 1974.

Vickery, C. "The Time-Poor: A New Look at Poverty." Journal of Human Resources, 12, 1977, 27-48.