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THE ECONOMICS OF PUBLIC SCHOOL CLOSINGS

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ABSTRACT

Public school closings have occurred with increasing frequency in the United States since the early 1970s. Faced with continuing enrollment declines and rising real education costs, school administrators throughout the country will be forced to consider further closings in the future. Although a closing decision will generally benefit many residents of the affected community, others (especially those in the immediate school neighborhood) will strongly oppose the closing. Thus it is necessary to tabulate the net benefit to the city in order to justify a closing on efficiency grounds.

This paper presents a model of a school closing, suggests five explanatory variables that may predict a school closing decision by administrators, and provides a practical test of their significance by examining a school closing decision in Madison, Wisconsin.

The Economics of Public School Closings

Public school closings have occurred with increasing frequency in the United States since the early 1970s. Faced with continuing enrollment declines and rising real education costs, school administrators throughout the country will be forced to consider further closings in the future (Bailey, 1977; Council of Educational Facility Planners, 1978; Illinois State Office of Education, 1975; Jensen, 1978).

While a closing decision will generally benefit many residents of the affected community, others (especially those in the immediate school neighborhood) stand to lose and will strongly oppose the closing. Thus it is necessary to tabulate the net benefit to the city from a closing in order to justify it on efficiency grounds.

In Section 1, a model of a school closing is developed. In this model, those residents near the site of the closing lose, in that they incur higher transportation costs and may suffer a reduction in educational consumption; residents further away may gain in that taxes are reduced because of the economies generated by the school closure. The strategy taken in this model is to redistribute the tax savings from the closing in such a way as to compensate the losers, in order to determine whether the closing results unambiguously in either a Pareto improvement or a Pareto decrease in utility for the community. This analysis yields a necessary condition for a Pareto improvement to result from a school closing. In Section 2, this condition and other conclusions from the model suggest five explanatory variables which may predict the school closing decision of optimizing school administrators. A practical test of the significance of these variables in predicting school closure decisions is set out in Section 3, which examines school closings in Madison, Wisconsin.

1. A MODEL OF A SCHOOL CLOSING

Suppose a city is closed (i.e., there is no migration into or out of it) and contains n individuals, each occupying one unit of housing on one unit of land. The city can therefore be represented by a line of length n. It is assumed that there are a fixed number of public schools located along this line. Prior to a proposed closing, this fixed number of schools is given by $\underline{x}$ and the city is divided into $\underline{x}$ neighborhoods, each of length $n/\underline{x}$ and containing a school at its center. If $\bar{d}$ represents the distance from a school to either boundary of its neighborhood, $\bar{d}$ must satisfy

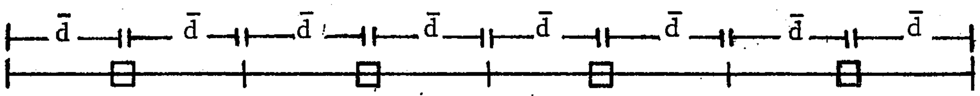
$$\frac{n}{\underline{x}} = 2\bar{d}. \tag{1}$$

Figure 1 offers a simple illustration.

Each individual in the city is assumed to attend the school in his neighborhood.[1] Let d_i represent the distance from residence to school of an individual i, and let $T(d_i)$ represent his annual transportation costs (both travel and opportunity costs of time) for school attendance, where $T' > 0$. The individual annually consumes a private (numeraire) good in the amount of z_i and education in an amount e_i equal to the quantity of education consumed by all individuals attending the same school. All individuals earn an identical annual

Figure 1

A Simple Illustration
of a City with Four Schools



□ School

| Neighborhood boundary

$\bar{d}$ = Distance from school to either boundary income of y and possess identical functions $u(z_i,e_i)$, where u is monotone increasing in both arguments.

All education is provided by local government (there are no private schools) and is financed by taxes which are assessed annually in the amount $r(\cdot)$ upon each individual. This tax function $r(\cdot)$ is set so that two conditions are met both before and after the school closing occurs:

1. Locational equilibrium holds; i.e., there is no incentive for any resident to move, on the margin, from his current location in the city.
2. The government budget constraint is satisifed; i.e., total tax receipts throughout the city exactly cover the operating costs of all schools in the city.

By requiring that the tax function be consistent with both of the above conditions, we may derive an expression for the tax assessed an individual which is of the form $r(d,\underline{x})$, where the subscript i is suppressed. The locational equilibrium condition implies that the tax function r satisfies

$$\frac{\partial r}{\partial d} = -T'(d). \qquad (2)$$

This condition, which implies that $\frac{\partial r}{\partial d} < 0$, assures that an individual who moves a small increment, dd, further away from his nearest school incurs additional transportation costs of T'dd, which just equal his reduction in tax payments of $-\frac{\partial r}{\partial d}$ dd. Integration of this condition yields

$$r = -T + c \qquad (3)$$

where c is a constant.[2] Using equation (3) and requiring that the government budget constraint holds, we may obtain an expression for c in terms of $\underline{x}$. Assuming that the marginal and average cost of operating each school is a constant,[3] p, we write the government budget constraint for a community with x schools as follows:

$$2x \int_0^{\bar{d}} r \, dd = px \qquad (4)$$

Let the transportation function T(d) be given by

$$T(d) = gd \qquad (5)$$

where g is a positive constant. Substituting equations (3), (5), and (1) into (4) and solving for c as a function of $\underline{x}$ yields

$$c(\underline{x}) = \frac{p\underline{x}}{n} + \frac{gn}{4\underline{x}}. \qquad (6)$$

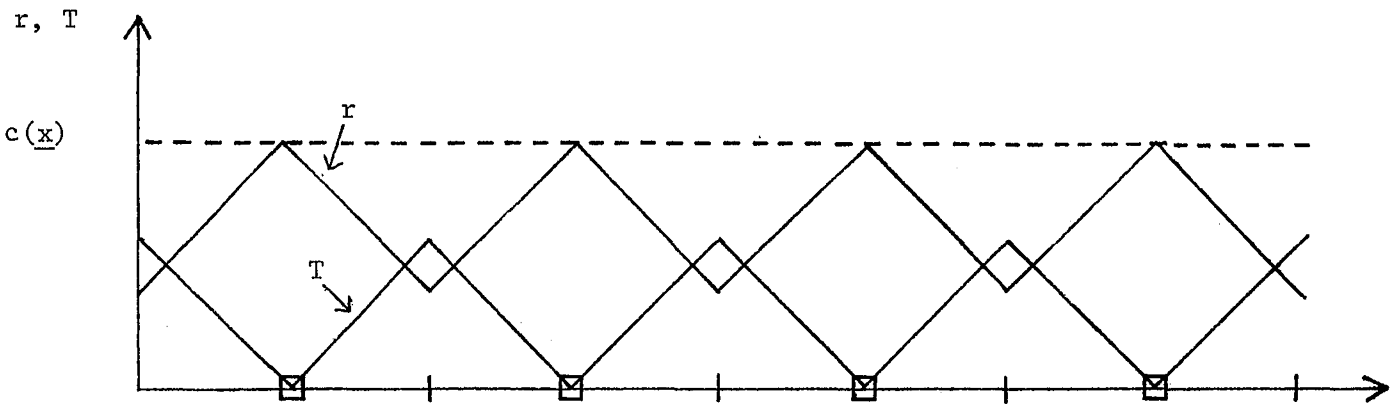
Thus the expression for the tax function, which is obtained by requiring that locational equilibrium and the government budget constraint hold simultaneously, is reached by substituting equations (5) and (6) into (3):

$$r(d,\underline{x}) = -gd + \frac{p\underline{x}}{n} + \frac{gn}{4\underline{x}} \qquad (7)$$

Figure 2 illustrates the locational distribution of r and T. It is easily seen that locational equilibrium holds throughout the city, while $c(\underline{x})$ may be set at a level which guarantees that total school operating costs $p\underline{x}$ will be collected in taxes.

Figure 2

Locational Distribution of r and T



The quantity of education consumed by an individual attending a particular school is assumed to be given by an educational production function of the form

$$e = e(\tau, k), \quad e_{\tau} > 0, \quad e_{K} > 0, \tag{8}$$

where e = quantity of education, τ = teacher-student ratio, K = classroom-student ratio.[4] Before the closing, all schools are assumed to have equal numbers of teachers, classrooms and students. Thus the preclosing quantity of education is equal to a constant e for all individuals in the city.

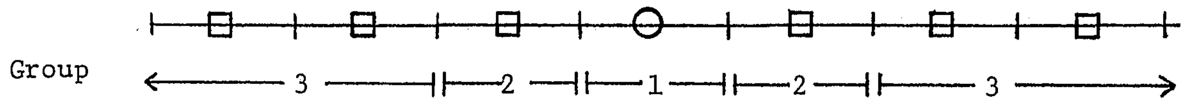
In the analysis, we divide the city into three groups of residents that differ in relative distance to the site of the closing (Figure 3 illustrates). Group 1 consists of those who live in the neighborhood (required to be an interior neighborhood of the city) of the school selected for closure. After this school is closed, Group 1 individuals attend the closer of the two adjacent schools. Group 2 individuals live in the preclosing neighborhoods of these two adjacent schools. Group 3 covers all other residents.

Secondly, the process of a school closing is divided into two stages (Figure 3 illustrates). Stage I will refer to the preclosing state of the city, in which there are x identical public schools equally spaced throughout the city. Stage II will indicate the state of the city after a school closing is implemented, when the (x-1) functioning public schools in the city are no longer all equidistant from one another. The two adjacent schools each annex half of the neighborhood of the closed school in Stage II, while all remaining schools retain their preclosing attendance boundaries.

Figure 3

Neighborhood Groups Before and After School Closing

a. Stage I. Before Closing



b. Stage II. After Closing

○ Closed School □ Open School

In the analysis to follow, some notation will be needed to distinguish the behavior of variables of the model for the different groups of individuals and for the different stages of the analysis. If v is a variable of the model, v_{iG}^{S} refers to the value in Stage S of v for the i^{th} individual, who is a member of Group G. If one of the subscripts is left out, it will be clear whether the individual or the group is being referred to, since only i will be used to represent the individual, while the group will always be designated by either 1, 2, or 3. If neither subscript is used, the individual and group are left unspecified; if there is no superscript, the stage is left unspecified.

The Stage I Utility Maximization Problem

The utility maximization problem for a representative individual in Stage I is the same for all groups:

$$\max_{d^I} u^I(z^I, e^I) \tag{9}$$

$$\text{s.t.} \quad y = z^I + r^I(d^I, \underline{x}) + T^I(d^I)$$

For any preclosing distance to school d^I, we may use the preclosing expressions for T and r given in equations (5) and (7) respectively to solve for z^I in the budget constraint equation:

$$\begin{aligned} z^I &= y - r^I(d^I, \underline{x}) - T^I(d^I) \\ &= y - c(\underline{x}) \\ &\equiv \underline{z} \end{aligned} \tag{10}$$

Thus the level of private good consumption in Stage I is independent of distance to school and group and equals a constant $\underline{z}$, as shown in Figure 4. Educational consumption and utility are also fixed throughout the city. That is,

$$e^{I} = \underline{e}$$

and $$u^{I} = u(\underline{z},\underline{e}) \equiv \underline{u}.$$

Thus, locational equilibrium holds in Stage I, since utility is constant throughout the city and hence there is no incentive to move.

Stage II

After the closure, the city is assumed to be characterized by the following:

1. There is no change in location of remaining schools or of any individuals.
2. All Group 1 students (those displaced by the closing) attend the closer of the two adjacent schools.
3. All Group 1 teachers and administrative staff are fired.[5]

With these assumptions on the nature of the closing, the Stage II values of some variables of the model vary with group; their locational distribution is presented below. These changes result in a postclosing distribution of utility across the city which will constitute either a Pareto increase or a Pareto decrease in welfare in the community.

Figure 5 shows the locational distribution of distance to school in Stage I (d^{I}) and in Stage II (d^{II}). It is easily seen that a Group

Figure 4

Private Good Consumption in Stage I

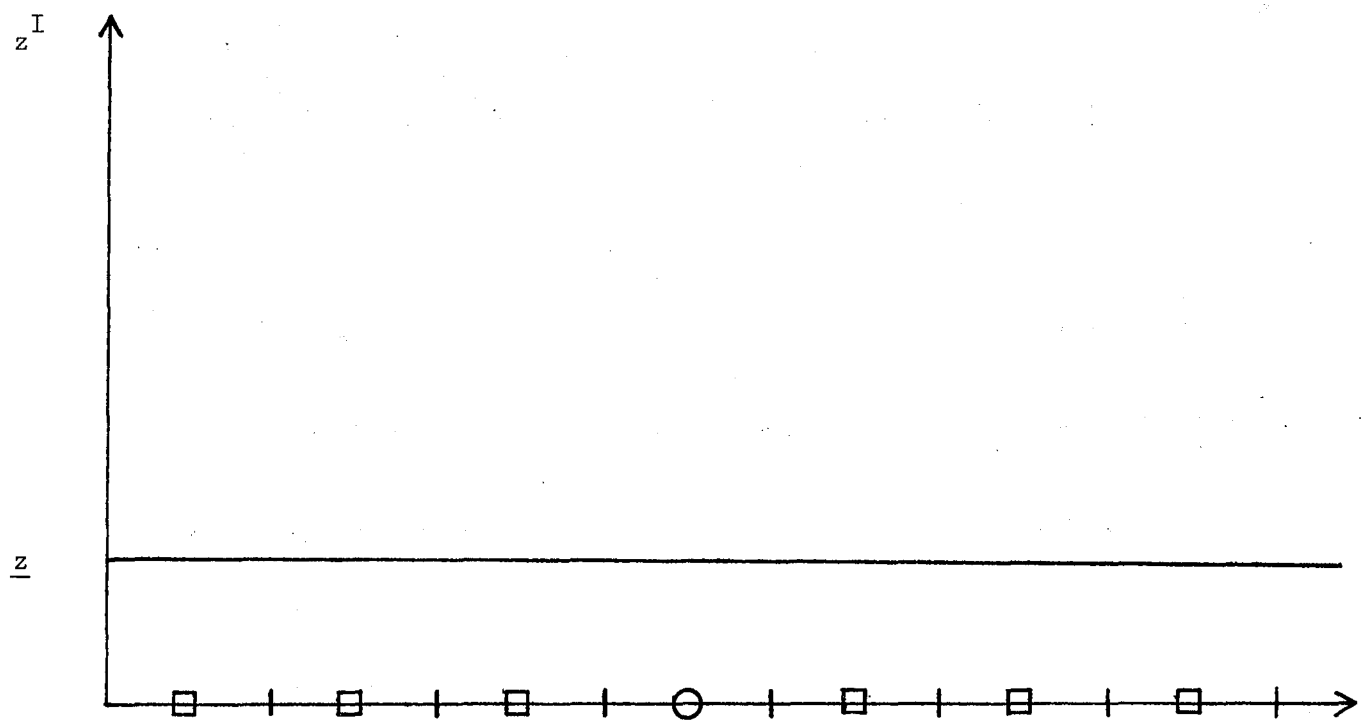
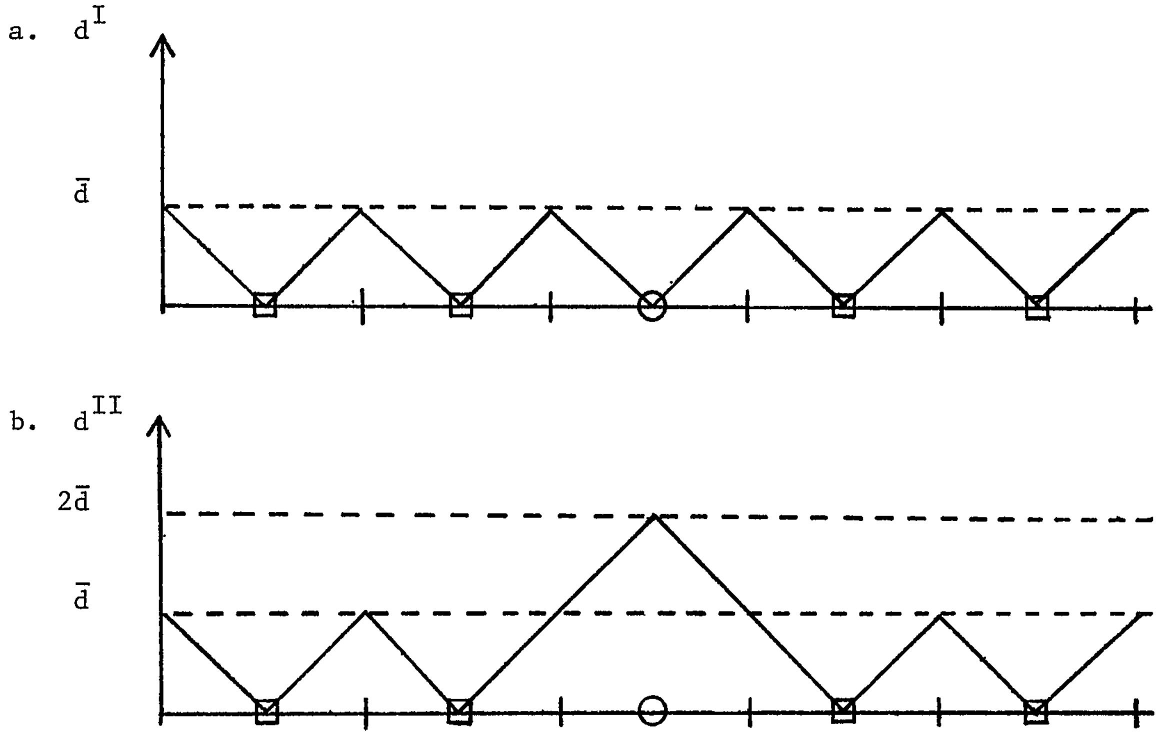


Figure 5

Locational Distribution of Distances to School



$$e_i^{II} = \begin{cases} \underline{e} - e' & \text{if i is in Group 1 or 2} \\ \underline{e} & \text{if i is in Group 3.} \end{cases} \tag{13}$$

Determining the locational distribution of private good consumption and utility in Stage II requires knowledge of the Stage II tax structure. It is shown in the Appendix that the following Stage II tax function satisfies both the government budget constraint (4) and locational equilibrium (2):

$$r_i^{II}(d^I,\underline{x}) = \begin{cases} gd^I - \frac{gn}{4\underline{x}} \text{ for i in Group 1} \\ -gd^I + \frac{p\underline{x}}{n} + \frac{gn}{4\underline{x}} \text{ for i in Group 2 and 3} \end{cases} \tag{14}$$

Using (14) and the individual's budget constraint given in (9), we may calculate the levels of private good consumption in Stage II for the three groups as follows:

$$z_i^{II} = \begin{cases} y - T_1^{II} - r_1^{II} = y - \frac{3gn}{4\underline{x}} = z_1^{II} \text{ for i in Group 1} \\ y - T_2^{II} - r_2^{II} = y - \frac{p\underline{x}}{n} - \frac{gn}{4\underline{x}} = \underline{z} \text{ for i in Group 2} \\ y - T_3^{II} - r_3^{II} = y - \frac{p\underline{x}}{n} - \frac{gn}{4\underline{x}} = \underline{z} \text{ for i in Group 3} \end{cases}$$

Figure 7 illustrates the distribution of z^{II} throughout the city for two cases: (A) $z_1^{II} \leq \underline{z}$ and (B) $z_1^{II} > \underline{z}$; the locational distribution of e^{II}; and the resulting utility $u^{II}(z^{II},e^{II})$ for individuals throughout the city, with A and B indicating alternative placement of u_1^{II} for cases A and b, respectively. In case A, it may unambiguously

Figure 6

Locational Distribution of
Quantity of Education in Stage II

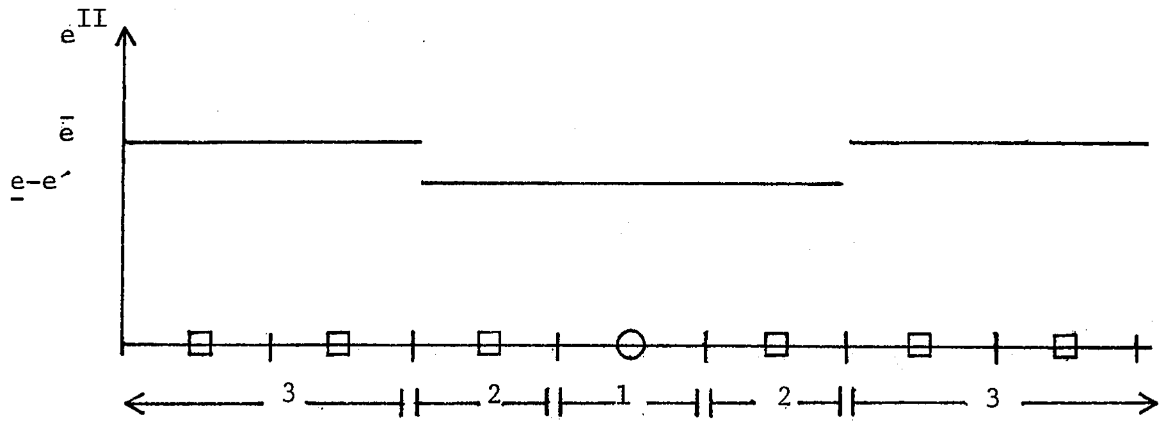
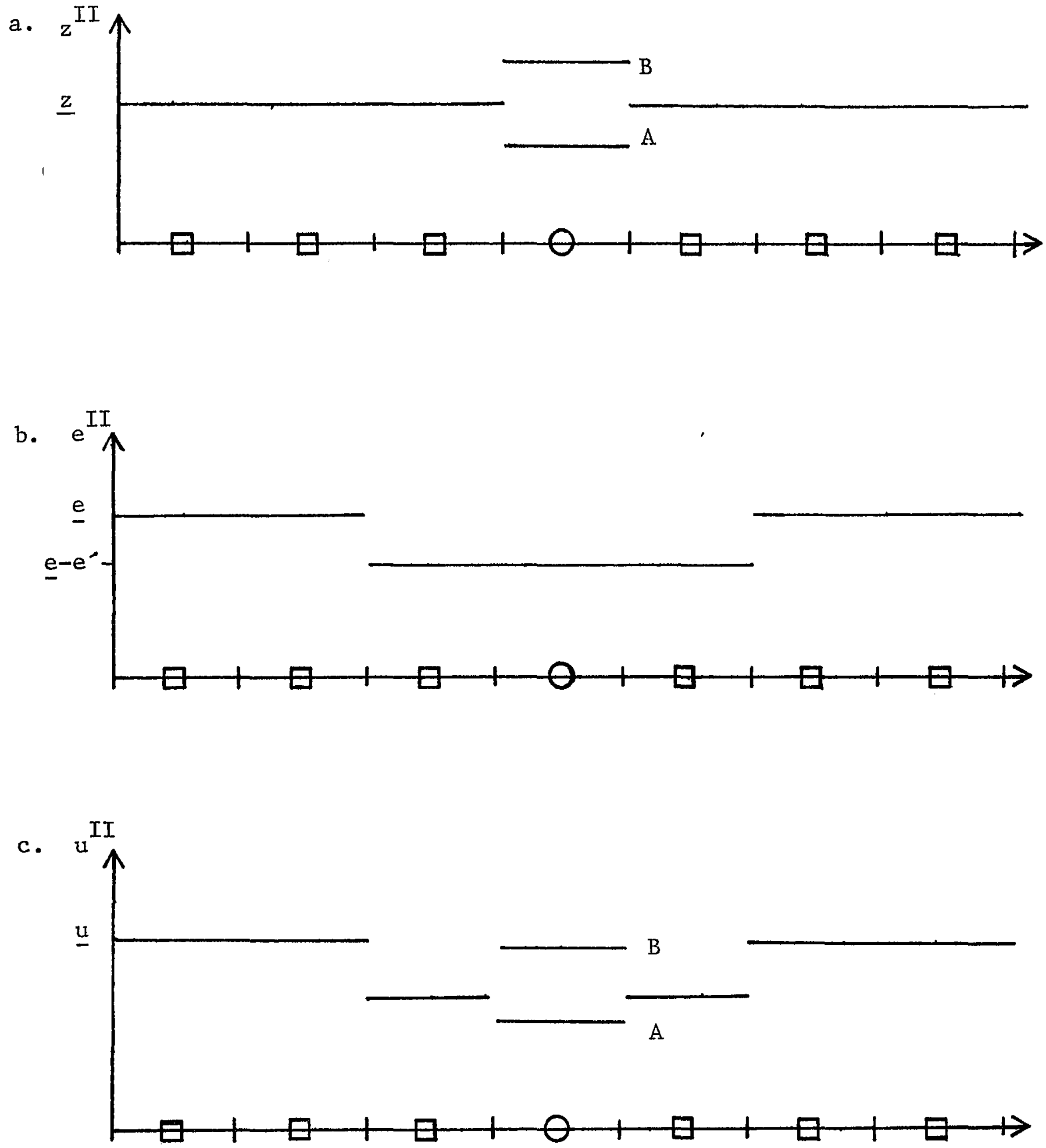


Figure 7

Locational Distributions of $\dot{z}^{II}$, e^{II}, and u^{II}



be stated that the closing has resulted in a Pareto welfare decrease since some individuals (those in Group 2 and Group 1) are worse off from the closing (i.e., $u^{II} < u^{I} = \underline{u}$ for these individuals), while others are no better off (Group 3 individuals: $u_3^{II} = \underline{u} = u^{I}$). That is, a sufficient condition for a closing to result in a Pareto welfare decrease is that case A holds, or that

$$z_1^{II} \leq \underline{z},$$

which may be shown to be equivalent to the inequality

$$p \leq 2g\bar{d}^2. \tag{16}$$

In case B ($z_1^{II} > \underline{z}$), a redistribution of the postclosing tax burden (which maintains both government budget balance and locational equilibrium) may bring about a Pareto welfare increase as follows. First collect in additional taxes from each Group 1 individual the amount

$$z_1^{II} - \underline{z} = \frac{p\underline{x}}{n} - \frac{gn}{2\underline{x}},$$

which reduces the private good consumption of each of the $\frac{n}{\underline{x}}$ Group 1 individuals to $\underline{z}$. This generates tax revenue in the amount

$$REV = \frac{n}{\underline{x}}\left(\frac{p\underline{x}}{n} - \frac{gn}{2\underline{x}}\right) = p - \frac{gn^2}{2\underline{x}^2}. \tag{17}$$

Now disburse the entire amount REV (this preserves government budget balance) equally to all of the $\frac{3n}{\underline{x}}$ Group 1 and Group 2 individuals so that their common level $z_{1,2}^{II'}$ of postclosing private good consumption is brought to

$$z_{1,2}^{II'} = \underline{z} + \frac{\frac{REV}{3n}}{\underline{x}} .$$

At this point the locational distribution of z and e are as displayed in Figure 8. If the common utility level $u_{1,2}^{II'} = u(z_{1,2}^{II'}, \underline{e} - e')$ of Group 1 and 2 individuals is greater than the preclosing level $\underline{u} = u(\underline{z}, \underline{e})$, then the closing may be said to have brought about a Pareto welfare increase, since some individuals (those in Group 1 and 2) are better off and all others (those in Group 3) are no worse off from the closing. However, if $u_{1,2}^{II'} \leq \underline{u}$, the closing will have brought about either no change or a Pareto decrease in utility in the community. All these cases are illustrated in Figure 8.

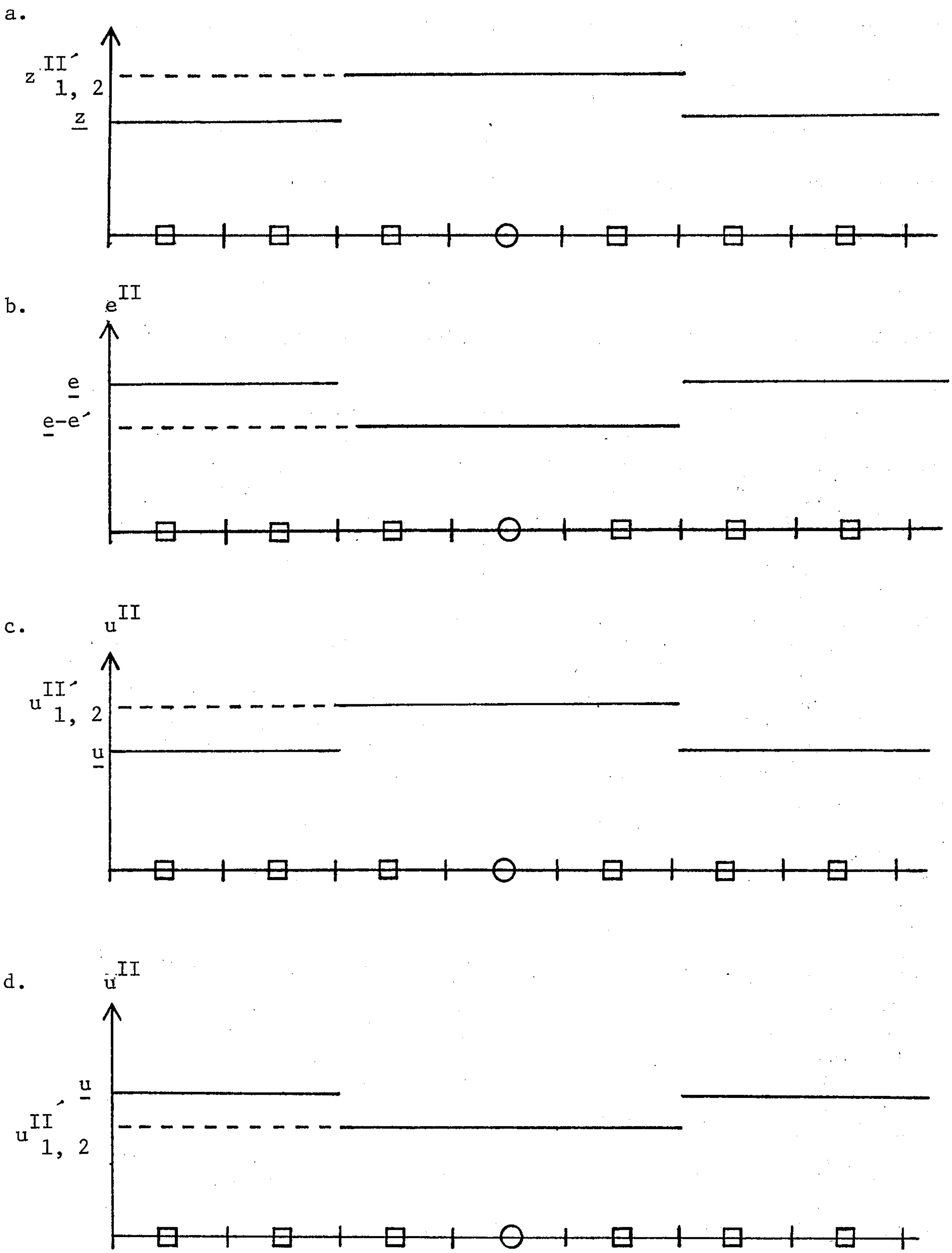
The above argument shows that case B represents a necessary (but not sufficient) condition for a closing to result in a Pareto improvement in utility in the community. This condition may be expressed by the inequality

$$p > 2g\bar{d}^2 . \tag{18}$$

If e' = 0 (which may hold, for example, if the adjacent schools were operating below capacity and if no teachers were fired), inequality (18) is both a necessary and sufficient condition for a closing to result in a Pareto improvement. In addition, the reverse of inequality (18) is a necessary and sufficient condition for a closing to bring about Pareto decrease in welfare if e' = 0.[6]

Figure 8

Various Resulting Distributions of z^{II}, e^{II} and u^{II}



Condition (18) may be given a cost-benefit interpretation. The left-hand side, p, represents the benefit of closing the school since the marginal cost of operating the school, p, will be saved if the school is closed.[7] The right-hand-side, $2g\bar{d}^{\,2}$, represents the cost of closing the school, arising from higher transportation costs.[8] Thus the necessary condition (18) may be viewed as a requirement of positive net benefit.[9]

2. CONCLUSIONS OF THE MODEL: WHICH SCHOOL SHOULD CLOSE?

Conditions (16) and (18) were derived from a model in which all schools and neighborhoods are identical and in which the relevant policy issue is whether or not to close any of the identical schools in the community. It may be shown that condition (18) [condition 16] also constitutes a necessary (sufficient) condition for a closing to bring about a Pareto improvement (decrease) in a city in which schools differ in enrollment, physical size, operating cost, and neighborhood characteristics such as size, population, and composition of population.[10] Schools which differ in these respects will upon closure generate different levels of net benefit as measured by the variables of the necessary condition (18). Thus a school board interested in maximizing community welfare may possess a rationale for deciding which school to close; namely, to close schools first which promise the greatest net benefit from closure. This suggests that the school closing decisions of optimizing school administrators may be explained in part by the following variables, which appear in condition (18) and thus affect the net benefit from closure:

1. The savings in school operating costs (p): The larger these savings are, the greater the net benefit from closure and the more likely that the necessary condition (18) for a Pareto improvement will be satisfied.
2. The increase in transportation costs ($2g\bar{d}^2$) which results from a school's closure: The larger this variable, the smaller the net benefit from closure and the less likely the necessary condition (18) for a Pareto improvement will be satisfied.

Other variables whose effect on school closing decisions may be inferred by the model include:

3. Average earnings capacity in a neighborhood: A constant average wage will be used to represent opportunity cost in measuring the time cost component of the increase in the transportation cost, variable 2. However, since earnings capacity varies from neighborhood to neighborhood, the opportunity cost of increased time spent traveling to school will vary. An average income variable is thus used to capture this difference. The higher the average income, the lower the net benefit from closure.[11]
4. The distance to the city center: In an extension of this model in Lerman (1980) in which population density is taken to fall with distance to a central business district (CBD), it is shown that the farther from the CBD that the closing takes place, the greater the likelihood that a Pareto improvement may result from the closing.[12]
5. The marginal rate of substitution of education for the private good: The larger the relative taste for education versus the

private good, the greater the negative impact that arises from reduced education consumption for Group 1 and 2 individuals when the school is closed, and thus the smaller the net benefit from closure.

The direction of the effects of variables 1-5 upon the net benefit from closure is summarized below.

Variable	Effect on Net Benefit from Closure
1. Savings in school operating costs	+
2. Increase in transportation costs	-
3. Earnings capacity	-
4. Distance to CBD	+
5. Taste for education	-

3. A TEST OF THE MODEL FOR THE MADISON METROPOLITAN SCHOOL DISTRICT

In Section 2 it was argued that the model will explain which school or schools optimizing administrators would close. In this section I test this hypothesis for the school district in Madison, Wisconsin. The test is to determine, through multivariate regression, whether the five explanatory variables of Section 2 exert statistically significant effects in the direction suggested by the model, on the Madison school administrators' decision about school closure for the 1979-1980 school year. In the remainder of this section, I first describe how the dependent and independent variables of the test regression are measured, and then present and discuss the regression results.

The Dependent Variable

In the actual process of deciding to close three of the 32 elementary schools in the Madison, Wisconsin, school district in the fall of 1979, Madison school administrators computed a composite score for each school based upon its performance in 12 separate criteria for closing.[13] The three top-ranking schools according to these composite scores were the schools which Madison school administrators choose to close.[14] Since the composite score of a school apparently determines the propensity of the school board to close the school, the composite scores are the measure used to represent the dependent variable "likelihood of closure," CLOSE.[15]

Independent Variables

1. Savings in school operating costs. In the model, all schools are assumed identical in enrollment and in total yearly operating cost. In fact, the 32 elementary schools in the Madison Metropolitan School District varied widely in enrollment and total operating cost. Thus it is necessary to control for the size of a school in measuring its contribution toward the total cost savings to the school district from a given target reduction in the school system's physical plant. The variable PSCH is defined as

$$\mathrm{PSCH} = \frac{S}{E}\,\frac{\overline{E}}{\overline{D}},$$

where

S = the absolute amount of the cost savings from closure (on an annual basis)

E = the total 1977-78 enrollment

$\bar{E}$ = the average 1977-78 enrollment

$\bar{D}$ = the number of school days per year.

Thus PSCH represents the value for cost savings per school day from a school's closure which would result if the school were of average enrollment. Data on E were available by school from the school district; from them, $\bar{E}$ was calculated. $\bar{D}$ was taken to equal 180.

Information on the budgetary expenditures for past years of each of the 32 schools was used in measuring S. Some budgetary items, such as textbooks and instructional equipment, would be transferred in full to the budgets of the adjacent schools and are thus not included in the measurement of S. However, budgetary items such as utility costs, which reflect the cost of maintaining the school building, would no longer have to be paid if the school were closed and were thus included in full in the measurement of S.[16] In addition, reductions in salary outlays on certain eliminated administrative and teaching staff positions were included in the measurement of S.[17] A fixed percentage of the budgetary cost savings generated by closure is calculated by state government and given to the school district as "secondary state aid."[18] The sum of the budgetary cost savings and the forthcoming state aid was the figure used to represent S. The values of S for each school were in turn used to calculate values of the independent variable PSCH.

2. Increase in transportation costs. In Section 1 it was shown that the total increase in transportation costs for all Group 1 individuals equals $2g\overline{d}^2$. Because in practice the lengths of school neighborhoods vary for different schools in the city, the parameter $\overline{d}$ is treated as a variable whose observed value $\hat{d}_i$ for school i depends upon the sum L_i of the lengths of the neighborhood of school i and the two adjacent neighborhoods and upon the estimated number of households HH_i in the neighborhood of school i and the two adjacent neighborhoods.[19] The schools which were chosen as being adjacent to the proposed closure were in general the schools which were nearest to the closed school, as measured by the shortest possible route along city streets.[20] Fixed values for automobile transportation costs per mile and for average hourly wage were used in measuring g, the round-trip transportation and time costs per unit distance.[21] The variable measuring $2g\overline{d}^2$, the increase in transportation costs from closure (TRANSP), is given for school i by

$$TRANSP_i = (9.61 \times 10^{-6})\, L_i \cdot HH_i. \tag{19}$$

Equation (19) captures into the measurement of TRANSP the interaction of two effects: (1) the degree to which schools are spread apart in the immediate vicinity of the closing (L_i), and (2) the degree of concentration of households sustaining transportation cost increases in the vicinity of the closed school (HH_i).[22] Measurements for L_i for each of the 32 schools were based upon the choice of the appropriate adjacent schools and upon the measured distances along city streets between the closed school and adjacent schools.[23] Measurements of

HH_i for each of the 32 schools were based upon enrollment figures for the proposed closing and its two adjacent schools and an estimated ratio of enrollment to number of households used for the entire city.[24]

3. <u>Earnings capacity</u>. Data on the number of pupils receiving free or reduced-price lunches for each school were gathered by the Madison school board. The percentage of these pupils to total enrollment was calculated for each school. This percentage is labeled POV; it is an inverse measure of earnings capacity in a school neighborhood.[25]

4. <u>Distance to CBD</u>. The distance to the CBD of a school (DCBD) was computed by measuring the distance, in inches on a street map, between the school and the state capitol building in Madison's CBD via the shortest possible route along city streets.

5. <u>Taste for education</u>. Several proxies for the taste for education of a school's neighborhood residents were measured; and hypotheses about the direction of their effects established.

(a) The proportion of college graduates in the neighborhood (COLL). It is conjectured that the larger COLL is, the greater will be the taste for education.

(b) The proportion of renters in the nieghborhood of a school (RENT). The greater the proportion of renters in a school's neighborhood, the more transient will be its population and the less its stake in local public education; i.e., the less its taste for education.

(c) The proportion of elderly persons (65 years and older) in the neighborhood of a school (ELD). The greater the proportion of elderly in a school's neighborhood, the smaller will be the school-age population, and thus the smaller the taste for education.

(d) The proportion of parochial school students to the total student population in the neighborhood of a school (PAROC). The greater the proportion of parochial school students in a neighborhood school, the smaller will be the taste for public education in the neighborhood.[26]

The above independent variables will affect the dependent variable CLOSE through their effect upon the net benefit to the city from closure. Since net benefit from closure and the likelihood of closure (CLOSE) should be positively related, the expected signs in a regression of these independent variables on CLOSE should be given by the respective effects of each independent variable upon the net benefit from closure. These effects are listed below.

Independent variable		Expected sign
1.	PSCH	-
2.	TRANSP	+
3.	POV	+
4.	DCBD	+
5.	COLL	-
	RENT	+
	ELD	+
	PAROC	+

POV, of course, is an inverse measure of earnings capacity, and its sign will be the opposite of that given for earnings capacity earlier. Also, the four proxies for "taste for education" are not all in the same direction.

Regression Results

The final regression model implied by the analysis is the regression of the dependent variable CLOSE on explanatory variables 1 through 5. Since there are four proxies to represent explanatory variable 5, the taste for education, the following four regressions were run to represent the final regression model:

	1	2	3	4
dependent variable	CLOSE	CLOSE	CLOSE	CLOSE
independent variables	TRANSP	TRANS	TRANSP	TRANSP
	PSCH	PSCH	PSCH	PSCH
	POV	POV	POV	POV
	DCBD	DCBD	DCBD	DCBD
	COLL	RENT	ELD	PAROC

A constant term was used in each regression.

The results for Regression 3 are displayed in Table 1.

Table 1

(Dependent variable: CLOSE)

Independent Variable	Estimated Coefficient	t-Statistic
constant	-145.955	-28.35**
TRANSP	-.015	-1.58*
PSCH	.017	9.19**
POV	.137	2.53**
DCBD	.870	2.13**
ELD	.154	.95
$R^2 = .862$		

*significant at .10 level

** significant at .05 level

In this regression, the signs of the estimated coefficients for each of the explanatory variables are as expected by the model. Furthermore, all explanatory variables except ELD were significant at the .10 level at least, with PSCH, POV, and DCBD being significant at the .05 level. Results were similar for regressions 1, 2, and 4; that is, only the variables representing the taste for education (COLL, RENT, and PAROC) failed to exert a statistically significant effect in the direction suggested by the model upon the dependent variable. Thus there is no evidence that the variable taste for education has a significant role in explaining the school closure decisions of school administrators.

With this conclusion, explanatory variable 5 is deleted and a regression of CLOSE on explanatory variables 1 through 4 is run. The results are displayed in Table 2.

Table 2

(Dependent variable: CLOSE)

Independent Variable	Estimated Coefficient	t-Statistic
constant	-142.778	-35.57**
TRANSP	-.015	-1.62*
PSCH	.017	9.23**
POV	.121	2.35**
DCBD	.591	2.09**
R^2 = .858		

*significant at .10 level

**significant at .05 level

In this regression, the signs of the estimated coefficient for each of the explanatory variables are as expected by the model. Furthermore, three of the four variables (excluding the constant term) are significant at the .05 level, while one variable (TRANSP) is significant at the .10 level. It is concluded that for the Madison case, each of the explanatory variables suggested by the school closing model, with the exception of the taste for education, exerts a significant effect in the direction predicted by the model upon the school closure decisions of school administrators.

Of the four schools who ranked first for the closure list using the fitted values of the regression in Table 2, three were schools actually closed by school administrators. (The order of schools 3 and 4 was reversed in the two lists). Thus the process of closing schools according to the fitted values of the regression in Table 2 closely replicated the actual school closing process. Moreover, the explanatory variables in the regression of Table 2 which generated this result were not selected randomly or on an ad hoc basis, but were derived in a model based upon the optimization behavior of school administrators. The significance of these explanatory variables in the regression provides support for the validity of a theory of school closing decisions based upon systematic behavior by the decision-makers.

4. FINAL COMMENTS

The results of the model may be generalized to apply to other local public goods which are located at various sites in a community--

for instance, parks, libraries, and railroad stations (where these are supplied publicly). For each of these goods, it is reasonable to assume that consumers must incur transportation and time costs which are proportional to distance from the facility in order to derive utility from these goods. Thus an individual whose nearest public facility closes suffers a greater loss (because of both higher transportation costs and greater congestion at the adjacent sites) than those whose nearest public facility remained open. Apparently, such multisite public facilities as parks, libraries, etc., fit into the framework of the school closing model. This suggests that some important determinants of decisions to close public facilities by optimizing local governments are the following:

(1) the increase in transportation and time costs associated with the closing of the facility;

(2) the cost savings which result from no longer having to operate the facility;

(3) earnings capacity in the neighborhood of the closed facility;

(4) the distance of the closed facility to the city center; and

(5) the marginal rate of substitution of the good supplied by the facility for the private good in the neighborhood of the closed facility.

However, even in this general form the model offers only a partial solution to the problem of public facility closings. Actually, the local government has control over the site location of many public goods (e.g., libraries, parks, and railroad stations) and public "bads" (e.g., prisons and halfway houses[27]). Any complete treatment of the problem of public facility closings should thus take into account the relative change in demand for the various public facilities, their mix of locations throughout the city, and the preferences of the public for each of these goods relative to one another. Such a treatment, for example, would account for the interdependence between a school closing in one part of town and a park closing in another part of town and could thus evaluate the net effect of both closures upon the community. An even more general treatment would consider the total mix of locational decisions, not just closings, to be made by a locality.

In addition, a true general equilibrium treatment of the problem would include the movement of individuals in the city following the closing. In this framework, a school closing (or other exogenous amenity change) would disturb the initial locational equilibrium by producing new rent gradients and capital losses for those near the closed school. These capital losses, which reflect the decline in the value of houses located near the closed school, occur because there is no market mechanism which would restore the utility of those near the closed school by compensating them for their previously-held "nearness to school," since the market recognizes no such property right. The wealth effect of these capital losses and the new rental structure may cause a shuffling of households throughout the city. For example, an individual (call him A) who resides near a closed school suffers a capital loss (producing an income effect) and faces a new rent gradient in the city (producing a substitution effect). All others in the city face possible income and substitution effects as well, producing various incentives to move and consequent upward- and downward-bidding of housing prices at various locations. The general equilibrium outcome of these forces will be a different pattern of individual location throughout the city. In this equilibrium, the utility of individual A has not been restored to its preclosing level (because of the income effect produced by the capital loss) but has increased by moving (unless indivisibilities, moving costs or other market imperfections preclude his moving). That is, if u_0 represents individual A's preclosing utility level, u_1 his utility immediately following the closing, and u_2 his utility after general equilibrium has been reached (i.e., after moving), $u_1 \leq u_2 < u_0$. The model of this paper implied that for a Pareto welfare improvement, compensation payments are required to bring individual A (among others) from utility level u_1 to u_0. In the general equilibrium treatment, restoring individual A to his preclosing utility level only requires that compensation measures be taken to bring his utility from u_2 to u_0 (which is less of a jump than from u_1 to u_0 unless $u_1 = u_2$, i.e., no general equilibrium adjustments occur for individual A). To the extent, then, that any general equilibrium adjustments in the city actually take place, the compensation measures of the model overestimate the degree of compensation necessary to guarantee a Pareto welfare improvement.

Limitations of the Compensation Approach

The use of the tax function in restoring equilibrium following a closing has its limitations. For one thing, unless proposed compensation measures are actually carried out, the conclusion that a closing increases net welfare in the community depends upon interpersonal comparisons. More concretely, a school closing which increases net welfare of the community according to the model but which is unaccompanied by the associated compensation measures may increase inequality if the closing occurs in a low-income neighborhood. In such a case, an Atkinson social welfare function or other social welfare function which includes society's aversion to inequality of income might well reverse a conclusion of the model that a school closing would increase net welfare (or a conclusion of the model that a school closing in a wealthy neighborhood would decrease net welfare).

Also, a policy of carrying out all prescribed compensation measures poses problems for the government administrator. Computing the appropriate compensation due different groups, given interdependent locational movement of various multi-site facilities, would seem quite difficult if not impossible. As an alternative approach, the city could dynamically adjust the mix of locational decisions on various public facilities in the spirit of the compensation measures, i.e., compensating losers. That is, the city might put a library or park in a neighborhood where it was forced to close a school or put a public "bad" (such as a prison or halfway house) in a neighborhood which was the beneficiary of earlier locational decisions.

FOOTNOTES

[1]In reality, of course, only school-age children attend public schools. The model could be reformulated with the assumption that the city contains n households, each with at least one child attending school. This approach would complicate the analysis without altering the conclusions of the model.

[2]In practice, local public education is financed by revenue from property taxes, not from taxes of the form $r(\cdot)$ in equation (3), which are directly dependent upon an individual's distance to school or transportation costs to school. However, the incidence of an educational tax such as r may be shown to be equivalent to the incidence of an *ad valorem* property tax under certain conditions: (1) the present value of all future transportation costs is reflected in the value of a house; (2) the property tax is imposed upon the value of an individual's house less the value of a house at the border of his neighborhood; and (3) the property tax rate equals the real interest rate. To the extent that these assumptions approximate reality, the educational tax r may be interpreted as an *ad valorem* property tax.

[3]The assumption that all schools are of identical cost may be relaxed without changing the conclusions of the analysis. See Lerman (1980, pp. 77-84).

[4]There is some evidence that smaller class sizes lead to greater educational output as measured by student performance (see Furno, 1967). This lends some support for the above specifications, since larger τ and K are associated with smaller class size, which in turn is associated with greater quantity of education.

[5]An alternative assumption, that all displaced staff either are placed in one of the two adjacent schools (with no more than half of the total going to either school) or are fired, would not change the conclusions of the model.

[6]These propositions are proven in Lerman (1980, pp. 58-63).

[7]In practice, the amount of cost savings from closing a school may be less than operating costs, since some of the closed school's operating costs (e.g., teacher's salaries) will be transferred to the adjacent schools and some minimum maintenance costs may be required for the closed building. If, however, the school district sells the building, the initial cost savings may exceed the operating costs. In Section 3, these considerations will be taken into account in measuring p.

[8]The increase in transportation cost from a closing for a Group 1 individual located at distance to school d is given by

$$\begin{aligned} T_1^{II}(d) - T_1^{I}(d) &= (2g\bar{d} - gd) - gd \\ &= 2g\bar{d} - 2gd. \end{aligned}$$

Since only Group 1 individuals suffer transportation cost increases, the total increase in transportation costs which is due to the closing is given by

$$2\int_0^{\bar{d}} (2g\bar{d} - 2gd)\,dd = 2g\bar{d}^2,$$

which is the right-hand-side of inequality (18).

[9]However, this net benefit figure neglects the costs in utility from reduced education consumption (such costs are zero if $e' = 0$).

[10]If all Group 3 schools are different in these respects while Group 1 and 2 schools are identical and possess operating cost p and neighborhood size $2\bar{d}$, it may be shown that a necessary condition for a closing to bring about a Pareto improvement is

$$p > 2g\bar{d}^2,$$

where per unit transportation cost g is assumed constant throughout the city. See Lerman (1980, pp. 77-84).

[11]The use of an earnings capacity variable is clearly at odds with equity goals. A significant relationship between earnings capacity and school closure decisions may thus indicate that school administrators have placed more importance on efficiency than on equity goals.

[12]This result holds, essentially, because neighborhoods farther from the CBD are sparser in population density, by assumption, and thus contain fewer people to compensate for a closure than neighborhoods near the CBD. See Lerman (1980, p. 109-126).

[13]The composite scores were obtained as follows. The data for each criterion were adjusted into normalized scores (i.e., scores whose distribution is approximately standard normal). For 5 of the 12 criteria, the higher the normalized score, the higher the likelihood of closure. The normalized scores for the remaining 7 criteria were all multiplied by -1 to make these scores increasing in the likelihood of closure also. After adding 10 to each (adjusted) normalized score to make each observation positive for convenience, the resulting scores were summed for each school, yielding 32 composite scores. The

12 criteria used were 1978 enrollment, projected 1979 enrollment, projected 1980 enrollment, projected 1981 enrollment, number of students within 1.5 miles of school, ease of reassignment, maintenance cost per square foot, energy cost per square foot, maintenance cost per student, energy cost per student, instructional cost per student, and number of potential new students. Details on the measurement, rationale, and data sources for these criteria are found in Madison Metropolitan School District (1978A).

[14]These rankings were obtained after eliminating from consideration a school equipped to handle handicapped students, which otherwise would have been ranked first, because of federal requirements on the number of such handicapped-equipped schools in a community the size of Madison.

[15]As an alternative, the dependent variable may be treated as dichotomous. Logit analysis was attempted but failed because of the small number of schools which were closed (three) relative to the number of observations (32). Using a dummy variable for the dependent variable (which equals 1 if the school is closed and zero otherwise) and running a least squares regression failed for the same reason.

[16]Some level of maintenance may be required for the empty school buildings and thus the savings in maintenance may be somewhat overstated. However, this overstatement should be uniform across all schools and thus should not affect the regression results.

[17]It was assumed that following any school closing, personnel would be shifted throughout the district so that one full-time position of lowest seniority in each of the following administrative categories would be eliminated: principals, librarians and counselors, and secretaries. In addition, it was assumed that savings from

teacher staff reductions would be equal, for each of the 32 schools, to an average of estimates on the reduced salary outlays from teacher staff reductions resulting from closure. These estimates were made for four schools by the Madison Metropolitan School District simulating the closure of these four schools in a 1978 study.

[18]The percentage varies from year to year and depends upon the pattern of educational expenditures in school districts throughout the state. For the 1979-80 school year, the figure used was 33%. A detailed description of the procedure for calculating state aid is contained in Wisconsin Department of Public Instruction (1978, pp. II-V).

[19]The number of households HH_i is needed to measure $\hat{d}_i$ in terms of housing units, defined as the unit of length which contains the average-sized residence of one household. Such units are used to conform with the model's assumption of unit population density throughout the city. See Lerman (1980, Chapter 5).

[20]A set of rules used to determine the adjacent schools to any proposed closing was formulated in Lerman (1980, Chapter 6). These rules were based upon a judgment on how school administrators would place former students of the closed school into the remaining schools in the general area of the closing.

[21]Details on the measurement of g are found in Lerman (1980, pp. 146-147).

[22]I made an attempt to measure the separate effects of L_i and HH_i in a comprehensive regression model, but these variables failed to register significant effects upon closure decisions.

[23]The boundaries of a school's neighborhood were assumed to be given by the midpoints between the school and the relevant adjacent schools on either side. Details on the measurement of L_i are provided in Lerman (1980, pp. 143-145).

[24]While enrollment data for all schools were available, data on number of households were not generally available. However, estimates for the number of households for two of the schools in the city were supplied by these schools' principals. The average of the two enrollment-to-number of household ratios for these schools was used to generate data on number of households for all schools in the city. See Lerman (1980, pp. 145-146).

[25]The median annual income level of a school's neighborhood was used as an alternative measure of earnings capacity. The source was 1974 U.S. Census data. For a given school, the median annual income of the census tract (or tracts) that was approximately coterminous with the actual school attendance areas was used. However, since the boundaries of school attendance areas were not coincident (and in some cases were widely divergent) with the boundaries of the associated census tract (or tracts), this measure was judged inferior to the POV measure, whose values <u>were</u> uniquely associated with the schools measured.

[26]The source for the data on PAROC was the Madison Metropolitan School District, October 1973. 1970 U.S. Census data were used for COLL and RENT, and October 1974 Census data for ELD.

[27]While these goods contribute to community welfare and thus should be regarded as public goods rather than "bads," individuals in the urban framework presented here would probably perceive prisons and halfway houses located near their residence as "bads" rather than goods.

APPENDIX

Derivation of Stage II Tax Function

The Stage II function r^{II} is formed by requiring that it satisfy both (4) and (2). Under the preclosing tax structure r^{I}, the government budget would be in surplus after the closing by p, the amount saved annually by no longer having to operate the closed school. To satisfy the government budget constraint in Stage II, it is necessary to set the State II tax function r^{II} so that p less taxes are collected than were collected in Stage I under r^{I}. This may be done by setting Stage II taxes so that

(a) the tax revenue collected from Group I individuals, which totaled p in Stage I (seen by dividing equation (4) by x), totals zero in Stage II; and

(b) the tax revenue collected from Group 2 and Group 3 individuals in Stage II is identical to that collected in Stage I.

From (a), r_1^{II} must satisfy

$$\int_0^{\bar{d}} r_1^{II}\, dd = 0. \tag{20}$$

To derive conditions on r_2^{II} and r_3^{II} for (b) to hold, first observe that satisfaction of the government budget constraint in Stage I under taxes r^{I} implies that the total tax revenue collected from any neighborhood in Stage I equals

$$2 \int_0^{\bar{d}} r^{I}\, dd = p. \tag{21}$$

Thus (b) holds if total tax revenue collected in Stage II from each neighborhood of Group 2 and 3 individuals equals p; i.e., (b) holds if r_2^{II} and r_3^{II} satisfy the following:

$$2 \int_0^{\bar{d}} r_2^{II}\, dd = p \tag{22}$$

$$2 \int_0^{\bar{d}} r_3^{II}\, dd = p \tag{23}$$

If the Stage II tax function, r^{II}, is set to satisfy equations (20), (22), and (23), then the government budget constraint is satisfied in Stage II. By also requiring that r^{II} be set so that locational equilibrium holds in Stage II, we may obtain a functional form for the tax function r^{II}. The condition for locational equilibrium in Stage II is that the tax function r^{II} be of the form given by equation (3):

$$r^{II}(d^I,\underline{x}) = -T^{II}(d^I) + c^{II}(\underline{x}) \tag{24}$$

Note that for Group 2 and 3 individuals, $T^{II}(d^I) = gd^I = T^I(d^I)$ [see equation (12)]. Thus,

$$r_2^{II}(d^I,\underline{x}) = -gd^I + c_2^{II}(\underline{x}) \tag{25}$$

$$r_3^{II}(d^I,\underline{x}) = -gd^I + c_3^{II}(\underline{x}) \tag{26}$$

Substituting (25) into (22) and (26) into (23) and solving for c_2^{II} and c_3^{II} respectively, we obtain the same solution for the constant terms as as obtained in Stage I [in equation (6)]:

$$c_2^{II}(\underline{x}) = c_3^{II}(\underline{x}) = \frac{p\underline{x}}{n} + \frac{gn}{4\underline{x}} \tag{27}$$

Substituting (27) into both (25) and (26), we may conclude that

$$r_2^{II} = r_3^{II} = r^{I} = -gd^{I} + c^{I}(\underline{x}). \tag{28}$$

For Group 1 individuals, equation (12) states that $T_1^{II}(d^I) = 2g\bar{d} - gd^I$. Thus the locational equilibrium condition (24) for Group 1 individuals is given by

$$r_1^{II}(d^I, \underline{x}) = -2g\bar{d} + gd^I + c_1^{II}(\underline{x}) \tag{29}$$

Substituting (29) into (20) and solving for $c_1^{II}(\underline{x})$ using equation (1) yields

$$c_1^{II}(\underline{x}) = \frac{3gn}{4\underline{x}}. \tag{30}$$

Thus

$$r_1^{II}(d^I, \underline{x}) = gd^I - \frac{gn}{4\underline{x}}.$$

This yields the Stage II tax function given by equation (14) of the text.

REFERENCES

Bailey, J. D., ed. 1977. Declining enrollments and school closings. Unpublished paper, Kansas University, Lawrence, Kansas.

Council of Educational Facility Planners, International. 1978. Surplus school space--the problem and the possibilities. Columbus, Ohio.

Furno, O. F. 1967. Class size and pupil learning. Unpublished paper, Baltimore City Public Schools.

Illinois State Office of Education. 1975. Facilities and declining enrollment. Department of Planning and Research, Springfield, Illinois, December.

Jensen, K. 1978. Survey of forty-three school districts in six mid-western state. Unpublished paper available at Madison Metropolitan School District office.

Lerman, D. 1980. The economics of public school closings. Unpublished Ph.D. dissertation, University of Wisconsin-Madison.

Madison Metropolitan School District. 1978A. 1978-79 elementary school rankings and simulated school closings.

________. 1977. 1977-78 Budget.

________. 1978B. 1978-79 Budget.

________. 1979. 1979-80 Budget.

U.S. Bureau of the Census. 1973. Census of population and housing: 1970 census tracts. Washington: GPO.

________. n.d. First count summary tape of the 1970 census of population and housing, Washington.

________. n.d. October 1974 population by age groups--City of Madison. Washington.

Wisconsin Department of Public Instruction. 1978. Distribution of Wisconsin public school state aid dollars for 1977-78. Bulletin No. 8372.

Wholeben, B. E. 1974. The design, implementation, and evaluation of mathematical modeling procedures for decisioning among educational alternatives. Unpublished Ph.D. dissertation, Department of Educational Administration, University of Wisconsin-Madison.