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EQUIVALENCE SCALES, HORIZONTAL EQUITY AND OPTIMAL TAXATION

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Equivalence Scales, Horizontal Equity and Optimal Taxation

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ABSTRACT

Families of various sizes all share the same preference maps over consumption and leisure where consumption has been appropriately scaled to reflect those variations. Under lump sum taxes, equating labor supplies and scaled consumptions is not the optimal way to achieve horizontal equity. Under an income tax structure, if there is sufficient diversity of ability for all family sizes, the opposite is true.

In particular, the marginal tax rates of the two ends of the ability scales are positive for small families and negative for large ones.

Equivalence Scales, Horizontal Equity
and Optimal Taxation

1. INTRODUCTION

In this paper we analyze the question of how differences in family size should be treated by the income tax system in order to achieve horizontal equity. The answer to this question depends on (a) how differences in family size manifest themselves in the consumption patterns (or, more generally, the preferences) of households of various sizes, and (b) how we define horizontal equity. One of the most common procedures of incorporating demographic variables in general and household size in particular into demand systems is called "demographic scaling" (see Pollak and Wales, 1978a, 1978b and their references). This method employs the idea of "equivalence scales" or "standard adults" in explaining the differences in demand patterns caused by household size differences.

For our purposes we can describe the procedure of scaling as follows. Suppose for the sake of simplicity that there are only two family sizes: one-member and two-member families. It is assumed that a family of i members ($i = 1,2$) contains c_i "standard adults." With no loss of generality, let $c_1 = 1$ (so that a family of one person has one standard adult) and write c for c_2. Each family has one wage-earner and consumes an aggregate consumption good (x) and provides labor services (y). Scaling amounts to assuming that the two-member family's preferences over bundles (x,y) are the same as the one-member family's preferences over

$(x/c,y)$. The rationale for this assumption is that when a family of two members has a consumption of x, then its per-standard-adult consumption is only x/c. Thus, the utility that a family with two standard adults derives from the bundle (x,y) is the same as the utility derived by a family with only one standard adult from the bundle $(x/c,y)$.[1]

In our model economy, families differ from each other by their size and also by their earning ability or skill. Horizontal equity is defined here as requiring that families with the same earning ability will enjoy the same level of utility, irrespective of their size. Accordingly, we define our objective as that of maximizing a utilitarian social welfare function, subject to the principle of horizontal equity, i.e., equal utilities for households with equal earning abilities.

One way of insuring equal utilities for families with equal skills is to equate their labor supplies and per-standard-adult consumptions, namely

$$y_1 = y_2 \text{ and } x_1 = x_2/c,$$

where the subscripts 1 and 2 refer to one-member and two-member families, respectively. However, this is certainly not the only way to guarantee horizontal equity. In fact, if the government can employ lump-sum taxes and transfers, then it never pays to equate the labor supplies and per-standard-adult consumptions of families of different size but the same earning ability. As might be expected, the larger household should provide a lower supply of labor and have a lower level of consumption per-standard-adult (this is shown in the following section).

However, one of the main findings of this paper is that if lump-sum taxes and transfers are excluded and the government has to rely on income taxation, then equating the labor supplies and the per-standard-adult consumptions is the only way to grant equal utilities for households of different size but the same earning ability (see section 3). This suggests that relying on income taxation in order to achieve horizontal equity causes some additional dead weight loss on top of the standard one which stems from the fact that an income tax creates a wedge between the marginal rate of substitution of consumption for leisure and the marginal product of labor. Section 5 illustrates the magnitude of this additional loss.

Optimal taxation theory says that when all families are of the same size, then the marginal tax rate should be nonnegative everywhere and equal to zero at the bottom and top ends of the income distribution (see, for instance, Mirrlees, 1971, 1976). Section 4 discusses the properties of the optimal tax schedules that face the two types of households in our model economy. We show that an appropriately defined average of these two schedules must have a marginal rate which is non-negative everywhere and equal to zero at both ends of the income distribution. But, each individual schedule need not have this property. Moreover, one of these two taxes will definitely have a marginal rate which is negative at sufficiently low and sufficiently high income levels; in fact, this rate could even be negative everywhere.

Throughout this paper the principle of horizontal equity is imposed on the utilitarian sum-of-utilities objective function, for utilitarianism by itself cannot guarantee such equity. In an appendix to the paper we investigate the question of whether a weighted sum of utilities, where higher weights are assigned to the larger families, can bring about horizontal equity.

2. HORIZONTAL EQUITY WITH LUMP-SUM TAXES

Let u and U be the utility functions of a one-member family and a two-member family, respectively. As explained in the introduction, it is assumed that these two utility functions are related to each other by

$$U(x,y) = u(x/c,y). \tag{1}$$

Recall that horizontal equity is understood in this paper to mean that families with the same earning ability should enjoy the same level of utility. The earning ability or skill of a household is identified with the wage rate of the single wage-earner in the household. Since lump-sum taxes and transfers are admissible in this section, it will suffice to consider only one wage level. Thus, we suppose that there are one one-member family and one two-member family, both facing the same wage rate, denoted by n.

The objective is to maximize a utilitarian social welfare function

$$W = u(x_1,y_1) + u(x_2/c,y_2), \tag{2}$$

where x_i and y_i are the consumption and labor supply, respectively, of the family of i members, $i = 1,2$. On this objective function we impose the horizontal equity principle

$$u(x_1,y_1) = u(x_2/c,y_2). \tag{3}$$

The first-best optimum is then obtained by maximizing the utilitarian social welfare function (2), subject to the horizontal equity principle (3), and the resource constraint

$$x_1 + x_2 + R \leq ny_1 + ny_2, \tag{4}$$

where R is some predetermined level of public consumption.

The Pareto-efficiency condition implied by our utilitarian objective requires us to equate each family's marginal rate of substitution of consumption for leisure to its wage rate. Since our two households face the same wage rate, then the marginal rates of substitution must be equated to each other:

$$\frac{u_y(x_1,y_1)}{u_x(x_1,y_1)} = \frac{cu_y(x_2/c,y_2)}{u_x(x_2/c,y_2)} = -n. \tag{5}$$

Since $c > 1$ (it is generally believed that $1 < c < 2$), then condition (5) rules out the possibility of achieving horizontal equity, condition (3), by equating labor supplies and per-standard-adult consumptions. In fact, assuming, as we do, that u is strictly concave, then (3) and (5) imply that the larger family should work and consume per-standard-adult less than the smaller family (see Figure 1). The economic explanation for this is quite straightforward: Since a dollar of consumption at the disposal of the larger family means only $1/c$ dollars per-standard-adult, then the smaller household is more efficient in consuming x. Thus, the smaller family should consume more x per-standard-adult and, in order to maintain horizontal equity, should also work harder.

3. HORIZONTAL EQUITY WITH INCOME TAXATION

The preceding section shows that when lump-sum taxes are available, equating the labor supplies and the per-standard-adult consumptions of households of different size but the same earning ability is not an optimal way to achieve horizontal equity. However, we show in this section that such an equality becomes the only possible way of maintaining horizontal equity when lump-sum taxes and transfers are replaced by income taxes.

In this section we assume that there is a continuum of households of each size. We denote by $F_i(n)$ the number of i-member households who earn a wage rate which is less than or equal to n. It is assumed that the range of wages is the interval $[o,\infty)$.[2] We define $f_i(n) = F_i'(n)$, $i = 1,2$. Let $x_i(n)$ and $y_i(n)$ be the consumption and labor supply of an i-member household whose wage is n. Then the utilitarian social welfare function becomes

$$W = \int u[x_1(n),y_1(n)]f_1(n)dn + \int u[x_2(n)/c,y_2(n)]f_2(n)dn. \qquad (2')$$

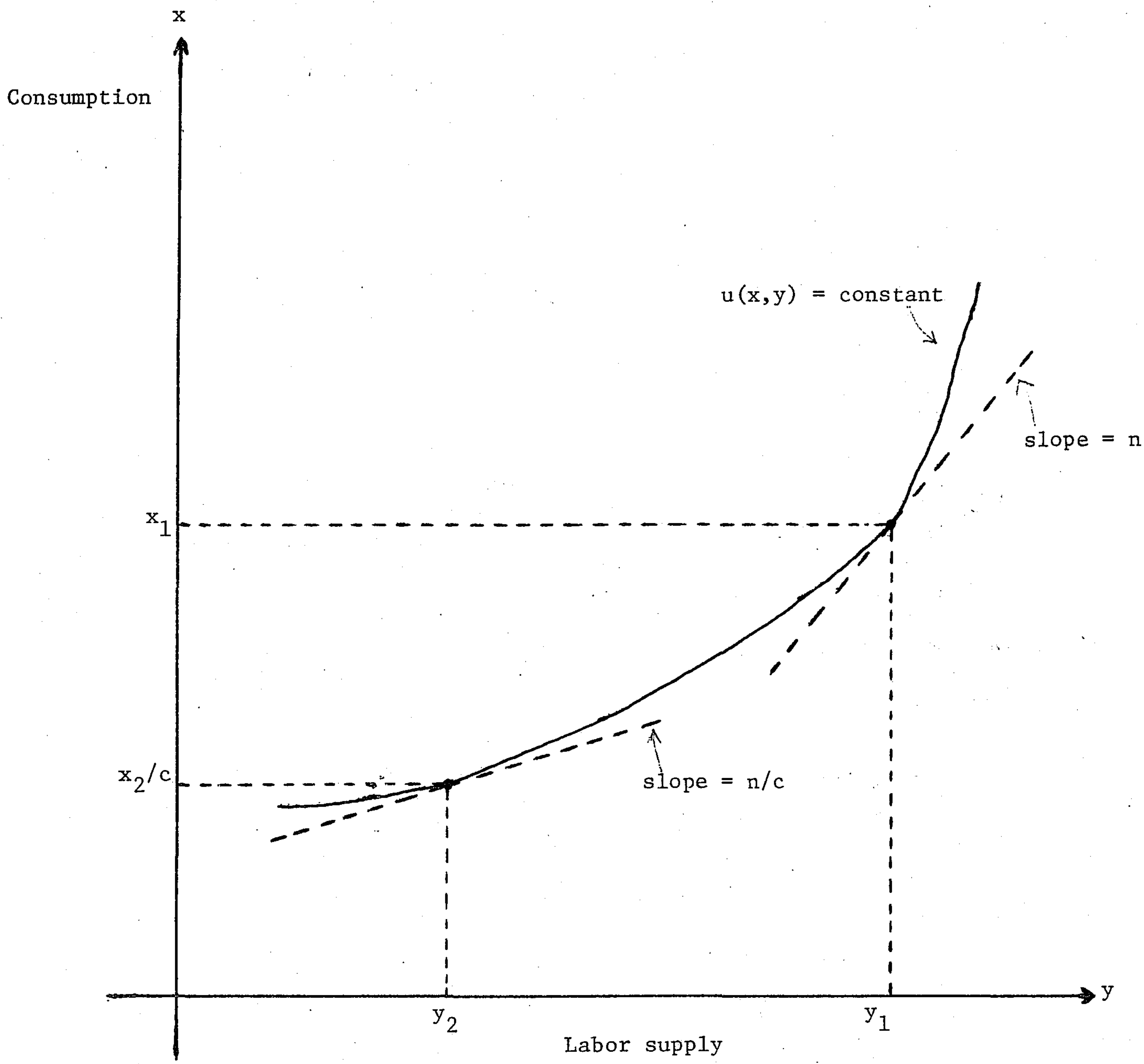

Figure 1. Indifference maps of the two households.

The horizontal equity condition becomes

$$u[x_1(n), y_1(n)] = u[x_2(n)/c, y_2(n)] \text{ for all } n, \tag{3'}$$

and the resource constraint becomes

$$\int x_1(n) f_1(n) dn + \int x_2(n) f_2(n) dn + R \leqq \int n y_1(n) f_1(n) dn + n y_2(n) f_2(n) dn. \tag{4'}$$

We assume that both assumption and leisure are normal goods and that $u(x,y)$ is strictly increasing in x and strictly decreasing in y.

The objective is to maximize (2') subject to (3'), (4'), and the constraint that only income taxation can be used. The latter constraint restricts our choice of allocations $(x_1(\cdot), y_1(\cdot))$ for one-member households and $(x_2(\cdot), y_2(\cdot))$ for two-member households. Specifically, these allocations have to be sustainable by income tax functions. Let A_i be the set of such allocations for i-member households, namely

$$A_1 = \{(x(\cdot), y(\cdot)) / \exists \text{ on income tax function } T \text{ such that for each } n, (x(n), y(n)) \text{ maximizes } u(x,y), \text{ subject to } x \leqq ny - T(ny)\}, \tag{6}$$

and

$$A_2 = \{(x(\cdot), y(\cdot)) / \exists \text{ an income tax function } T \text{ such that for each } n, (x(n), y(n)) \text{ maximizes } u(x/c,y), \text{ subject to } x \leqq ny - T(ny)\}. \tag{7}$$

(Note that these sets are determined exclusively by the underlying utilities, u for A_1 and U for A_2, and not the distribution functions F_1 and F_2.) Then the restriction to employ only income taxation is complied with by adding the constraint

$$(x_i(\cdot),y_i(\cdot)) \in A_i, \ i = 1,2. \tag{8}$$

The main finding here then is that when horizontal equity, condition (3'), must be attained by income taxation, condition (8), then one has to equate the per-standard-adult consumptions and the labor supplies of households with the same wages:

$$x_1(n) = x_2(n)/c \text{ and } y_1(n) = y_2(n) \tag{9}$$

for all except, at most, countably many n's.

The proof of this may be done in a few steps.[3]

(a) We first show that $(x(\cdot),y(\cdot)) \in A_2$ if and only if $(\bar{x}(\cdot),y(\cdot)) \in A_1$, where $\bar{x}(n) \overset{\text{def.}}{=} x(n)/c$. Suppose $(x(\cdot),y(\cdot)) \in A_2$. Then there exists an income tax function T such that for each n, $u[x(n)/c,y(n)] \geq u(x/c,y)$ whenever $x \leq ny - T(ny)$. Hence, $u[\bar{x}(n),y(n)] \geq u(x/c,y)$ whenever $x/c \leq [ny - T(ny)]/c$. Put $\bar{x} = x/c$ and define another tax function $\bar{T}$ by

$$z - \bar{T}(z) = [z - T(z)]/c \text{ (implying } \bar{T}(z) = T(z)/c + z(c - 1)/c). \tag{10}$$

Then, $u[\bar{x}(n),y(n)] \geq u(\bar{x},y)$ whenever $\bar{x} \leq ny - \bar{T}(ny)$. Thus, $\bar{T}$ sustains the allocation $(\bar{x}(\cdot),y(\cdot))$ and hence $(\bar{x}(\cdot),y(\cdot)) \in A_1$. The converse statement is proved in a similar way.

(b) We state here some properties of allocations which can be supported by income tax functions. Let v denote the household utility function and suppose that $(x(\cdot),y(\cdot))$ is sustainable by some income tax function, say T. Then

(i) The pre-tax income function $z(n) \overset{\text{def.}}{=} ny(n)$ is nondecreasing in n (see Mirrlees, 1971). Thus, if $(x_i(\cdot),y_i(\cdot)) \in A_i$, then $z_i(n) = ny_i(n)$ is nondecreasing in n.

(ii) For all except, at most, countably many n's, there is a unique solution to the household utility-maximization problem: $\max v(x,y)$ subject to $x \leq ny - T(ny)$ (see Sadka, 1976).

(iii) With no loss of generality we may assume that if $z \geq 0$, then there exists $n \in [o,\infty)$ such that $z(n) = z$ (see Sadka, 1976).

(c) Next we show that if (i) $(x(\cdot),y(\cdot))$ and $(\bar{x}(\cdot),\bar{y}(\cdot))$ are two allocations which can be supported by income tax functions, say T and $\bar{T}$, respectively, and (ii) $v[x(n),y(n)] = v[\bar{x}(n),\bar{y}(n)]$ for all n, then $T = \bar{T}$ and $(x(n),y(n)) = (\bar{x}(n),\bar{y}(n))$ for all except, at most, countably many n's. Suppose to the contrary that $T \neq \bar{T}$, i.e., there exists $z_o \geq 0$ such that $T(z_o) > \bar{T}(z_o)$. By (b) (iii) above, there exists $n_o \in [o,\infty)$ such that $z(n_o) = n_o y(n_o) = z_o$. Since $T(z_o) > \bar{T}(z_o)$, it follows by revealed preference that $v[\bar{x}(n_o),\bar{y}(n_o)] > v[x(n_o),y(n_o)]$, which is a contradiction. Hence, $T = \bar{T}$, and it follows from (b) (ii) that $x(n) = \bar{x}(n)$ and $y(n) = \bar{y}(n)$ for all except, at most, countably many n's.

Now we are in a position to prove the main finding of this section, namely that (3') and (8) imply (9). Since $(x_2(\cdot),y_2(\cdot)) \varepsilon A_2$, then it follows from (a) above that $(x_2(\cdot)/c,y_2(\cdot)) \varepsilon A_1$. We also know that $(x_1(\cdot),y_1(\cdot)) \varepsilon A_1$ and that $u[x_2(n)/c,y_2(n)] = u[x_1(n),y_1(n)]$ for all n. Therefore, it follows from (c) that $x_2(n)/c = x_1(n)$ and $y_2(n) = y_1(n)$ for all except, at most, countably many n's.

Furthermore, if we denote the taxes which support the allocations $(x_1(\cdot),y_1(\cdot))$ and $(x_2(\cdot),y_2(\cdot))$ by T_1 and T_2, respectively, then (a) (and especially equation (10)) and (c) can determine the relationship between T_1 and T_2. Since $(x_2(\cdot),y_2(\cdot)) \varepsilon A_2$, then $(\bar{x}_2(\cdot),y(\cdot)) \varepsilon A_1$, where $\bar{x}_2(n) = x_2(n)/c$. Equation (10) shows that the tax function $\bar{T}_2$ defined by

$$\bar{T}_2(z) = T_2(z)/c + z(c - 1)/c \qquad (11)$$

supports $(\bar{x}_2(\cdot),y_2(\cdot))$. By (c), $\bar{T}_2 = T_1$. Thus,

$$T_1(z) = T_2(z)/c + z(c - 1)/c \qquad (12)$$

and

$$T_1'(z) = T_2'(z)/c + (c - 1)/c \text{ or } T_2'(z) = cT_1'(z) - c + 1. \qquad (13)$$

Using (13), we can see that $1 - T_2'(z) = c[1 - T_1'(z)]$ and hence, since $c > 1$, it follows that

$$T_1'(z) > T_2'(z). \qquad (14)$$

The group of one-member families faces higher marginal tax rates than the group of two-member families.

4. OPTIMAL TAX RATES

The optimal tax problem under consideration is that of maximizing the social welfare function (2'), subject to the horizontal equity principle (3'), the resource constraint (4'), and the income taxation constraint (8). However, in view of the results of the preceding section, this problem reduces to the following:

$$\max \int u[x_1(n), y_1(n)] f(n)\, dn \tag{15}$$

s.t.:

$$\int [n y_1(n) - p(n) x_1(n)] f(n)\, dn \geqq R \tag{16}$$

and

$$(x_1(\cdot), y_1(\cdot)) \varepsilon A_1, \tag{17}$$

where

$$p(n) = [f_1(n) + c f_2(n)]/[f_1(n) + f_2(n)] \tag{18}$$

and

$$f(n) = f_1(n) + f_2(n). \tag{19}$$

(Note that when the distribution of wages within each family size group is the same, i.e., when $f_1 = \alpha f_2$ for some constant $\alpha > 0$, then p becomes a constant, independent of n.)

The above problem is a standard optimal tax problem analyzed in the public finance literature (see, for instance, Mirrlees, 1976), except for one minor difference: p in our problem depends on n. However, whether p is a constant or varies with n plays no role in establishing the qualitative properties of the optimal marginal tax rates (again, see Mirrlees, 1971, 1976).

Thus, we can still claim that optimality requires that for each n, the marginal rate of substitution of leisure for consumption, namely $-u_y/u_x$, should be smaller than or equal to the social marginal rate of transformation of consumption into leisure, which is, by (16), equal to n/p. (This is known as nonnegativity of optimal marginal tax rate.) Let T_1 be the income tax function which supports the optimal allocation $(x_1(\cdot), y_1(\cdot))$. Since each household equates its marginal rate of substitution of leisure for consumption to its net wage rate, which is $n(1 - T_1')$, it follows that

$$n(1 - T_1') \leqq n/p$$

or

$$T_1'[ny_1(n)] \geqq \frac{p(n) - 1}{p(n)} = \frac{f_2(n)(c - 1)}{f_1(n) + cf_2(n)}. \tag{20}$$

We can then conclude from (13) that

$$T_2'[ny_2(n)] \geqq - \frac{f_1(n)(c - 1)}{f_1(n) + cf_2(n)}. \tag{21}$$

Hence,

$$f_1(n)T_1'[ny_1(n)] + f_2(n)T_2'[ny_2(n)] \geq 0. \tag{22}$$

This last result states that the weighted average of the two marginal tax rates, T_1' and T_2', should be nonnegative, where the weights are f_1 and f_2, respectively. Since, by (14), we already know that $T_1' > T_2'$, it follows that $T_1' > 0$, as (20) indeed confirms. Thus, the marginal tax rate imposed on one-member families should be strictly positive. However, two-member households may well face negative marginal tax rates.

Similarly, a number of authors showed that the marginal tax rate in the standard optimal tax model should be zero at the bottom and top ends of the income distribution.[4] This result too holds here only with respect to the weighted average of the two taxes, T_1' and T_2'. Thus, if we let the interval of wages be $[N_1,N_2]$, then (20), (21), and (22) hold as equalities for $n = N_1$ and $n = N_2$:

$$f_1T_2' + f_2T_2' = 0$$

$$T_1' = f_2(c - 1)/(f_1 + cf_2) > 0$$

and

$$T_2' = -f_1(c - 1)/(f_1 + cf_2) < 0.$$

Thus, at the end points N_1 and N_2, the marginal tax rate imposed on one-member families is positive, the marginal tax rate imposed on two-member families is negative and their weighted average is zero.[5]

5. THE DEADWEIGHT LOSS OF HORIZONTAL EQUITY WITH INCOME TAXATION

We have seen that when horizontal equity must be achieved via income taxation we must have equal per-standard-adult consumptions and equal labor supplies for households with equal wages. On the other hand, such equality is nonoptimal when lump-sum taxes can be used. This suggests that when horizontal equity is to be maintained income taxation has some extra deadweight loss over and above the usual deadweight loss which stems from the divergence between consumer and producer prices. In this section we attempt to illustrate the magnitude of this additional loss, which we call the horizontal equity deadweight loss of income taxation.

We do this by comparing the cost of achieving horizontal equity via lump-sum taxes with the cost of achieving it via income taxation. Recall that the horizontal equity principle was imposed on, rather than implied by, the utilitarian social welfare objective. This suggests that horizontal equity imposes some cost on the utilitarian sum-of-utilities. By comparing this cost when lump-sum taxes are employed with the cost incurred when income taxation is used, we can get some idea about the magnitude of what we termed the horizontal equity deadweight loss of income taxation.

Specifically, we first find the optimal lump-sum taxes and transfers without imposing the principle of horizontal equity. This is done by maximizing the social welfare function (2') subject to the resource constraint (4'). Denote the optimal level of W by $\bar{W}$. We next ask what is the minimum amount of revenue that we have to lose if we try to maintain that same level of social welfare (namely, $\bar{W}$) while imposing horizontal equity. This loss is the cost of maintaining horizontal equity via lump-sum taxes. In a similar fashion we find the cost of maintaining horizontal equity via income taxation. The difference between the latter and the former costs illustrates the magnitude of the horizontal equity deadweight loss of income taxation.

Consider a very simple poor-rich model: There are only two wage levels, n_P ("poor") and n_R("rich"), where $n_P < n_R$. There are only two families at each wage level, one small and one large. The utility function u of one-member families is specified as a Cobb-Douglas

$$u(x,y) = [x^{\beta}(K - y)^{1-\beta}]^{1-\varepsilon}/(1 - \varepsilon),$$

where K is the endowment of leisure, and ε is a measure of inequality-aversion (increasing in ε). The method that can be used to calculate optimal taxes in such discrete models is discussed at length in Balcer and Sadka (1979). Here we will present only the results. The values for the parameters of the model are taken from Sadka, Garfinkel, and Moreland (1979) who employ the 1976 Current Population Survey:

R = \$6184 per year

n_P = \$3.56 per hour[6]

n_R = \$6.63 per hour[7]

β = 0.75

K = 3120 hours per year (60 hours per week, 52 weeks per year)

c = 1.2.[8]

Two values of ε are considered: $\varepsilon = 0.5$ and ε 2.0. Table 1 presents optimal tax rates, labor supplies, pre-tax incomes, consumptions, and utilities, when horizontal equity is not imposed. Table 2 presents data for these variables when we try to achieve the same welfare levels as in Table 1, while imposing the principle of horizontal equity.[9]

Table 3 presents the costs of horizontal equity in absolute terms and as percentages of government revenues (R). The horizontal equity deadweight loss of income taxation is not significant: Only \$15 or 0.24% of R when $\varepsilon = 0.5$, and \$16 or 0.26% of R when $\varepsilon = 2.0$.

6. CONCLUSION

The results regarding the structure of the optimal income tax are surprising at first. It is shocking to realize that the mere fact of being two instead of one and being the most able should guarantee a wage subsidy at the margin; as mentioned earlier, this stems from the fact that the marginal tax rate of the average family must be zero at that point and horizontal equity is imposed. More realistically, consumption scales should depend on consumption levels also, not just on family size; if this were the case, and the scale was one for an increment in consumption at the levels contemplated by the most able, then we conjecture that the tax would be the same on all most able workers regardless of the size of their families.

Table 1

No Horizontal Equity

	LUMP-SUM TAXATION				INCOME TAXATION			
	"Poor" Families		"Rich" Families		"Poor" Families		"Rich" Families	
	One-Member Family	Two-Member Family	One-Member Family	Two-Member Family	One-Member Family	Two-Member Family	One-Member Family	Two-Member Family
$\varepsilon = 0.5$								
Marginal Tax Rate	0	0	0	0	.106	.117	0	0
Average Tax Rate	-.776	-.440	.382	.474	-.226	.011	.176	.296
Annual Labour Supply	1960	2108	2587	2655	2141	2271	2448	2527
Pre-Tax Income	6977	7505	17,150	17,602	7622	8086	16,229	16,753
Consumption	12,389	10,806	10,606	9250	9342	8001	13,369	11,795
Per-Standard-Adult Consumption	12,389	9005	10,606	7708	9342	6667	13,369	9829
Utility	6853	5214	5022	3820	5315	3982	6331	4872
$\varepsilon = 2.0$								
Marginal Tax Rate	0	0	0	0	.204	.193	0	0
Average Tax Rate	-.285	-.419	.368	.313	-.066	-.186	.274	.215
Annual Labour Supply	2184	2118	2577	2539	2157	2094	2512	2473
Pre-Tax Income	7777	7541	17,086	16,831	7678	7456	16,654	16,395
Consumption	9991	10,698	10,799	11,563	8186	8844	12,096	12,872
Per-Standard-Adult Consumption	9991	8915	10,799	9636	8186	7370	12,096	10,727
Utility	5527	5162	5113-	4776	4795	4501	5728	5316

Table 2

Horizontal Equity

	LUMP-SUM TAXATION				INCOME TAXATION			
	"Poor" Families		"Rich" Families		"Poor" Families		"Rich" Families	
	One-Member Family	Two-Member Family	One-Member Family	Two-Member Family	One-Member Family	Two-Member Family	One-Member Family	Two-Member Family
$\varepsilon = 0.5$								
Marginal Tax Rate	0	0	0	0	.192	.030	0	0
Average Tax Rate	-.450	-.789	.472	.378	-.012	-.214	.297	.166
Annual Labour Supply	2104	1954	2653	2584	2201	2201	2528	2441
Pre-Tax Income	7489	6958	17,588	17,133	7837	7837	16,761	16,186
Consumption	10,857	12,447	9294	10,655	7930	9516	11,775	13,500
Per-Standard-Adult Consumption	10,857	10,373	9294	8879	7930	7930	11,775	11,250
Utility	6006	6006	4401	4401	4626	4626	5576	5576
$\varepsilon = 2.0$								
Marginal Tax Rate	0	0	0	0	.271	.125	0	0
Average Tax Rate	-.223	-.491	.394	.284	-.026	-.231	.307	.178
Annual Labour Supply	2217	2084	2596	2519	2124	2124	2535	2449
Pre-Tax Income	7891	7419	17,209	16,700	7561	7561	16,804	16,235
Consumption	9650	11,064	10,430	11,958	7758	9309	11,644	13,350
Per-Standard-Adult Consumption	9650	9220	10,430	9965	7758	7758	11,644	11,125
Utility	5338	5338	4938	4938	4644	4644	5514	5514

Table 3

The Cost of Horizontal Equity

	LUMP-SUM TAXATION		INCOME TAXATION	
	Dollars	Percentage of R	Dollars	Percentage of R
$\varepsilon = 0.5$	269	4.35	284	4.59
$\varepsilon = 2.0$	67	1.08	83	1.34

APPENDIX

Throughout this paper the principle of horizontal equity is imposed on the utilitarian sum-of-utilities social welfare function, for utilitarianism by itself does not guarantee such equity. In this appendix we discuss the question of whether a weighted sum of utilities, where higher weights are assigned to larger families, can enhance horizontal equity.

For this purpose it suffices to consider only two households: a one-member household and a two-member household, both having the same earning ability, which is denoted by n. We ask whether there exists a constant $\lambda > 0$ such that the social welfare function

$$W = u(x_1,y_1) + \lambda u(x_2/c,y_2) \tag{24}$$

shows a preference towards horizontal equity in the sense that the optimal way to divide the national pie must preserve horizontal equity. Formally, we ask whether there exists $\lambda > 0$ such that if (x_1,y_1) and (x_2,y_2) maximize (24) subject to the resource constraint

$$p(x_1 + x_2) + R \leq n(y_1 + y_2), \tag{25}$$

then they must satisfy the horizontal equity principle

$$u(x_1,y_1) = u(x_2/c,y_2). \tag{26}$$

For reasons which will be clear later we denote here the producer price of consumption by p rather than normalize it to a unity, as we did in the paper.

Here it will be easier to work with the indirect utility function and with leisure ($\ell = K - y$) rather than labor (y). Let $V(p,n,I)$ and $\bar{V}(p,n,I)$, where I denotes full-income, be the indirect utility function of a one-member household and a two-member household respectively, i.e.,

$$V(p,n,I) = \max u(x, K - \ell) \qquad (27)$$
$$\text{s.t. } px + n\ell \leqq I$$

and

$$\bar{V}(p,n,I) = \max u(x/c,\ K - \ell) \qquad (28)$$
$$\text{s.t. } (cp)(x/c) + n\ell \leqq I.$$

Then it is clear from (27) and (28) that

$$\bar{V}(p,n,I) = V(cp,n,I). \qquad (29)$$

With the aid of the indirect utility functions, the optimization problem of maximizing (24) subject to (25) reduces to the unconstrained maximization problem

$$\max_{T}[V(p,n,I - T) + \lambda V(cp,n,I + T - R)]. \qquad (30)$$

T is a lump-sum tax imposed on the small family; T-R is the lump-sum tax imposed on the large family. The horizontal equity principle becomes now

$$V(p,n,I - T) = V(cp,n,I + T - R). \qquad (31)$$

The first-order condition for (30) which is both necessary and sufficient (note that since u is strictly concave in (x,ℓ), then V is strictly concave in I) is

$$V_I(p,n,I - T) = \lambda V_I(cp,n,I + T - R). \tag{32}$$

Thus, we ask the question whether there exists a constant $\lambda > 0$ such that (32) implies (31).

We can show that such λ exists for a Cobb-Douglas utility function. More generally, we can state sufficient conditions for such λ to exist. Suppose that $u(x, K - \ell)$ is homothetic in (x,ℓ). In this case V takes the form

$$V(p,n,I) = G[h(p,n)I]. \tag{33}$$

Since V is homogeneous of degree zero in (p,n,I), it follows that h has to be homogeneous of degree -1 in (p,n). G is strictly increasing and strictly concave. Suppose further that h is of the form

$$h(p,n) = h_1(p)h_2(n). \tag{34}$$

Under these conditions, the required λ exists. If we define

$$\lambda = h_1(1)/h_1(c), \tag{35}$$

then (32) reduces in this case to

$$G'[h(p,n)(I - T)]h(p,n) = G'[h(cp,n)(I + T - R)]h(cp,n)h_1(1)/h_1(c). \tag{36}$$

Employing (34) and the homogeneity of degree -1 of h, we see that

$$\frac{h(cp,n)h_1(1)}{h_1(c)} = \frac{h(c,n/p)h_1(1)}{ph_1(c)} = \frac{h_1(1)h_2(n/p)}{p} = \frac{h(1,n/p)}{p} \tag{37}$$
$$= h(p,n).$$

Thus, (36) and (37) imply that

$$G'[h(p,n)(I - T)] = G'[h(cp,n)(I + T - R)]$$

and hence, by the strict concavity of G,

$$h(p,n)(I - T) = h(cp,n)(I + T - R).$$

In view of (33), this last equality implies (31).

The reader can verify that (33) and (34) hold for a Cobb-Douglas utility function. However, these two conditions, and especially (34), seem very strong to us. A CES utility function, for instance, does not satisfy (34). We could not find weaker conditions than (33) and (34) and we conjecture that they are necessary as well as sufficient conditions.

NOTES

[1]Some may argue that, although scaling is a good way to explain consumption patterns across families with different sizes, one should not use scaling for welfare comparisons among such families (see, for instance, Pollak and Wales, 1978c). In other words, one should not attempt to say, as we did, that a family of two persons derives the same utility from (x,y) as a family of one person derives from $(x/c,y)$, for such a statement ignores the "utility from children." Therefore it is concluded that no special attention should be paid by the tax laws to the family size. Although the argument that parents derive utility from their children is not without merit, we nevertheless cannot accept the position that the size of the family should be ignored by the tax laws. We do not think that children can be simply treated as a consumption by their parents. Children are, after all, human beings.

[2]This assumption is much stronger than we actually need. For instance, it will suffice to assume that the range of wages is an interval of the form $[o,\bar{n})$ where $\bar{n} < \infty$ (and even this is stronger than we need). Roughly speaking, all we need is that this interval not be very small. Recall that in the extreme case where this interval shrinks to just one point (Section 2), horizontal equity does not necessarily mean equal per-standard-adult consumptions and labor supplies.

[3]The reader can skip the remainder of this section with no loss of continuity.

[4]The result for the bottom end holds only if the poorest person works.

[5]The result that the group of one-member families faces a positive marginal tax rate everywhere, including at N_2, implies that the society's insistence on horizontal equity prevents it from making Pareto-improvement changes. By setting the marginal tax rate facing this group at zero beyond the income level of $z_2(N_2)$, members of this group (namely those with wages sufficiently close to N_2) will be made better-off and no one will be made worse-off (see Sadka, 1976). Of course, the principal of horizontal equity will then be violated.

[6]This is the average wage of the second lowest quintile of the population of the nonretired heads of households.

[7]This is the average wage of the second highest quintile of the population of nonretired heads of households.

[8]This figure is taken from van der Gaag and Smolensky (1980).

[9]Recall that we have here a discrete distribution of wages. This explains why the marginal income tax rate is zero at the top end of the income distribution, in contrast to the results of the preceding section which are valid only in the continuous case.

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