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THE WELFARE APPROACH TO MEASURING INEQUALITY

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ABSTRACT

The authors review the economic literature on measuring inequality. They argue that traditional measures of inequality are not conceptually well founded and that different measures will often lead to very different results. The basic theory of measuring inequality is presented and generalizations of the theory are discussed. They examine a particular measure suggested by Atkinson (1970) which, they argue, is superior to previous measures that have been used by sociologists.

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1. INTRODUCTION

A mistaken and unchecked assumption made by most sociologists is that different measures of inequality will rank order distributions in the same way. Where differences in the orderings are recognized, they are viewed as minor and best ignored. The typical procedure in sociology is to choose a single index, such as the Gini coefficient or the coefficient of variation, as a measure of inequality and then to analyze one's data with it. Examples of this technique in the recent literature are Gartrell (1977), Rubinson and Quinlan (1977), Blau (1977), Jencks (1972), and Chase-Dunn (1975).

Different measures, however, often do not give the same results, and the discrepancy can be considerable. We demonstrate this by analyzing Kuznets' (1963) classic data on the distribution of individual income for twelve countries, circa 1950. Table 1 presents the Spearman rank order correlations of four commonly used measures of inequality for the Kuznets data (The data from which the correlations are derived are found in Appendix A.) The Spearman rank order correlations are a conservative test of the degree of agreement among different measures. Measures must agree only on the rank order of countries from least equally distributed to most equally distributed. If Pearson correlations were used the lack of agreement would, in general, be even greater. We would not only be testing whether the rank orderings were similar, but also whether the measures represented the same interval scale. (In this paper we will be concerned only with the degree to which measures rank order distributions in the same way).

The four measures used are the Gini coefficient, the coefficient of variation, the standard deviation of the logarithm of income, and the mean relative deviation. (Formulas for these measures are given in Appendix B.) The first three are commonly used to measure income or other types of inequality; the mean relative deviation is less frequently used for this purpose. It is, however, the principal index used to measure segregation. In this context it is known as the index of dissimilarity. Duncan and Duncan (1955) show that measuring segregation is structurally similar to measuring economic inequality. Our comments about measures of inequality will therefore pertain equally to measures of segregation. We will, however, limit our specific discussion to the problem of measuring inequality.

The correlations in Table 1 are surprisingly low. The correlations of the standard deviation of the logarithm of income with each of the other measures are the lowest, with .608, .287, and .566 representing substantial disagreement. Even the correlation of .874 found between the Gini coefficient and the coefficient of variation is not particularly high. This correlation represents the fact that out of 66 pairs of countries, the two measures ranked ten of the pairs differently. The correlation of .28 between the coefficient of variation and the standard deviation of the logarithm of income represents a lack of agreement in 28 of the 66 pairs.

The differences in the rank orders assigned by the different measures is illustrated dramatically by a comparison of India and Sweden. India is ranked eleventh, ninth, third, and eleventh by the Gini, coefficient of variation, the standard deviation of the logarithm, and the mean relative

Table 1

Spearman Rank Order Correlations between Different Measures of Inequality

	Gini	Coefficient of Variation	Standard Deviation of the Logarithm of Income	Mean Relative Deviation
Gini	1	.874	.608	.972
Coefficient of Variation		1	.287	.874
Standard Deviation of the Logarithm of Income			1	.566
Mean Relative Deviation				1

Source: Data is from Kuznets (1963).

deviation respectively. Sweden is ranked fourth, sixth, eleventh, and fourth by each of these respective measures.

The Kuznets data is not unusual. Similar results have been found by Aigner and Heins (1967) and Yntema (1933). In our own recent work (as yet unpublished) analysis of other data sets has shown that correlations of this order are the rule rather than the exception.

If measures of inequality do not generally agree, how are we to choose among the various measures? Similarly, how are we to rank distributions in terms of inequality?

In the last eight years a considerable literature has developed in economics that attempts to answer these questions. We have termed it "the welfare approach to measuring inequality." The literature has appeared primarily as articles by a number of authors in The Journal of Economic Theory. It also is represented in other sources by Aigner and Heins (1967), Kolm (1969), Kondor (1975), and Sen (1973). The theoretical roots of this work are found in Lorenz (1905), Pigou (1912; 1920), and Dalton (1920; 1925), but the cornerstone of this body of literature is Dalton's (1920) article, "The Measurement of Inequality of Incomes."

The purpose of this paper is to provide an exegesis of this literature. Our reasons for doing this are several. First, although much of the important work was done in the early 1970s, sociologists seem to be unaware of it. Economists have not only suggested a number of new and important measures of inequality, they have also clarified many of the conceptual issues involved in determining whether one distribution is more equally distributed than another. Finally, they have provided a strong critique of traditional methods used by social scientists.

We will not provide an exhaustive review of all the findings and ideas of the welfare approach; this would take more than a single paper. Nor will we critique this approach. (That will be done in another paper.) We do hope, however, to convince the reader that traditional measures of inequality such as the Gini coefficient, the coefficient of variation, the standard deviation of logarithms, and the mean relative deviation can no longer be used uncritically.

The theory that has developed in economics has two components: (1) a basic theory that exists independent of welfare economics; and (2) a generalization of that theory that relies heavily on welfare economics. The basic theory enjoys considerable consensus among economists. We suspect that sociologists will find little that is objectionable and many ideas that are already familiar. Although well developed, the basic theory is incomplete in that it allows us to determine only in certain special cases whether one distribution is more equally distributed than other.

The next section provides a detailed description of the basic theory. Subsequent sections discuss the theoretical and empirical shortcomings of this approach; the major ideas of the welfare economics theory; and traditional approaches to the measurement of inequality. The final section discusses the implications of economic theory for empirical research on inequality.

2. THE BASIC THEORY

We assume that all inequality measures share a number of properties. First, they are zero when incomes are distributed equally and positive otherwise. Second, they are impartial in that they are independent of the identity of who specifically possesses what income. The four traditional measures discussed above all exhibit these properties.

The basic theory has three axioms or assumptions. We expect that the first axiom, which is central to any notion of inequality, is universally acceptable. The second and third axioms enjoy considerably less consensus, but we would expect that they are acceptable to the majority.

Principle of Transfers

Basic to any notion of inequality is the idea that inequality is reduced if we transfer income from a rich person to a poorer one. Of course the transfer should not be so large that after the transfer the poor person has become richer than the rich person. This concept has become known as the Pigou-Dalton principle of transfers (hereafter referred to as the transfers principle).

The transfers principle allows us to compare distributions involving the same number of people and the same mean income. If one distribution of income can be obtained from another by transferring income from the rich to the poor then we know that the former is less equal than the latter. (A specific empirical test for deciding whether such a transformation can take place will be given later.) In order to make comparisons between populations with differing numbers of people and differing mean incomes, however, we will need two additional axioms.

Population Symmetry Axiom

If we have two populations of equal size, and income is identically distributed in both, then income inequality is the same in both. Additionally, it seems reasonable to assume that inequality in the two combined populations should be the same as inequality in each of the two separate populations. Sen (1973) has labelled this the symmetry axiom for populations. The symmetry axiom allows us to compare distributions for groups of unequal size but with the same mean income. Given two populations with differing numbers of people (m and n) we only need add the first population n times to itself and the second population m times to itself to obtain two populations with the same total number of people and same mean income. We can then compare one population with the other by using the transfers principle.

An argument has been made against this axiom. Assume that we have two populations of equal size and total income. In each, one person has all the income. This represents maximal inequality for each population. In the combined population two people will each have half of the total income. Inequality could be increased if only one person had all the income. Martin and Gray (1971) have argued that all situations of maximal inequality represent the same degree of inequality. Since inequality in the combined population is not maximal, it should be smaller than in each of the individual populations.

We do not accept Martin and Gray's position. It is a very different situation for one person to have all the income if there are only four others, than for one person to have all the income if there are one thousand others; thus the second situation represents much greater inequality than the first. One would think that the events leading one person in five to have all the income of a group would not be nearly as unusual as the events that would lead to one person in a thousand having all the income. Additionally, for large populations (which is what we usually compare) the maximum that a measure can take will not be very different from one population to another.

Intensity Axiom

The symmetry axiom for populations allows us to deal with populations of different size. But how are we to deal with populations with different mean levels of income? The usual assumption is that if we increase every individual's income by the same proportion then income inequality will remain unchanged. In other words, the size of the "pie" to be divided has no bearing on the degree of inequality--it is only the relative share that each person receives that is important in determining inequality.

We suspect that most people would find this axiom acceptable, but that a substantial minority might not. Dalton (1920), for instance, believed that an addition of the same amount of income to each person decreases inequality, but proportionate additions increase inequality. Research to date has not produced a satisfactory conclusion about the acceptability of this axiom. The best discussion thus far is Kolm (1976a and 1976b). For the present we accept the axiom (which we will term the intensity axiom) and make no further comment.[1]

Lorenz Criterion

Our three axioms are intimately related to the Lorenz criterion. Before we can discuss this relationship, however, we need to define this concept which, for the reader who is unfamiliar with it, is explained in the two paragraphs that follow.

In order to understand the Lorenz criterion it is first necessary to know what a Lorenz curve is. A Lorenz curve is constructed by ordering people from the poorest to the richest. The Lorenz curve is then the graph of the percentage of the total income (the Y coordinate) possessed by the X poorest percentage of the population. Figure 1 shows the Lorenz curve from Kuznets's (1963) data for Great Britain and Mexico.

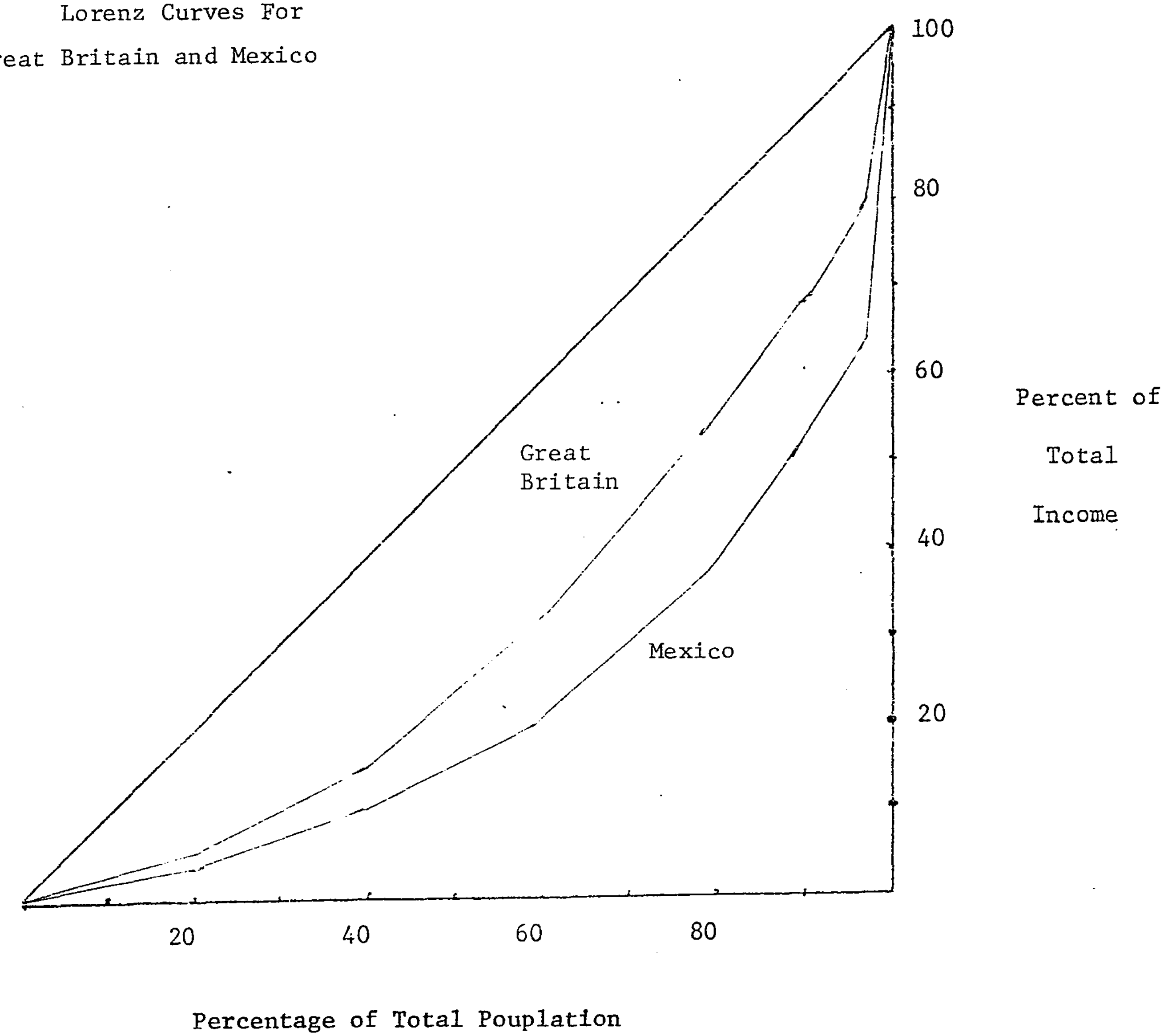
The Lorenz criterion states that a distribution A is more equally distributed than another distribution B if the Lorenz curve for A is nowhere below the Lorenz curve for B. Thus in Figure 1 income is more equally distributed in Great Britain than it is in Mexico. One justification for the Lorenz criterion is that in the distribution with the higher curve, the poorest X percent of the population always has an equal if not a larger share of the total income than the poorest X percent of the population in the other distributions for all X between zero and 100%.

The Lorenz criterion has a special relationship with our three axioms. If we have two populations of the same size and the same mean income, then accepting the Lorenz criterion is identical to assuming the transfers principle. For populations with different numbers of people but the same mean incomes, accepting the Lorenz criterion is identical to assuming the transfers principle and the symmetry axiom. The symmetry axiom allows us to express the X-axis in terms of percentages

Figure 1

Lorenz Curves For

Great Britain and Mexico



of the total population rather than actual numbers of people. For populations with differing numbers of people and differing mean incomes, acceptance of all three axioms is identical to acceptance of the Lorenz criterion. The intensity axiom allows us to express the Y-axis in terms of percentages rather than total dollars.

Proofs of these results are not given here. The interested reader is referred to any of a number of articles and books (Atkinson, 1970; Dasgupta et al., 1973; Sen, 1973; Rothschild and Stiglitz, 1973; Kolm, 1976b). Sen. (1973) is probably the easiest to follow.

The Lorenz criterion provides a means of empirically testing whether, according to our three axioms, one distribution is more equally distributed than another.

3. INCOMPLETENESS OF THE BASIC THEORY

The transfers principle and its generalization to the Lorenz criterion provide ways of comparing distributions. The basic theory is, however, incomplete. If the Lorenz curves for two different distributions cross, there is no way of determining which distribution is more equally distributed. Such is the case in Figure 2, which presents the Lorenz curves for the United States and India. If the Lorenz curves for given sets of data cross often, then the basic theory and the Lorenz criterion are of limited usefulness.

Using the Kuznets data, Table 2 presents the number of times that Lorenz curves for different pairs of countries cross each other. Of the 66 possible pairs of curves, 50 pairs (or 76%) cross each other.

Figure 2

Lorenz Curves for the

United States and India

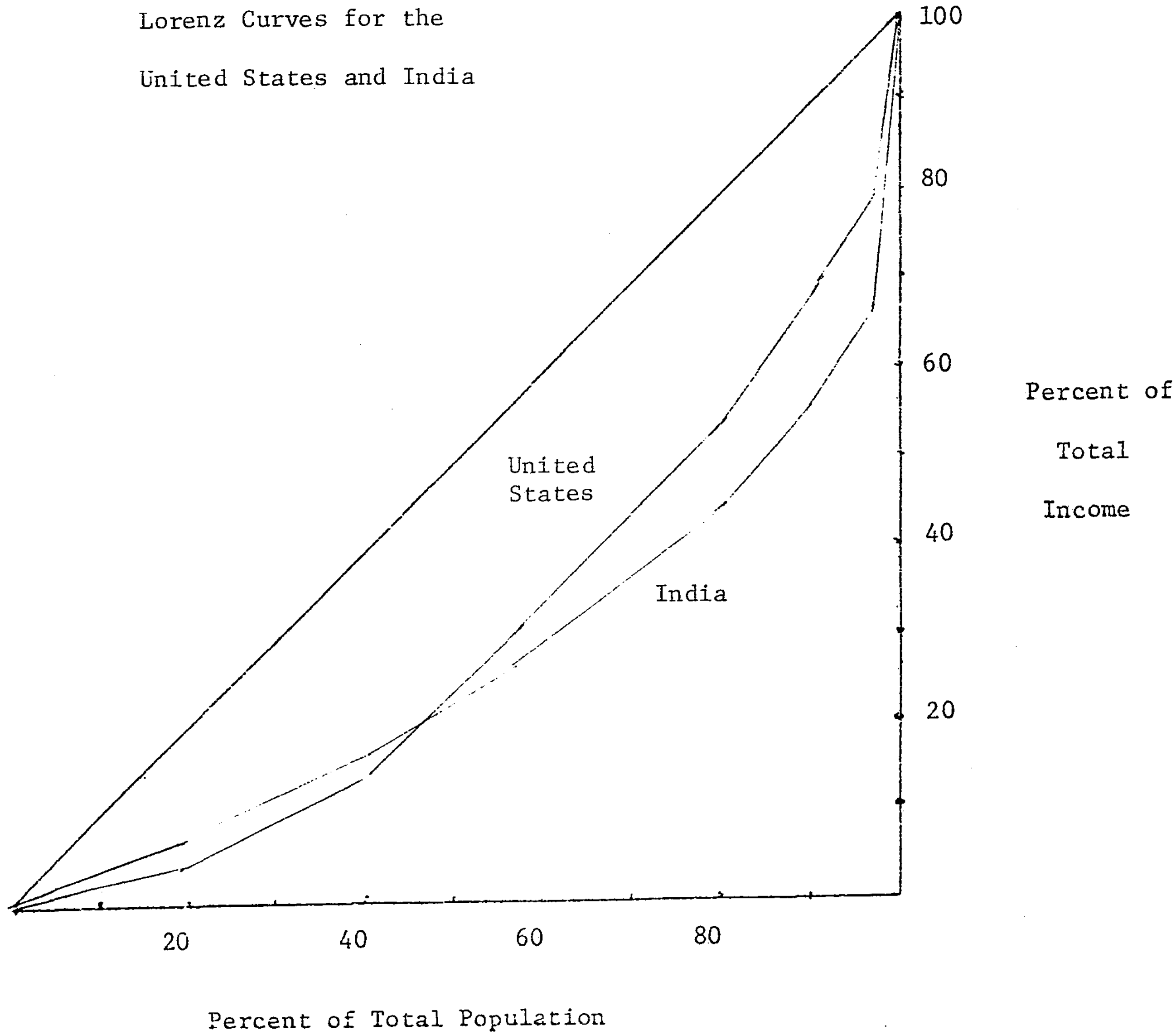


Table 2

Number of Times Pairs of Lorenz Curves Cross

Country	United States	Italy	Puerto Rico	Denmark	Sweden	Netherlands	West Germany	India	Barbados	Ceylon	Mexico
Great Britain	1	1	1	1	1	0	0	1	0	0	0
United States	–	1	1	1	1	0	0	1	0	1	0
Italy	–	–	1	1	1	0	1	1	1	0	0
Puerto Rico	–	–	–	1	1	2	2	1	1	0	0
Denmark	–	–	–	–	2	1	1	1	1	1	1
Sweden	–	–	–	–	–	1	1	1	1	1	1
Netherlands	–	–	–	–	–	–	1	1	1	1	1
West Germany	–	–	–	–	–	–	–	1	3	1	1
India	–	–	–	–	–	–	–	–	1	1	0
Barbados	–	–	–	–	–	–	–	–	–	1	1
Ceylon	–	–	–	–	–	–	–	–	–	–	0
Mexico	–	–	–	–	–	–	–	–	–	–	–

In only 24% of the cases can we use the Lorenz criterion to determine which country has the more equally distributed distribution of income.[2] Our experience has been that the Kuznets data is in no way unusual in this respect. Thus, if we are to have a theory that is of practical use the basic theory must be extended or a more general theory must be developed.

4. THE WELFARE APPROACH

Economists have attempted to develop a general theory which is both consistent with the basic theory and would allow us to deal with all situations where Lorenz curves cross. Their approach has been to base the measurement of inequality on a theory of social and individual welfare.

Dalton (1920) was perhaps the first to argue that economists were interested not in inequality per se, but rather in the effects of inequality on economic welfare. As he put it, "The objection to great inequality of incomes is the resulting loss of potential economic welfare."

This argument has been used to justify developing a general theory based on notions of individual and societal welfare. Dalton goes on to suggest that the degree of inequality in a distribution should be measured by the loss in welfare that results from it. The formalization of this idea will be presented later.

A concern about the relationship between welfare and inequality is certainly not new to sociologists. Most sociologists, however, would probably find economists' treatment of welfare quite foreign.

By individual welfare an economist means an individual's sense of well-being, his happiness or satisfaction with life.[3] In the literature on income inequality a standard theoretical assumption is that every individual has the same welfare function; that is, the relationship between income and well-being is the same for everyone. (No economist would claim that this assumption is true in reality. Rather, it is used as a heuristic device.) Economists also assume that increasing a person's income increases his welfare. Additionally, it is assumed that the effect of income on an individual is independent of other resources the individual might possess. This is equivalent to assuming that the individual well-being function is of the form $g(X) + f(Y)$, where Y represents the income possessed by the individual and X represents his/her other resources. This assumption, too, has been made in order to make the theory tractable. Finally, it is assumed that the level of well-being that an individual possesses is determined by the amount of his/her income, independent of the amount of income possessed by others.[4]

Besides using a notion of individual welfare, economists also use a notion of societal or social welfare. Social welfare is measured by a function S, which represents society's notion of how fair, just, or desirable a particular distribution is. S may be a function of individual welfares--$g(\underset{\sim}{X}) + f(Y)$, the part of individual welfare due to income--$f(I)$ or $\underset{\sim}{Y}$--the incomes that individuals receive. In the first case S is a measure of the fairness, justness, or desirability of complete individual welfare, in the second that of the welfare due to income, and in the third that of income. Only the last two formulations have been extensively considered in the literature. It is assumed that S increases in income. That is, if we increase everyone's income, social welfare is increased. This implies that for distributions where income is distributed equally, S ranks the distributions in the same order as their mean incomes.

A specific form of S is of particular interest to economists: the additive welfare function $S = \sum_{i \epsilon P} f(Y_i)$. Societal welfare is just the sum total of individual welfare (more precisely, welfare due to the part of individual welfare derived from income). This is a particularly simple form of S. It assumes that the welfare gained by society from each individual's welfare is independent of the welfare of other individuals. This is a generalization (to the societal level) of the individual independence assumption. The desirability of a particular distribution has nothing to do with fairness or justice. Desirability is defined only in terms of maximizing total individual welfare.[5]

Many sociologists may not accept a social welfare function. It is well known that there are severe, if not insurmountable, problems in constructing a general social welfare function from individual welfare functions (where individual welfare functions might reflect attitudes about how income ought be distributed, as well as the well-being received from the particular income possessed by an individual. (See Arrow [1963] for a discussion of his "impossibility theorem.") Hamada (1973) has shown that these same problems exist in the specific case of income inequality.

One solution is to take an idealist or Kantian point of view and assume that there is a S that measures the real level of social welfare for different distributions of income, and that it is only because of a lack of knowledge that individuals cannot agree on what the function S should be. (See Arrow [1963, Chapter 7] for an excellent discussion of this viewpoint.) For the practitioner, however, this still leaves the problem of how to discover the true S.

5. MEASURES OF INEQUALITY

We are now in a position to define measures of inequality based on functions for individual and social welfare. Our procedure will be first to define the measures and then to discuss what properties are needed for them to be consistent with the basic theory.

Dalton (1920) suggests measuring inequality as the loss in welfare that results from inequality. Let S(Y) be the amount of welfare that exists when income is distributed as the vector Y and let $S^*(Y)$ be the amount of welfare when the income in Y is distributed equally. We make the important assumption that total income remains constant when income is redistributed. Dalton's measure of inequality is $I^* = 1 - S/S^*$. We can interpret this measure as the percentage of total potential welfare that is lost due to income inequality. As an example, Dalton suggests that S might be an additive function of individual welfare, and that individual welfare might be a linear function of the logarithm of income. That is, if I is our inequality measure, then

$$I = 1 - \sum_{i \varepsilon P} \frac{a + b \log Y_i}{n(a + b \log \bar{Y}_i)} . ^6$$

This measure will be zero when income is equally distributed and positive otherwise. The upper limit of the measure is one.

Atkinson (1970) points out that Dalton's measure makes very strong assumptions about the measurability of social and individual welfare. We must be able to measure both social and individual welfare with a ratio scale. With respect to the above example, we would have to know not only that individual well-being is linear in the log of income, but also what the ratio of a to b is. This may not be possible.

Atkinson (1970) provides us with a way to make weaker assumptions about the measurability of welfare. He suggests measuring the ratio in Dalton's formula in income units (which is a ratio level variable) rather than in welfare units.

It was noted earlier that distributions where income is equally distributed are ranked in the same order by their means as they are by S. If S is continuous in income, then we can use these mean incomes as an indicator of the level of welfare. This is the idea behind Atkinson's notion of equally distributed income equivalents. We identify the level of welfare of a distribution with the mean income of that equally distributed income that has the same level of welfare, W' (that is, the equally distributed income equivalent of a distribution). Y' is equal to the solution of the equation $S(Y) = S(Y')$ where Y is vector of incomes from the population and Y' is a vector of incomes all equal to the same thing. When income is equally distributed, the Y' of a distribution will just be equal to the mean of the distribution, $\bar{Y}$. Dalton's measure can now be redefined as $I' = 1 - Y'/\bar{Y}$. The numerator is the amount of welfare associated with a distribution measured in income units. The denominator is the amount of potential welfare that would result from distributing income equally, again measured in income units. Our new measure may be interpreted as that percentage by which we could reduce current total income and still maintain the same level of welfare if income were equally distributed in the process. Our measure will be equal to zero if income is equally distributed and will approach one the more unequally income is distributed and the larger the total population.

The advantage of using equally distributed income equivalents is that it allows us to make weaker assumptions about the measurability of welfare. Social welfare need only be ordinally measurable; that is, we need only be able to rank order societies in terms of their level of welfare. If social welfare is additive, then we need only be able to measure individual welfare in terms of an interval scale; that is, we need only know the relationship between income and welfare to within a linear transformation. Making these weaker assumptions about measurability may be important if we think that there are problems in measuring different levels of social and individual welfare.

5. CONSISTENCY WITH THE BASIC THEORY

In the last section we discussed how different functions relating social and individual welfare might be used to develop measures of inequality. We presented three basic types of measures: Dalton's measure, $I^* = 1 - S/S^*$; Atkinson's redefinition of Dalton's measure in terms of equally distributed income equivalents, $I' = 1 - Y'/\bar{Y}$; and the special case where S is an additive function of individual welfare $I^a = 1 - \Sigma\, f(Y_i)/f(\bar{Y})$. What properties must S, Y', and f(Y) possess in order for the above measures to be consistent with the basic theory?

We will examine properties necessary and/or sufficient for satisfying (1) the transfers principle, (2) the population symmetry axiom, and (3) the intensity axiom.

Consistency with the Transfers Principle

In order for I^* and I' to satisfy the transfers principle it is necessary and sufficient that S, Y' satisfy a very weak concavity property. The mathematical term is strict Schur-concavity (see Kolm, 1976a). Rothschild and Stiglitz (1973) have termed this property "locally equality preferring." The definition (taken from Rothschild and Stiglitz, 1973) is:

A function S(Y) is strictly locally equality preferring if, for every vector Y,

$$S(Y) < S(aZ + (1 - a) Y) \quad \text{for } 0 < a < 1$$

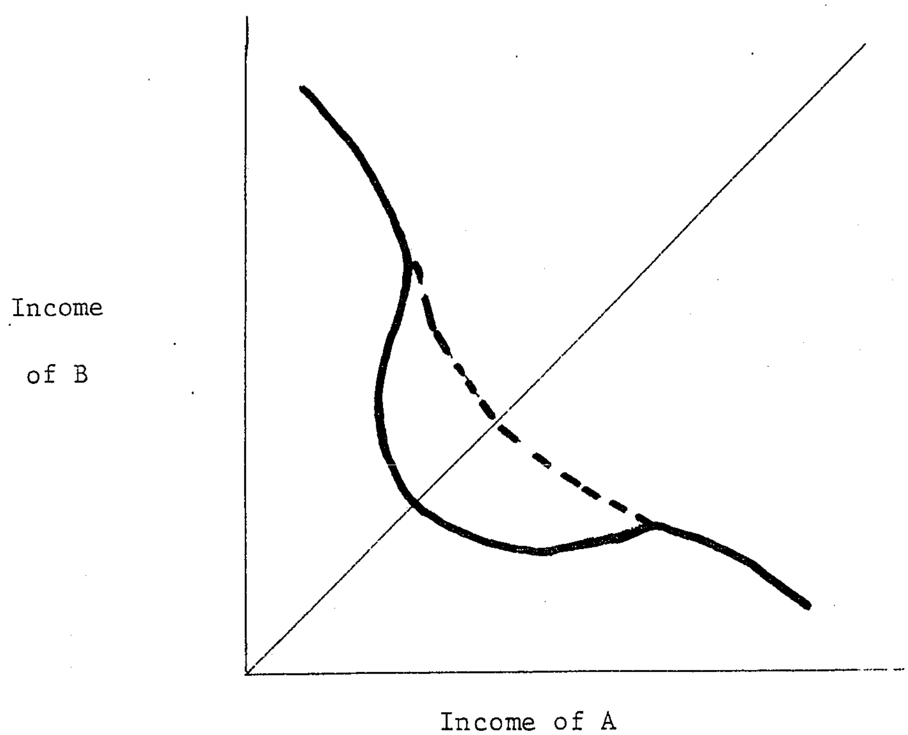
where $Z_i = Y_i \quad i \neq j,k$

$$Z_j = Z_k = (Y_j + Y_k)/2$$

Z is thus just a vector in which j and k have equalized their incomes; that is, there has been a transfer of income from one to another. The terms in parentheses on the right side of the inequality represent the case where there is some transfer between j and k (from the richer one to the poorer one) and where everyone else's income has remained the same. A better understanding of strict Schur-concavity can be gotten by examining Figure 3, adopted from Rothschild and Stiglitz for the case with two people. Schur-concavity means that the isoquants representing

Figure 3

The Difference Between Schur-Concavity
and Strict Concavity and Quasi-Concavity



levels of social welfare must be increasing from the origin and that any line perpendicular to the 45-degree line can cross an isoquant only twice (Rothschild and Stiglitz, 1973). The more traditional notions, of strict quasi-concavity and strict concavity[7] are stronger conditions, and are sufficient, but not necessary conditions for I^* and I' to satisfy the transfers principle (proofs are found in Rothschild and Stiglitz (1973) for I^* and in Kolm (1976b for I'). Each of these two conditions would imply that the isoquant could not curve out as it does in Figure 1, but rather that it would have to follow the dotted line. A necessary condition for strict concavity and a necessary and sufficient condition for strict quasi-concavity is that any line connecting two points on the isoquant always be above the isoquant.

If S is an additive social welfare function then the second derivative of individual welfare, $f(Y_i)$ must be negative in order to satisfy the transfers principle (previously we assumed that the first derivative is positive). This implies that each additional dollar of income increases welfare less. The sum of all individual welfare is increased by reducing inequality, because in taking a dollar away from a rich person and giving it to a poorer person, we decrease the rich man's welfare by less than we increase the poorer person's welfare.[8]

Consistency with the Population Symmetry Axiom

A sufficient condition for I^* and a necessary and sufficient condition for I' to satisfy the symmetry axiom is the following: If we have populations 1 through r and incomes $Y_{11}, Y_{12} \ldots . Y_{21} \ldots . Y_{rn}$ = $\underline{Y}$, with n people in each population and income identically distributed in each population, then S must have the property $S(\underline{Y}) = rS(Y_1)$, when $Y_1 = Y_{11}, Y_{12} \ldots Y_{1n}$.

Proof is straightforward. Since additive S necessarily satisfies this property, I^a will always satisfy the symmetry axiom for populations.

<u>Consistency with the Intensity Axiom</u>

A sufficient condition for I^* to be consistent with the intensity axiom is that S be homogeneous of any degree. A function S is homogeneous of degree P if $S(\lambda X) = \lambda^P S(X)$. Homogeneity implies two things. First, if we increase an individual's income by a factor λ, welfare will increase by λ^P. If $p > 1$ then welfare will increase faster than income. If $P < 1$ then welfare will increase more slowly. Homogeneity also implies homotheticity. A function is homothetic if its isoquants are always parallel. This means that if $S(X) = S(Y)$, then $S(\lambda X) = S(\lambda y)$ for any scalar λ.

If I' is to satisfy the intensity axiom then Y' must be linear homogeneous (i.e., of degree one). This follows directly from the fact that $\bar{Y}$ is linear homogeneous. (Multiplying all incomes by a constant multiplies the mean income by the same constant.) If Y' is linear homogeneous, this implies that S is homothetic (see Kolm, 1976b, for further discussion).

If I^a is to satisfy the intensity axiom, f must have a very particular form[9]: $f = bY^{1-e}$. Note that e must be greater than 1 in order for Y^a to satisfy the transfers principle. (Proof of this is given in Kolm [1976a]). This is a very strong result. We have not had to make any additional assumptions to those found in the basic theory other than to assume that welfare is additive or, equivalently, that income inequality should be measured by a sum of functions of the individual incomes. This form of I^a is known as Atkinson's measure (see Atkinson, 1970). Because of its special properties and because of its increasing use in empirical analysis, we discuss the measure in detail in the next section.

Atkinson's Measure

In the last section we pointed out that Atkinson's measure was the only measure based on an additive social welfare function that was consistent with our basic theory. We can think of Atkinson's measure as having two forms. Earlier we represented the measure in terms of a ratio of actual, total individual welfare to potential total individual welfare:

$$1 - \frac{(\Sigma\, Y_i^{1-e})}{n\, \bar{Y}^{1-e}} \qquad \text{for } e > 0 \text{ and } e \neq 1 \tag{A}$$

$$1 - \frac{(\Sigma \log \bar{Y})}{n \log \bar{Y}} \qquad \text{for } e = 1 \tag{B}$$ [10]

Alternatively we may express it in terms of distributed income equivalents. The two versions give the same rank ordering since one is just a strictly increasing monotonic function of the other:

$$1 - \left(\frac{\Sigma\, Y_i^{1-e}}{n\, \bar{Y}^{1-e}}\right)^{1/1-e} \qquad e \neq 1 \text{ and } e > 0 \tag{C}$$

$$1 - \exp \frac{\Sigma \log \bar{Y}}{n \log \bar{Y}} \qquad \text{for } e = 1. \qquad \text{(D)}$$

The core of Atkinson's measure is the ratio between a generalized mean and the standard arithmetic mean for a distribution. Thus in formula (D) Atkinson's measure is just the ratio between the geometric mean and the arithmetic mean. As long as e > 0 the generalized mean will always be smaller than the arithmetic mean except where income is distributed equally, in which case they will be equal.

How else are we to think of e? One way is as a measure of inequality aversion. As e increases, the value of Atkinson's measure will also increase, indicating a bigger difference between equality and the actual level of inequality in the distribution. Thus for Kuznets's data for the United States, for values of e of .5, 1, 2, 4 (the equally distributed income equivalent version) Atkinson's measure takes on the values .1296, .2425, .4204, .605. Another way to understand e is that as e increases, more and more weight is put on the share of income possessed by the bottom portion of the population. Let us compare the United States and India (see Figure 2). Since the bottom part of the population in India has a larger share of the total income than in the United States, for large enough e we will find that India has a more equal distribution of income. For India Atkinson's measure has the values .1878, .295, .3973, .4751 for e's of .5, 1, 2, and 4. Atkinson's measure has the same value for India and the United States when e equals approximately 1.75. For values below this the United States is considered to have a more equal distribution of income; for values greater than 1.75 India is considered to have a more equal distribution of income. The fact that as e gets larger incomes at the bottom are weighted more heavily is brought out most dramatically by letting e go to infinity. In this case pairs of distributions will be rank ordered by the shares possessed by the poorest individual in each distribution, who do not have identical shares of the total income (Hammond, 1975).

How are we to choose e? (If no Lorenz curves cross then the choice of e is irrelevant. Atkinson's measure will rank order the distributions in the same order no matter what value e takes. If the Lorenz curves for different distributions do cross, then in general Atkinson's measure will order the distributions differently, depending on the value of e.) There are two ways. First, we may choose e so as to represent our attitudes towards inequality. The more averse we are to inequality, or alternatively the more we are concerned with the share that the bottom part receives, the greater e should be. If we take this approach we may want to use a number of values of e in order to judge the sensitivity of our results to a particular choice. Alternatively, we might try to estimate an equation that relates individual welfare to income in terms of the functional form $W = a + bY^p$. Crude attempts in this vein that have been made (e.g., see Stevens, 1959; Schwartz, 1974; Winship, 1976) suggest that e should be between two-thirds and one-half.

6. CRITICISMS OF THE TRADITIONAL APPROACH

In the introduction we noted that one of the problems with the traditional measures of inequality is that they do not provide rank orderings of distributions that are consistent with one another.

Futhermore, there seem to be no criteria for deciding which measure is the correct or appropriate one.

The economics literature has postulated that a measure of inequality should be based on some appropriate notion of welfare. Stretching the point further, we may state that any measure of inequality either explicitly or implicitly incorporates some notion of social welfare. How do our traditional measures fare in this respect? One way to show that the traditional measures imply specific conceptions of social welfare is to compare them with Atkinson's measure using different values of e. For the Kuznets data, the rank order given by the standard deviation of the logarithm of income is identical to the ordering produced by Atkinson's measure, with e equal to anything between 1.81 and 1.84. The mean relative deviation and Gini coefficient correspond well to Atkinson's measure for values of e between .55 and .95. Never are more than two pairs of countries rank ordered differently. The coefficient of variation corresponds most closely to very small values of e. When e = .01 there are three pairs of countries rank ordered differently.[11]

It is, however, important that we take a much closer look at each of these measures, and try to interpret them directly in terms of notions of social and individual welfare. Are the notions they imply sensible? Are our measures consistent with the basic theory outlined above?

Two of our measures are not consistent with the basic theory. The other two have peculiar properties when interpreted from the welfare perspective.

Neither the standard deviation of the logarithm of income nor the mean relative deviation are consistent with the transfers principle.

The standard deviation of the logarithm of income will not rank distributions correctly if they are extremely skewed. This is easily illustrated. Assume that we have ten people, nine of whom have one dollar apiece and one of whom has one million dollars. We transfer half of this last person's money to one of the other people. Before the transfer the standard deviation of the logarithm of income was 1.8, and after the transfer it will be 2.28, indicating that income inequality has increased. Clearly in real terms it has not.

The mean relative deviation fails to satisfy the transfers principle for other reasons. The mean relative deviation is only sensitive to transfers from people who have incomes above the mean to people who have incomes below the mean. Again we can illustrate by example. Assume that we have four people with incomes of 0, 25, 50, and 75 dollars. Assume that the second and fourth persons each transfer 12½ dollars to the first and third persons, respectively, so that the incomes are now 12½, 12½, 62½, and 62½ dollars. Both before and after the transfer the mean relative deviation is equal to .5. No change in inequality is indicated, though inequality has certainly decreased.

The fact that the standard deviation of the logarithm of income and the mean relative deviation are not consistent with even the transfers principle would seem to rule them out as measures of inequality. Are our other two measures, the Gini coefficient and the coefficient of variation, consistent with the transfers principle? Yes, though they have other undesirable, though less damning, properties.

The coefficient of variation has been critized for giving equal weight to transfers at all levels. This is because the derivative expressing the change in the coefficient of variation due to a transfer is linear: $dC.V./dt = 2(Y_i - Y_j)$. The effect of the transfer is proportional to the difference in income between the person giving the money and the person receiving it. As long as money is transferred from a rich person (j) to a poorer person (i) the coefficient of variation will decrease, thus satisfying the transfers principle. But the effect of the transfer will be the same independent of the absolute amounts of income the two people have. Only the difference between their incomes will be important. Thus a transfer from a person who has 125,000 dollars to a person who has 100,000 dollars will decrease the coefficient of variation by the same amount as transferring money from a person who has 25,000 dollars to a person who has no money. Atkinson (1970), Kolm (1976b), and others have argued (and we would agree) that the second transfer should have a greater effect. Kolm has labeled this property the principle of diminishing transfers.

It is obvious that the Gini coefficient satisfies the transfers principle. This is easily seen by viewing the graphic interpretation of the Gini coefficient as the ratio of the area under the Lorenz curve to the total area under the diagonal (see Appendix B, Figure B.1). The Gini index has an undesirable property. A change in the Gini due to a small transfer will be proportional to (i - j), where i and j are the rank orders of the two people in the income distribution.[12] Atkinson (1970) notes, consequently, that if the distribution of income is unimodal then transfers among people in the middle of the distribution will be given more weight than transfers at either end. Thus, like the coefficient of variation, the Gini coefficient does not satisfy Kolm's principle of diminishing transfers. This problem is also illustrated by the fact that as a rich person transfers money to a poorer person, the effect of each additional dollar on the Gini diminishes only if the rich person or the poor person changes his/her position in the rank ordering. This seems to be a particularly peculiar property for a measure of inequality. Further criticisms of the Gini are given in Rothschild and Stiglitz (1973) and Theil (1967).

7. IMPLICATIONS AND CONCLUSION

We suspect that few sociologists will be influenced by economists' theory of the relationship between social welfare and inequality--the differences between these two fields on this subject are just too great (see note 4). The literature does, however, have two lessons to teach sociologists--one critical and negative in character, and the other suggestive and positive.

First, the findings of much sociological work that has used various measures of inequality may be completely dependent on the choice of the inequality measure used by the investigator. If a different measure had been used completely different results might have been obtained.[13] This is a strong claim, and its validity needs to be investigated. We hope that sociologists will be aware of its implications.

Besides being critical, the literature is also suggestive of how one should go about measuring inequality. Very conservative data analysts will probably want to limit themselves to comparing the Lorenz curves for different distributions. In this way they will not be making assumptions about inequality that go beyond the basic theory. Unfortunately there will also probably be many cases in which the Lorenz curves cross, making it impossible to state which distribution is more equally distributed.

The less timid may want to test the robustness of their results by using a number of different measures. The obvious technique would be to use the class of measures defined by Atkinson (1970). Varying the value of e over a range of, let us say, .25 to 2.5 or even greater allows this test. As e increases, more and more weight is being put on the share of income received by the lower part of the distribution. In some cases it may also be possible to determine what is a reasonable value or range for the value of e.

In conclusion, we reiterate our warning to the analyst who insists on using a single measure of inequality. Use of a different, equally reputable measure might well yield different results.

Appendix A

Income Data for Twelve Countries

Source: Kuznets, 1963, Table 3.

Table A.1

Percentage of Total Income Received by Ranked Cohorts of Population

Country	Year	0-20	20-40	40-60	60-80	80-90	90-95	95-100
India	1950	7.82	9.22	11.4	16	12.4	9.62	33.5
Ceylon	1952-53	5.1	9.3	13.3	18.4	13.3	9.6	31
Mexico	1957	4.4	6.9	9.0	17.4	14.7	9.7	37
Barbados	1951-52	3.6	9.3	14.2	21.3	17.4	11.9	22.3
Puerto Rico	1953	5.6	9.8	14.9	19.9	16.9	9.5	23.4
Italy	1948	6.09	10.5	14.6	20.4	14.4	9.99	24.1
Great Britain	1951-52	5.4	11.3	16.6	22.2	14.3	9.3	20.9
West Germany	1950	4	8.5	16.5	23	14	10.4	23.6
Netherlands	1950	4.2	9.6	15.7	21.5	14	10.4	24.6
Denmark	1952	3.4	10.3	15.8	23.5	16.3	10.6	20.1
Sweden	1948	3.2	9.6	16.3	24.3	16.3	10.2	20.1
United States	1950	4.8	11	16.2	22.3	15.4	9.9	20.4
Average		4.8	9.61	14.6	10.9	15.0	10.1	25.1

Table A.2

Percentage of Total Income Received by Poorest "x" Percent of Population

Country	20	40	60.	80	90	95	100
India	7.82	17	28.5	44.5	56.9	66.5	100
Ceylon	5.1	14.4	27.7	46.1	59.4	69	100
Mexico	4.4	11.3	21.2	38.6	53.3	63	100
Barbados	3.6	12.9	27.1	48.4	65.8	77.7	100
Puerto Rico	5.6	15.4	30.3	50.2	67.1	76.6	100
Italy	6.09	16.6	31.2	51.5	65.9	75.9	100
Great Britain	5.4	16.7	33.3	55.5	69.8	79.1	100
West Germany	4	12.5	29	52	66	76.4	100
Netherlands	4.2	13.8	29.5	51	65	75.4	100
Denmark	3.4	13.7	29.5	53	69.3	79.9	100
Sweden	3.2	12.8	29.1	53.4	69.7	79.9	100
United States	4.8	15.8	32	54.3	69.7	79.6	100
Average	4.801	14.41	29.03	49.88	64.83	74.92	100

Appendix B

Formulas for Traditional Measures of Inequality

The Gini coefficient is defined as the average of the absolute differences between all pairs of relative incomes $(x_i/\bar{x})$:

$$G = (1/2n^2) \sum_n \sum_n \left| \frac{x_i}{\bar{x}} - \frac{x_j}{\bar{x}} \right|$$

The Gini coefficient is directly interpretable in terms of the Lorenz curve. It is the ratio of the area between the Lorenz curve and the diagonal of equality to the total area under the diagonal. This is illustrated in Figure B.1. The Gini is equal to the area of segment A divided by the sum of the areas of segments A and B.

The coefficient of variation is simply the standard deviation of income divided by its mean:

$$\frac{\sqrt{\sum_n (Y_i - \bar{Y})^2/n}}{\bar{Y}} \quad .$$

In terms of the Lorenz curve, the coefficient of variation is equal to the standard deviation of the slope of the curve.

The standard deviation of the logarithm of income is given as

$$\sqrt{\frac{\sum_n (\log x_i - Z)^2}{n}}$$

where Z is equal to $\sum_n (\log x_i)/n$.

The mean relative deviation is given by the formula

$$\Sigma \mid Y_i - \bar{Y} \mid / \bar{Y} \; .$$

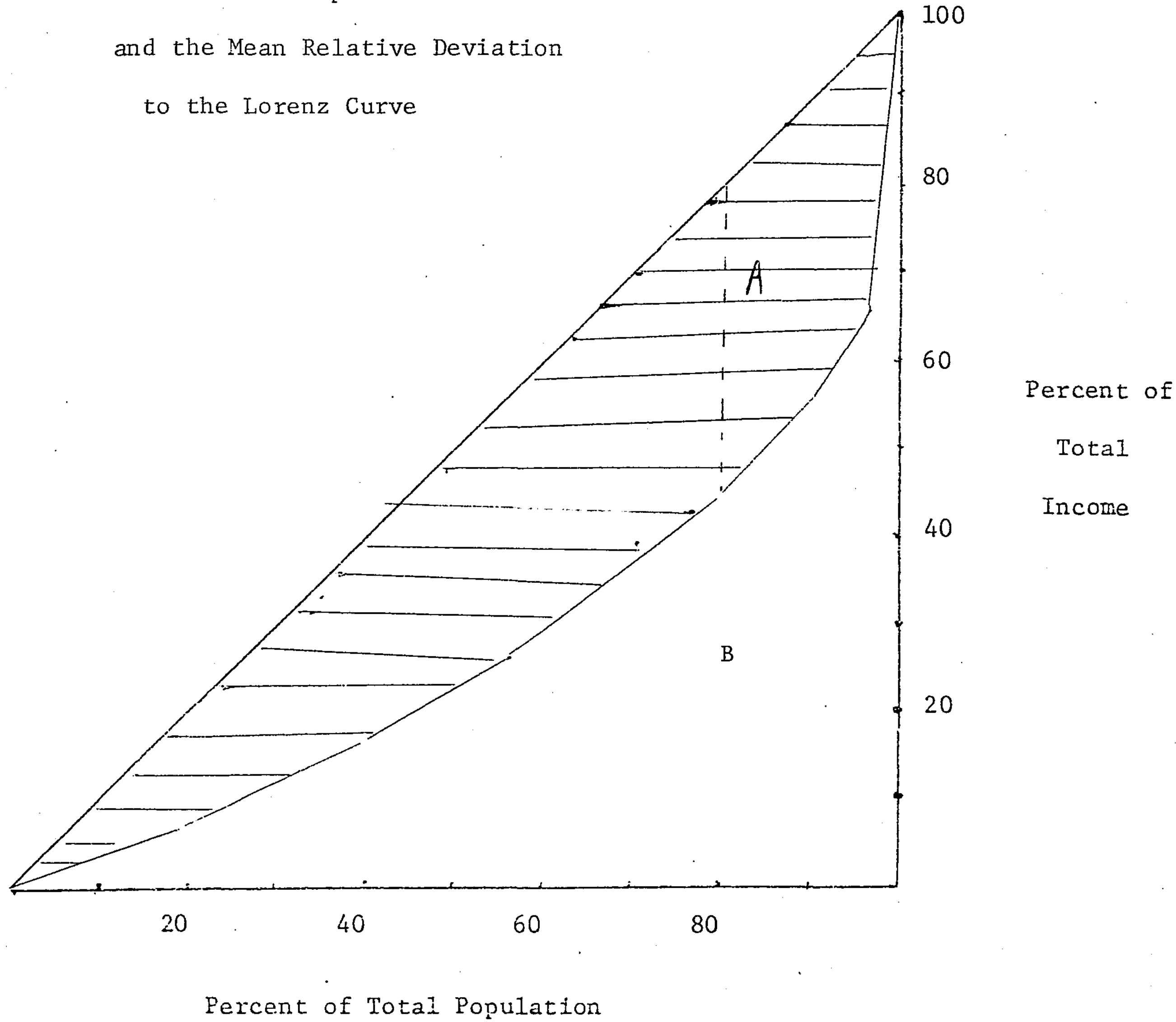
The mean relative deviation is equal to the maximum distance between the Lorenz curve and the diagonal of equality. This is represented by a dotted line in Figure B.1.

Figure B.1

The Relationship of the Gini

and the Mean Relative Deviation

to the Lorenz Curve



NOTES

[1]One reason that people may find this axiom unacceptable is that they are confusing measures of overall inequality with measures of income inequality. We feel, for instance, that this is true of Dalton's reasoning. The basic point to understand is that increasing everyone's income proportionately may leave income inequality unchanged but increase overall inequality. This will occur if income is more unequally distributed than other resources, and income inequality is a large part of overall inequality. By increasing everyone's income we increase the importance of income in overall inequality and thus increase overall inequality. Income inequality, however, is the same. A resulting implication is that decreasing income inequality may be considered objectionable if it means an increase in overall social inequality.

[2]Atkinson (1970) shows that, given two distributions with crossing Lorenz curves, we can always find two measures of inequality satisfying our three axioms that rank them differently.

[3]Most modern economists no longer equate the concepts of individual welfare and utility. For an excellent discussion of the history of the tension between these two concepts in economics, see Schumpter (1954).

[4]Although we expect that many sociologists will find this assumption objectionable we will leave analysis of this issue to another paper. Sociological criticism of this absolutist theory of the relationship between welfare (well-being) and income has a long history, going back to Marx, de Tocqueville, and Durkheim. Probably the most developed criticism

is the theory of relative deprivation (Merton, 1968; Stouffer, et al., 1949). For a recent discussion of the issue see Easterlin (1974).

[5]For a discussion of the independence assumption and its substantive importance for the distribution of income see Harsanyi (1955), Strotz (1958, 1961) and Fisher and Rothenberg (1961, 1962).

[6]In this example Dalton has confused measuring overall inequality with measuring income inequality. The constant term (a) is individual welfare due to nonincome sources, which Dalton assumes is the same for all individuals. Dalton's measure is <u>the loss in total welfare due to income inequality</u>. A more appropriate measure is the loss in welfare due to income from income inequality. This would be represented by $1 - \sum_{i \varepsilon P} \left(\frac{\log Y_i}{N \log \bar{Y}} \right)$. This is the type of measure that Atkinson arrives at via the route of equally distributed income equivalents. Note that Dalton's measure changes when all individuals' incomes are multiplied by the same amount. If basic welfare is positive ($a > o$), then increasing everyone's income proportionately increases overall inequality.

[7]A function $f(X)$ is strictly concave if $df^2/d^2Y_i < 0$ for all Y_i. A function $f(Y)$ is strictly quasi-concave if for two distributions Y^1, Y^2, if $f(Y^1) \geq f(Y^2)$ then $f(Y^1) > f(Y^i)$ for $Y^i = aY^1 + (1-a)\ Y^2$ for $0 < a < 1$.

[8]Proof of this statement is obtained via a simple application of the calculus of optimization using Lagrangian multipliers.

[9]If we formulate Atkinson's measure in terms of equally distributed income equivalents, f can be of the form: $a + b\ I^{1-e}$.

[10]As C approaches 1 the limiting behavior of y_i^{1-e} is log y_i.

[11]The coefficient of variation orders distributions in the reverse order to Atkinson's measure where e = -1.

[12]This follows from noting that the Gini coefficient can be expressed as (Rothschild and Stiglitz, 1973):

$$\frac{1}{\bar{Y}\, n^2} \sum_{i=1}^{n} (2i - n - 1)\, Y_i$$

where i is the rank order of the individual in the income distribution.

[13]This conclusion may also apply to the work that has been done by sociologists using measures of segregation.

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