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PARALLEL PREFERENCE STRUCTURES

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ABSTRACT

This paper covers two interrelated topics. The first is a comment on a model of family labor supply developed by Ashenfelter and Heckman. The second is a discussion of the properties of demand models based on "Parallel" or "Absolutely Homothetic" preference structures. The latter models represent a generalization of one form of the Ashenfelter-Heckman parameterization to forms that are flexible and consistent with utility maximization over a broad range.

The parameterization of family labor supply responses developed by Ashenfelter and Heckman (A-H) provides for direct local estimates of income and substitution effects as coefficients of small changes in wage rates and real income. In their empirical implementation, they replace changes in real income with changes in market consumption, the numeraire good in the three good model. Our analysis of their model shows that constant income and substitution parameters, while convenient for local approximation, are generally inconsistent with utility maximization over a range of income and wage rates. The A-H <u>empirical</u> formulation is shown to imply variable income and substitution effects in which their estimated coefficients appear as parameters. As a consequence, revised estimation restrictions are necessary to assure local symmetry of substitution effects. Except for degenerate cases, the model is not globally consistent with utility maximization.

The general parallel preference model incorporates indifference surfaces that are identical in shape and scale (absolutely homothetic). These surfaces may be represented by compensated demand functions that are

additively separable in real income and functions of prices. The full preference structure is generated by the translation of the homothetic surfaces throughout the consumption space along parallel expansion paths. These structures lead to demand relationships (identified herein as expansion-path forms) expressing equilibrium consumption of (n - 1) goods as functions of consumption of the excluded (numeraire) good and prices. If the expansion paths are linear, the dependence on the numeraire is linear and separable as in the Ashenfelter-Heckman model. The linear models can be solved for demand functions that are explicit functions of income. Explicit indirect utility functions may also be derived for the linear parallel model.

The price dependence of the demand relationships follows directly from the form of the compensated demand functions. Since the price dependence of those functions is independent of the level of utility it is comparatively easy to generate specific functional forms that can incorporate very flexible substitution effects. The price parameterization for a simple specific case is derived as an example.

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Parallel Preference Structures

In a recent article in **Econometrica** Ashenfelter and Heckman [1] (henceforth A-H) introduced a novel parameterization for empirical implementation of the classical demand theory model of family labor supply. These authors also presented a similar model with slightly different empirical implementation in the Cain and Watts volume of labor supply studies [2]. In both papers the authors are candid in stating that the parameterization was chosen as an empirically convenient approximation and was not intended to be globally consistent with any particular utility function.

In this paper we attempt to clarify some of the theoretical properties of the A-H model. We also discuss the properties of a general class of models of which their empirical model can be a special case. In section 1 we review the A-H parameterization and seek to establish two points: 1) the two basic features assumed in the A-H parameterization, namely constant income effects and constant substitution effects, are suitable for local approximation but, in general, are mutually inconsistent properties of utility maximizing labor supply functions over a finite range of income and wage rates, and 2) the A-H model, as empirically implemented does not yield consistent estimates of the income and substitution effects directly, but instead yields estimates of parameters of functions defining those effects.

Our more general discussion in section 2 concerns the properties of the class of "parallel" or "absolutely homothetic" preference structures of which the A-H empirical model can be a special case. We use two terminological alternatives because any given preference structure in this class is characterized both by parallel income expansion paths[1] and by identically shaped indifference surfaces throughout the consumption space. Subject to these restrictions, the general parallel model is highly flexible since it can accommodate any desired shape of indifference surface and, independently, any desired relative magnitudes of income effects. The parallel model thus provides a basis for a flexible parameterization of labor supply functions or commodity demand functions that are theoretically consistent over a large range. These properties are particularly valuable in the labor supply context given the widespread use of estimated supply parameters to simulate responses to the large net wage and income changes resulting from proposed income maintenance policies.[2] A model based on a consistent preference structure that is defined over the full choice range is also necessary for an integrated treatment of labor force participation and continuous labor supply.

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[1]We use the term "expansion paths" to refer to loci of points in n-dimensional consumption space having constant marginal rates of substitution. As such, they are analogous to the expansion paths of production functions.

[2]This is indeed the context of the 1973 version of the A-H model.

1. PROPERTIES OF THE ASHENFELTER-HECKMAN PARAMETERIZATION

Ashenfelter and Heckman follow the classical model in defining general labor supply functions[3]

(1) $R_i = R_i(W_m, W_f, P, Y) \qquad i = m,f$

that represent the levels of labor supply that maximize a classical utility function

(2) $U = U(T-R_m, T-R_f, X)$

subject to the budget constraint

(3) $W_m R_m + W_f R_f + Y = PX$.

The variables R_m and R_f are levels of labor supply of the male and female household members, W_m and W_f are their respective wage rates, Y is non-labor income, T is the total time available to each member, X is a Hicksian composite of market consumption goods, and P is the price of that composite. Without loss of generality we may choose P as the numeraire and eliminate it from the budget constraint (3'). Wage rates and nonwage income are then expressed relative to the price of market goods for which we introduce lower case notation:

(3') $w_m R_m + w_f R_f + y = X$

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[3]We adopt their notation with simple modifications noted in the text. We abbreviate their presentation and thus do not maintain a correspondence of equation numbers.

The supply equations are correspondingly simplified, (1'). Equation (1") represents the demand function for market goods and completes the system:

$$(1') \quad R_i = R_i(w_m, w_f, y) \qquad i = m,f$$

$$(1'') \quad X = X(w_m, w_f, y)$$

At any equilibrium point the partial derivatives of the general labor supply function (1') are subject to the familiar set of general restrictions based on the Slutsky relationship (4):

$$(4) \quad \left.\frac{\partial R_i}{\partial w_j}\right|_u = \frac{\partial R_i}{\partial w_j} - R_j \cdot \frac{\partial R_i}{\partial y} \qquad \begin{array}{l} i = m,f \\ j = m,f \end{array}$$

The terms on the left-hand side of (4) are the compensated wage effects or "substitution effects". Those on the right-hand side are uncompensated wage and income effects. For infinitesimal changes about a given equilibrium point all of the elements of (4) will be effectively constant, whatever the functional form of the supply functions. In general, however, all of the elements of the Slutsky relationship will be interrelated, non-degenerate functions of the budget variables. One aspect of the interrelationship of wage rate and income derivatives is evident from differentiation of (4) with respect to income:

$$(5) \quad \frac{\partial}{\partial y}\left(\left.\frac{\partial R_i}{\partial w_j}\right|_u\right) = \frac{\partial}{\partial y}\left(\frac{\partial R_i}{\partial w_j}\right) - R_j \frac{\partial}{\partial y}\left(\frac{\partial R_i}{\partial y}\right) - \frac{\partial R_j}{\partial y}\ \frac{\partial R_i}{\partial y} \qquad \begin{array}{l} i = m,f \\ j = m,f \end{array}$$

Interchanging the order of differentiation of the first term on the R.H.S. and introducing the A-H notation of B_i and S_{ij} for income and substitution effects, respectively, we obtain the form in (6):

$$(6)\quad \frac{\partial}{\partial y} S_{ij} = \frac{\partial B_i}{\partial w_j} - \frac{R_j \partial B_i}{\partial y} - B_i B_j \qquad \begin{array}{l} i = m,f \\ j = m,f \end{array}$$

It is clear from (6) that the income and substitution effects cannot simultaneously be constant over finite ranges of income and wage rates except in the degenerate case where the former are zero. Our first point is thus established. Note, however, that if the income effects are small, the use of consistent estimates of the income and substitution effects for a central equilibrium point will result in only small approximation errors over a substantial income range.

The approximation errors inherent in the assumed constancy of income and substitution effects are not present in the A-H model <u>as empirically implemented</u>. This is because the parameters they identify as B_i and S_{ij} are related to, but not identical to, conventionally defined income and substitution effects. In the following discussion asterisks are added to the parameters of their empirical model to maintain the distinction.

The labor supply equations estimated by Ashenfelter and Heckman, after translation from derivation form are shown in (7):

$$(7)\quad R_i = R_{io} + S^*_{im} w_m + S^*_{if} w_f + B^*_i F \qquad i = m,f$$

The parameter R_{io} is an appropriate initial constant and the variable, F, also translated from deviation form, is defined by (8):

$$(8)\quad F = w_m R_m + w_f R_f + y.$$

Reference to equation (3') reveals that the variable F is identical to X, the third endogenous variable in the set of demand (or supply) equations. The equations, (7), thus represent interrelationships among endogenous variables and are clearly not conventional supply functions of the form of (1'). At any given values of wage rates the equations (7) define parallel loci in consumption space. If they represent utility maximizing values at those wage rates, the loci may be interpreted as income expansion paths. To investigate the consistency with utility maximization we solve (7) and (8) for the conventional labor supply and market goods demand functions whence we may derive expressions for the implied substitution effects. The conventional supply (demand) functions are shown in (9a, 9b),

(9a) $R_i(w_m, w_f, y) = \phi^i + (B^*_i/\xi)[w_m\phi^m + w_f\phi^f + y]$ $\qquad i = m,f$

(9b) $X(w_m, w_f, y) = (1/\xi)\ [w_m\phi^m + w_f\phi^f + y]$

where $\xi(w_m, w_f) = (1 - B^*_m w_m - B^*_f w_f)$ and $\phi^i = R_{io} + S^*_{im}\, w_m + S^*_{if} w_f$.

The relationship between the B^*_i and conventionally defined income effects is given by the derivatives of (9a, 9b) shown in (10a, 10b)

(10a) $\dfrac{\partial R_i}{\partial y} = B^*_i \Big/ \xi$

(10b) $\dfrac{\partial x}{\partial y} = 1/\ \xi$

The function $1/\xi$, the income effect on market consumption, will be equal to one only in the degenerate classes of zero wage rates or zero B^*. In the general case, some portion of an exogenous income increase will be spent on leisure so that the income effect on market goods will be less than unity.

It is straightforward to derive the substitution effects shown in equation (11) using (4).

$$(11) \quad S_{ij}(w_m, w_f) = S^*_{ij} + \frac{B^*_i}{\xi}(w_m S^*_{mj} + w_f S^*_{fj}) \qquad \begin{array}{l} i = m,f \\ j = m,f \end{array}$$

Except in the degenerate cases noted above, the substitution effects from the A-H estimation model will be functions of the S^*_{ij}, B^*_i and wage rates. Furthermore, equality of the parameters S^*_{mf} and S^*_{fm} is not sufficient for satisfaction of the symmetry condition. If the A-H parameterization is to be employed as a local approximation, then the parameter restrictions to assure symmetry should be based on the expressions (11) evaluated at appropriate wage values. For a sample with limited wage variations the model should then provide highly satisfactory point estimates of the theoretical parameters.

The A-H model with non-zero income effects can satisfy the symmetry conditions over a range of wage rates, but only if the parameters satisfy the three restrictions (12):

$$(12) \quad \frac{B^*_f}{B^*_m} S^*_{mm} = S^*_{mf} = S^*_{fm} = \frac{B^*_m}{B^*_f} S^*_{ff}$$

These restrictions leave only three of the six parameters independent and also imply that the determinant, $(S_{mm} S_{ff} - S_{mf}^2)$, is identically zero. The model thus yields plausible results only when used as a local approximation.

In concluding our comments on the specific Ashenfelter-Heckman model we want to emphasize that they apply to interpretation of the results rather than the estimation technique. The direct estimation of the expansion path relationships using instrumental variables does yield consistent local estimates of the S^* and B^* parameters which define the conventional income and substitution effects. In the current model the expansion path form is linear in parameters. In the more flexible models introduced in section 3, that linearity is lost but the expansion path form may still be useful for estimation.

2. GENERAL PARALLEL PREFERENCE STRUCTURES

The general parallel model represents an addition to a growing number of flexible demand models that are consistent with utility maximization. One class of these models is exemplified by the transcendental logarithmic (Translog) models of Christensen, Jorgensen and Lau [3] and the generalized Cobb-Douglas model of Diewart [6]. These models may be interpreted as second order approximations to general utility functions and provide highly flexible parameterizations of substitution response over a moderate range of prices. They may be less satisfactory over a large range of equilibrium marginal prices as is suggested by the results of Wales and Woodland [10] who apply both models in a labor supply context.

A second class of models, such as Hanoch's implicitly additive models [8], are based on an initial assumption of additivity or separability among goods. These models are relatively more economical in parameters than those above and are thus particularly well suited for large systems of demand functions. However, the assumed separability limits the models so that they cannot accomodate specific substitution or complementarity relationships among closely related goods.

The general parallel preference model incorporates a different form of separability. We begin with compensated demand functions in which the form of the price dependence is independent of the level of real income or utility. The separable price sub-functions determine the shape of the indifference surfaces which are then the same at all levels of utility (absolute homotheticity). Since we need to choose the functional form for only one indifference surface it is comparatively easy to select parameterizations that allow for very flexible substitution responses over a broad range of marginal prices. The absolute homotheticity is also limiting, however, since it implies that the pattern of substitution responses is the same at all levels of real income.

In this discussion of the general parallel model we use the notation for commodity demand. For labor supply applications, translation is accomplished by simple sign reversal for the appropriate parameters. In part (i) we develop the expansion path form of the general parallel model from the basic formulation of the preference structure. In part (ii) we discuss the choice of parameterization for the indifference surfaces and provide a simple example. Part (iii) then includes a derivation of explicit conventional demand functions and an indirect utility function for the linear parallel model.

(i). The Expansion Path Form of the Parallel Model

Any well behaved preference structure underlying a set of demand functions may be characterized by equations in indifference surfaces having the general form of (13):[4]

$$(13) \quad q_i = \Psi^i \, (\underline{p}, U) \qquad i = 1....n$$

The equilibrium marginal prices, $\underline{p}' = [p_2 ... p_n]$ are expressed relative to the price of the first good, which is chosen as the numeraire. The argument U is any chosen utility index.

Parallel preference structures may be represented by the separable form (14):[5]

$$(14) \quad q_i = q_{io}(U) + \tilde{\Psi}^i \, (\underline{p}) \qquad i = 1...n$$

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[4] See Samuelson [9] p. 114. The Ψ^i may be interpreted as Hicks-compensated demand curves.

[5] The form of the Ψ^i function for parallel or absolutely homothetic functions, shown in (14) may be compared with those for conventional homothetic structures. In the latter, the utility index enters through a proportional term--$\Psi^i = q_{io}(U) * \overset{\Delta}{\Psi}(p_2....p_n)$ - rather than through a translational term as in (14).

The shapes of the indifference surfaces, determined by the $\tilde{\Psi}^i(\underline{p})$ are independent of the level of utility. For any given level of U, the q_{io} (U), i = 1....n are the coordinates of a reference point on the indifference surface corresponding to the base set of marginal prices $(\underline{p}_o)$, at which all $\tilde{\Psi}_i(\underline{p}_o)$ are equal to zero.

As U varies the q_{io} (U) trace out a locus in consumption space that may be interpreted as a reference expansion path for the base prices $\underline{p}_o$. The n equations, $q_{io} = q_{io}$ (U), are a parametric representation of that path. The utility index, U, is arbitrary up to a monotonic transformation and, for any normal good, i, q_{io} (U) is such a transformation. That is, ascending values of any normal good along the reference expansion path provide an unambiguous ordering of the indifference surfaces intersecting that path. For convenience we choose the reference value of the numeraire good as the utility index (15):

(15) $U \equiv q_{1o}\ (U)$

The presentation of the reference expression path then has the general form (16a), with (16b) being the form for linear parallel structures:

(16a) $q_{io} = q_{io}\ (q_{1o})$ $\qquad i = 2....n$

(16b) $q_{io} = K_{io} + D_i\ q_{1o}$ $\qquad i = 2....n$

The D_i are constants closely related to income effects[6] and the K_{io} are initial constants that will depend on the parameterization of the $\tilde{\Psi}^i$ and the choice of $\underline{p}_o$.

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[6] The D_i are the general counterparts of the B_i^* in equation (7) of the A-H model. The D_i are positive for normal goods while the B_i^* applied to the negative good, labor supply, and were negative.

The relationships (15) and (16a,b) allow us to reexpress the full parallel preference structure (14) in terms of observable variables. The utility index, as a function of equilibrium prices and observed consumption of the numeraire, follows from (15) and the first equation of (14):

$$(17) \quad U(q_1, \underline{p}) = q_{1o}\,(q_1, \underline{p}) = q_1 - \tilde{\Psi}^1(\underline{p})$$

The remaining (n - 1) equations of (14), together with (15) and (16a, b) then yield the expansion path forms of the parallel model (18a, b):

$$(18a) \quad q_i(q_1, \underline{p}) = q_{io}(q_i - \tilde{\Psi}^1(\underline{p})) + \tilde{\Psi}^i(\underline{p}) \qquad i = 2 \ldots n$$

$$(18b) \quad q_i(q_1, \underline{p}) = K_{io} + D_i(q_1 - \tilde{\Psi}^1(\underline{p})) + \tilde{\Psi}^i(\underline{p}) \qquad i = 2 \ldots n$$

In geometric interpretation, the equations (18a, b) describe the preference structure that is generated when a given indifference surface, described by the $\tilde{\Psi}^i(\underline{p})$, is translated through the consumption space by moving a reference point on that surface along the expansion locus described by (16a) or (16b).

The form (18b) is separable with respect to the numeraire good, q_1, and a functions of prices, $\phi_i(\underline{p}) = K_{io} - D_i\ \tilde{\Psi}^i(\underline{p}) + \tilde{\Psi}^i(\underline{p})$, and is linear in q_1. It is thus a generalization of the Ashenfelter-Heckman model with respect to substitution possibilities allowed by the general form of the $\tilde{\Psi}_i$. Its tractability as an estimation form will depend on the parameterization of the $\tilde{\Psi}^i(p)$. The form may be linear in coefficients but will generally be nonlinear in theoretical parameters and parameter restrictions. A specific example of a parameterization is provided in part (ii). The linear parallel structure (22b) may also be solved to yield conventional demand functions. These functions, derived in part (iii), are intrinsically nonlinear in income and prices.

The general parallel form (18a) with nonlinear expansion loci is attractive with respect to its flexibility. In addition to the substitution possibilities allowed by general $\tilde{\Psi}^i(p)$ it would admit the possibility of a good passing from a luxury to an inferior good as utility levels increase.[7] The flexibility comes at the expense of estimation problems with nonlinear form and the difficulty or impossibility of solving for explicit conventional demand functions.

(ii) The parameterization of $\tilde{\Psi}^i(\underline{p})$

The functions $\tilde{\Psi}^i(\underline{p})$ determine the shape of the homothetic indifference surface for the parallel model. The various substitution effects are given by the price derivatives of the $\tilde{\Psi}^i$ functions. Our basic procedure in generating the $\tilde{\Psi}^i$ is to select a parameterization for the substitution effects that satisfies the integrability requirements and that embodies the desired flexibility for the demand system being studied. We then integrate to obtain the functions, $\tilde{\Psi}^i$, and the resulting estimation forms. The choice of parameterization for the $S_{ij}(\underline{p})$ clearly determines both the plausibility and the tractability of the model.

For our purposes it is convenient to choose the parametric form of the (n-1) own-substitution effects and (n-1)(n-2)/2 cross-substitution effects for goods other than the numeraire. The remaining terms are then generated by the symmetry condition and by the Cournot aggregation condition expressed in compensated form (19 a,b):[8]

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[7] A potential problem with inferior goods in the parallel model is that indifference surfaces would intersect at the (usually very high) price level $p_i = -1/D_i$ or $p_i = -1/q_{io}'$.

[8] See Goldberger [7] p. 21.

(19a) $\sum_{i=1}^{n} p_i \, S_{ij} \, (\underline{p}) = 0$ $\qquad j = 2 \ldots . n$

(19b) $S_{1j}(\underline{p}) = - \sum_{i=2}^{n} p_i \, S_{ij} \, (\underline{p})$ $\qquad j = 2 \ldots . n$

As a simple example we solve for the form of the model in which the own-substitution effects for goods 2....n are linear functions of the inverse price and all cross substitution effects for those goods are constants. In (20) γ_i is the price parameter and $\delta_{ij} = 1$ for i=j and 0 otherwise:

(20) $S_{ij} \, (\underline{p}) = \frac{\partial \tilde{\Psi}^i}{\partial p_j} = S_{ij} + \frac{\gamma_i}{p_i} \delta_{ij}$ $\qquad i,j = 2 \ldots . n$

From the aggregation condition (19b) it then follows that the cross substitution effects for the numeraire good have the form (21):

(21) $S_{1j} = \frac{\partial \tilde{\Psi}^1}{\partial p_j} = - \sum_{i=2}^{n} p_i \, S_{ij} - \gamma_j$ $\qquad j = 2 \ldots . n$

Upon integration we obtain the $\tilde{\Psi}^i$ functions (22a, b):

(22a) $\tilde{\Psi}^1 = - \left[\sum_{j=2}^{n} \gamma_j p_j + \frac{1}{2} \sum_{i=2}^{n} \sum_{j=2}^{n} p_i p_j \, S_{ij} \right] \Bigg|_{\underline{p}_o}^{\underline{p}}$

(22b) $\tilde{\Psi}^i = \left[\sum_{j=2}^{n} S_{ij} p_j + \gamma_i \ln p_i \right] \Bigg|_{\underline{p}_o}^{\underline{p}}$ $\qquad i = 2 \ldots . n$

The functions (22a, b) will be consistent with a quasiconcave utility function if $S_{ii} < 0$, $\gamma_i < 0$, $S_{ij} = S_{ji}$ and $S_{ij}^2 < S_{ii} S_{jj}$. Somewhat less restrictive conditions will yield a locally acceptable function.

This model yields the estimation form (23),

(23) $q_i = a_i + D_i q_1 + \gamma_i \ln p_i + \sum_{j=2}^{n} b_{ij} p_j + \sum_{j=2}^{n} \sum_{k=2}^{n} c_{ijk} p_j p_k$ $\qquad i = 2 \ldots . n$

where $b_{ij} = S_{ij} + D_i\gamma_j$, $c_{ijk} = 1/2\ (D_i S_{jk})$

and $a_i = K_{io} - \sum_j b_{ij} - \sum_j \sum_k d_{ijk}$ given $\underline{p}_o = [1....1]$.

The parameterization developed above allows for any degree of specific complementarity or substitutability among pairs of goods. Relative magnitudes of income effects among goods are also unconstrained. The simple constancy of parameters does imply, of course, that any given pattern of effects is the same at all consumption points.

The presence of the inverse price term in the own-substitution effects assures nonsatiety. If one or more of the δ_i are constrained to zero, those own-substitution effects are constant. Any compensated price reduction, specifically a reduction to zero price, results in a finite consumption increase $\Delta q_i = S_{ii}\ \Delta p_i$. The resultant satiation level of consumption would be plausible in some but not all models. For example, the A-H estimates of the substitution effect for male labor supply[9] would, if assumed globally constant, imply leisure satiation at a level of labor supply not far below standard full time work. The parameterization of the own-substitution effects with inverse price terms provides the flexibility to avoid such implications.

For some applications it may be desirable to specify the cross substitution effects as nondegenerate functions to relative prices. This would allow the degree of complementarity or substitutability to vary over the consumption space. For example, a specification that allows strong cross substitution effects for similar prices and weaker effects for disparate prices would be desirable for some pairs of goods. Under these circum-

[9]Estimates by numerous other authors including the present one yield similar results, see Dickinson [4], [5].

stances the choices of parametric form of different substitution effects are necessarily interrelated, since the S_{ij}, j = 2....n are all partial derivatives of a common $\Psi^i(\underline{p})$. The substitution functions must thus be chosen so that

$$\frac{\partial(S_{ij})}{\partial p_k} = \frac{\partial^2 \Psi^i}{\partial p_j \partial p_k} = \frac{\partial^2 \Psi^i}{\partial p_k \partial p_j} = \frac{\partial(S_{ik})}{\partial p_j}$$

for all pairs of substitution effects for a given good. These interrelationships then extend among goods by virtue of the symmetry conditions. Subject to these conditions, choices of functional form appear to be further limited only by mathematical and empirical tractability.

(iii) Explicit Conventional Demand Functions and Indirect Utility Functions for Linear Parallel Models

The expansion path form of the linear parallel model (18b) includes the endogenous variable q_i on the right hand side and is not an explicit function of income. Conventional demand functions may be obtained by simultaneous solution of the (n-1) equations (18b) together with the budget constraint (24):

$$(24) \quad q_1 + \sum_{i=2}^{n} p_i q_i = y$$

The expansion path relations (18b) are conveniently reexpressed in (25)

$$(25) \quad q_i = D_i q_1 - \phi_i(\underline{p}) \qquad i = 2....n$$

where $\phi_i(\underline{p}) = K_{io} - D_i \Psi^1(\underline{p}) + \Psi^i(\underline{p})$.

In matrix form, the system of an equation then becomes

$$(26) \quad \left[\begin{array}{c|c} 1 & \underline{p} \\ \hline -\underline{D} & \underset{(n-1)}{I} \end{array}\right] \left[\underline{q}\right] = \left[\begin{array}{c} y \\ \underline{\phi} \end{array}\right]$$

where $\underline{p}$, $\underline{D}$, $\underline{\phi}$ are (n-1) vectors with elements p_i, D_i, ϕ_i for $i = 2....n$, and $\underline{q}$ is the n-vector of consumption quantities.

The system (26) has the solution (27)

$$(27)\quad \left[\underline{q}\right] = \frac{1}{\xi}\left[\begin{array}{c|c} 1 & -\underline{p}' \\ \hline \underline{D} & \xi I - \underline{D}\,\underline{p}' \end{array}\right]\left[\begin{array}{c} y \\ \underline{\phi} \end{array}\right]$$

where $\xi = 1 + \underline{p}'\underline{D}$.

In algebraic form the solutions are conveniently reexpressed in terms of the $\tilde{\Psi}_i$

$$(28)\quad q_i = k_{io} + \tilde{\Psi}^i + \frac{D_i}{\xi}\left[y - \sum_{j=1}^{n} p_j(k_{jo} + \tilde{\Psi}^j)\right] \qquad i = 1....n$$

where the form (28) applies the the numeraire good as well if we define $k_{1o} = 0$, $D_1 = 1$, and $p_1 = 1$.

Each of the equations (28) includes all the parameters in the system and is intrinsically nonlinear. For a small number of goods and economical parameterizations of the $\tilde{\Psi}^i$ functions, the system appears to be roughly comparable to the translog model with respect to tractability as an estimation form. Practical experience will provide further information on the relative usefulness of the explicit demand functions as compared with the expansion path form (18b). The Ashenfelter-Heckman results indicate that the latter can be implemented successfully, at least with very economic parameterizations.

An explicit form of the indirect utility function is easily derived from the mixed utility index (17) once the form of the demand function for the numeraire good is known. Substitution in (17) from the first equation of (28) yields the indirect utility function (29):

(29a) $U(q_1(y,\underline{p}),\ \underline{p}) = q_1(y,\ \underline{p}) - \tilde{\Psi}^1(\underline{p})$

(29b) $V(y,\ \underline{p}) = \frac{1}{\xi}\ [y - \sum_j p_j(K_{.o} + \tilde{\Psi}^j)]$

The price normalization in the parallel model differs from that conventionally used in indirect utility functions. The indirect utility function and the demand functions can be reexpressed in terms of $v_i = P_i/y$, though the $\tilde{\Psi}_i$ will continue to be functions of the price ratios $p_i = v_i/v_1$. The asymmetry of the formulation of the parallel model thus persists despite a symmetric price normalization.

The formulation of the parallel model in terms of an explicit indirect utility function suggests potential extensions and generalizations based on duality theory. These aspects of the model will be explored in future work.

3. CONCLUDING REMARKS

Ashenfelter and Heckman estimated a model of labor supply in which labor supply (or leisure demand) is a separable function of prices and observed consumption of the numeraire good (a composite of market goods). Their model is linear with respect to prices and the numeraire good. Our analysis has shown that their model is suitable for local approximation of a preference structure but that the linear coefficients do not correspond directly to income and substitution effects except in degenerate cases.

Our subsequent discussion concerned the properties of "Parallel" or "Absolutely Homothetic" preference structures. All indifference surfaces in this model are identical in shape and scale. These homothetic surfaces may incorporate any desired complexity of substitution possibilities. The full preference structure is generated by the systematic translation of a single surface through the consumption space. In the course of this translation, points on the indifference surface, each corresponding to a given set of marginal rates of substitution, trace out parallel expansion paths in the consumption space.

If the expansion paths are linear, the parallel model gives rise to demand relations for the n-1 goods other than the numeraire, that are separable with respect to a function of equilibrium prices and a linear function of observed consumption of the numeraire good. The linear parallel model thus generalizes the Ashenfelter-Heckman model to allow general and consistent substitution responses. The linear parallel model may also be solved for explicit conventional demand functions. These functions are nonlinear in all parameters.

The linear parallel model also yields explicit indirect utility functions. These functions may provide a basis for extensions and generalizations of the parallel model.

As currently formulated, the parallel model is asymmetric between the numeraire good and all others in the system. As such, it would appear to be most useful for the study of demand relations for a small number of related goods with the composite "other goods" taken as the numeraire. The power of the model for parameterizing flexible substitution and complementarity relationships is also likely to be most useful in such circumstances. The model appears to be particularly well suited to models of family labor supply, which incorporate a natural numeraire and also require consistent and plausible properties over a wide range of marginal prices.

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