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Efraim Sadka

UNIVERSITY OF WISCONSIN ~ MADISON

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The author, Lecturer of Economics, Tel-Aviv University, is currently a visiting Associate Professor of Economics at the University of Wisconsin-Madison. This research was supported by funds granted to the Institute for Research on Poverty, at the University of Wisconsin-Madison, by the Department of Health, Education and Welfare pursuant to the Economic Opportunity Act of 1964.

Abstract

This paper studies the distributional effects of taxes and subsidies on consumption externalities. It is well known that a Pigouvian excise tax need be imposed on external diseconomies if the competitive equilibrium is to be Pareto-optimal. The focus of this paper is an examination of whether these taxes enhance a Pareto-improvement with respect to the pretax equilibrium.

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I. Introduction

In the presence of external diseconomies a Pigouvian tax schedule is needed to sustain a Pareto-optimal allocation.[1] An interesting question with respect to distribution of income is whether such a schedule results in an allocation which is Pareto-superior to the pretax competitive equilibrium. Clearly, the answer depends on what is done with the tax revenues.

To illustrate this point, consider the following example. In recent years many economists have suggested raising the excise tax imposed on gasoline in order to reduce the U.S. dependence on foreign oil supplies. One way to view this problem is to consider gasoline to be a good which generates external diseconomies, where the form of the externality appears as the aggregate consumption of that good (namely, the dependence on foreign oil supply). In order to undo the distributioanl effects of raising the excise tax on gasoline, it was suggested that the increased tax payments by various income groups be estimated and that the income tax schedule be correspondingly reduced.

This example raises the following question. Suppose that a Pigouvian tax is imposed on a good which generates external diseconomies and that the tax revenues are distributed back to the consumers, each one getting a lump-sum transfer which is exactly equal to his tax payment. Does such a tax and distribution scheme necessarily result in a Pareto-superior allocation (compared to the pretax competitive equilibrium)? This question is the focus of this paper, and it will be seen that the answer is in the negative: while some (sufficiently small) excise tax on an external diseconomy may indeed induce a Pareto-superior allocation, this need not be the case with a Pareto-optimal tax. We may be tempted to attribute the failure of the combined tax and distribution scheme to enhance a Pareto-superior allocation to the income effect embodied in the distribution scheme. However, it will be shown that the income effect alone cannot explain this failure; rather, it is due to the dead-weight loss associated with commodity taxation.

2. A Two-Good Model

Consider an economy with two goods, X and Y, where Y is externality-free and X causes external diseconomies. There are H individuals (indexed h = 1, . . . , H) in this economy with utility functions $u^h(x^h, y^h; x)$ where $x = \sum_{h=1}^{H} x^h$. The externality is thus working through the aggregate consumption of X. Suppose, for the sake of simplicity, that the producer prices of X and Y are fixed at (p,1). The price of Y is thus normalized to 1. Denote the consumer price of X by $q = p + t$, where t is the excise tax imposed on X. Denote the income of individual h by $R^h = I^h + T^h$, where I^h is a (given) pretransfer income and T^h is the lump-sum transfer. Define $R = (R^1, . . . , R^H)$ as the income distribution vector.

Given the price level (q), the income (R^h), and the externality level (x), individual h chooses x^h and y^h so as to maximize his utility $u^h(x^h, y^h; x)$ subject to his budget constraint $px^h + y^h \leq R^h$.

Denote the solution by $\bar{x}^h(q; R^h; x)$ and $\bar{y}^h(q; R^h; x)$. Suppose that the equation

$$x = \sum_{h=1}^{H} \bar{x}^h (q; R^h; x) \tag{1}$$

can be solved explicitly for x as a function of (q; R). Denote the solution by x(q; R). This function is the aggregate demand function and, unlike the $\bar{x}^h$'s and the $\bar{y}^h$'s, it takes into account the effect of changes in (q;R) on the externality level. Similarly, such is the difference between $\bar{x}^h$ and x^h, which is defined as

$$x^h (q; R) \equiv \bar{x}^h [q; R^h; x(q; R)], \; h = 1, \ldots, H. \tag{2}$$

Then from (1) we have $\sum_{h=1}^{H} x^h (q; R) = x(q; R)$.

Denote by

$$\bar{v}^h(q; R^h; x) \equiv u^h [\bar{x}^h(q; R; x), \bar{y}^h(q; R; x); x], \; h = 1, \ldots, H \tag{3}$$

the indirect utility function of individual h and define

$$v^h (q; R) \equiv \bar{v}^h [q; R^h; x(q; R)], \; h = 1, \ldots, H. \tag{4}$$

The difference between v^h and $\bar{v}^h$ is again that v^h takes into account the effect of changes in (q; R) on the externality level.

The tax payment of individual h which is equal to the transfer that he receives is defined implicitly by the following system of equations:

$$T^h = (q-p) \cdot \bar{x}^h [q; I^h + T^h; x (q; I^1 + T^1, \ldots, I^H + T^H)] \tag{5}$$
$$h = 1, \ldots, H.$$

Denote the solution to (5) by $T(q) = [T^1(q), \ldots, T^H(q)]$.[2]

Given this distribution scheme, suppose that there is $q^* > p$ (namely, a tax $t^* = q^* - p$) which supports a Pareto-optimal allocation. The problem is to determine whether this Pareto-optimal allocation is superior to the pretax competitive allocation. Define

$$V^h\ (q) = v^h \left[q;\ I + T(q)\right],\ h = 1, \ldots, H. \tag{6}$$

We thus wish to determine whether or not

$$V^h\ (q^*) \gtrless V^h(p)\ \text{for all } h. \tag{7}$$

If we calculate $\frac{dV^h}{dq}$ and find that it is negative everywhere on the interval $\left[p,\ q^*\right]$ for at least one h, we can definitely say that our tax and distribution scheme do not enhance a Pareto-superior allocation. Employing (6) and (4), we can write

$$V^h(q) \equiv \bar{v}^h \left\{q;\ I^h + T^h\ (q);\ x[q;\ I + T(q)]\right\},\quad h = 1, \ldots, H.$$

Hence,

$$\frac{dV^h}{dq} = \frac{\partial\bar{v}^h}{\partial q} + \frac{\partial\bar{v}^h}{\partial R^h}\frac{dT^h}{dq} + \frac{\partial\bar{v}^h}{\partial x}\left(\frac{\partial x}{\partial q} + \sum_{j=1}^{H}\frac{\partial x}{\partial R^j}\frac{dT^j}{dq}\right),\quad h = 1, \ldots, H. \tag{8}$$

By Roy's identity, we have

$$\frac{\partial\bar{v}^h}{\partial q} = -\frac{\partial\bar{v}^h}{\partial R^h}\bar{x}^h\ ,\qquad h = 1, \ldots, H. \tag{9}$$

Employing (3) and Samuelson's envelope theorem, we find that

$$\frac{\partial\bar{v}^h}{\partial x} = \frac{\partial u^h}{\partial x}\ ,\qquad h = 1, \ldots, H. \tag{10}$$

Define $MRS^h \equiv -\frac{\partial u^h}{\partial x}\Big/\frac{\partial u^h}{\partial y^h} \gtrless 0$ as the marginal rate of substitution of y^h for x.

Since the price of Y is normalized to one, we have $\frac{\partial \bar{v}^h}{\partial R^h} = \frac{\partial u^h}{\partial y^h}$ in this case. Hence we can rewrite (10) as

$$\frac{\partial \bar{v}^h}{\partial x} = -\frac{\partial \bar{v}^h}{\partial R^h} \, MRS^h \quad , \qquad h = 1, \ldots, H. \tag{11}$$

Substituting (9) and (11) into (8), we obtain

$$\frac{dV^h}{dq} = -\frac{\partial \bar{v}^h}{\partial R^h}\left[\bar{x}^h - \frac{dT^h}{dq} + MRS^h\left(\frac{\partial x}{\partial q} + \sum_{j=1}^{H} \frac{\partial x}{\partial R^j}\frac{dT^j}{dq}\right)\right], \tag{12}$$

$$h = 1, \ldots, H.$$

The interpretation of this last equation is very simple and is left to the reader.

The derivative $\frac{dT^h}{dq}$ can be computed by differentiating (5) with respect to q. Its sign cannot be determined. Likewise, we cannot determine the signs of $\frac{\partial x}{\partial q}$ and $\frac{\partial x}{\partial R^j}$. Thus, we cannot show that (7) holds in general. In fact, turning to a special case in the next section, it is even possible to show that (7) is false.

3. The Zero-Income Elasticity Case

We may attribute our failure to determine the sign of the derivative $\frac{dV^h}{dq}$ in (12) to the existence of an income effect (due to our distribution scheme). This effect shows up explicitly in $\frac{\partial x}{\partial R^j}$ and implicitly in $\frac{dT^h}{\partial q}$ (through (5)) and in $\frac{\partial x}{\partial q}$. However, the reason that our tax and distribution scheme fails to enhance a Pareto-superior allocation cannot be attributed to this income effect. It seems more appropriate to attribute this failure to the well-known deadweight loss involved in commodity taxation.

To demonstrate this we will employ a special form of the utility functions:

$$u^h(x^h, y^h; x) = A^h(x^h; x) + y^h \quad , \qquad h = 1, \ldots, H. \qquad (13)$$

In this case any additional income available to the consumer will be spent on Y (ignoring corner solutions) and will have no effect on $\bar{x}^h$ or x. Hence, $\frac{\partial x}{\partial R^h} = 0$ and $\frac{\partial \bar{v}^h}{\partial R^h} = 1$, h = 1, . . . , H. Also, from (5):

$$\frac{dT^h}{dq} = \bar{x}^h + t\,\frac{\partial \bar{x}^h}{\partial q} + t\,\frac{\partial \bar{x}^h}{\partial x}\,\frac{\partial x}{\partial q} \quad , \qquad h = 1, \ldots, H \qquad (14)$$

Thus, (12) becomes

$$\frac{dV^h}{dq} = t\,\frac{\partial \bar{x}^h}{\partial q} + t\,\frac{\partial \bar{x}^h}{\partial x}\,\frac{\partial x}{\partial q} - MRS^h\,\frac{\partial x}{\partial q} \quad , \qquad h = 1, \ldots, H. \qquad (15)$$

Since, by (2), $\frac{\partial x^h}{\partial q} = \frac{\partial \bar{x}^h}{\partial q} + \frac{\partial \bar{x}^h}{\partial x}\,\frac{\partial x}{\partial q}$, it follows that

$$\frac{dV^h}{\partial q} = t\frac{\partial x^h}{\partial q} - MRS^h\,\frac{\partial x}{\partial q} \quad , \qquad h = 1, \ldots, H. \qquad (16)$$

Recalling that $\sum_{h=1}^{H} x^h = x$, we can rewrite (16) as

$$\frac{dV^h}{\partial q} = (t - MRS^h)\,\frac{\partial x^h}{\partial q} - MRS^h \sum_{j \neq h} \frac{\partial x^j}{\partial q} \quad , \qquad h = 1, \ldots, H \qquad (17)$$

In general, we may expect the signs of $\frac{\partial x^j}{\partial q}$ and $\frac{\partial x}{\partial q}$ to be nonpositive, but counterintuitive results in the presence of externalities are known to exist (see, for instance, Buchanan and Kafoglis [1963]). However, it seems more natural to assume that $\frac{\partial x^h}{\partial q} < 0$ for all h and hence $\frac{\partial x}{\partial q} < 0$. In this case, as long as $0 \leq t \leq MRS^h$, we have $\frac{dV^h}{\partial q} > 0$, h = 1, . . ., H. Thus, some taxation of the externality with our distribution scheme results in a Pareto-superior allocation.[3] The reason for this conclusion is our assumption that each utility-maximizing individual ignores his own contribution to the aggregate level of the externality. Hence, as long as the tax does not exceed his own evaluation of the externality (namely, MRS^h), he is better off.

However, as the tax is increased to its Pareto-optimal level (namely, $\sum_{h=1}^{H} MRS^h$), we can no longer claim that $\frac{dV^h}{\partial q} > 0$ for all h; for then individual h is made to bear the cost which his contribution to the level of the externality inflicts upon all the other individuals. By summing (16) over all h, we obtain

$$\sum_{h=1}^{H} \frac{dV^h}{\partial q} = t \sum_{h=1}^{H} \frac{\partial x^h}{\partial q} - \frac{\partial x}{\partial q} \sum_{h=1}^{H} MRS^h = (t - \sum_{h=1}^{H} MRS^h) \frac{\partial x}{\partial q}. \qquad (18)$$

Hence, at the optimum level of t, $\sum_{h=1}^{H} \frac{dV^h}{\partial q} = 0.$[4] Thus, it is impossible to have $\frac{dV^h}{dq} > 0$ for all h. Furthermore, if $\frac{dV^h}{\partial q} > 0$ for some h, we must then have $\frac{dV^h}{\partial q} < 0$ for some other h. Thus, the movement from a no-tax state to the optimal tax state need not be monotonic in the sense that everyone is made better off everywhere along the way.

We can even establish stronger negative results. It is well-known that, in the absence of externality, an excise tax combined with a lump sum transfer which is equal to the tax payment results in a reduction in utility (see, for instance, the analysis of the deadweight loss of taxation in Diamond and McFadden [1974]). Hence, if there is an individual whose evaluation of the externality is sufficiently low (namely, he suffers very little from the external diseconomy), then this individual will be worse off with our tax and distribution scheme. To see this, consider a limiting case where some individual h does not suffer at all from the externality, namely $\frac{\partial u^h}{\partial x} = 0$ and hence MRS^h x 0 and $\frac{\partial \bar{x}^h}{\partial x} = 0$. In this case, (15) reduces to

$$\frac{dV^h}{\partial q} = t \frac{\partial \bar{x}^h}{\partial q} . \tag{19}$$

Since the income effect on $\bar{x}^h$ is zero, it follows that $\frac{\partial \bar{x}^h}{\partial q}$ reflects a substitution effect only, and hence $\frac{\partial \bar{x}^h}{\partial q} < 0$. Thus, $\frac{dV^h}{\partial q} < 0$ everywhere along the way from a no-tax state to the optimal tax state. Such an individual is made worse off by our tax and distribution scheme.[5]

NOTES

[1]To avoid needless repetition, I discuss the case of external diseconomies only. The results, however, may be extended in a very straightforward manner to the case of external economies as well.

[2]Indeed, each T^h depends on all the I^h's. But since the latter are given throughout, I ignore this dependence.

[3]For this result to hold, it is also necessary to assume that $MRS^h > 0$ for all h.

[4]This result can be alternatively derived by observing that when the utility functions have the form (13), then a noncorner Pareto-optimal allocation maximizes the sum of individual utilities (see also Diamond and Mirrlees [1973]).

[5]In fact, this result does not even depend on the assumption of a zero income elasticity.

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