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ABSTRACT

Extending the classical twin method, A. R. Jensen (1974) claims to find unique estimates of the variances, and covariance, of the genetic and environmental components of IQ. But his claim is based on faulty arithmetic. A wide range of estimates are in fact consistent with his model and data.

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1. INTRODUCTION

Jensen (1974) extends the classical twin method to allow for correlation between genes and environment. In the last section of the paper (pp. 22-27), he uses observed MZ and DZ correlations along with an independent estimate of test reliability and plausible sign restrictions. From these, he claims to obtain unique estimates of the variances, and covariance, of the genetic and environmental components of IQ.

Lest his claim be taken seriously, we show that his arithmetic is wrong.

2. SPECIFICATION

Jensen's equations (33)-(35) may be written as

$$\sigma_G^2 + \sigma_E^2 + 2r\,\sigma_G\sigma_E = 213.75 \tag{1}$$

$$\sigma_G^2 + \rho\,\sigma_E^2 + 2r\,\sigma_G\sigma_E = 195.75 \tag{2}$$

$$g'\,\sigma_G^2 + \rho'\sigma_E^2 + 2r\,\sigma_G\sigma_E = 126.00, \tag{3}$$

where

g' = genetic correlation for DZs

ρ = environmental correlation for MZs

ρ' = environmental correlation for DZs

and

σ_G^2 = genetic variance

σ_E^2 = environmental variance

r = correlation between genotype and environment.

The numbers on the right-hand side of (2) and (3) are observed IQ covariances for MZs and DZs respectively. The number on the right-hand side of (1) is the estimated variance of true IQ, i.e. the observed variance reduced by 5% for measurement error ($213.75 = .95\ (15)^2$).

Jensen tells us that, for each combination of the following assigned values,

$g' = .50, .54, .58, .60, .70$

$\rho = .90, .80, .70$

$\rho' = .90, .80, .70, .60,$

he solved the system (1)-(3) for the unknowns σ_G^2, σ_E^2, r. He reports that for only one of the sixty combinations was an admissible solution obtained -- that is a solution with σ_G^2, σ_E^2, r all positive. This unique combination was:

$g' = .50, \qquad \rho = .70, \qquad \rho' = .70,$

which produced the solution:

$\sigma_G^2 = 139.50, \quad \sigma_E^2 = 60.00, \qquad r = .078.$

Translated into proportions, this allocates true IQ variance into:

h^2 = heritability = 139.50/213.75 = .65

e^2 = environmentability = 60.00/213.75 = .28

2rhe = covariance effect = $2(.078)\sqrt{(.65)(.28)}$ = .07.

Jensen goes on to emphasize the uniqueness of this solution, concluding:

> "... only one possible solution (when restricted by the assumption that all the estimated variance components should be positive) emerged. It attributes 65% of the variance to genetic factors, 28% to environmental factors, and 7% to the covariance between genetic and environmental factors. The analysis also indicates equal environmental correlations for MZ and DZ twins (with respect to environmental influences on IQ) and suggests (but cannot precisely estimate) a substantial amount of dominance in the genetic determination of IQ differences."

His conclusion that environments of MZs and DZs are equally correlated rests on the finding $\rho = \rho' = .70$. His conclusion that dominance is substantial rests on the finding $g' = .50$, a value too low to be accounted for by purely additive effects when assortative mating is strong.

3. ARITHMETIC VERIFICATION

But Jensen's combination is not unique. As the reader can easily verify, setting

$$g' = .50, \qquad \rho = .90, \qquad \rho' = .80$$

produces the solution

$$\sigma_G^2 = 31.50, \qquad \sigma_E^2 = 180.00 \qquad r = .0149.$$

Translated into proportions this says

$$h^2 = .15, \qquad e^2 = .84, \qquad 2rhe = .01,$$

an allocation which is quite different from Jensen's. Alternatively, setting

$$g' = .58, \qquad \rho = .80, \qquad \rho' = .60$$

yields

$$h^2 = .58, \qquad e^2 = .42, \qquad 2rhe = 0.$$

Here $\rho \neq \rho'$, so that the environments of MZs and DZs are not equally correlated, and g' is large enough that no dominance effects are indicated.

Jensen's claim and conclusions are clearly unfounded.

4. ALGEBRAIC ANALYSIS

Evidently a wide range of parameter values are consistent with Jensen's data and model. The extent of indeterminancy can best be exhibited via some elementary algebra. For that purpose, it is convenient to standardize

the system by dividing (1)-(3) through by $\sigma_Y^2 = 213.75$. We then have

(4) $$h^2 + e^2 + 2rhe = 1$$

(5) $$h^2 + \rho\ e^2 + 2rhe = .916$$

(6) $$g'h^2 + \rho'\ e^2 + 2rhe = .589$$

The numbers on the right-hand side of (5) and (6) are the estimated MZ and DZ correlations, i.e., the observed correlations corrected for reliability.

Subtracting (5) from (4), and (6) from (5), yields

(7) $$(1 - \rho)\ e^2 = .084$$

(8) $$(1-g')\ h^2 + (\rho - \rho')\ e^2 = .327.$$

From (7) we see that the value of e^2 is determined by the value for ρ. The requirement $0 \leq e^2 \leq 1$ will be satisfied if we restrict attention to $0 \leq \rho \leq .916$; as ρ varies from 0 to .916, e^2 varies from .084 to 1. For any such value of e^2, we can choose h^2 arbitrarily in the interval $(1 - e)^2 \leq h^2 \leq 1 - e^2$, and be assured that the value of r implied by (4) will satisfy the requirement $1 \geq r \geq 0$. Finally, with ρ, e^2, h^2 in hand, equation (8) determines g' as a linear function of ρ'. We can choose ρ' arbitrarily in the interval $0 \leq \rho' \leq 1$, and calculate the implied value of g'. Provided that the resulting value of g' falls within the genetically relevant interval, the entire exercise has provided an admissible solution.

The range of possibilities is suggested by the following tabulation:

Line	ρ	r	e^2	h^2	ρ'	g'
1	0	0	.084	.916	0	.643
2	0	0	.084	.916	1	.551
3	0	1	.084	.504	0	.351
4	0	1	.084	.504	1	.184
5	.916	0	1.000	0	.589	0 to 1
6	.916	1	1.000	0	.589	0 to 1

With $e^2 = 1$ and $h^2 = 0$ in lines 5 and 6 of the table, equation (8) implies $\rho-\rho' = .327$, leaving the value of g' indeterminate. While the values of g' in lines 3 and 4 may be genetically implausible, that objection can be removed by reducing the value of r. Thus, for example, we can take r = 1/2 and obtain:

Line	ρ	r	e^2	h^2	ρ'	g'
3'	0	.500	.084	.677	0	.517
4'	0	.500	.084	.677	1	.393.

Now, Jensen did limit the range of values entertained for the assigned parameters. We construct another table to suggest the possibilities which remain when the assigned environmental correlations are confined to his intervals $.7 \leq \rho \leq .9$, $.6 \leq \rho' \leq .9$:

Line	ρ	r	e^2	h^2	ρ'	g'
1	.7	0	.280	.720	.6	.585
2	.7	0	.280	.720	.9	.468
3	.7	.25	.280	.529	.6	.435
4	.7	.25	.280	.529	.9	.275
5	.9	0	.840	.160	.6	.531
6	.9	0	.840	.160	.62	.426

Interested readers may easily locate other feasible parameter combinations.

6. REMARKS

Our analysis has shown that, contrary to Jensen's contention, a wide variety of parameter estimates are compatible with his data and his model. But the extent of indeterminancy is even wider.

For, his data are not immutable: the population IQ correlations may well differ from his .916 and .589. Readers can easily determine -- by perturbing the right-hand sides of (7) and (8) -- how sensitive the implied parameter estimates are to slight changes in the data.

Furthermore, his model is not immutable. In particular we may question his assumption that a DZ twin's environment is as highly correlated with his brother's genotype as with his own genotype. This assumption was implicitly introduced by Jensen when he defined r as "the genotype-environment correlation" without specifying <u>whose</u> genotype and <u>whose</u> environment are involved. Relaxing this assumption will again widen the indeterminancy.

It should now be clear that the twin method cannot be used to extract meaningful estimates of the variances, and covariance, of the genetic and environmental components of human intelligence. Any plausible model for the resemblance between the twins will have so many more unknown parameters than observations that the task is futile.

REFERENCE

A. R. Jensen (1974), "The problem of genotype-environment correlation in the estimation of heritability from monozygotic and dizygotic twins," University of California, Berkeley: Institute of Human Learning (mimeographed), 29 pp. To appear in Acta Geneticae Medicae et Gemellologiae.