303-75

INSTITUTE FOR RESEARCH ON POVERTY DISCUSSION PAPERS

LOCATION AND THE PRICE OF HOUSING

John Yinger

UNIVERSITY OF WISCONSIN ~ MADISON

LOCATION AND THE PRICE OF HOUSING

John Yinger

Institute for Research on Poverty

September 1975

This paper has benefited greatly from the comments and teaching of Edwin S. Mills. Helpful suggestions have also been made by James Ohls, Jonathan Dickinson, and the participants in the Noon Seminar at the Institute for Research on Poverty. Valuable research assistance was provided by Harriet Newburger, and the data were generously supplied by John M. Quigley. The author is grateful for all of these contributions.

The research reported here was supported by funds granted to the Institute for Research on Poverty at the University of Wisconsin-Madison by the Department of Health, Education, and Welfare pursuant to the provisions of the Economic Opportunity Act of 1964. The opinions expressed are those of the author.

ABSTRACT

This paper has two purposes: first, to review and extend the basic theory on housing prices and location; and second, to present some precise empirical tests of that theory--that is, tests that make use of theoretically determined functional forms. This paper is organized around six models of the relationship between housing prices and location. These models are defined in section II. The theory of location and the price of housing is incorporated into these models in section III, and procedures for estimating several of the models using ordinary least squares (OLS) are described in section IV. The results of an application of these procedures to data for single-family, owner-occupied housing in St. Louis in 1967 are presented in section V. A summary and some conclusions about testing the relationship between location and the price of housing are given in section VI.

LOCATION AND THE PRICE OF HOUSING

I. Introduction

The proposition that the price of housing varies with location is central to the economics of urban residential structure. Theoretical statements about land rents, population density gradients, the spatial distribution of income classes, and other aspects of urban residential structure are based on the relationship between housing prices and location. In addition, this relationship has several important practical applications. For example, for policy purposes it is important to determine how much of the observed black-white price differential in housing is due to discrimination against blacks and how much to the fact that blacks are concentrated in the center of the city where the equilibrium price of housing is higher than in other locations.

A large theoretical literature exists on the relationship between housing prices and location. But despite the theoretical and practical importance of the topic, the conclusions in that literature have not been subjected to detailed empirical testing. The empirical studies that do exist are of two types. The first type simply includes distance from the central business district (CBD) as an independent variable in a regression of house values (or rentals) on housing characteristics.[1] The second type provides an indirect test of the relationship between housing prices and location by estimating equations, such as land rent and population density gradients, that are related theoretically to that relationship.[2]

This paper has two purposes: first, to review and extend the basic theory on housing prices and location; and second, to present some precise empirical tests of that theory--that is, tests that make use of theoretically determined functional forms. This paper is organized around six models of the relationship between housing prices and location. These models are defined in section II. The theory of location and the price of housing is incorporated into these models in section III, and procedures for estimating several of the models using ordinary least squares (OLS) are described in section IV. The results of an application of these procedures to data for single-family, owner-occupied housing in St. Louis in 1967 are presented in section V. A summary and some conclusions about testing the relationship between location and the price of housing are given in section VI.

II. Six Models of Location and the Price of Housing

Location is one aspect of the relationship between the price of housing and housing characteristics. This relationship has two different interpretations. First, the market value (a price or rental) of a dwelling unit can be thought of as the product of a price per unit of housing services and a quantity of housing services. This interpretation is derived from some of the recent literature on housing,[3] and is implicit in the mathematics of urban models. Briefly stated, this view is that there exists a commodity called housing services and that every dwelling unit yields some quantity of this commodity per unit of time. In addition, this commodity is assumed to measure everything about housing that is valued by consumers and to sell for a constant unit price. Second, the relationship between housing characteristics and the market value of housing can be viewed as an implicit (or hedonic) price relationship. According to this interpretation, which is derived from the literature on hedonic prices,[4] the interaction of supply and demand in the market determines a market value for a commodity with any given set of characteristics and thereby implicitly reveals the contribution of each characteristic to market value.

Both of these interpretations of the market value of housing (V) can be stated algebraically as

$$V = (P)H(X_1, \ldots, X_n) \tag{1}$$

Following the first interpretation, P is a unit price and H is the number of units of housing services (a function of the housing characteristics X_1 to X_n). The second interpretation assumes that P is a hedonic price index and H is an implicit price function relating housing characteristics to the market value of housing.

In general, there is no way to determine the precise form of the H-function in the relationship between housing characteristics and the market value of housing. According to both of the interpretations given above, this form is determined by the interaction of supply and demand in the housing market; however, the theory of housing is not well enough developed to derive this market relationship from assumptions about production and utility. As a result, we will simply choose a functional form that has several desirable properties. In particular, we will use a multiplicative form, since it implies that the implicit price of a housing characteristic depends on the quantity of other characteristics and that the marginal valuation of each housing characteristic is a declining function of its quantity.

Each of the interpretations of the market value relationship leads to a linear form in a different special case. When the market value of housing is viewed as the product of a unit price times a quantity of housing services, and when the marginal valuation of each characteristic is constant, then the H-function is linear. This case is less plausible than the multiplicative case (is the second kitchen really worth as much as the first?), but it is considered here because marginal valuations may be constant in the range of characteristics observed in most houses (how many houses have a second kitchen?). Furthermore, the implicit price function determined in the market may be linear. In this case, the market value of a dwelling unit is simply the sum of all housing characteristics in that unit times their implicit prices. Consumers can force the implicit price function to be linear if there are no barriers to the combining of different packages of housing characteristics into dwelling units (Rosen, 1974, p. 38). To take a simple example, if consumers can purchase rooms separately and combine them to form dwelling units, then the market value of a dwelling unit will be a linear function of the number of rooms it contains. As before, this special case is less plausible than the multiplicative form.

There are also two ways to look at the relationship between location and the market value of housing. Location can be considered a neighborhood characterisic of housing, like the quality of local public schools or the level of air pollution, or it can be considered one of the variables that determine the price of housing, that is, the unit price of housing services or the implicit prices of housing characteristics. The former view is based on the belief that there may be some intrinsic value in particular locations that is not measured by other neighborhood characteristics of housing. The latter view, which we will discuss in some detail in the next section, is based on the belief that people must be compensated for higher commuting costs in certain locations by lower prices for housing in those locations.

The preceding discussion leads to six models of the relationship between house value and location, as illustrated in Table 1. These six models can be expressed algebraically as follows, where u denotes location and ln denotes a natural logarithm.

Model I: $$V = (P)H(u,X_1,\ldots,X_n) = P\,u^{a_o}X_1^{a_1}\ldots X_n^{a_n} \qquad (2)$$

or

$$\ln(V) = \ln(P) + a_o\ln(u) + \sum_i a_i\ln(X_i) \quad . \qquad (3)$$

Model II: $$V = P(a_o u + \sum_i a_i X_i) \qquad (4)$$

or

$$V = P_o u + \sum_i P_i X_i \quad . \qquad (5)$$

Model III: $$V = P(u)(X_1^{a_1}\ldots X_n^{a_n}) \qquad (6)$$

or

$$\ln(V) = \ln(P(u)) + \sum_i a_i\ln(X_i) \quad . \qquad (7)$$

Model IV: $$V = P(u)\Sigma a_i X_i = \Sigma a_i P(u) X_i \quad . \qquad (8)$$

Table 1

Six Models of Location and the Price of Housing

Type of Relationship Between the Market Value of Housing and Housing Characteristics	Role of Location: Housing Characteristic	Role of Location: Unit Price Variable	Role of Location: Implicit Price Variable
Multiplicative	Model I	Model III	Model V
Additive	Model II	Model IV	Model VI

Model V: $V = X_1^{P_1(u)} \ldots X_n^{P_n(u)}$ (9)

or

$$\ln(V) = \sum_i P_i(u)\ln(X_i) \quad . \tag{10}$$

Model VI: $V = \sum_i P_i(u)X_i$. (11)

Given a set of data containing observations for V, u, and X_is, Models I and II can be estimated exactly as they are written in equations (3) and (5). Models III and IV, on the other hand, cannot be estimated until a form is provided for the P(u) function, and Models V and VI require a form for the $P_i(u)$ function. The derivation of these price functions is the subject of the next section.

III. Locational Equilibrium

The basic theoretical notion that links location to the price of housing is that of locational equilibrium, a state that exists on the demand side when no household has an incentive to change its location. The variable that establishes locational equilibrium for households is the price of housing; thus the problem is to find the price of housing, expressed as a function of location, that gives no household an incentive to move.

Deriving a Price-Distance Function

The standard derivation of a price-distance function begins with the assumption that a household maximizes its utility over a composite consumption good and housing subject to a budget constraint that includes commuting costs to the CBD. This household maximization problem, which is the demand side of a simple long-run urban model,[5] leads to a form for the $P(u)$ function in Models III and IV. A household attempts to

$$\text{Maximize} \quad U(Z,H) \tag{12}$$
$$\text{Subject to} \quad Y = P_z Z + P(u)H + T(Y,u)$$

where

Z = the composite consumption good (with price P_z);

H = units of housing services;

Y = income;

u = miles from the CBD;

$P(u)$ = the price of a unit of housing services at location u; and

$T(Y,u)$ = round-trip commuting costs from location u to the CBD.

The first-order condition for this problem with respect to u is the locational equilibrium condition for a single household; that is, it determines the location from which the household will not have an incentive to move. This condition is

$$P'(u)H + \partial T/\partial u = 0 \ . \tag{13}$$

At the market level, the problem is to choose a $P(u)$ function that will lead to locational equilibrium for all households. The desired $P(u)$ function is the one that guarantees that equation (13) holds at every u, so that a household will not have an incentive to move no matter where it locates. Thus the market problem is to solve the differential equation (13) for $P(u)$.

An important case of this household maximization problem occurs when households have a Cobb-Douglas utility function and face constant per-mile commuting costs. This case can be written

Maximize $U = b_1 \ln(Z) + b_2 \ln(H)$ (14)

Subject to $Y = P_z Z + P(u)H + tu$.

The first-order conditions for this problem lead to the following demand function for H

$$H = (b_2/(b_1+b_2))(Y-tu)/P(u) = k(Y-tu)/P(u) \ . \tag{15}$$

Substituting this demand function into the household locational equilibrium condition and integrating the result, we find the price-distance function

$$P(u) = K(Y-tu)^{1/k} \tag{16}$$

where K is a constant of integration. Now by assuming that

$$P(\bar{u}) = \bar{P} \tag{17}$$

where $\bar{u}$ is the outer edge of the city and $\bar{P}$ is the opportunity cost of a unit of housing services,[6] we find that

$$P(u) = \bar{P}\,[(Y-tu)/(Y-t\bar{u})]^{1/k} \ . \tag{18}$$

Equation (18) can be substituted into the third model of housing prices and location. The result is

$$V = \bar{P}[(Y-tu)/(Y-t\bar{u})]^{1/k} X_1^{a_1} \cdots X_n^{a_n} \ . \tag{19}$$

Given an exogenous estimate of t ($=t^*$), equation (19) can be estimated using ordinary least squares and the equation

$$\ln(V) = C + \alpha \ln(Y-t^*u) + \beta \ln(Y-t^*\bar{u}) + \sum_i a_i \ln(X_i) \tag{20}$$

where

$$C = \ln(\bar{P})$$

$$\alpha = -\beta = 1/k \ .$$

Alternatively, some nonlinear procedure can be used to estimate t as well as the other parameters of equation (20).

It is possible to derive market equilibrium conditions for other assumptions about utility functions or demand functions. However, other assumptions do not result in price-distance functions that can be estimated with linear regression techniques.[7] Furthermore, the substitution of equation (18)--or any similar price-distance function--into Model IV (equation (8)) results in a complicated nonlinear equation, which we will make no attempt to estimate.

Price-Distance Functions with More than One Income Class

The preceding derivation can easily be extended to the case of more than one income class. Every income class j has a price-distance function given by

$$P^j(u) = P_j^* [(Y_j - t_j u)/(Y_j - t_j u_j^*)]^{1/k_j} \tag{21}$$

where P_j^* is the price at the outer edge of the area inhabited by income class j ($=u_j^*$). Since housing at a given location will be sold to the highest bidder, this form implies that each income class will inhabit a ring around the CBD. Equation (21) also implies that higher income classes will live farther from the CBD.[8]

To estimate Model III with m income classes, we divide a city into m rings and define a dummy variable, D_j, for each ring, j. The estimating equation is

$$\begin{aligned} \ln(V) = C + \sum_{j=2}^{m} \gamma_j D_j + \alpha_1 \ln(Y_j - t_j^* u) + \beta_1 \ln(Y_j - t_j^* u_j^*) \\ + \sum_{j=2}^{m} \alpha_j [D_j \ln(Y_j - t_j^* u)] + \sum_{j=2}^{m} \beta_j [D_j \ln(Y_j - t_j^* u_j^*)] + \sum_i a_i \ln(X_i) \end{aligned} \tag{22}$$

where

$$C = P_1^*$$

$$\alpha_j = P_j^* - P_1^*$$

$$\alpha_1 = -\beta_1 = 1/k_1$$

$$\alpha_j = -\beta_j = 1/k_j - 1/k_1 \quad .$$

Equation (22) can also be used to estimate the price-distance function in a monocentric city in which there are rings of employment around the CBD. In such a city, u is reinterpreted to be distance to place of employment, and the subscript j refers to members of a given income class who work in a given employment ring.

Amenities

These simple models do not recognize that certain neighborhood characteristics of housing, such as the quality of the public schools and the level of air pollution, vary with location. Such characteristics, often referred to in the literature as amenities, change the mathematical statement of the household's maximization problem and complicate the derivation of a price-distance function.

Let us return to the case of a Cobb-Douglas utility function and constant per-mile commuting costs. In our presentation it will prove useful to distinguish between total housing services received by a household (=H) and the housing services not associated with amenities (=X). Thus if amenities are labeled A(u), total housing services are some function of amenities and nonamenities or

$$H = H[X, A(u)] \quad . \tag{23}$$

The household's maximization problem is now (14) with equation (23) substituted into the utility function and X, instead of H, in the budget constraint.

It is important to note that X replaces H in the budget constraint of this problem. It is well known that in the long run the marginal valuation of a housing characteristic is equal to its cost of production. This conclusion applies both to the physical characteristics of housing and to the amenities associated with a given house. Furthermore, if neighborhoods with a certain amenity can be reproduced in the long run, then for houses built in such neighborhoods, no marginal cost will be associated with that amenity.[9] Note that this argument only applies to long-run models such as the ones in this paper; amenities do affect the budget constraint in the short run.

In order to find the effect of amenities on the price-distance function, let us assume that the H-function is multiplicative,[10] so that

$$H = Xf[A(u)] \quad . \tag{24}$$

In this case the household's utility function is

$$U = b_1 \ln(Z) + b_2 \ln(H) = b_1 \ln(Z) + b_2 \ln(X) + b_2 \ln[f(A(u)] . \tag{25}$$

A procedure that is mathematically equivalent to the above is to simply include A(u) as a third argument in a Cobb-Douglas utility function. This procedure is used by Polinsky and Rubinfeld (1975). However, if A(u) is a measured level of some amenity, such as air pollution, then the use of an f-function allows more flexibility in specifying how the measured level of the amenity affects the quantity of housing services.

When equation (25) is substituted into problem (14), the household locational equilibrium is

$$\{b_2/f[A(u)]\}[\partial f/\partial A(u)]A'(u) - \lambda[P'(u)X + t] = 0 \qquad (26)$$

where λ is a Lagrangian multiplier. The other first-order conditions of the problem lead to equations for X and λ. Substituting these equations into (26) and integrating yields[11]

$$P(u) = K(Y-tu)^{1/k}f[A(u)] \ . \qquad (27)$$

Using the initial condition (17), equation (27) becomes

$$P(u) = \bar{P}[(Y-tu)/(Y-t\bar{u})]^{1/k}\{f[A(u)]/f[A(\bar{u})]\} \ . \qquad (28)$$

The theory leading to equation (28) does not change the procedure used to estimate Model III; the market value of housing is still a function of the variables in equation (21) and of housing characteristics, including amenities. But this theory may provide some assistance in choosing a functional form for the relationship between amenities and the market value of housing,[12] and it does change the interpretation of the amenity variables: They are now part of the price-distance function instead of part of the H-function.

Implicit-Price-Distance Functions in Model V

Model V combines a multiplicative relationship between housing characteristics and the price of housing with the assumption that the implicit prices of housing characteristics vary with location. In this model, households

$$\text{Maximize} \quad U(Z, X_1, \ldots, X_n) \tag{29}$$

$$\text{Subject to} \quad Y = P_z Z + V + tu$$

$$= P_z Z + \prod_i X_i^{P_i(u)} + tu \quad .$$

The household locational equilibrium condition associated with this problem is

$$\sum_i [\ln(X_i)\ P_i'(u)\ V] + t = 0 \quad . \tag{30}$$

Now given a Cobb-Douglas utility function of the form

$$U = c_o \ln(Z) + \sum_i c_i \ln(X_i) \quad , \tag{31}$$

the first-order conditions of problem (29) can be used to show that[13]

$$P_i(u) = c_i P^*(u) \tag{32}$$

and

$$P^*(u) = [\bar{P} + (1/c_o)][(Y-tu)/(Y-t\bar{u})]^{c_o/\Sigma c_i \ln(X_i)} - (1/c_o) \quad . \tag{33}$$

Unfortunately, the substitution of (32) and (33) into equation (10) results in a complicated nonlinear equation, which we will make no attempt to estimate. As an alternative, we will simply assume two simple forms, linear and quadratic, for the $P_i(u)$ functions in equation (10) and estimate the model using OLS. Such estimations should be regarded not as tests of implicit-price-distance functions but as preliminary attempts to determine the usefulness of Model V.

Implicit-Price-Distance Functions in Model VI

According to Model VI, the market value of housing is the sum of housing characteristics times their implicit prices, which are functions of location. Thus households

$$\begin{aligned} &\text{Maximize} \quad U(Z, X_1, \ldots, X_n) \\ &\text{Subject to} \quad Y = P_z Z + V + tu \\ &\qquad\qquad\quad = P_z Z + \sum_i P_i(u) X_i + tu \; . \end{aligned} \tag{34}$$

The first-order condition of problem (34) with respect to u is

$$[\sum_i P_i'(u) X_i] + t = 0 \; . \tag{35}$$

This equation cannot be solved for the individual $P_i(u)$ functions; therefore, it is not possible to derive implicit-price-distance functions from this household maximization problem.

Since theory does not provide a form for the $P_i(u)$ functions, it seems reasonable to perform, as we did for Model V, OLS estimation of this model using linear and quadratic $P_i(u)$ functions.

Once a form is assumed for the $P_i(u)$ functions, the household maximization problem (34) with the Cobb-Douglas utility function (31) can be used to incorporate the notion of locational equilibrium into Model IV. The first-order conditions for this problem lead to the following demand function for X_i:

$$X_i = [Y - tu - \sum_{j \neq i} P_j(u) X_j] / P_i(u) c_i^* \tag{36}$$

where

$$c_i^* = (c_o / c_i + 1) \; .$$

Plugging this demand function into the locational equilibrium condition, and noting that

$$\sum_{j \neq i} P_j(u)X_j = V - P_i(u)X_i \ , \tag{37}$$

we can derive

$$\sum_i P_i'(u)[(Y - tu - V + P_i(u)X_i)/P_i(u)c_i^*] + t = 0 \tag{38}$$

or

$$V = Y - tu + t/[\sum_i P_i'(u)/P_i(u)c_i^*] + \sum_i [P'(u)X_i/c_i^*]/\sum_i [P_i'(u)/P_i(u)c_i^*]. \tag{39}$$

Given a form for the $P_i(u)$ functions, one can estimate this market value equation using nonlinear methods. For example, if the price functions are exponential, or

$$P_i(u) = a_i e^{-b_i u} \ , \tag{40}$$

and commuting costs are the sum of operating costs (t_o) and time costs ($t_y Y$), or

$$t = t_o + t_y Y \ , \tag{41}$$

then equation (39) becomes

$$V = t_o/k_o + (1 - t_y/k_o)Y - t_o u - t_y(Yu) + \Sigma k_i e^{-b_i u} X_i \tag{42}$$

where

$$k_o = \sum_i [b_i c_i/(c_o + c_i)]$$

$$k_i = a_i b_i c_i / k_o(c_o + c_i) \ .$$

The significance of the coefficients of Y, u, and (Yu) in an estimate of equation (42) can be interpreted as a test of the locational equilibrium concept.

IV. Estimating Procedure

In sections II and III we defined six models of location and the price of housing and derived equations for estimating five of those models. In this section we will describe the data and the procedure used to estimate these equations.

The Data

The data cover 266 single-family, owner-occupied houses in St. Louis in 1967. For a detailed description of these data, see Kain and Quigley (1970). For each house, information is available on owner-estimated market value, an extensive list of structural and neighborhood characteristics, and several characteristics of the household. In addition, the census tract of each house is identified, so the original data can be supplemented with data from the 1970 Census. Table 2 lists the variables considered in this study.

Estimating the H-Function

The H-function in equation (1) was defined to be either a multiplicative or an additive function of housing characteristics. Two alterations in these simple functional forms were used in the estimations.

First, the housing characteristics that enter into the market value equations may be transformations of measured housing characteristics. For example, the logarithm of the number of rooms, instead of the actual number of rooms, may enter into the market value equations. Therefore, X_i in the estimating equations should be

Table 2

List of Variables

Type	Name	Description
Dependent	VALUE	Owner-estimated market value of house
Structural	ROOMS	Number of rooms
	BATHS	Number of bathrooms plus one
	FIRST	First floor area (hundreds of square feet)
	PARCEL	Parcel area (hundreds of square feet)
	MAQUAL	Material quality (assessor's data; 1=best, 4= worst)
	AGE	Age of house (in years)
	WATER	Dummy variable for hot water
	HEAT	Dummy variable for central heat
	FAC2[a]	Dwelling unit quality (Kain and Quigley's second factor)
Neighborhood (=Amenities)	FAC1	Basic residential quality (Kain and Quigley's first factor)
	FAC3	Quality of proximate properties (K & Q's third factor)
	FAC4	Nonresidential usage (K & Q's fourth factor)
	FAC5	Average structure quality (K & Q's fifth factor)
	MATH	Average eighth-grade math achievement score in local public school
	EDUC	Median years of schooling of adult population (1970 Census)
	FINCOM	Median income of families (1970 Census)
	PSAME	Percent of families in the same house in 1965 (1970 Census)
	POLD	Percent of population over 65 years old (1970 Census)
	POVFAM	Percent of families below the poverty line (1970 Census)
	PBLACK	Percent of population that is black (1970 Census)
Location	CBDDIS	Distance to CBD (in miles)
Household	RACE	Dummy variable for race of household (1= nonwhite)

[a]The variables FAC1-FAC5 are factors determined by factor analysis from a set of 39 structural and neighborhood characteristics, none of which are included separately in this list of variables. For details see Kain and Quigley (1970).

interpreted as

$$X_i = g_i(X_i^*) \qquad (43)$$

where X_i^* is the measured value of the ith housing characteristic.

Second, in the multiplicative specification a zero value for any housing characteristic in a given house has the unacceptable implication that there are no housing services in that house. To avoid this implication, we will assume that any housing characteristic, X_i^*, whose range includes zero affects the quantity of housing services exponentially, or, in symbols, that

$$X_i = g_i(X_i^*) = e^{X_i^*} . \qquad (44)$$

Estimating the Price-Distance Function

Five complications arise in estimating the price-distance function.

1. In the multi-income-class model in section III, each income class lives in a ring around the CBD. This model is operationalized by dividing a city into rings and considering every observation in a given ring to be in the same income class. Although this procedure results--as the theory predicts--in rings with average incomes that increase with distance from the CBD (see Table 3), there is considerable variation in income within each ring. Because of this variation, the median income of the tract in which an observation is located is used for the income term (Y) in the estimating equations. The use of census tract income does not depart significantly from the view that there is a single income class in each ring; the results reported below are virtually identical to results obtained using the average

Table 3

Description of Rings

Ring (=j)	Outer Edge of Ring ($=u_j^*$, in miles)	Average Income in Ring[a]	Percent of Workers Who Work in the CBD[b]	Number of Observations in Ring
1	2	5137	2.86	14
2	3	6980	4.51	32
3	4	7887	5.86	41
4	5	8443	6.21	60
5	6	8961	6.18	69
6	7	10534	6.04	50

[a] Average income is defined to be the population-weighted mean of the median incomes of tracts in the ring. Tracts not largely in one ring are assumed to have their populations divided evenly between the two rings.

[b] Mean of census tract percentages for the observations in the ring.

income in the ring instead of census tract income.[14]

2. Among the parameters to be estimated in Model III are the price constants, P_j^*. These constants are derived from the "anchoring" assumptions that $P(\bar{u}) = \bar{P}$ and that $P^j(u_j^*) = P_j^*$. According to the specification in equation (22), the price constant for the outer ring ($=\bar{P}$) is the sum of the constant term and the coefficient of the dummy variable for that ring. Similarly, the constant term can be interpreted as the unit price of housing at the outer edge of the inner ring--a number that has no particular significance. An alternative specification is to anchor the price-distance function at the inner edge of each ring instead of at the outer edge, so that the constant term can be interpreted as the price at the innermost part of the residential area of the city. To operationalize this alternative, we simply need to replace u_j^* with u_j'--the inner edge of ring j.

3. As indicated in section III, yearly round-trip communting costs per mile (t) must be either determined exogenously or estimated using a nonlinear technique. The estimate of t in this paper is the value that minimizes the sum of squared errors (SSE) from an OLS regression of equation (22). If the error terms are normally distributed, this is a maximum likelihood procedure (Goldfeld and Quandt, 1972, p. 58). In practice, the SSE-minimizing value of t was found by making various assumptions about its components, operating costs and time costs (see equation (41)). Thus if c_o is per-mile operating costs, MPH is average commuting speed, and w is the fraction of the wage rate at which travel time is valued, then t_j (the value of t for income class j) is equal to the sum of

$$t_o = (2)x(\text{working days per year})x(c_o)$$
$$= (2)x(250)x(c_o) = 500c_o \tag{45}$$

and

$$t_yY_j = (2)x(\text{working days per year})x(\text{minutes per mile of commuting})\ x(\text{dollar value of a minute spent commuting})$$
$$= (2)x(250)x(60/MPH)x(wY_j/\text{minutes worked per year})$$
$$= (30000/MPH)(wY_j/(60x8x250)) = (.25w/MPH)Y_j \quad . \tag{46}$$

The values of c_o, MPH, and w that result in a minimum SSE for all of the Model III regressions reported below are

$$c_o = .25$$
$$MPH = 4$$
$$w = 1 \quad .$$

These estimates are considerably higher than other estimates of commuting costs;[15] for reasons to be discussed in the next section, they should not be regarded as precise. Furthermore, note that for any given income class there are infinitely many combinations of the three parameters that lead to the same value of t_j. The combination given above is, in this author's judgment, the most plausible of the combinations that lead to the SSE-minimizing values of t_j.

4. One of the terms in the price-distance functions derived in section III is $A(u_j^*)$--the level of amenities at the edge of ring j. This term equals the term A(u) when $u = u_j^*$, so that, as shown by equation (28), $P^j(u_j^*)=P_j^*$. In practice, however, the level of any given amenity varies greatly at a ring boundary. For example, the racial composition of census tracts at a given distance from the CBD can--and often does--vary from 0 to almost 100 percent black. Thus the average value

of A for all neighborhoods along the edge of ring j--that is, $\bar{A}(u_j^*)$--will not necessarily equal A(u) when $u = u_j^*$. As a result, our estimating procedure does not guarantee that $P^j(u_j^*) = P_j^*$. Furthermore, even if meaningful measures of $A(u_j^*)$ could be obtained, they would be perfectly collinear with the ring dummy variables and therefore could not be estimated separately.

This inability to measure $A(u_j^*)$ is somewhat troublesome for our estimating procedure, because the coefficient of the jth ring dummy variable will be an estimate of $P_j^*/A(u_j^*)$ instead of an estimate of P_j^*. Since $A(u_j^*)$ varies a great deal within a given ring, we should expect some imprecision in our estimates of these coefficients.

5. Finally, it should be recognized that the "amenity" effect of racial composition is not the only aspect of the relationship between housing prices and race. Because blacks are restricted to certain areas of a city, the unit price of housing services may be higher in black and integrated areas than in white areas. In addition, price discrimination may result in higher prices for black households than for white households in neighborhoods with a given racial composition. Therefore, we will include in our regressions a dummy variable for nonwhite households and dummy variables for integrated and largely black neighborhoods.[16] The complete set of racial variables is given in Table 4. These variables are defined so that the PBL variables capture the amenity effect of racial composition in each type of neighborhood and the BL variables indicate the percentage by which unit housing prices in each type of neighborhood shift above the price in largely white neighborhoods.

Table 4

Racial Variables

Variable	Definition
BLACK	Dummy variable for race of household (1 = nonwhite)
BL4080	Dummy variable for integrated neighborhoods (=1 in census tracts with populations that are 40 to 80 percent black)
BL8099	Dummy variable for largely black neighborhoods (=1 in census tracts with populations that are 80 to 100 percent black)
PBL0040	Racial composition in largely white neighborhoods = PBLACK x (1 - BL4080 - BL8099)
PBL4080	Racial composition in integrated neighborhoods = (PBLACK - 40) x BL4080
PBL8099	Racial composition in largely black neighborhoods = (PBLACK - 80) x BL8099

Integrated and largely black neighborhoods are defined by their racial compositions. The levels of racial composition chosen as boundaries between the two types of neighborhoods are the levels that minimize the sum of squared errors in Models I and II. These levels (40 percent black for the boundary between largely white and integrated neighborhoods and 80 percent black for the boundary between integrated and largely black neighborhoods) are then used in the other models. In principle, this iterative procedure should be performed simultaneously with the iterative procedure used to determine the value of t in Model III. However, in practice there appears to be no interaction between the two iterations: The estimate of t is the same regardless of the racial compositions used as neighborhood boundaries, and the estimated boundary percentages are the same regardless of the value used for t.

Hypotheses About Price-Distance Functions

The conformity of the coefficients of a price-distance function estimated using equation (22) with the predictions of our theory can be tested using the hypotheses in Table 5. Our theory indicates that $\alpha_j = -\beta_j = 1/k$ where k is the proportion of income (net of commuting costs) that is spent on housing (about .2). Hence, our theory predicts that the first two null hypotheses in Table 5 will be rejected and the second two null hypotheses will be accepted. Tests of these hypotheses are presented in the next section.

The third null hypothesis can also be included as a restriction in the estimating equation (22) by replacing the two independent variables $\ln(Y_j - t_j u)$ and $\ln(Y_j - t_j u_j^*)$ with the single independent variable

Table 5

Hypotheses About Price-Distance Functions

Number	Null Hypothesis	Alternative Hypothesis
1	H_o: $\alpha_j \leq 0$	H_1: $\alpha_j > 0$
2	H_o: $\beta_j \geq 0$	H_2: $\beta_j < 0$
3	H_o: $\alpha_j + \beta_j = 0$	H_3: $\alpha_j + \beta_j \neq 0$
4	H_o: $\alpha_j = -\beta_j = 5$	H_4: $\alpha_j, \beta_j \neq 5$
5	H_o: $P^j(u_j^*) = P^{j+1}(u_j^*)$	H_5: $P^j(u_j^*) \neq P^{j+1}(u_j^*)$

<u>Note</u>: α_j and β_j refer to the coefficients in equation (22).

$(\ln(Y_j-t_ju)-\ln(Y_j-t_ju_j^*))$. The significance of the set of such restrictions for all rings can be determined using an F-test. (See Johnston, 1972, pp. 192 ff.) This set of restrictions has the advantage of helping to make sense of the coefficients of the ring dummy variables. If the coefficient of $\ln(Y_j-t_ju)$ is not equal to the absolute value of the coefficient of $\ln(Y_j-t_ju_j^*)$, then the coefficient of the jth ring dummy will not represent the unit price of housing at u_j^*--indeed, the coefficient will have no obvious interpretation.

The assumption of competition in our models also implies that the price-distance functions of bordering rings will meet at the boundary between the two rings. This implication is expressed as null hypothesis 5. Although it is not possible to set up a t-test for this hypothesis, we can include the hypothesis as a restriction on the price constants in the estimating equation for model III. From null hypothesis 5 and equation (21) it follows that

$$P^1(u_1^*) = P_1^*[(Y_1-t_1u_1^*)/(Y_1-t_1u_1^*)]^{1/k_1} = P_1^*$$

$$= P_2(u_1^*) = P_2^*[(Y_2-t_2u_1^*)/(Y_2-t_2u_2^*)]^{1/k_2} . \qquad (47)$$

Equation (47) can now be solved for the price constant in ring two:

$$P_2^* = P_1^*[(Y_2-t_2u_2^*)/(Y_2-t_2u_1^*)]^{1/k_2} . \qquad (48)$$

Similarly, the restriction that $P^2(u_2^*) = P^3(u_3^*)$ can be used to solve for P_3^* , and it can be shown that in general

$$P_j^* = P_1^* \prod_{i=2}^{j} \left[[(Y_i-t_iu_i^*)/(Y_i-t_iu_{i-1}^*)]^{1/k_i}\right] . \qquad (49)$$

Substituting equations (21) and (49) into equation (6) yields

$$V = P_1^* \left\{ \prod_{i=2}^{j} \left[[(Y_i - t_i u_i^*)/(Y_i - t_i u_i^*)]^{1/k_i} \right] \right\}$$

$$\cdot \; [(Y_j - t_j u)/(Y_j - t_j u_j^*)]^{1/k_j} [X_1^{a_1} \ldots X_n^{a_n}] \tag{50}$$

In order to estimate equation (50), define, for i = 2 to m,

$$\delta_i = 1 \text{ if } i \leq j, \tag{51}$$

$$= 0 \text{ otherwise,}$$

where j is the ring in which an observation is located. Now the estimating equation is[17]

$$\ln(V) = C + \alpha_1 \Big(\ln(Y_j - t_j u) - \ln(Y_j - t_j u_j^*)$$

$$+ \sum_{i=2}^{m} \delta_i \left[\ln(Y_i - t_i u_i^*) - \ln(Y_i - t_i u_{i-1}^*) \right] \Big)$$

$$+ \sum_{i=2}^{m} \alpha_i \Big(D_i \left[\ln(Y_i - t_i u) - \ln(Y_i - t_i u_i^*) \right]$$

$$+ \delta_i \left[\ln(Y_i - t_i u_i^*) - \ln(Y_i - t_i u_{i-1}^*) \right] \Big) + \sum_{i=1}^{n} a_i \ln(X_i) \tag{52}$$

where

$$C = \ln(P_1^*)$$

$$\alpha_1 = 1/k_1$$

$$\alpha_i = 1/k_i - 1/k_1 \; .$$

The significance of this set of restrictions can be determined using an F-test.

As discussed earlier, it is possible to anchor the price-distance functions at the inner edge of each ring instead of at the outer edge. In this case the fifth null hypothesis is written

$$P^{j}(u'_{j+1}) = P^{j+1}(u'_{j+1}) \tag{53}$$

where u'_j is the inner edge of ring j. Equation (53) can be used to derive an inner-anchor estimating equation analogous to equation (52).

V. Empirical Results

In this section we will present estimates of our models obtained using data from St. Louis.

Choosing the Housing Characteristics

The first problem in estimating any of our models is to choose a set of housing characteristics. The following procedure was used: Each of the structural and neighborhood characteristics in Table 2 was included in Models I and II unless both (a) its coefficient had a t-value less than one either as measured or in logarithmic form, and (b) there was no strong theoretical reason for including it. All of the variables chosen by this method for Model I were then included in Models III and V, and the variables chosen for Model II were included in Models IV and VI. This procedure resulted in the elimination of HOTWAT, CENHEAT, FAC3, and FAC5 from all of the models and of POVFAM from the multiplicative models (I, III, and V).

The insignificance of the excluded variables might be due to their collinearity with one or more of the other variables. To test for this possibility, the simple correlation coefficients between each of the excluded variables and each of the included variables were examined; none was found to be particularly high.[18] There did appear to be collinearity between EDUC and FINCOM, however. The simple correlation between the two variables is fairly high (.58), and neither of the variables is significant at the 10 percent level in either Model I or Model II when they are both included. Since the two variables perform about equally when included separately (FINCOM is somewhat more significant in Model I, and EDUC is slightly more significant in Model II), and since the exclusion of either one had little impact on the performance of the other variables in the regressions, FINCOM was dropped from the final set of housing characteristics.

Models I and II

The estimates of Models I and II are presented in the first two columns of Table 6. Almost all of the housing characteristics are highly significant with the predicted sign. The set of housing characteristics explains about 80 percent of the variation in the dependent variable in both models, and the two models have similar implications about the effect of individual housing characteristics on house values. Since the coefficients of any housing characteristic, X_i, in the Model I regression can be interpreted as the percentage increase in house value associated with a unit increase in X_i,[19] the similarity between the two models can be seen by translating the Model II coefficients into percentages--that is, by dividing them by the mean house value ($14,596).

Table 6

Estimates of the H-Function

Variable	Model I		Model II		Model III: Inner Anchor		Model III: Outer Anchor	
	Coefficient	t-statistic	Coefficient	t-statistic	Coefficient	t-statistic	Coefficient	t-statistic
CONSTANT*	6.6565	13.141	288.184	.057	7.1684	12.604	6.7651	12.893
CBDDIS*	.00479	.324	202.024	.856	–	–	–	–
FAC1	.0850	3.333	1112.668	2.606	.0619	2.183	.0605	2.123
FAC2	.0483	2.682	726.586	2.532	.0447	2.306	.0443	2.273
FAC4	.0476	2.409	502.458	1.591	.0443	2.198	.0425	2.102
LROOMS	.1591	3.209	1479.381	1.994	.1399	2.783	.1390	2.764
LBATHS	.0998	1.460	1343.122	1.231	.0862	1.236	.0869	1.244
LFIRST[a]	.2879	4.887	4.473	4.648	.2726	4.540	.2676	4.449
LPARCEL[a]	.1911	4.691	80.668	10.285	.2038	4.953	.2057	4.984
AGE	-.00776	-8.215	-110.792	-7.555	-.00827	-8.415	-.00833	-8.459
LMAQUAL[a]	-.2825	-3.167	-1494.456	-3.319	-.3034	-3.298	-.3063	-3.326
PSAME	-.00324	-1.441	-60.658	-1.744	-.00375	-1.622	-.00369	-1.573
EDUC	.0106	.719	482.482	1.918	.0118	.781	.0109	.715
POLD*	.00748	2.695	134.269	3.090	.00833	2.901	.00822	2.845
POVFAM*	–	–	91.258	1.829	–	–	–	–
MATH	.0501	1.686	844.098	1.821	.0297	.9483	.0318	1.020
RACE	.1515	1.941	2608.403	2.141	.1651	2.007	.1603	1.940
BL4080	.2586	1.528	8342.902	3.051	.2351	1.372	.2437	1.423
BL8099	.2739	2.173	3537.416	1.704	.3090	2.277	.299	2.189
PBL0040	-.00617	-2.680	-86.994	-2.323	-.00642	-2.678	-.00620	-2.589
PBL4080*	-.00880	-1.501	-287.111	-3.014	-.00900	-1.523	-.00899	-1.522
PBL8099*	-.0184	-2.465	-313.390	-2.690	-.0211	-2.447	-.0200	-2.291
R^2	.7754		.8162		.7882		.7881	
Estimating Equation	(3)		(5)		(22)[b,c]		(22)[b]	

Notes to Table 6

A first letter "L" indicates that a variable is expressed as a natural logarithm.

The one-tailed 10 percent (1 percent) significance level is 1.282 (2.326). The two-tailed 10 percent (1 percent) significance level is 1.645 (2.576). Variables for which a two-tailed test is appropriate are marked with an asterisk (*).

[a]Expressed in logarithmic form only in Model I.

[b]Estimated with the restriction that $\alpha_j = \beta_j$.

[c]The estimating equation is (22) with u_j^* redefined to be the inner edge of ring j.

For most characteristics, the percentages obtained for Model II by this method are close to the coefficients for Model I.

Several aspects of these estimates are worth emphasizing. First, the coefficient of CBDDIS is not significant, indicating that if location does have an effect on housing prices, the specification used for Models I and II does not capture that effect. Second, the coefficients of the racial variables are very significant. The racial variables for which signs are predicted (RACE, BL4080, BL8099, and PBL0040) all have the predicted signs and are significant at the 10 percent level or above. The coefficients of these variables indicate that both white aversion to blacks and discrimination against blacks are reflected in house values. For example, the coefficient of PBL0040 in Model I indicates that house values will be 6 percent lower in neighborhoods with populations that are 10 percent black than in neighborhoods that are all white. The coefficient of BL4080, on the other hand, indicates that house values are 25.9 percent higher in integrated neighborhoods than in white neighborhoods. Finally, the coefficient of RACE indicates that, in any given neighborhood, house values are 15 percent higher for blacks than for whites.

Model III

Model III estimates the parameters of a price-distance function as well as the contributions of housing characteristics to housing services. The coefficients of the price-distance function variables are presented in the first column of Tables 7 and 8. Regardless of whether an inner anchor (Table 7) or an outer anchor (Table 8) is used, the ring dummy

Table 7

Estimates of the Price-Distance Function Using An Inner Anchor

Variable	(1) Coefficient	(1) t-statistic	(2) Coefficient	(2) t-statistic	(3) Coefficient	(3) t-statistic	(4) Coefficient	(4) t-statistic
CONSTANT	5.9077	1.594	7.1684	12.604	6.6688	12.129	7.4099	12.200
RING 1[a]	-	-	-	-	-	-	-3.459	-2.72
RING 2	.4330	.113	-.1366	-.747	-	-	-	-
RING 3	-2.0911	-.523	-.2316	-1.242	-	-	-	-
RING 4	3.7894	.953	-.1803	-.999	-	-	-	-
RING 5	2.1133	.515	-.1917	-1.061	-	-	-	-
RING 6	1.3399	.262	-.0989	-.526	-	-	-	-
NETINC[b]	4.9216	1.902	4.7127	1.836	-2.148	-.116	4.4914	1.746
NETINC2	-5.4353	-1.854	-5.6052	-1.962	-.2759	-.113	-3.7763	-1.381
NETINC3	-4.1619	-1.444	-5.0829	-1.785	.8261	.425	-4.1824	-1.571
NETINC4	-5.4484	-1.965	-5.1484	-1.884	-.0482	-.024	-4.7132	-1.778
NETINC5	-5.5511	-2.029	-5.3049	-1.966	-.0691	-.035	-4.8575	-1.848
NETINC6	-4.2901	-1.532	-4.1490	-1.521	.5006	.243	-4.1896	-1.569
NIEDGE[c]	-4.7665	-1.837	-	-	-	-	-	-
NIEDGE2	5.3674	1.845	-	-	-	-	-	-
NIEDGE3	4.3771	1.517	-	-	-	-	-	-
NIEDGE4	4.9858	1.777	-	-	-	-	-	-
NIEDGE5	5.2785	1.903	-	-	-	-	-	-
NIEDGE6	4.1169	1.407	-	-	-	-	-	-
R^2	.7949		.7882		.7768		.7834	
Estimating Equation	(22)		(22)[d]		(52)[e]		(52)[e]	

Notes to Table 7

A two-tailed test is appropriate for all variables. The 10 percent (1 percent) significance level is 1.645 (2.576).

[a]RINGj is the dummy variable for ring j (= D_j in equation (19)).

[b]The definitions for NETINC in the various regressions are as follows.

(1): NETINC = $\ln(Y_j - t_j u)$

(2): NETINC = $\ln(Y_j - t_j u) - \ln(Y_j - t_j u_j^*)$.

[Note that in regressions (1) and (2)

NETINCj = (NETINC)x(RINGj)].

(3) and (4):

$$\text{NETINC} = \ln(Y_j - t_j u) - \ln(Y_j - t_j u_j^*)$$

$$+ \sum_{i=2}^{m} \delta_i [\ln(Y_i - t_i u_i^*) - \ln(Y_i - t_j u_{i-1}^*)]$$

and

$$\text{NETINCj} = D_j [\ln(Y_j - t_j u) - \ln(Y_j - t_j u_j^*)]$$

$$+ \delta_j [\ln(Y_j - t_j u_j^*) - \ln(Y_j - t_j u_{j-1}^*)] \quad .$$

[c]The definition of NIEDGE is

NIEDGE = $\ln(Y_j - t_j u_j^*)$

and

NIEDGEj = (NIEDGE)x(RINGj) .

[d]The estimating equation is (22) with the restriction that $\alpha_j = \beta_j$.

[e]The estimating equation is the one for an inner anchor analogous to equation (52), which is based on an outer anchor.

Table 8

Estimates of the Price-Distance Function Using an Outer Anchor

Variable	(1) Coefficient	(1) t-statistic	(2) Coefficient	(2) t-statistic	(3) Coefficient	(3) t-statistic	(4) Coefficient	(4) t-statistic
CONSTANT	4.5591	1.245	6.7651	12.893	6.6671	12.709	6.9734	13.133
RING 1[a]	-	-	-	-	-	-	-.329	-2.64
RING 2	1.9474	.502	.3710	3.002	-	-	-	-
RING 3	-.839	-.201	.2515	1.953	-	-	-	-
RING 4	5.2214	1.339	.2853	2.214	-	-	-	-
RING 5	3.659	.911	.3022	2.264	-	-	-	-
RING 6	2.443	.506	.2497	1.737	-	-	-	-
NETINC[b]	4.9988	1.926	4.4996	1.775	-.00794	-.004	4.2630	1.677
NETINC2	-5.4989	-1.874	-5.3353	-1.921	-.4966	-.203	-3.5701	-1.332
NETINC3	-4.2684	-1.477	-5.2420	-1.876	.4973	.248	-4.0751	-1.549
NETINC4	-5.5305	-1.990	-4.7891	-1.778	-.1858	-.089	-4.3989	-1.683
NETINC5	-5.6081	-2.044	-5.0154	-1.881	-.2653	-.130	-4.6177	-1.775
NETINC6	-4.3636	-1.556	-3.9208	-1.450	.2275	.108	-4.0270	-1.529
NIEDGE[c]	-4.7361	-1.862						
NIEDGE2	5.3107	1.866						
NIEDGE3	4.3828	1.551						
NIEDGE4	4.9535	1.803						
NIEDGE5	5.2121	1.918						
NIEDGE6	4.1012	1.434						
R^2	.7949		.7881		.7764		.7828	
Estimating Equation	(22)		(22)[d]		(52)		(52)[e]	

See notes to Table 7.

variables are not significant,[20] and the two net income variables, NETINC and NIEDGE, are significant. In four of the six rings, the two net income variables are significant at the 10 percent level, and the t-statistic is greater than 1.4 for the coefficients of these variables in the other two rings.

The addition of a price-distance function has little impact on the magnitudes or significance levels of the coefficients of the housing characteristics. This result is true whether an inner or an outer anchor is used and whether or not restrictions are included in the estimating procedure. The coefficients of the housing characteristics estimated using the restriction that $\alpha_j = \beta_j$ (see pages 25-26) are listed in the last columns of Table 6. These estimates are extremely close to the estimates obtained from other versions of Model III. It should be pointed out that the amenity variables in Table 6 (that is, the last 11 variables) are interpreted as price variables in Model III. One of the important implications of the finding that Model III does not change the coefficients of the housing characteristics is that the black-white price differential in St. Louis is not the result of blacks living in the city center where the equilibrium price of housing is higher than elsewhere.

As indicated on page 11, the coefficients from our Model III regression must be transformed to determine the estimated parameters of the price-distance function. The results of such transformations are presented in Table 9. For example, using an inner anchor for regression (1), the estimated price constant in ring 5 ($=P_5^*$) is the exponential of the sum of the coefficients of the constant term and RING5, or

Table 9

Estimated Parameters of the Price-Distance Function

Ring	Regression (1) P^*_j [a]	Regression (1) $1/k_j$ [b]	Regression (1) $1/k_j$ [c]	Regression (2) P^*_j [a]	Regression (2) $1/k_j$ [b]	Regression (4) P^*_j [d]	Regression (4) $1/k_j$ [b]
				Inner Anchor			
1	267.86	4.9216	4.7665	1297.77	4.7127	1169.11	4.4914
2	567.19	-.5137	-.6009	1132.07	-.8925	1085.55	.7151
3	45.45	.7597	.3894	1029.47	-.3702	1014.13	.3090
4	16270.36	-.5268	-.2193	1083.66	-.4357	982.63	-.2218
5	3044.22	-.6295	-.5120	1071.38	-.5922	1007.31	-.3661
6	1404.73	.6315	.6496	1175.56	.5637	1050.18	.3018
				Outer Anchor			
1	95.50	4.9988	4.7361	867.05	4.4996	768.47	4.2630
2	669.48	-.5001	-.5746	1256.52	-.8357	999.71	.6929
3	41.27	.7304	.3533	1114.99	-.7424	980.71	.1879
4	17685.49	-.5317	-.2174	1153.32	-.2895	995.73	-.1359
5	3707.45	-.6093	-.4760	1172.98	-.5158	1040.50	-.3547
6	1098.94	.6352	.6349	1112.98	.5788	1007.74	.2360

Notes to Table 9

In the following notes, "exp" stands for an exponential function and "coef" stands for "coefficient".

[a] $P_1^* =$ exp (coef of the constant);

$P_j^* =$ exp (coef of the constant plus coef of RINGj).

[b] $1/k_1 =$ coef of NETINC;

$1/k_j =$ coef of NETINC plus coef of NETINCj.

[c] $1/k_j =$ coef of NIEDGE;

$1/k_j =$ coef of NIEDGE plus coef of NIEDGE_j.

[d]For the outer anchor, P_j^* is calculated using equation (49)--with the inclusion of the coefficient of RING1 for P_1^*. For the inner anchor, P_j^* is calculated using an analogous formula.

Table 10

Tests of Null Hypotheses About Price-Distance Functions (Test Statistics in Parentheses)

	Hypothesis 1	2	3	4[a]	5	5[b]
Theoretical prediction	REJECT	REJECT	ACCEPT	ACCEPT	ACCEPT	ACCEPT
Results for inner anchor in ring						
1	REJECT (t=1.902)	REJECT (t=-1.837)	ACCEPT (t=.362)	ACCEPT (t=.030)	-	-
2	ACCEPT (t=-.370)	ACCEPT (t=.457)	ACCEPT (t=.435)	REJECT (t=-3.967)	-	-
3	ACCEPT (t=.583)	ACCEPT (t=-.315)	REJECT (t=1.703)	REJECT (t=-3.253)	-	-
4	ACCEPT (t=-.605)	ACCEPT (t=.237)	ACCEPT (t=-1.118)	REJECT (t=-6.351)	-	-
5	ACCEPT (t=-.799)	ACCEPT (t=.605)	ACCEPT (t=-.454)	REJECT (t=-7.149)	-	-
6	ACCEPT (t=.672)	ACCEPT (t=-.549)	ACCEPT (t=-.040)	REJECT (t=-4.648)	-	-
All rings			ACCEPT (F=1.236)		REJECT (F=1.839)	ACCEPT (F=1.277)
Results for outer anchor in ring						
1	REJECT (t=1.926)	REJECT (t=-1.862)	ACCEPT (t=.616)	ACCEPT (t=.0004)	-	-
2	ACCEPT (t=-.362)	ACCEPT (t=.451)	ACCEPT (t=.352)	REJECT (t=-3.988)	-	-
3	ACCEPT (t=.559)	ACCEPT (t=-.292)	REJECT (t=1.671)	REJECT (t=-3.268)	-	-
4	ACCEPT (t=-.611)	ACCEPT (t=.241)	ACCEPT (t=1.172)	REJECT (t=-6.355)	-	-
5	ACCEPT (t=-.772)	ACCEPT (t=.578)	ACCEPT (t=-.530)	REJECT (t=-7.110)	-	-
6	ACCEPT (t=.677)	ACCEPT (t=-.556)	ACCEPT (t=.0009)	REJECT (t=-4.654)	-	-
All rings			ACCEPT (F=1.269)		REJECT (F=1.876)	REJECT (F=1.356)

Notes to Table 10

The t-tests in this table are of the form

$$t(D) = (c'b - r)/\left(s = \sqrt{c'(X'X)^{-1}c}\right)$$

where $c'\beta = r$ is the hypothesis being tested, b is the vector of estimated coefficients, s is the standard error of the regression, and $(X'X)^{-1}$ is the variance-covariance matrix. (See Johnston, 1972, p. 155). All these tests have 229 degrees of freedom (=D), and are evaluated at the two-tailed 10 percent level (=1.645).

The F-tests are of the form

$$F(R,D) = (\Delta SSE/R)/(SSE/D)$$

where SSE is the sum of squared errors from the unrestricted regression, ΔSSE is the change in SSE that occurs when the restriction is added, R is the number of restrictions, and D is the degrees of freedom in the unrestricted regression (=229). (See Johnston, p. 198.) The relevant numbers of restrictions and 10 percent significance levels for the F-tests are

Hypothesis	R	10% level
3	6	1.77
5	11	1.57
5*	10	1.60

[a]The tests of hypothesis 4 presented here are based on the coefficients of NETINCj; the results are the same for tests based on the coefficients of NIEDGEj.

[b]This hypothesis is the same as hypothesis 5, without the restriction that the price-distance functions in the first two rings meet at the boundary between the two rings.

exp(5.9077 + 2.1133) = 3044.22. Similarly, the first estimate of $1/k_5$ is the sum of the coefficients of NETINC and NETINC5, or 4.9216-5.511 = -.6295. The second estimate of $1/k_5$ is determined like the first using the negative of NIEDGE instead of using NETINC.

Tests of our hypotheses about price-distance functions are summarized in Table 10. Support for our first four hypotheses comes from two findings: All of our predictions are upheld in ring 1, and null hypothesis 3 cannot be rejected at the 10 percent level in any ring except the third, where it cannot be rejected at the 5 percent level. Except for these two findings, however, our hypotheses are not supported. The signs of the coefficients of the net income terms in rings 2, 4, and 5 are the opposite of the predicted signs, and the magnitudes of these coefficients in rings 2 to 6 are much smaller than expected. These findings imply that, except in the first ring, the price-distance function is much flatter than expected and is actually upward sloping in some rings.

The most likely explanation of these results is that, as shown in the last column of Table 3, few people actually commute to the CBD. Paradoxically, the first ring, where our hypotheses are upheld, is also the ring with the smallest percentage of workers commuting to the CBD. However, many of the workers in the first ring probably work near the CBD. In any case, more accurate estimates of price-distance functions clearly require a better measure of commuting distance than CBDDIS. If we had data on actual miles commuted (as well as on miles to the CBD), then, as suggested on page 11, our estimating procedure would still be valid if we assumed that people worked in rings around the CBD. In this case, we would still divide the city up into rings around the CBD, but we would redefine the variable "u" to be "distance to place of employment." Note that the scarcity of commuting to the CBD casts some doubt on our estimates of t; if people do not work in the CBD then $(t)\cdot(u)$ is not a measure of their commuting costs.

The restriction that $\alpha_j = \beta_j$ (hypothesis 3) is included in regression (2) in Tables 7 and 8. The coefficients of the net income variables in this restricted regression are very similar in magnitude and significance to the net income variables in the unrestricted regression. Using either an inner or an outer anchor, five of the six rings have income variables that are significant at the 10 percent level. But as predicted on page 27, this restriction adds precision to the estimates of the price constants, which are determined from the ring dummy variables. This increase in precision is particularly striking using an outer anchor; in that case all of the ring dummy variables are significant at the 10 percent level. In addition, the appropriate F-test (given in the "all rings" rows in Table 10) does not allow us to reject the hypothesis that the restriction is met.

The price-distance function parameters implied by restricted regression (2) are given in Table 9. Since P_j^* is the unit price of housing at the edge of ring j, the estimated price constants can be used to plot an estimated price-distance function. To be specific, the estimated price at the inner edge of ring j is shown in the first panel of Table 9 and the estimated price at the outer edge of that ring is shown in the second panel. These estimated unit prices are plotted in Figure 1.[21] This figure clearly illustrates our main conclusion from Model III: The estimated price-distance function, as predicted, is sharply declining in ring 1, but, contrary to prediction, is essentially flat beyond two miles from the CBD. In addition, Figure 1 shows that the price-distance function shifts upward at the outer edge of the first ring.

The restriction that the price-distance functions meet at ring boundaries is included in regression (3) in Tables 7 and 8. This restriction eliminates the significance of the net income variables, but it has little effect on the coefficients of the housing characteristics. As indicated in Table 10, we can reject at the 10 percent level the hypothesis that this restriction is met, regardless of whether an inner or an outer anchor is used. Since there appears to be a large difference between the heights of the price-distance function in the first ring and elsewhere, a dummy variable for the first ring was added to this restricted regression. This dummy variable lifts the restriction that the price-distance functions must meet at the boundary between the first and second rings. As shown under regression (4) in Tables 7 and 8, the inclusion of this dummy variable greatly improves the performance of this restricted regression. Using either an inner or an outer anchor, the dummy variable is significant at the 1 percent level and the net income variable is significant in three of the six rings. Furthermore, the F-tests for the two anchoring methods do not allow us to reject (at the 10 percent level) the hypothesis that the

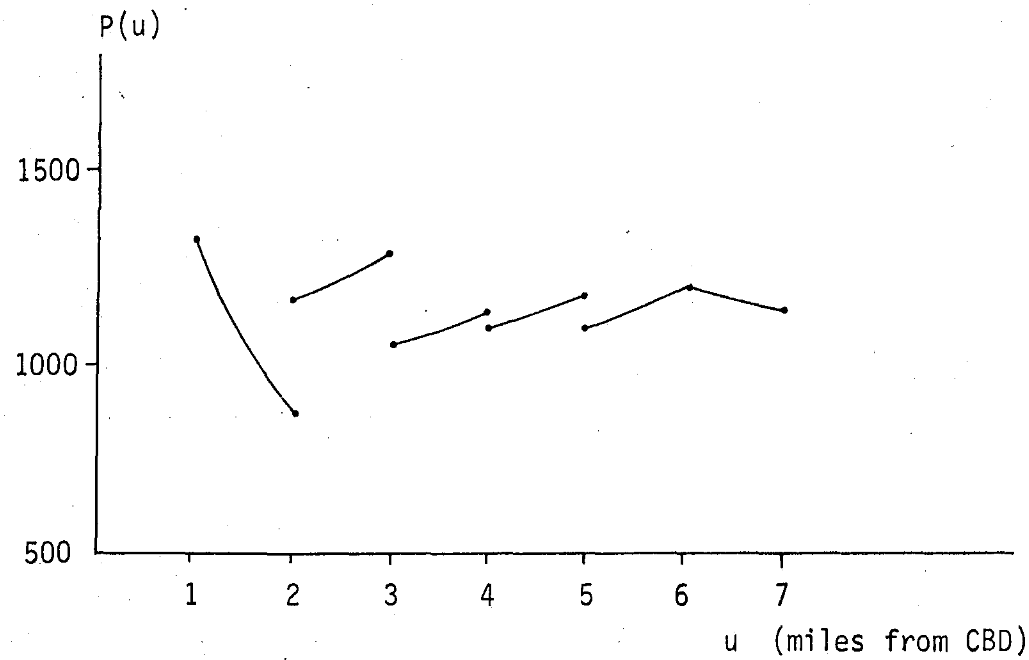

Figure 1.

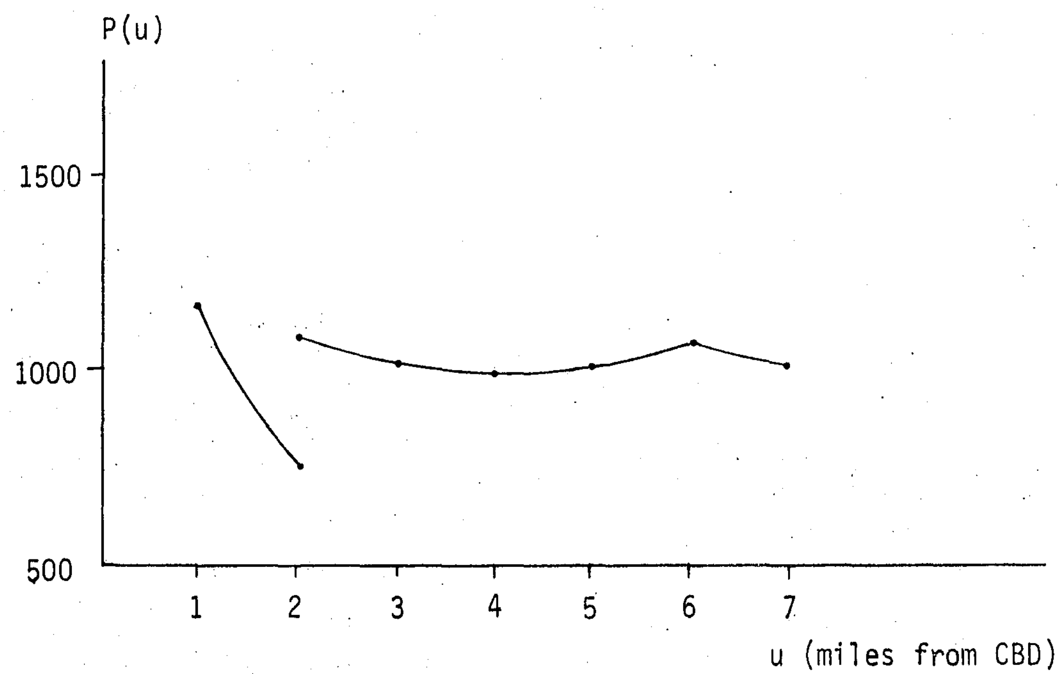

Figure 2.

Table 11

Estimates of Implicit-Price-Distance Functions

Variable	Model V					Model VI				
	Linear		Quadratic			Linear		Quadratic		
	a_i	b_i	a_i	b_i	c_i	a_i	b_i	a_i	b_i	c_i
CONSTANT	5.3213**	.2554	4.0992	-.0351	.1037	8010.41	-1009.44	-93156.92	42839.93	-4424.71
FAC1	.2201**	-.0357*	.4329*	-.1800*	.0196	1885.59	-231.60	10496.73**	-5376.06**	662.57**
FAC2	.0407	-.00066	.1181	-.0553	.00750	-242.68	180.95	3342.19	-1869.02	257.70
FAC4	.1443*	-.0246*	.3269*	-.1472*	.0166*	1401.10	-209.07	5018.79*	-2409.11*	282.39*
LROOMS	-.1299	.0627	-1.2342*	.6270*	-.0662*	-2061.67	801.56	-10032.48	4851.77	-479.16
LBATHS	.4596*	-.0879	1.4984*	-.6200*	.0627	5951.30	-1185.33	13355.51	-4996.77	450.50
LFIRST[a]	.1729	.0267	-.4766	.4391*	-.0539*	1.567	.754	-2.160	3.988	-.483
LPARCEL[a]	.2806*	-.0164	.8972*	-.3499*	.0399*	59.09	3.848	86.00	-16.89	2.924
AGE	-.00208	-.00128*	.00567	-.00553	.00051	6.592	-23.57*	205.61	-135.15*	13.47*
LMAQUAL[a]	.3655	-.1335*	.6251	-.1895	-.00038	1387.62	-634.11*	413.07	61.63	-95.28
PSAME	-.00199	-.00006	-.00292	.00264	-.00046	-31.46	-.022	-76.51	32.63	-4.998
EDUC	-.0161	-.00528	.3072	-.1380	.0150	832.08	228.97	4182.06	-2036.54	242.11
POLD	.0142	-.00084	.0338	-.00939	.00082	76.46	14.91	96.31	31.64	-4.786
POVFAM	-	-	-	-	-	-211.78*	56.09	338.06	-247.31	36.52
MATH	.1653	-.0247	.2281	-.0487	.00213	572.24	16.76	6946.68	-2757.70	283.44
RACE	.1273	-	.1161	-	-	1907.12	-	2645.73*	-	-
BL4080	.4704**	-	.6360**	-	-	10534.20**	-	2685.39**	-	-
BL8099	.8919**	-	1.1321**	-	-	18926.07**	-	23222.16**	-	-
PBL0040	-.00399	-	-.00273	-	-	-33.98	-	-22.78	-	-
PBL4099	-.0152**	-	.0198*	-	-	-335.83**	-	-418.22**	-	-
R^2	.7947		.8137			.8415		.8558		
Estimating Equation	(10)[b]		(10)[c]			(11)[b]		(11)[c]		

Notes to Table 11

The symbol "*" indicates significance at the 10 percent level, and "**" indicates significance at the 1 percent level. All tests are two-tailed.

[a] Logged only in Model V.

[b] This equation was estimated by assuming that

$$P_i(u) = a_i + b_i u \quad .$$

[c] This equation was estimated by assuming that

$$P_i(u) = a_i + b_i u + c_i u^2 \quad .$$

restriction is met. Following the procedure given above for regression (2), the price-distance function implied by regression (4) is plotted in Figure 2.

Models V and VI

The OLS estimates of our approximation to Models V and VI are presented in Table 11. These estimates are obtained by assuming that the $P_i(u)$ functions in equations (10) and (11) are either linear or quadratic. For example, Model V with quadratic $P_i(u)$ functions is estimated by adding to Model I interaction terms between CBDDIS and all of the housing characteristics as well as interaction terms between $(\text{CBDDIS})^2$ and those characteristics. Two simplifications are made in this procedure. (1) the PBL4080 and PBL8099 variables are combined, and (2) the coefficients of the racial variables are assumed not to vary with location. The first simplification reflects the finding that in Models I and II we cannot reject the hypothesis that the coefficients of PBL4080 and PBL8099 are the same. The second simplification is included because the racial variables are already defined with an implicit spatial component that is highly correlated with distance from the CBD. Inferences made about the $P_i(u)$ functions for nonracial housing characteristics are not affected by these simplifications.

Although the linear and quadratic $P_i(u)$ functions are not derived from the theory of locational equilibrium, that theory does help us to interpret the coefficients of those functions. To be specific, the theory predicts that the price of land--which is closely related to the price of housing (see note 6)--will decline with distance from the CBD, so that land will be substituted for capital at locations far from the CBD. Accordingly, the implicit prices of housing characteristics produced mainly with land (such as PARCEL!) and the implicit prices of neighborhood characteristics--which represent economic rent to land with certain characteristics--will decline with distance from the CBD. With a constant price of capital throughout the urban area, the implicit prices of characteristics produced largely with capital (such as ROOMS) will also decline, but to a lesser degree.

The coefficients of the housing characteristics (a_i in Table 11) represent implicit prices in the CBD; therefore, we expect these coefficients to have the same signs as the coefficients of the same housing characteristics in Models I and II. We also expect that each b_i in Table 11, reflecting the declining price of land, will have the opposite sign from the corresponding a_i. Finally, due to the existence of suburban employment centers, the price of land is likely to rise near the outer edge of the city, so that each c_i in the quadratic $P_i(u)$ functions will have the opposite sign from the corresponding b_i.

The pattern of signs in Table 11 indicates that Model V and the quadratic $P_i(u)$ functions conform more closely to our expectations than Model VI and the linear $P_i(u)$ functions. In the quadratic version of Model V, 10 of the 14 estimated $P_i(u)$ functions have the expected sign pattern, and in 4 of these 10 cases (FAC1, FAC4, BATHS, and PARCEL) at least two of the three coefficients are significant at the 10 percent level. In addition, 33 of the 42 coefficients of the $P_i(u)$ functions have the expected signs and 10 of these coefficients are significant at the 10 percent level. The quadratic version of model VI conforms somewhat less well to our expected sign pattern. Using linear $P_i(u)$ functions, 7 of the 14 estimated $P_i(u)$ functions in Model V and only 4 of the 15 functions in Model VI have the expected signs.

Although these results are intriguing--at least in the quadratic case--they clearly do not provide a satisfactory explanation of the variation in implicit prices with location. In the best regression (the quadratic Model V) fewer than 40 percent of the price coefficients are significant at the 10 percent level and several of these significant coefficients do not have the expected sign. In part, the pervasive insignificance of the coefficients is due to severe multicollinearity in the data. For example, the simple correlation between the CBDDIS interaction term and the $(\text{CBDDIS})^2$ interaction term is about .95 for most characteristics, and the correlation between the characteristic and the CBDDIS interaction term is, in many cases, almost as high. Unfortunately, there is no way to eliminate this problem. Another reason for the imprecision of our results is that the simple forms for the $P_i(u)$ functions do not accurately reflect the trade-off between housing costs and commuting costs. It may therefore prove useful to estimate the more precise, but nonlinear, versions of these models derived in section III.

VI. Summary and Conclusions

The empirical results in the previous section suggest that if one is interested simply in explaining the variation in the price of housing, then Models I and II, which treat location as a housing characteristic, are satisfactory; each explains about 80 percent of the variation in house values--only slightly less than our more complicated models. If, on the other hand, one is interested in explaining the relationship between location and the price of housing, then Models I and II are not sufficient; the location variable in these models is not statistically significant, and the significance of the location variables in our other models suggests that there are better alternatives available.

Our estimates of Model III, which make use of a price-distance function from a simple urban model, are particularly promising. The coefficients of the price-distance function variables are usually statistically significant, all of our hypotheses about these variables are upheld in the first ring, and two of our hypotheses (that $\alpha_j = \beta_j$ and that the price-distance functions meet at ring boundaries) are strongly supported by the data. However, probably because of the inadequacy of our measure of commuting distance, our hypotheses about the signs and magnitudes of the coefficients of the net income variables in the price-distance function are not upheld at locations more than two miles from the CBD.

Finally, the consistent sign pattern of the coefficients in the quadratic versions of Models V and VI suggests that it may be fruitful to estimate models that allow for different implicit-price-distance functions for each housing characteristic. For the following three reasons, however, this suggestion is very tentative: (1) The theory behind Model V indicates that different implicit-price-distance functions may not represent an equilibrium; (2) severe multicollinearity precludes precise estimation of the coefficients; and (3) linear and quadratic approximations do not accurately reflect the trade-off between housing costs and commuting costs.

The results in this paper indicate that the theory of locational equilibrium can be useful in determining the relationship between location and the price of housing. However, two factors have prevented us from obtaining completely satisfactory estimates of that relationship. First, many of the equations derived in this paper cannot be estimated using linear regression techniques. In principle, of course, this obstacle can be overcome by using nonlinear techniques, but only with a considerable increase in cost. Second, the use of CBDDIS as a measure of commuting distance is clearly inadequate. As we have said, it is possible to incorporate a measure of actual commuting distance into the estimating procedure in this paper; in the opinion of this author, our results are sufficiently interesting to warrant such an extension. It may turn out, however, that in some cities commuting to places of employment other than the CBD eliminates the usefulness of the monocentric assumptions on which the models used here are based. If so, a more general treatment of the theory of locational equilibrium will be required to determine the appropriate estimating equations for the relationship between location and the price of housing.

NOTES

[1]Examples of this type of study include Grether and Mieszkowski (1974), Kain and Quigley (1970), King and Mieszkowski (1973), Muth (1969), and Ridker and Henning (1967). Siegel (1975) estimates rents and driving time to the CBD simultaneously, but does not have many housing characteristics in his data. Straszheim (1973) deals with the theoretical presumption that house values vary with location by dividing his sample into several locations and estimating separate house value equations for each location.

[2]Mills (1969) estimates land rent gradients using a theoretically determined functional form that is closely related to our equation (18). For estimates of population density gradients, see Mills (1972a) and Harrison and Kain (1974) and the references cited therein.

[3]See especially Muth (1960) and Olsen (1969).

[4]See Rosen (1974) and the references cited therein.

[5]These models were developed by Alonso (1964), Mills (1967), and Muth (1969). For a clear exposition of the type of model considered here, see Mills (1972b, ch. 5).

[6]When the supply of housing is added to a model such as this one, the price of housing is functionally related to the price of land. In this case, the city will extend to the point where land rent is equal to the opportunity cost of land (= the agricultural rental rate), and the price of housing at that point (= $\bar{u}$) will then be determined by the function relating the price of housing to the price of land. See Mills (1972b, ch. 5). Note that $P(u)$ is a price per unit of housing services per year. Thus if V is the market value of a house, $V = P(u)H/r$ where r is the interest rate. In this case, $\bar{P}$ needs to be reinterpreted as a yearly price constant divided by the interest rate.

[7]See, for example, the market equilibrium conditions derived in Mills (1972, ch. 5) and Yinger (1975). Some of the conditions derived by Yinger cannot even be solved for the price-distance function.

[8]It can easily be seen that the slope of equation (18) decreases in absolute value as income increases. This finding implies that higher-income classes will have flatter price-distance functions and will therefore live farther from the CBD. For a more general treatment of this point, see Muth (1969) or Mills (1972b).

[9] For more on this point, see Hamilton (1972) and Yinger (1974).

[10] Market locational equilibrum conditions can also be derived for other forms of the H-function. For example, Yinger (1975) derives such conditions for an additive form; however, the additive form cannot be solved explicitly for a price-distance function, so it is not considered here.

[11] Equation (27) has been independently derived by Polinsky and Rubinfeld (1975, equation 3.6) using an indirect utility function. In their model, the price-distance function is anchored using a level of utility.

[12] For a discussion of the appropriate form for the f-function for the amenity "racial composition," see Yinger (1974; 1975).

[13] Equation (32) requires some comment. The first-order condition of problem (29) with respect to X_i (when the utility function is Cobb-Douglas) can be written

$$\lambda = c_i/P_i(u)V \quad .$$

Thus a solution to the problem requires that $c_i/P_i(u) = c_j/P_j(u)$ for all X_i and X_j in the utility function. Equation (32) is an expression of this result.

[14] Census tract income is used instead of ring income because ring income leads to perfect collinearity between the ring dummies and the $\ln(Y_j - t_j u_j^*)$ terms in equation (22), so that the latter terms cannot be estimated separately. However, the coefficients of the $\ln(Y_j - t_j u)$ terms and of the housing characteristics are virtually the same regardless of which definition of income is used.

[15] For example, estimates by Meyer, Kain and Wohl (1965) of the speed of commuting on public transportation range from 6 to 16 MPH. Commuting speeds by car are probably somewhat faster. In addition, Beesley (1965) estimates that commuters value their commuting time at from one-third to one-half the wage rate. Per-mile operating costs vary greatly depending on the mode of transport; one estimate of such costs is suggested by the fact that business trips by car can be deducted at a rate of 15 cents per mile on federal income tax returns.

[16] For a more detailed discussion of this racial specification, see Yinger (1974).

[17]One problem that comes up in estimating equation (52) is that the terms in the brackets that are multiplied by δ_i and δ_j only make sense if ring income (as opposed to census tract income) is used. Therefore, the income terms in the brackets are defined to be ring income and the other income terms are defined to be tract income. The results are essentially the same if all income terms are defined to be ring income.

[18]It is possible that collinearity would not show up in simple correlation coefficients (Johnston, 1972, p. 163). Since the exclusion of these variables did not have much impact on the coefficients of the other variables, no attempt was made to find more complicated collinearities.

[19]Coefficients of double log regressions are usually interpreted as elasticities, not percentages. But a coefficient that is an elasticity with respect to X is a percentage with respect to ln(X). Thus the comparison between the two regressions applies only to variables that are either logged or not logged in both models.

[20]Actually, the insignificance of the price constants is expected. See the arguments on pages 23 and 27.

[21]Price-distance functions can also be calculated by plugging the estimated parameters and the ring incomes in Table 3 into equation (18). However, this procedure leads to essentially the same conclusions as the simpler one described in the text.

REFERENCES

Alonso, William. 1964. Location and Land Use. Cambridge, Mass.: Harvard University Press.

Beesley, M.E. 1965. "The Value of Time Spent in Travelling: Some New Evidence." Economica 37: 174-185.

Goldfeld, S.M., and Quandt, R.E. 1972. Nonlinear Methods in Econometrics. Amsterdam: North-Holland Publishing Company.

Grether, D.M., and Mieszkowski, Peter. 1974. "Determinants of Real Estate Values." Journal of Urban Economics 1: 127-146.

Hamilton, Bruce. 1972. "The Effects of Property Taxes on Local Public Spending and Property Values: A Theoretical Comment." Working Paper 1207.13. Washington, D.C.: Urban Institute.

Harrison, David, and Kain, John F. 1974. "Cumulative Urban Growth and Urban Density Functions." Journal of Urban Economics 1: 61-98.

Johnston, J. 1972. Econometric Methods. 2nd Edition. New York: McGraw-Hill Book Company.

Kain, John, and Quigley, John M. 1970. "Measuring the Value of Housing Quality." Journal of the American Statistical Association 65: 532-548.

King, A. Thomas, and Mieszkowski, Peter. 1973. "Racial Discrimination, Segregation, and the Price of Housing." Journal of Political Economy 81: 590-606.

Meyer, J.R., Kain, J.F., and Wohl, M. 1965. The Urban Transportation Problem. Cambridge, Mass.: Harvard University Press.

Mills, Edwin S. 1967. "An Aggregative Model of Resource Allocation in a Metropolitan Area." American Economic Review 57: 197-210.

_______. 1969. "The Value of Urban Land." In The Quality of the Urban Environment, edited by Harvey S. Perloff. Washington, D.C.: Resources for the Future.

_______. 1972a. Studies in the Structure of the Urban Economy. Baltimore: The Johns Hopkins University Press.

_______. 1972b. Urban Economics. Glenview, Ill.: Scott, Foresman and Company.

Muth, Richard F. 1960. "The Demand for Nonfarm Housing." In The Demand for Durable Goods, edited by Arnold C. Harberger. Chicago: University of Chicago Press.

_______. 1969. Cities and Housing. Chicago: University of Chicago Press.

Olsen, Edgar O. 1969. "A Competitive Model of the Housing Market." American Economic Review 59: 612-622.

Polinsky, A. Mitchell, and Rubinfeld, Daniel L. 1975. "Property Values and the Benefits of Environmental Improvements: Theory and Measurement." Discussion Paper No. 67. Ann Arbor: University of Michigan, Institute of Public Policy Studies.

Ridker, Ronald G., and Henning, John A. 1967. "The Determinants of Residential Property Values with Special Reference to Air Pollution." Review of Economics and Statistics 49: 246-257.

Rosen, Sherwin. 1974. "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition." Journal of Political Economy 82: 34-55.

Siegel, Jay. 1975. "Intrametropolitan Migration: A Simultaneous Model of Employment and Residential Location of White and Black Households." Journal of Urban Economics 2: 29-47.

Straszheim, Mahlon. 1973. "Estimation of the Demand for Urban Housing Services from Household Interview Data." Review of Economics and Statistics 55: 1-8.

Yinger, John. 1974. "Three Essays on the Economics of Discrimination in Housing." Ph.D. Dissertation Princeton University.

_______. 1975. "Racial Prejudice and Locational Equilibrium in an Urban Area." Discussion Paper No. 251-75. Madison: University of Wisconsin, Institute for Research on Poverty.