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ON MODELS OF RACIAL PREJUDICE AND URBAN
RESIDENTIAL STRUCTURE

Paul N. Courant and John Yinger



UNIVERSITY OF WISCONSIN - MADISON

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Paul N. Courant and John Yinger*

June 1975

*Department of Economics, University of Michigan and Institute for Research on Poverty, University of Wisconsin. This paper is the result of several years work, both independent and collective, by the two authors. Much of that work took place under the wise and stimulating guidance of Edwin S. Mills. We gratefully acknowledge his many contributions to this paper. We are also grateful for helpful comments received at various times from James Ohls and Alan Deardorff.

This Discussion Paper is being issued jointly by the Institute for Research on Poverty, as no. 283-75, and the Institute of Public Policy Studies, University of Michigan, as no. 73.

ABSTRACT

Economists have studied the effects of racial prejudice on urban residential structure using a set of models that focus on conditions at the border between the black and white areas. This paper is a review of the theoretical literature on these border models and an investigation of their generality. The main result derived in the paper is that border models are logically inconsistent without unrealistic assumptions either about the incomes of blacks relative to the incomes of whites or about the extent of white prejudice. The paper concludes with several suggestions for more satisfactory modeling of prejudice and urban structure.

ON MODELS OF RACIAL PREJUDICE AND URBAN RESIDENTIAL STRUCTURE

Introduction and Summary

Understanding the effects of racial prejudice on urban residential structure is important for (at least) two reasons. First, prejudice is a powerful and pervasive attitude that affects the residential location decisions of many families. Second, it is important for policy purposes to separate the effects of prejudice per se, which does not necessarily imply discrimination, from the effects of discriminatory behavior.¹ In particular, it is necessary that we determine how much (if any) of observed residential segregation and observed racial differentials in housing prices can be explained simply by attitudes rather than by discrimination.

A major analytical tool used by economists to study these issues has been a set of models that we call "border models."² As this name implies, these models apply to completely segregated cities (with blacks assumed to locate in the city center) in which price and locational adjustments are made with reference to conditions at the border between black and white areas. The purpose of this paper is to review and extend the theoretical literature on border models. In particular, we are concerned with determining the generality of these models and with suggesting alternative approaches to studying those sets of situations for which the models are shown to be inapplicable.

To these ends, the paper is organized as follows. In section I, we consider the "granddaddy" of border models, developed by Bailey

(1959). We show that without substantially stronger assumptions than were made in its original statement, the Bailey model is internally inconsistent as an equilibrium model of residential structure.

In section II, we consider a general equilibrium border model developed independently by Courant (1973) and Rose-Ackerman (1975). The results of that work are briefly summarized and the model is explicitly solved for the case of Cobb-Douglas utility functions.

In section III, the two types of border models presented earlier are amended to allow for the possibility of differences in income between and within the racial groups. Given this amendment, it is shown that the original Bailey model cannot be made internally consistent without patently contrafactual assumptions about income distributions. Furthermore, it is proved that the assumption that blacks inhabit a central annulus, an assumption that is fundamental to all of the border models, is not generally consistent with distributions of income in which some blacks have substantially higher incomes than some whites. Finally, the case of Cobb-Douglas utility with different incomes is presented as an example, and it is shown that border models are internally consistent only under very high levels of white prejudice.

In section IV, the implications of these findings for the appropriateness of border models are discussed in some detail, and a number of suggestions are made for alternative ways of modeling the effect of racial prejudice on urban structure.

I. Bailey's Border Model

The original border model was presented by Martin Bailey (1959). It has been used and extended somewhat by Muth (1969 and 1975). The Bailey border model is based on the assumptions that the population of a city is divided into two groups, X and Y; that Group X prefers to live near Group Y; and that Group Y prefers to live away from Group X. Although he does not identify these two groups, it is clear that Bailey intends Group X to represent blacks and Group Y to represent whites.³ Our subsequent discussion will refer to blacks and whites instead of Group X and Group Y.

Bailey assumes that blacks and whites are completely segregated with blacks living on blocks A, B, C, and D, and whites living on blocks E, F, G.... Those people on adjoining or border blocks, D and E, are considered to be near to the other group; everyone else is considered to be far from the other group, that is, in their own "interior" area.

These assumptions lead directly to the conclusion that unit housing prices are higher at D than in the black interior, and lower at E than in the white interior. The equilibrium relationship between the housing prices for the two groups depends, according to Bailey, on the nature of the housing industry. If blocks D, E, and F are owned by a single firm, then, in equilibrium, prices in the black interior will be equal to prices in the white interior. If, on the other hand, the housing industry is made up of many small firms, an equilibrium will be reached when the two border prices are the same.

These conclusions can be further explained with reference to Figure 1.⁴ The Bailey model operates under perfect (but segregated) competition, so the price that can be charged by any firm for a single house is given by BB' in the black area and by WW' in the white area. If a single firm owned blocks D, E, and F, a shift of the boundary one block to the right would bring an increase in its revenue on block E equal to $(B'-W)N$, where N is the number of houses on one block. Such a move would also bring a loss on block D equal to $(B'-P)N$ and a loss on block F equal to $(P-W)N$. Thus, assuming that a one-block boundary shift does not change supply in the two areas enough to shift the BB and WW curves, the single owner would clearly not benefit from a one-block move to the right.

If there were many housing firms, however, each of which owned a single house, every firm on block E would have an incentive to sell to blacks since it would increase its revenue by an amount equal to $(B'-W)$. Thus the border would move to the right. As it moved, the supply of houses in the (growing) black area would increase and the BB' curve would shift downward. An equilibrium would be reached when the price in the black border area, (B') equaled the price in the white border area (W) . At such an equilibrium, the price in the white interior would be higher than the price in the black interior.

In short, Bailey's border model predicts that buyer tastes will lead to either higher prices in the white interior than in the black interior or higher prices for blacks at the border, depending on the nature of the housing industry.

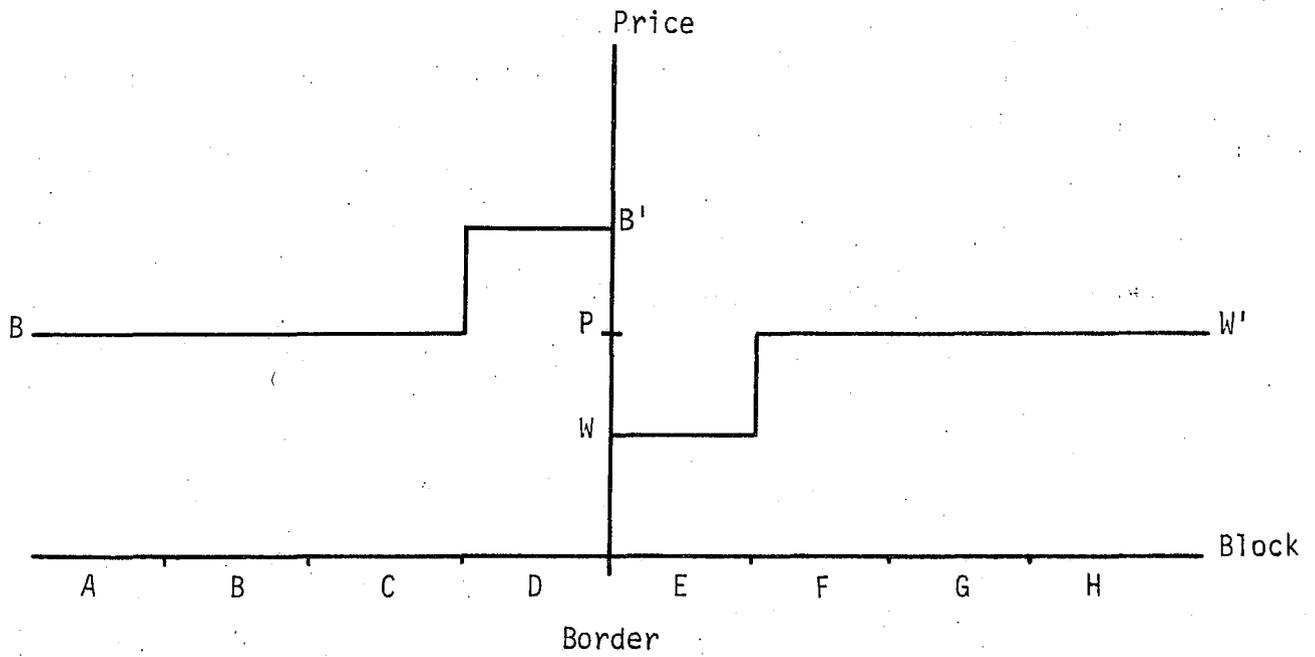


Figure 1.

Closer examination reveals that Bailey's border model does not have an equilibrium either in the case of large firms or in the case of individual owners. Let us begin with the case of large firms. If a firm owned blocks A-F, a one-block move to the right would not benefit the firm, but a two-block move would increase its revenue by pushing the low-rent houses on the black border into the next owner's territory.⁵ The next owner would then benefit from yet another move to the right because such a move would bring the high-rent houses on the black border into his territory. This process would continue, as it would in the case of many small firms, until the price at the black border equaled the price at the white border.

The above argument is still incomplete, however, because it assumes that firms are unaware of the shifting of the BB' and WW' curves that accompanies the rightward progress of the border. If firms have foresight, an owner of both border blocks might want to prevent the border from shifting to the right in order to avoid losses from the downward shifting of the BB' curve. In fact, such an owner might maximize his profits by moving the border to the left, thereby raising the BB' curve. In this case, prices would be higher in the black interior than in the white interior--a contradiction of Bailey's main result.⁶

In short, the case of large firms is inconclusive unless further assumptions are made about the way the BB' and WW' curves shift and about the foresight of housing firms. As a result, Bailey's border model cannot determine the effect of prejudice on the pattern of housing prices in the case of large housing firms. We will henceforth

concentrate on the case of many small firms, since it appears to be more realistic (see, for example, the evidence presented by Sternlieb (1969, ch. 6)).

The Bailey model indicates that when there are many small housing firms prices will be higher in the white interior than in the black interior. However, this result does not represent an equilibrium unless one makes the additional assumption that city size is fixed. If city size were not fixed, housing firms would attempt to capture the economic rent associated with housing in the white interior by building new all-white housing at the outer edge of the city. Thus, competition would drive down the price of housing in the white interior. If the black-white border responded to such a downward shift in the white price curve, as the Bailey logic indicates that it would, then the city would continue to grow and the black-white border would continue to move outward. This movement would stop only when the city reached some set of physical barriers to further expansion--that is, when it reached some fixed size.

Note that the existence of nonresidential use for land, such as agriculture, does not lead to an equilibrium in the Bailey model. If competition lowered the price of land in the white interior to the nonresidential rental rate, and if a Bailey "equilibrium" were obtained with border prices equal, then nonresidential users would be willing to pay more for land than owners of housing in the black interior or at the black-white border. Thus nonresidential activities would move into the center of the city, the black price curve would shift upward, and the rightward movement of the black-white border would continue.

II. General Equilibrium Border Models

Both Courant (1973, 1974) and Rose-Ackerman (1975) have extended Bailey's border model concept by introducing racial prejudice into a general equilibrium model of urban residential structure as developed by Alonso (1964), Mills (1967, 1972), and Muth (1969). These extensions not only lead to an equilibrium in a border model (by tying a city together with commuting cost); they also lead to several precise statements about the effect of prejudice on urban structure.

The Courant and Rose-Ackerman models of prejudice and urban structure assume, like Bailey's model, that blacks and whites are completely segregated with blacks concentrated in the city center.⁷ They also assume that white utility is affected by distance from blacks—an assumption in the spirit of the Bailey model if somewhat different in its specification. On the other hand, they assume that blacks have no preferences with regard to the race of their neighbors. The white utility function is

$$U_w = U_w(Z, H, D) \quad , \quad (1)$$

where Z is a composite consumption good, H is housing services and D is "social distance" from blacks. All the partial derivatives of this function are assumed to be positive. In addition, social distance is an increasing function of physical distance. Thus,

$$D = D^*(u-u^*) = D(u) \quad , \quad (2)$$

where u is the distance from the CBD at which the white family lives, and u^* is the location of the black-white border (in miles from the CBD).

Since white utility increases with distance from the border, $D'(u)$ is positive. It is also reasonable to assume that $D''(u)$ is negative and, indeed, that $D'(u)$ reaches zero at some large value of $(u-u^*)$.

Finally, whites face the budget constraint

$$Y = P_z Z + P_w(u)H + T(Y,u) \quad , \quad (3)$$

where Y is income, P_z is the price of Z , $P_w(u)$ is the price paid by whites per unit of H (a function of u), and T is round-trip commuting costs. The maximization of (1) subject to (3) results in the following locational equilibrium condition for whites:

$$\frac{\partial U_w}{\partial D(u)} D'(u) - \lambda(P'_w(u)H + T'_u) = 0$$

or⁸

$$P'_w(u) = -T'_u/H + \frac{\partial U_w}{\partial D(u)} D'(u)/\lambda H \quad . \quad (4)$$

This equation can be interpreted as a market equilibrium condition--that is, it defines the $P_w(u)$ function that makes whites indifferent to their location.

Equation (4) reveals that $P'_w(u)$ is ambiguous in sign, and in particular that $P'_w(u)$ may be increasing near the black-white border where $D'(u)$ is large. An example of such a white price-distance function is presented in Figure 2.

By assuming some form for the utility function, one can solve this type of model explicitly for the price-distance function, $P_w(u)$. For example, suppose that per-mile commuting costs (t) are constant and that whites have the following Cobb-Douglas utility function:

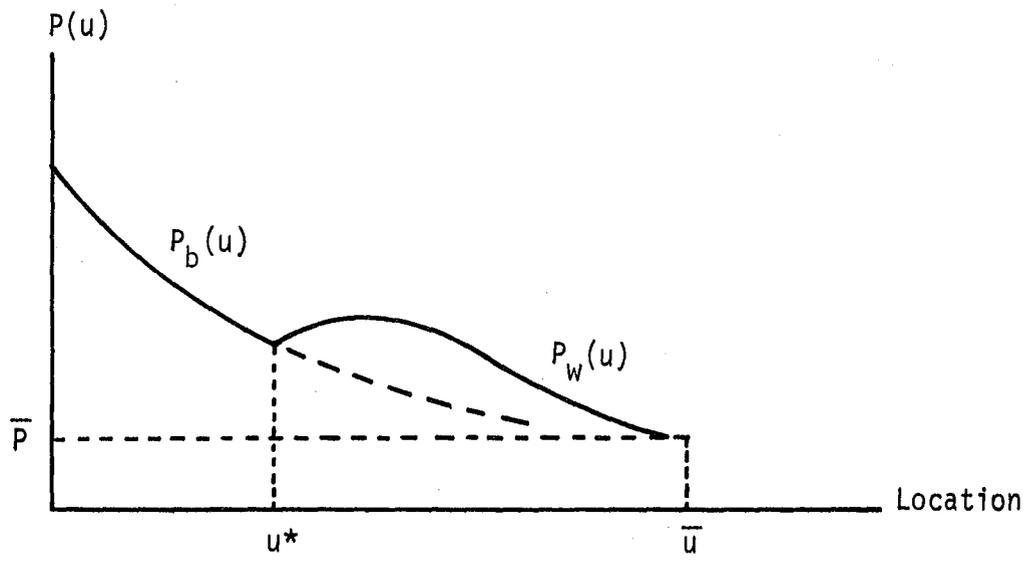


Figure 2.

$$U_w = a_1 \log Z + a_2 \log H + a_3 \log D . \quad (5)$$

In this case it can easily be shown that the demand function for H is

$$H = (a_2 / (a_1 + a_2)) (Y - tu) / P_w(u) = k(Y - tu) / P_w(u) . \quad (6)$$

It can also be shown that

$$\lambda = a_2 / P_w(u) H . \quad (7)$$

Substituting (6) and (7) into (4) yields

$$P_w'(u) = -tP_w(u) / k(Y - tu) + a_3 D'(u) P_w(u) / a_2 D(u) \quad (4')$$

or

$$P_w'(u) / P_w(u) = -t / k(Y - tu) + a_3 D'(u) / a_2 D(u) . \quad (8)$$

Integrating both sides, we find that

$$P_w(u) = K(Y - tu)^{1/k} D(u)^{a_3/a_2} , \quad (9)$$

where K is a constant of integration. By anchoring this price-distance function at the outer edge of the city (\bar{u}) using the equation⁹

$$P_w(\bar{u}) = \bar{P} \quad (10)$$

we obtain

$$P_w(u) = \bar{P} ((Y - tu) / (Y - t\bar{u}))^{1/k} (D(u) / D(\bar{u}))^{a_3/a_2} . \quad (11)$$

The price-distance function will, of course, take on a different form if different assumptions are made about the utility function.

The terms in this price-distance function that reflect white prejudice can be given a simple interpretation: they indicate the proportion by which the unit price of housing, as determined by commuting costs, must be lower at u in order to compensate whites for their nearness to blacks. It will prove useful to define the inverse of these terms, evaluated at $u = u^*$, as

$$\bar{D} = [D(\bar{u})/D(u^*)]^{a_3/a_2} . \quad (12)$$

This expression is an indicator of the strength of white prejudice. It gives the proportional increase in the unit price of housing that whites would be willing to pay (if there were no transportation costs) in order to live $(\bar{u} - u^*)$ miles away from blacks instead of right next to blacks.

Five main results about urban structure can be derived from this type of general equilibrium border model:¹⁰

1. The white price-distance function is flatter when whites have racial prejudice than when they do not (because moving farther from the CBD leaves whites farther from blacks) and may be upward-sloping near the black-white border. (See Courant, 1973, p. 56 and Rose-Ackerman, 1975, p. 91). Courant points out that, in models of the type under consideration, higher housing prices imply higher land prices and thus higher capital-land ratios in housing production. This has the testable implication that there will be capital substitution near the black-white border—that is, that there will be a belt of relatively high-rise buildings at some distance from the border (Courant, 1973, p. 70).

2. Blacks will pay less for housing and live at lower densities when whites are prejudiced than when whites are not prejudiced (Courant, 1973, p. 61; Rose-Ackerman, 1975, p. 92). This is consistent with the results of many nonspatial competitive models in which whites "pay for their prejudice."

3. Most whites, but not those near the black-white border, will pay more for housing and live at higher densities than they would in a city without white prejudice (Rose-Ackerman, 1975, p. 92).

4. Under certain values of the parameters of the white price-distance function, there will exist a zone of nonresidential land use between the black and the white residential areas. In this zone, land used for housing has a marginal value product less than the nonresidential rental rate, and thus no housing is produced (Courant, 1973, p. 56). This condition, referred to as a "greybelt" in section III, occurs when whites offer less for housing at u^* than at \bar{u} .

5. A city will be larger in area, for a given population size, when it contains prejudiced whites than when it contains no prejudiced whites (Courant, 1974, p. 11; Rose-Ackerman, 1975, p. 92).

III. Border Models with More Than One Income Class

The logic of border models depends on the assumption of a single income class. In this section we will show that when more than one income class exists in a city, both Bailey's and the general equilibrium border models apply only to a very restricted set of cities.

To understand why the single-income-class assumption is so important, it is helpful to emphasize one characteristic of the Bailey model: Blacks are assumed to prefer living with whites but to always end up living apart from whites. This combination of assumptions is somewhat disturbing. If blacks prefer to live with whites, why do they not simply move into white neighborhoods? Muth answers this question by adding a further assumption to the model: "If B-types [that is, blacks] prefer integration with A-types [whites], . . . it is assumed that they are willing to offer less of a premium to live among A-types than other A-types" (1975, p. 87). To put this assumption another way, all whites must be willing to pay more to live in a white neighborhood than are any blacks. Muth does not offer any evidence to support this assumption, but it does make the Bailey model consistent; that is, it describes a situation in which blacks prefer integration but do not achieve it.

However, Muth's assumption is not plausible when there is a range in black incomes. The amount a family is willing to pay to live in a white area is a function of its income as well as of its attitudes. Therefore, for any given amount that a white is willing to pay to live in a white area, there is some income that will lead a black to be willing to pay even more. So if there is a range in black incomes, the Bailey model is consistent only if yet another assumption is made: Not only must the black taste for integration be less strong than the white taste for segregation, but the income of the richest black must be sufficiently low relative to the income of the poorest white that the richest black will not outbid the poorest white for housing in a white neighborhood.

In our view, this second additional assumption is so strong that it leaves the Bailey model without practical interest. Table 1 presents some evidence to support our view: It indicates that in a variety of cities, about one-quarter of the black families have incomes above the mean income for white families.

By introducing transportation costs, the Courant and Rose-Ackerman models lead to equilibrium in a Bailey-like world and enrich our understanding of the effect of prejudice on urban structure. We will proceed to show, however, that these models are also unsatisfactory when there is more than one income class. In particular, we will show that, if some blacks are significantly richer than some whites, then the models are logically inconsistent unless there is a great deal of white prejudice. Furthermore, we will show that when the models are consistent it is possible that there will be a greybelt between the black and white areas.

By way of review, general equilibrium border models combine several assumptions about perfect competition in the housing market with several Bailey-like assumptions about white prejudice. Of particular interest for what follows is the assumption that blacks and whites each live in one and only one region of a city so that there is a single black-white border.

Four properties of the price-distance functions in these models are also important for the discussion that follows.

1. Whenever the income elasticity of demand for housing is unity or greater, both the black and the white price-distance functions

Table 1

Summary of Income Distributions of Black and White
Families in Selected Cities

City	% of Families That Are Black	Mean Black Income	Mean White Income	% of Blacks over \$10,000	% of Blacks over \$15,000	% of Whites under \$10,000	% of Whites under \$5,000
Los Angeles	16.98	8055	13619	31.05	10.45	42.51	15.89
Chicago	29.60	8733	12455	35.77	13.54	41.79	13.54
Baltimore	41.66	8065	11394	27.41	9.32	48.06	16.67
Boston	16.50	7182	12004	25.10	6.53	52.50	18.53
Detroit	40.49	9292	12051	40.22	15.80	42.93	15.67
Mobile	30.43	5420	11451	11.22	2.64	50.72	16.85
Orlando	25.49	5561	10995	13.85	3.33	55.43	20.68
Rochester	14.56	8240	11139	32.45	10.51	47.04	16.23
Tulsa	8.93	5899	12077	16.29	3.76	47.80	14.94
Chattanooga	31.79	5646	9566	13.37	3.65	62.66	25.32
Atlanta	47.72	7287	13689	24.76	7.90	46.40	16.21
Trenton	33.79	7829	10679	31.15	8.66	53.11	18.42
Columbus, O.	17.18	8268	11010	33.68	10.60	49.09	15.75
Wilmington, N.C.	23.35	5072	10533	10.30	2.96	76.25	19.93
Omaha	8.89	6939	12028	22.81	4.71	45.76	13.53
Sacramento	8.95	7715	11344	29.44	9.63	50.03	18.93

Source: U.S. Bureau of the Census, 1970 Census of Population.

become flatter as income rises. (See Muth, 1969, and Mills, 1972.) This result can easily be derived in the Cobb-Douglas case by differentiating equation (4') with respect to income.

2. Unless there is a greybelt, the black and white price-distance functions meet at u^* ; that is, $P_w(u^*) = P_b(u^*)$. This property is a product of competition; unless the black and white prices are equal at u^* , either blacks or whites will be willing to pay more than the other group on either side of the border, and the border will move.

3. The white price-distance function is flatter than it would otherwise be, because of white prejudice, and may be upward sloping near u^* .

4. At distances far from the black-white border, white prejudice has no effect on the slope of the price-distance function. It is assumed that the slope of the social distance function (that is, $D'(u)$) equals zero at distances far from u^* ; this property therefore follows directly from equation (4).

These four properties are sufficient to prove that if black incomes are higher than white incomes, the black and white price-distance functions will cross at some \hat{u} greater than u^* . It follows from properties (2) and (3) that just outside u^* the white price-distance function is above the black price-distance function.¹¹ Furthermore, it follows from properties (1) and (4) that at locations far from u^* the black price-distance function will be flatter than the white price-distance function whenever black incomes are higher than white incomes. Therefore, the white price-distance function will eventually fall to a point at which the black price-distance function intersects

it from the left. It should be clear that the higher black incomes are relative to white incomes, the lower will be the value of \hat{u} at which the two price-distance functions cross.

If the point of intersection between the two price-distance functions occurs within the urban area (that is, if \hat{u} is less than \bar{u}), then blacks will be willing to pay more for housing than will whites both inside u^* and outside \hat{u} . Under these conditions rich blacks will "hop" over poorer whites and the equilibrium solution to the model will involve two black areas--thereby contradicting one of the assumptions of the model. In this case, in other words, the border model is logically inconsistent. Figure 3 gives an illustration of price-distance functions that lead to this inconsistency. This contradiction is important because the assumptions about white prejudice depend on the existence of a single black-white border. The model provides no way to determine the effect that white prejudice will have on the equilibrium price-distance function if blacks live in two areas--so that there are two black-white borders.

If the black price-distance function intersects the white price-distance function outside the urban area (that is, if \hat{u} is greater than \bar{u}), then the general equilibrium border model is logically consistent; in equilibrium, there will be only one black area and one white area, and blacks will live in the city center. It does not follow, however, that there will literally be a black-white border. If prejudice has a strong effect on the white price-distance function, then whites may bid more for housing at \bar{u} than at u^* (that is, $P_w(\bar{u})$ may be greater

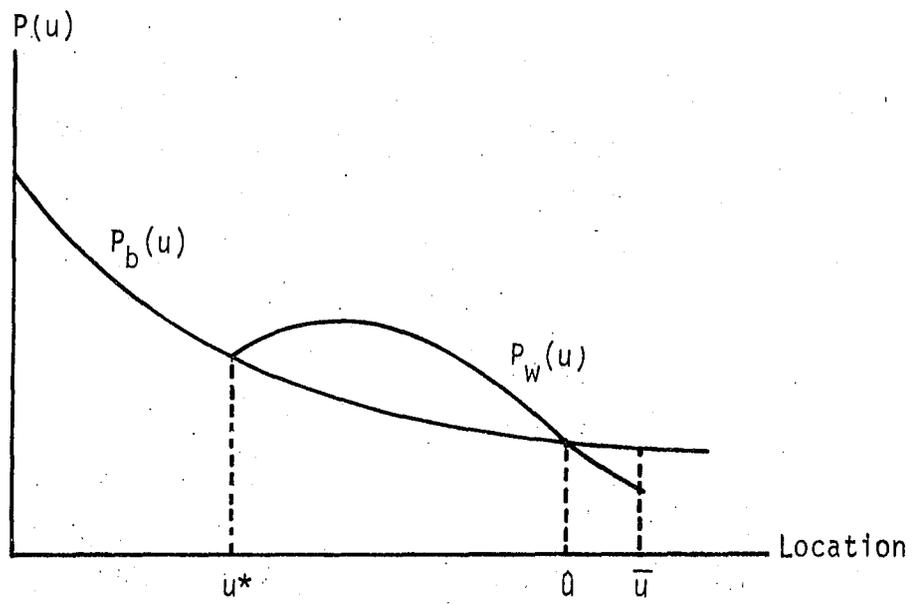


Figure 3.

than $P_w(u^*)$). In this case, which is illustrated in Figure 4, non-residential users of land will outbid both whites and blacks for land near u^* and, in equilibrium, there will be a greybelt of nonresidential land use between the black and white areas.

Only if $P_w(\bar{u})$ is less than $P_w(u^*)$ and \hat{u} occurs beyond \bar{u} is there a logically consistent border model that actually involves a black-white border. As we will see, this case is possible even if black incomes are infinite, but it appears to involve very high levels of white prejudice. This case is illustrated in Figure 5.

These results are summarized in the following theorem, which is already proved.

THEOREM. Given the assumptions of general equilibrium border models, and assuming that some blacks have higher incomes than some whites, equilibrium in the location of blacks vis-a-vis lower-income whites requires that one of the following cases occur:

1. Blacks are willing to pay a higher unit price than are whites for housing beyond some \hat{u} (where $u^* < \hat{u} < \bar{u}$), so that, in equilibrium, there will be more than one black area. In this case the pattern of racial segregation assumed by border models is not an equilibrium and the models are logically inconsistent.

2. White prejudice is so strong that whites are willing to pay a higher unit price for housing at \bar{u} than at u^* . In this case, the pattern of segregation assumed by the models is an equilibrium and, in addition, there will exist, in equilibrium, a zone of nonresidential land use between the black and white areas. In this case, therefore, border models are logically consistent but do not involve a black-white border.

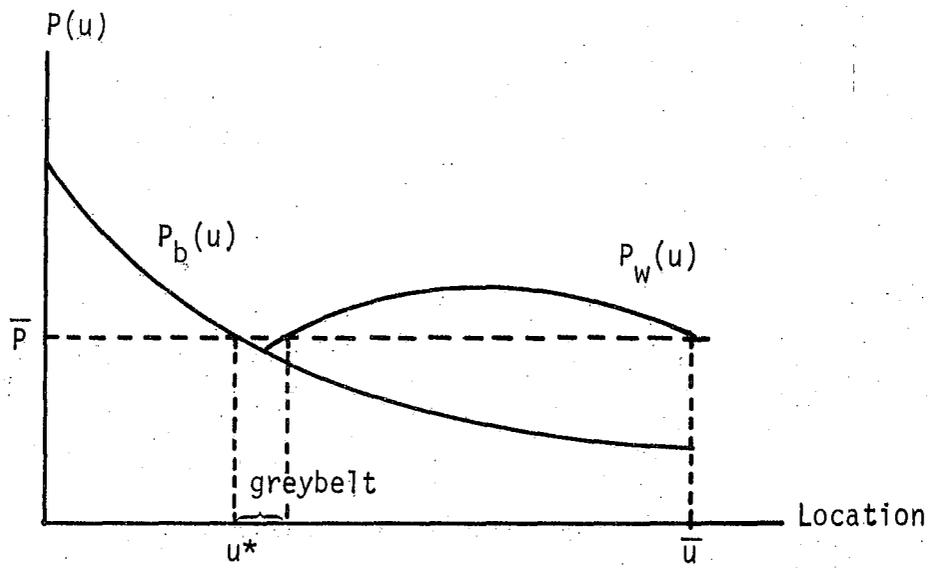


Figure 4.

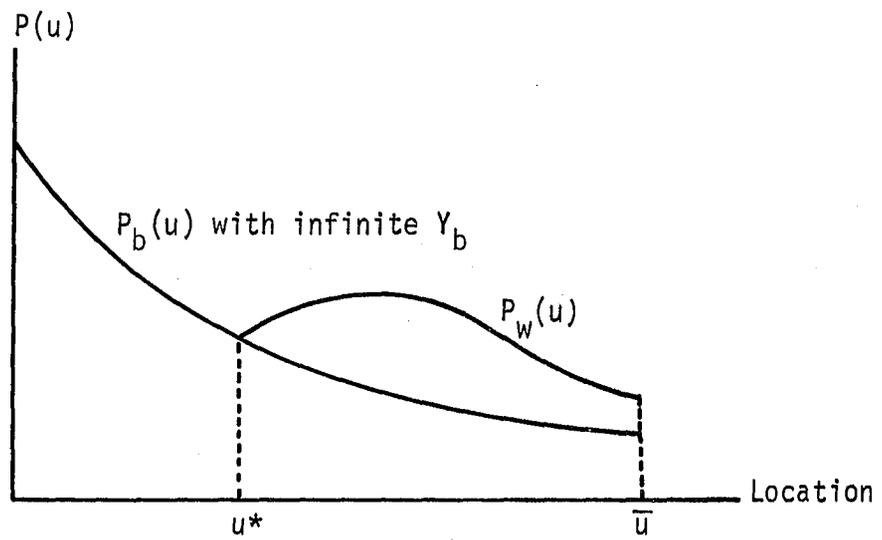


Figure 5.

3. If the black and white price-distance functions do not intersect between u^* and \bar{u} and the white price-distance function is lower at \bar{u} than at u^* , then the pattern of segregation assumed by border models is an equilibrium and there exists a black-white border. This is the case that most closely coincides with the spirit of the original Bailey model.

The reasoning behind this theorem is complicated somewhat by the introduction of several white income classes, but the above statement of the theorem is still valid. Since price-distance functions are more downward-sloping at lower incomes, the introduction of white low-income classes near u^* makes larger the range of parameters under which blacks hop. Similarly, the introduction of white high-income classes in the suburbs lessens the downward slope of the white price-distance function and makes smaller the range of parameters under which hopping occurs.

It is also possible to extend the model to include the attitudes of blacks. If, as surveys indicate, many blacks prefer to live in integrated neighborhoods, then blacks may be willing to offer more to live in white neighborhoods than the models presented here assume. If this is so, the black price-distance functions will be flatter and the likelihood of hopping will be greater.

Although the logic behind our theorem is perfectly rigorous, it is appropriate to state the results in more mathematical terms. The following mathematical derivation of the theorem assumes a Cobb-Douglas utility function and linear commuting costs, but the theorem does not depend on these somewhat restrictive assumptions.

As shown earlier, the equilibrium condition for prejudiced whites is:

$$P_w(u) = \bar{P} [(Y_w - t_w u) / (Y_w - t_w \bar{u})]^{1/k} [D(u) / (D(\bar{u}))]^{a_3/a_2} . \quad (13)$$

The analogous condition for blacks is¹²

$$P_b(u) = P_w(u^*) [(Y_b - t_b u) / (Y_b - t_b u^*)]^{1/k} . \quad (14)$$

To determine whether or not blacks will have an incentive to hop over whites, we need to determine whether or not¹³

$$P_b(\bar{u}) > P_w(\bar{u}) = \bar{P} \quad (15)$$

If inequality (15) holds, then blacks will bid more than whites for housing at \bar{u} and therefore will not be in equilibrium in the city center. Now from equations (13) and (14) we find that

$$P_b(\bar{u}) = \bar{P} \left(\frac{D(u^*)}{D(\bar{u})} \right)^{a_3/a_2} \left(\frac{Y_w - t_w u^*}{Y_w - t_w \bar{u}} \right)^{1/k} \left(\frac{Y_b - t_b \bar{u}}{Y_b - t_b u^*} \right)^{1/k} . \quad (16)$$

Thus inequality (15) will hold if

$$\left(\frac{D(u^*)}{D(\bar{u})} \right)^{a_3/a_2} \left(\frac{Y_w - t_w u^*}{Y_w - t_w \bar{u}} \right)^{1/k} \left(\frac{Y_b - t_b \bar{u}}{Y_b - t_b u^*} \right)^{1/k} > 1 \quad (17)$$

Linear commuting costs for group i can be expressed in the form

$$t_i = t_o + t_y Y_i ,$$

where t_o is the per-mile operating cost and t_y is the per-mile time cost of a round trip to the CBD. Substituting this expression (for both blacks and whites) into inequality (17), we have

$$\left(\frac{1}{\bar{D}W}\right) \left(\frac{Y_b(1 - t_y \bar{u}) - t_o \bar{u}}{Y_b(1 - t_y u^*) - t_o u^*}\right)^{1/k} > 1 \quad (19)$$

where

$$W = \left(\frac{Y_w(1 - t_y \bar{u}) - t_o \bar{u}}{Y_w(1 - t_y u^*) - t_o u^*}\right)^{1/k} \quad (20)$$

and \bar{D} is defined by equation (12).

As indicated on page 12, the value of \bar{D} is the proportional increase in the unit price of housing that whites are willing to pay, for racial reasons, to live at \bar{u} instead of at u^* . Thus if \bar{D} has a value of 1.10, whites are willing to pay a 10 percent higher price to live far away from blacks. The only convincing estimate of \bar{D} of which we are aware is the estimate by King and Mieszkowski (1973), who found that white apartment rentals were 7 percent lower in the black-white border area than in the white interior. This estimate implies a value of \bar{D} of $(1/(1-.07)) = 1.075$.

In analyzing inequality (19), it is useful to begin by determining the highest level of white prejudice at which hopping by blacks can occur. Now since as Y_b approaches infinity the ratio of $(Y_b(1 - t_y \bar{u}) - t_o \bar{u})$ to $(Y_b(1 - t_y u^*) - t_o u^*)$ approaches $(1 - t_y \bar{u})/(1 - t_y u^*)$, it follows from inequality (19) that hopping is logically possible as long as

$$(1/\bar{D}W) [(1 - t_y \bar{u})/(1 - t_y u^*)]^{1/k} > 1 \quad (21)$$

The level of \bar{D} at which inequality (21) holds as an equality can be called the no-hop point; that is, it is the level of white prejudice above which blacks, no matter how high their incomes, will never have an incentive to hop over whites. In symbols, if D_h is the no-hop point, then

$$D_h = (1/W) [(1 - t_y \bar{u}) / (1 - t_y u^*)]^{1/k} . \quad (22)$$

It is also possible to determine when greybelts will form. As indicated earlier, greybelts will form if

$$P_w(u^*) < P_w(\bar{u}) = \bar{P} \quad (23)$$

or

$$\bar{P} [(Y_w - t_w u^*) / (Y_w - t_w \bar{u})]^{1/k} [D(u^*) / D(\bar{u})]^{a_3/a_2} < \bar{P} \quad (24)$$

or

$$1/\bar{D}W < 1 . \quad (25)$$

The level of \bar{D} above which greybelts will form will be referred to as the greybelt point and labeled D_g . Thus

$$D_g = 1/W . \quad (26)$$

Finally, we can determine the minimum level of income at which blacks will have an incentive to hop over whites. By making inequality (19) into an equality and solving for Y_b , we obtain

$$Y_b = \frac{t_o [\bar{u} - (W\bar{D})^k u^*]}{1 - t_y \bar{u} - (W\bar{D})^k (1 - t_y u^*)} . \quad (27)$$

Further insight into the conditions under which hopping will occur and greybelts will exist can be gained by differentiating D_h and D_g with respect to the parameters of the model. The signs of the resulting partial derivatives are presented in Table 2.

These results indicate that the higher white incomes, the larger the black area, the greater the proportion of income spent on housing, the smaller the city, and the smaller the costs of commuting, the less white prejudice is required to eliminate the possibility of black hopping. Similarly, the lower white incomes, the smaller u^* and k , and the greater \bar{u} , t_o , and t_y , the less white prejudice is required to lead to greybelts.

These results can easily be extended in several ways. Two ways will be described briefly here. First, if there are three white income classes referred to by the superscripts H, M, and L, the price-distance functions for all white income classes must meet at the boundaries between the classes. Thus,

$$P_b(u^*) = P_w^L(u^*) \quad (28)$$

$$P_w^L(u_1) = P_w^M(u_1) \quad (29)$$

$$P_w^M(u_2) = P_w^H(u_2) \quad (30)$$

$$P_w^H(\bar{u}) = \bar{P} \quad (31)$$

where u_1 and u_2 refer to borders between income classes. From these conditions (and the assumption that white prejudice does not vary by income class), we find that $P_b(\bar{u})$ will exceed \bar{P} if¹⁴

$$(1/\bar{D}W^*)[(1 - t_y\bar{u})(1 - t_y u^*)]^{1/k} > 1 \quad (32)$$

Table 2

Signs of the Parital Derivatives of D_h and D_g

Parameter	∂D_h	∂D_g
Y_w	-	-
u^*	-	-
\bar{u}	+	+
t_o	+	+
t_y	+	+
k	-	-

where

$$W^* = W^H W^M W^L \quad (33)$$

$$W^H = [(Y_w^H(1 - t_y \bar{u}) - t_o \bar{u}) / (Y_w^H(1 - t_y u_2) - t_o u_2)]^{1/k} \quad (34)$$

$$W^M = [(Y_w^M(1 - t_y u_2) - t_o u_2) / (Y_w^M(1 - t_y u_1) - t_o u_1)]^{1/k} \quad (35)$$

$$W^L = [(Y_w^L(1 - t_y u_1) - t_o u_1) / (Y_w^L(1 - t_y u^*) - t_o u^*)]^{1/k} \quad (36)$$

Inequality (32) is identical to inequality (21) except that W has been replaced by W^* . It follows that formulas (22) and (26) for D_h and D_g are still valid in the three-income-class case if W is replaced by W^* .

These formulas can easily be extended to any number of white income classes or to the case in which white prejudice varies with income class.

Second, black attitudes can be introduced in a manner analogous to that of white prejudice. In this case the black price-distance function becomes

$$P_b(u) = P_w(u^*) [(Y_b - t_b u) / (Y_b - t_b u^*)]^{1/k} [D_b(u) / D_b(u^*)]^{a_3/a_2} \quad (37)$$

where D_b is the social-distance function perceived by blacks. Since many blacks prefer integration, it is assumed that blacks, like whites, gain utility by moving outside the black-white border. By substituting equation (37) for equation (14) it can easily be shown that the no-hop point is now

$$D_h = [\bar{D}_b / W] [(1 - t_y \bar{u}) / (1 - t_y u^*)]^{1/k} \quad (38)$$

where

$$\bar{D}_b = [D_b(\bar{u})/D_b(u^*)]^{a_3/a_2} \quad (39)$$

Not surprisingly, a black preference for integration increases the level of white prejudice required to eliminate the possibility of hopping by blacks.

The results of this section can be illustrated by some numerical examples. Let us assume that operating costs are 15 cents per mile, that commuting proceeds at 12 MPH and travel time is valued at one-half the wage rate, that whites earn \$10,000 per year, and that people spend one-fifth of their income on housing. Translated into daily terms, these assumptions imply that

$$t_o = .3$$

$$t_y = .0104$$

$$Y_w = 40$$

$$k = .2 \quad .$$

Now let us examine two cities with the dimensions shown in Table 3. Note that in an urban model these dimensions are determined by the sizes of the total and of the black populations; however, if there is hopping or a greybelt, these assumed values for \bar{u} and u^* are not equilibrium values.¹⁵

Using equations (22) and (26), it is now possible to calculate D_h and D_g . The results are presented in Table 4. This table indicates that for the possibility of black hopping to be eliminated in city A,

Table 3

Dimensions of Cities A and B

	City	
	A	B
\bar{u}	15	10
u^*	2	3

Table 4

The No-Hop Point and the Greybelt
Point in Cities A and B

	City	
	A	B
D_h	1.8928	1.3766
D_g	3.9788	2.0344

whites must be willing to pay 89 percent more for their housing in order to avoid blacks. The analogous figure for city B is 38 percent. The table also indicates that there will be a consistent border model with a black-white border in city A(B) only if whites are willing to pay between 89 and 298 percent (38 and 103 percent) more for housing in order to live as far from blacks as possible.

It is also possible to calculate, using equation (27), how high the incomes of the richest blacks would have to be at various levels of \bar{D} in order for those blacks to have an incentive to hop over whites. Such calculations for cities A and B are presented in Table 5. This table shows that at low levels of white prejudice blacks will have an incentive to hop if their incomes are only slightly greater than white incomes. As white prejudice approaches D_h , blacks will not have an incentive to hop unless their incomes are many times those of the poorest whites.

This example can be further extended in several ways. First, additional white income classes can easily be added by making use of inequality (32). Take, for example, two cities with white income classes earning \$5000, \$10,000, and \$20,000, and with the dimensions shown in Table 6. In light of the data presented in Table 1, these distributions of white income appear quite realistic. Calculations of D_h and D_g for cities A* and B* are given in Table 7. The addition of white higher-income classes to this example decreases the range of parameters for which blacks have an incentive to hop, and the addition of white lower-income classes increases this range. The overall effect is to slightly increase D_h and D_g .

Table 5

Levels of Black Income Above Which Black Hopping Will Occur,
for Various Levels of White Prejudice

\bar{D}	City			
	Daily Y_b	Yearly Y_b	Daily Y_b	Yearly Y_b
1	\$40.00	\$10,000	\$40.00	\$10,000
1.05	43.06	10,765	46.82	11,705
1.10	46.49	11,623	56.08	14,020
1.20	54.78	13,623	90.26	22,565
1.30	65.80	16,450	213.49	53,376
1.3766	77.09	19,273	∞	∞
1.40	81.22	20,305	-	-
1.50	104.42	26,105	-	-
1.60	143.36	35,840	-	-
1.70	222.55	55,638	-	-
1.80	472.03	118,008	-	-
1.8928	∞	∞	-	-

Table 6

Dimensions of Cities A* and B*

	City	
	A*	B*
u^*	2	3
u_1	6	5
u_2	11	8
\bar{u}	15	10

Table 7

The "No-Hop" Point and the Greybelt
Point in Cities A* and B*

	City	
	A*	B*
D_h	2.0105	1.4309
D_g	4.2262	2.1147

Finally, the effect of black attitudes can be calculated using equation (39). Table 8 describes calculations of D_h for cities A* and B*--the level of white preference that eliminates black incentive to hop--for various values of black preference for integration. For example, the table indicates that if blacks are willing to pay 5 percent more for housing in white than in black neighborhoods, the no-hop point in city B* goes from 43 to 50 percent. The introduction of black preferences does not change the greybelt point.

It should be noted that in all of these cases the level of prejudice necessary to achieve the no-hop point, and thus to render the border model internally consistent, is much larger than that found by King and Mieszkowski (1973) and larger than is easy to believe. The implications of this finding are discussed in section IV.

IV. Implications of the Analysis

The preceding discussion casts serious doubt on the appropriateness of both general equilibrium models and simple Bailey models as frameworks for the study of the effects of racial prejudice on urban structure. When realistic assumptions about the distribution of income are added to the models, the Bailey model simply collapses, and the general equilibrium models are logically inconsistent unless there are extremely high levels of white prejudice. In particular, if some blacks have higher incomes than some whites, the assumption that blacks live in a single ghetto is contradicted by the logic of the models. In the Cobb-Douglas case investigated in section III, the level of white prejudice necessary for consistency in the general

Table 8

No-Hop Points in Cities A* and B*, for
Various Levels of Black Preference for
Integration

Level of Black Preference for Integration	City	
	A*	B*
1.05	2.1110	1.5025
1.10	2.2116	1.5740

equilibrium models was, under the weakest set of assumptions made, over five times as high as the level reported in the best empirical study of how much whites are willing to pay to live far from blacks (King and Mieszkowski, 1973). We thus conclude that since many blacks have much higher incomes than many whites, one of the most attractive features of the border models--their assumption of one ghetto in a world where one ghetto is the rule rather than the exception--is inconsistent with the models themselves. Having concluded this, we suggest that the following areas of research hold some promise for better modeling of the questions the border model was designed to address.

1. It may be possible, although it looks very difficult, to create models of urban structure in which borders themselves are endogenous. Having established that equilibrium solutions to border models will require, given sufficient dispersion of income, that not all blacks live in one annulus, it must be true that any segregated equilibrium solutions to competitive monocentric models of urban structure must involve spatial allocations of residences such that blacks of different incomes are separated by whites of different incomes. The construction of models permitting such solutions, however, requires that rather than assuming a specified number of black rings the modeler permit the model to solve for the equilibrium configuration of spatial sorting by race and income, given an assumption about the distribution of income. While the development of a model capable of solving for endogenous borders might "save" the border model concept, we know that it will not generate single ghettos, which are what we observe. Further, the procedures involved in designing such a model

will be much more complicated than those involved in the simple border models heretofore developed, and we know of nothing in the literature that tells us where or how to begin.

2. A related line of inquiry involves the construction of models of cities in which the ghetto is not circular. Suppose, for example, that the ghetto is wedge-shaped, thus permitting one continuous area of black location in which members of high-income classes have access to distant locations without hopping over whites. Again, we know nothing about how to build such models, except that preliminary attempts on our part to model the relationships at the borders of a wedge strongly suggest that the set of conditions under which competitive equilibria at these borders exist is very small, if not empty.

3. Another possibility is that the effects of prejudice on a competitive housing market can be modeled in ways other than those implicit in the border models. In particular, Courant (1975) has shown that if there are positive costs of search for housing, and if some whites are averse to dealing directly with blacks, blacks rationally may choose not to search for housing in white parts of town even if they are willing to pay more than the going price of housing in those parts of town. Thus, there may be a barrier to black hopping due to search costs. However, this will not, in general, be an impermeable barrier, and Courant also suggests that it will be most permeable for higher-income blacks.

4. Finally, it may well be that competitive models are simply not the appropriate vehicle for analysis of this problem. One of the clear implications of our analysis is that it is very much in the interest of prejudiced whites, as a group, to organize housing markets in a manner that prevents high-income blacks from hopping even when the logic of the border model suggests that they will do so. To see why this is true, note that after hopping takes place all whites have additional disutility from nearness to blacks. Thus, the competitive models presented here have within them a strong suggestion that housing markets in fact may not be competitive—that there are strong incentives for the larger, richer, and more powerful elements of society to collude. A similar conclusion has been reached by Yinger (1975b) using a different specification of racial prejudice in an urban model. Yinger shows that if whites prefer not to live with blacks and if some blacks prefer to live in integrated neighborhoods, then competition cannot generate a stable equilibrium distribution of blacks and whites in an urban area. In this situation it is in the interest of whites to buy neighborhood stability by restricting the areas into which blacks can move.

Kain, in a number of works with a number of collaborators,¹⁶ has suggested that whites do organize housing markets to artificially restrict the range of locations available to blacks. Yinger (1975b) and Courant (1973) document a number of ways in which two important institutions in the market, real estate brokers and bankers, find it in their interest to promote racial segregation through their market behavior. For the case of realtors, Helper (1969) finds a great deal of evidence to support the contention that collusive, discriminatory behavior does indeed take place.

Economists have tended to ignore what they perceive to be the essentially sociological question of whether or not a society in which racial prejudice is pervasive might organize itself so that the shared attitude is reflected in its institutions. And the sociological literature strongly suggests that prejudice does pervade institutional and individual behavior.¹⁷ To ignore these findings in studying the effect of racial prejudice on urban structure is to leave unturned what may be a very large stone.

NOTES

¹For one statement showing that prejudice does not imply discrimination see Becker (1957). For a more complete discussion see Simpson and Yinger (1972).

²See for example Bailey (1959, 1966); Courant (1973, 1974); Rose-Ackerman (1975); King and Mieszkowski (1973); Muth (1969, 1975); Daniels (1975).

³A note on these assumptions about tastes is in order. Surveys reveal that most whites prefer not to live with blacks and that most blacks prefer to live in integrated neighborhoods. (See Pettigrew, 1973.) These results do not imply, however, that whites are prejudiced and that blacks have "reverse" prejudice, since the surveys cannot separate purely racial attitudes from attitudes about the public service levels in neighborhoods with different racial compositions.

⁴The diagram can be found in Courant (1973); Yinger (1974); and Muth (1975). Note that BB' and WW' are price curves determined by the intersection of demand curves and vertical supply curves.

⁵We are grateful to Robert Dennis for pointing this out.

⁶Bailey recognized the possibility of a leftward movement of the border in the case where a large firm gained control of the border blocks when the border prices were equal (1959, p. 289); however, he did not recognize either the possibility that such a leftward movement might not be profitable or the possibility that the leftward movement beyond the point where the interior prices were equal might be profitable. It is not difficult to think of cases in which shifts in the BB' and WW' curves lead to either of these results.

⁷Rose-Ackerman (1975, p. 90) justifies this assumption by arguing that blacks have lower incomes than whites, on average, and that it is well known that in urban models of the type under consideration higher-income groups locate farther from the city center than lower-income groups. Courant proves that if incomes are equal or if all blacks have lower incomes than any white the only equilibrium solution to the model will be one in which blacks inhabit the central annulus of the city. (1973, p. 68; 1974, p. 16.) In section III of this paper it will be shown that the assumption that blacks inhabit only the central annulus is not, in general, consistent with a situation in which some blacks have significantly higher incomes than some whites. The average incomes of the races are irrelevant to the question.

⁸Although the notation is different, our equation (4) is the same as Rose-Ackerman's equation (7), except that she neglected to include λ .

⁹Given the production function for housing, a unique \bar{P} will be implied by \bar{R} , the opportunity cost of agricultural land. See Mills (1972, ch. 5) for complete discussion of the model.

¹⁰Note that these results do not depend on the functional form used in the above exposition.

¹¹If there is a greybelt, the black price at the inner edge of the greybelt equals the white price at the outer edge of the greybelt (equals \bar{P}). In such a case, therefore, this sentence should conclude: "just outside the greybelt the white price-distance function is above the black price-distance function." This restatement does not affect the following argument.

¹²There are two differences between the white and black functions: (1) Since blacks are assumed to be indifferent to the race of their neighbors, social distance does not affect equation (13). (2) The black price-distance function is anchored to the white price-distance function at u^* ; hence, $P_w(u^*)$ in equation (14) is analogous to \bar{P} in equation (13). If there is a greybelt, u^* is the outer edge of the black area and the black price-distance function is anchored by the equation $P_b(u^*) = \bar{P}$. Finally, note that subscripts to denote black and white have been added to the right-hand sides of equations (13) and (14).

¹³In general, if there are many white income classes, and borders between white income classes are denoted u , we need only determine if $P_b(\hat{u}) > P_w(\hat{u})$, for any \hat{u} . The logic of the argument is most easily followed, however, if the discussion takes place in terms of \bar{u} . In doing this, we are not arguing that in order for the theorem to hold the richest blacks must outbid the richest whites.

¹⁴Note again that it is still possible for blacks to hop over some, but not all, whites. As before, one can compute the condition for hopping to an arbitrary \hat{u} , a location where a white income class poorer than blacks and one richer than blacks have a border.

¹⁵In an urban model of this type, either population or the dimensions of the city must be exogenous. Here we are assuming that population is given and that \bar{u} and u^* adjust so that there is room for the given population. It is also possible to assume that \bar{u} and u^* are fixed and let net migration occur until population just fills up the area of the city.

¹⁶See, for example, Kain (1969) and Kain and Quigley (1970).
See also Quigley (1974).

¹⁷See, for example, Chapter 4 in Simpson and Yinger (1972)
and the references cited therein.

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