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EQUILIBRIUM IN AN URBAN AREA

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ABSTRACT

In this paper, racial prejudice is introduced into an urban model and results about racial discrimination and residential segregation are derived. To be specific, a household maximization problem is used to determine the market price-distance function that gives no household an incentive to move. Prejudice is introduced by assuming that the racial composition of a location affects a household's utility and by deriving, for both blacks and whites, price-distance functions that reflect racial composition. These price-distance functions imply that if whites prefer segregation and some blacks prefer integration, no stable locational equilibrium exists for both races without discrimination.

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INTRODUCTION

Racial prejudice strongly influences the locational decisions of households in an urban area, but the relationship between racial prejudice and the pattern of residential location is not well understood. In this paper, therefore, we will introduce racial prejudice into a model of an urban area and derive several results about residential location. This exercise is useful not only because it helps explain the pattern of residential segregation, but also because it sheds some light on the relationship between prejudice and discrimination in housing.

The distinctions among several terms are important for what follows. Prejudice is defined to be an attitude--an inflexible, deeply felt attitude toward a particular group of people. Discrimination, on the other hand, is behavior that denies one group of people rights or opportunities given to others, and segregation is the actual physical separation of different groups of people.[1] Although logically separate, these three concepts are closely related in the structure of American society. It should be pointed out that price discrimination is one--but by no means the only--type of discrimination of interest to economists.[2]

The basic long-run model of an urban area developed by Alonso (1964), Mills (1967, 1972), Muth (1969), and others adds a locational dimension to a model of the housing market under perfect competition. The solution to such a model is a set of prices and quantities that, in addition to satisfying the usual profit- and utility-maximization conditions, insures that no firm or household will have an incentive to change its location. The main theoretical contribution of these models is therefore a locational equilibrium condition, which is the price per unit of housing services, expressed as a function of location, that insures that no one will have an incentive to move.

In this paper we will examine the demand side of this type of model in some detail and show how a simple formulation of racial prejudice affects the locational equilibrium condition. In particular, we will derive a housing-price function that leads to locational equilibrium for prejudiced whites and one that leads to locational equilibrium for prejudiced blacks. These two housing functions will be combined to obtain a condition for racial equilibrium such that neither blacks nor whites will have an incentive to move. A careful examination of this racial equilibrium condition provides some insight into the relationships between prejudice and both segregation and discrimination in housing.

The long-run perspective of this paper should be emphasized from the beginning. Factors that will be eliminated by the entry of housing firms or the movement of households will not be considered here. This is not, of course, to say that these factors are unimportant. My goal in this paper is to isolate some of the forces that affect residential location in the long run. I hope that an understanding of these forces will provide a useful complement to the analysis of the factors that affect residential location in the short run.

1. THE DEMAND SIDE OF AN URBAN MODEL

On the demand side of an urban model, consumers maximize their utility over a composite consumption good and housing, subject to a budget constraint that includes commuting costs. Consumers are all assumed to work in the central business district (CBD) and to choose a residential location (as measured by distance from the CBD) as part of their maximization problem. In symbols, a household will

$$(1) \quad \begin{array}{ll} \text{Maximize} & U(Z,X) \\ \text{Subject to} & Y = P_z Z + P(u)X + tu \end{array}$$

where

U = the household's utility function;

Z = the composite consumption good;

X = housing (measured in units of housing services);

Y = income;

P_z = the price of Z (which does not vary with location);

$P(u)$ = the price of a unit of housing services at distance u;

t = the cost per mile of a round trip to the CBD.

The Lagrangian expression for problem (1) is

$$L = U(Z,X) + \lambda(Y - P_z Z - P(u)X - tu)$$

and the first-order conditions are

$$(2.1) \quad \partial L/\partial Z = \partial U/\partial Z - \lambda P_z = 0$$

$$(2.2) \quad \partial L/\partial X = \partial U/\partial X - \lambda P(u) = 0$$

$$(2.3) \quad \partial L/\partial u = -\lambda[P'(u)X + t] = 0$$

$$(2.4) \quad \partial L/\partial \lambda = Y - P_z Z - P(u)X - tu = 0 \quad .$$

This set of four conditions can be simplified to two conditions with more straightforward interpretations. The first two conditions can in general be used to eliminate λ and Z so that Equation (2.4) can be written

$$(3) \quad X = D[(Y-tu), P(u)] \quad .$$

Equation (3) is the demand function for housing. Without a precise form for the utility function, the demand function cannot be derived explicitly.

Condition (2.3) can be rewritten as

(4) $P'(u)X + t = 0$.

This equation is the locational equilibrium condition for a household. It indicates that a household will have an incentive to move farther from the CBD until it arrives at the location where savings in the cost of housing are just offset by higher commuting costs. With any given P(u) function, households with different tastes will choose different quantities of X and, according to Equation (4), different locations.

In an urban model, Equation (4) becomes a market condition. The locational requirement of market equilibrium is that no household have an incentive to change its location; therefore, the solution to an urban model includes the P(u) function that will make households indifferent to their location. On the basis of the assumption that all households with a given income have identical tastes, the desired P(u) function is the solution to the differential equation given by Equation (4); that is, the equilibrium P(u) is a function that guarantees that (4) is met at every location. Hence, a market interpretation of the locational equilibrium condition for a single household indicates that for households to be indifferent among all locations in an urban area, the higher transportation costs incurred by living farther from the CBD must be just offset by a decrease in the amount spent on housing. If we find that this condition is met, we will say that households are in locational equilibrium.

In order to derive a locational equilibrium condition, Equation (3) is substituted into Equation (4) and the resulting differential equation is solved for $P(u)$. The differential equation obtained in this manner takes the form

$$(5) \quad f[P(u), P'(u), (Y-tu)] = -t \quad .$$

The solution to this equation, which is called the price-distance function, consists of the price per unit of housing services that would make consumers indifferent to their location.

Note that to find a definite solution to the differential equation (5), an initial condition is required. In the Mills (1967, 1972) version of the model, the initial condition comes from the supply side. In particular, Mills shows that with a Cobb-Douglas production function for housing, a perfectly elastic supply of capital, and a supply of land that is proportional to distance, the price of housing and the rental price of land are related by

$$(6) \quad P(u) = AR(u)^a$$

where A is a constant and a is the coefficient of land in the production function for housing (1967, p. 117, eq. 9; 1972, p. 82, eq. 5-11). Since a city will extend to the location where the price of land for use in housing equals the agricultural rental rate, the desired initial condition is

$$(7) \quad R(\bar{u}) = \bar{R}$$

where $\bar{u}$ is the outer edge of the city and $\bar{R}$ is the agricultural rental rate (1967, p. 119, eq. 15; 1972, p. 81, eq. 5-9).

In order to make use of this initial condition, Equation (6) and its derivative with respect to u are substituted into Equation (5) to obtain a differential equation of the form:

$$(8) \quad g[R'(u), R(u), (Y-tu)] = -t \quad .$$

The initial condition is the particular solution that is used in solving (8) for the rent-distance function, R(u). Note that although $\bar{u}$ is determined endogenously in an urban model, the results presented here hold for any value of $\bar{u}$.

It should be emphasized that every income class will have a different rent-distance function. The income class that lives at a given distance from the CBD will be the one that has the highest rent-distance function at that location, and it is typically true in urban models that groups with higher incomes live farther from the CBD.[3] Note also that the introduction of more than one income class complicates the initial condition (since only one group can live at the edge of the city) but does not change the substance of the preceding analysis.

2. UTILITY FUNCTIONS AND RENT-DISTANCE FUNCTIONS

In order to solve the differential equation (5) [or (8)], one must have a specific form for the demand function (3). The usual procedure in urban models is to assume a form for (3), instead of assuming a form for the utility function and deriving the demand function from it. The former procedure is followed because the demand functions that can be derived from utility functions are either not operational or not sufficiently general for empirical purposes. This problem is not, of course, unique to urban models. Although the use of specific utility functions does reveal something about the form the demand function should take, such information has not, in my opinion, been sufficiently utilized; indeed, I believe that the forms usually assumed for the demand function in an urban model are not generalizations of any demand functions that can be derived from a utility function.

The simplest form for a utility function is the Cobb-Douglas form (written here after a monotonic transformation into natural logarithms):

$$(9) \quad U = c_1 \log(Z) + c_2 \log(X) \quad .$$

As Green (1971) points out, this utility function leads to demand functions with several undesirable properties. In particular, the Engle curves associated with such demand curves are straight lines that pass through the origin; that is, the income elasticities are unitary. In an urban model, this result means that at any given distance the proportion of income spent on Z and X will not change as income changes.

A generalization of (9) is

$$(10) \quad U = c_1 \log(Z-s_1) + c_2 \log(X-s_2)$$

where s_1 and s_2 are what Green calls "survival quantities." In this case, the Engle curves are straight lines that pass throught the point (s_1,s_2), and the proportion of income spent on Z and X can change with income.

Substituting Equation (10) into the maximization problem (1), we have[4]

$$\begin{aligned} &\text{Maximize} \quad && U = c_1 \log(Z-s_1) + c_2 \log(X-s_2) \\ &\text{Subject to} \quad && Y = P_z Z + P(u)X + tu \quad . \end{aligned}$$

The relevant Lagrangian expression is

$$L = U + \lambda(Y-P_z Z-P(u)X-tu)$$

and the first-order conditions are

$$(11.1) \quad \partial L/\partial Z = c_1/(Z-s_1) - \lambda P_z = 0$$

$$(11.2) \quad \partial L/\partial X = c_2/(X-s_2) - \lambda P(u) = 0$$

$$(11.3) \quad \partial L/\partial u = -\lambda(P'(u)X + t) = 0$$

$$(11.4) \quad \partial L/\partial \lambda = Y - P_z Z - P(u)X - tu = 0 \quad .$$

By substituting the first two conditions into the fourth, we obtain the demand function

$$(12) \quad X = k_1 s_2 + k_2(Y-P_z s_1-tu)/P(u)$$

where

$$k_1 = c_1/(c_1 + c_2)$$

$$k_2 = c_2/(c_1 + c_2) \quad .$$

To derive the rent-distance function, we take the derivative of (6) with respect to u, or

$$(13) \quad P'(u) = aAR(u)^{a-1}R'(u) \quad ,$$

then substitute (6), (12), and (13) into (11.3) to obtain

$$aAR(u)^{a-1}R'(u)[k_1 s_2 + k_2(Y-P_z s_1-tu)/(AR(u)^a)] + t = 0$$

or

$$(14) \quad dR[k_1 s_2 aAR(u)^{a-1} + k_2 a(Y-P_z s_1-tu)/R(u)] + du(t) = 0$$

where d indicates a differential. Now making use of the integrating factor

$$R(u)^{-ak_2} \quad ,$$

we find that the general solution to the differential equation (14) is

$$R(u)^{ak_1} s_2 A - R(u)^{-ak_2}(Y-P_z s_1-tu) = K$$

where K is a constant of integration.[5] The initial condition can then be used to obtain the definite solution

$$(15) \quad s_2 A[R(u)^{ak_1} - \bar{R}^{ak_1}] - (Y-P_z s_1-tu)R(u)^{-ak_2} + (Y-P_z s_1-t\bar{u})\bar{R}^{-ak_2} = 0 \quad .$$

Since Equation (15) cannot be solved explicitly for R(u), it cannot be transformed back into P(u) or any of the other variables in an urban model. Nevertheless, we can examine the properties of the R(u) in (15). Differentiating (15) with respect to u yields

$$R'(u) = -t/D < 0$$

where

$$D = k_1 s_2 aAR(u)^{a-1} + (Y-P_z s_1-tu)k_2 aR(u)^{-1} > 0 \quad .$$

It can also be shown that[6]

$$R''(u) = t(\partial D/\partial u)/(D^2) > 0 \quad .$$

In short, although Equation (15) differs in form from other rent-distance functions that have appeared in the literature,[7] its basic properties--the signs of its slope and curvature--are the same as those of other rent-distance functions.

Without a survival quantity for X (the survival quantity for Z causes no analytical difficulties and is retained), we simply replace

(12) with

$$(16) \qquad X = k_2(Y-P_z s_1-tu)/P(u)$$

and follow the same steps as before to derive

$$R'(u)/R(u) = (1/ak_2)[-t/(Y-P_z s_1-tu)] \quad .$$

Integrating and taking the exponential of this equation and making use of the initial condition, we obtain

$$(17) \qquad R(u) = \bar{R}[(Y-P_z s_1-tu)/(Y-P_z s_1-t\bar{u})]^{1/ak_2} \quad .$$

It is easily seen that in this case, as before,

$$R'(u) < 0$$

$$R''(u) > 0 \quad .$$

As mentioned earlier, only certain types of demand functions can be derived from utility functions. One way to generalize our results without referring to a utility function is to include non-unitary price and income elasticities in the demand function (16). This is the type of demand function used, for example, by Mills, with the major difference that (Y-tu) instead of Y is now the income term. This generalized demand function takes the form

$$(18) \qquad X = k(Y-tu)^{\theta_1}P(u)^{\theta_2} \quad .$$

By combining Equation (18) with Equations (4), (6), (7), and (13), it can be shown[8] that if $\theta_2 \neq -1$,

$$(19) \quad R(u) = \left\{ \bar{R}^b + [bE/(1-\theta_1)]\,[(Y-tu)^{1-\theta_1} - (Y-t\bar{u})^{1-\theta_1}] \right\}^{1/b} ,$$

and if $\theta_2 = -1$,

$$(20) \quad R(u) = \bar{R} \exp \left\{ [E/(1-\theta_1)]\,[(Y-tu)^{1-\theta_1} - (Y-t\bar{u})^{1-\theta_1}] \right\}$$

where

$$E = (aA^{1-\theta_2}k)^{-1}$$

$$b = a(1+\theta_2) \quad .$$

In summary, the rent-distance functions given by Equations (15), (17), (19), and (20) are based either on demand functions that can be explicitly derived from a utility function or on simple generalizations of such demand functions. These rent-distance functions, like those of Muth and Mills, have negative slopes and positive curvatures; however, our analysis reveals that (Y-tu)--not simply Y as has been previously assumed--is the income term that should appear in the demand function for housing. The substitution of (Y-tu) for Y significantly changes the form of the rent-distance function (if not its basic properties), and will affect the implications of rent-distance functions in specific applications, such as the analysis of prejudice that follows.

3. RACIAL PREJUDICE AND LOCATIONAL EQUILIBRIUM

The type of prejudice considered in this paper can be thought of as a disutility of whites or blacks from living with or near members of the other race. There are two simple ways to include such prejudice in the analysis of the locational equilibrium of households in an urban area. The first method, which is found in the work of Courant

(1974), begins with the assumption that there is complete segregation in an urban area with one race living in the city center and the other living in the doughnut-shaped rest of the city. If whites get disutility from living near blacks, then some function of distance from the black-white border appears in the white utility function. The price-distance (or rent-distance) function that leaves whites in locational equilibrium can then be derived as described earlier.

Using this kind of "border model," Courant shows that households will be in locational equilibrium only if blacks live in the city center. He also shows that if there is more than one income group, rich blacks will have an incentive to "hop" over poor whites. Unfortunately, this result undermines the original assumption that all blacks live in the city center, so that the model must be re-solved with a new assumption about the pattern of racial segregation. Not only does this simultaneity between locational equilibrium and the pattern of segregation make the model unwieldy, it also undermines the single assumption about prejudice on which the model is based; since there will be many black-white borders when there are many income classes, it is no longer clear what to include in the utility function of whites.

An alternative approach, which is followed in this paper, is to assume that both blacks and whites get disutility from living with or near members of the other race--without making any assumption about the pattern of racial segregation--and then to investigate the factors that affect the locational decisions of whites and blacks. The key to this approach lies in the formulation of prejudice. As we have said, the utility of a prejudiced household will be lower if it has to live with or near members of the other race; therefore, let us begin by defining a variable that measures the degree to which a household will be with or near members of the other race at any particular location. To be specific, let us define r(u) to be a measure of the proportion of the population at and around location u that is black. The choice of race here is arbitrary; a symmetrical argument could be made using the proportion of the population that is white.

One way to define r(u) more completely is to say that it consists of a weighted sum of the racial compositions of the neighborhoods within a certain distance of u (say u*). For example, we might write

$$r(u) = \int_{u-u^*}^{u+u^*} W(u'-u)B(u')du'$$

where W is some weighting function and B(u) is the proportion of the population at u that is black; indeed, it might be desirable to use the right-hand side of the above equation in the analysis that follows, were it not for the difficulty such a procedure would add to the mathematics. In any case, we will assume that r(u) is some measure of the racial composition of a location--and in particular a proportional measure of its "blackness"--that appears in the utility functions of both blacks and whites.

For white households, the utility function takes the form

$$(21) \qquad U_w = U_w(Z_w, X_w, r(u))$$

where r(u) is the variable defined above and the "w" subscript indicates "white." It is clear that if whites are prejudiced the marginal utility of r(u) is negative.

It will prove useful to express Equation (21) in a somewhat different form in order to isolate the relationship between X and r(u). According to the view of the housing market used in this paper, the quantity of housing services that appears in a household's utility function depends on the quantitative and qualitative characteristics of the household's dwelling unit and neighborhood.[9] For a prejudiced household, r(u) is one of the neighborhood characteristics that affect housing services; consequently, we can write

$$(22) \quad H_w = H_w(X_w, r(u)) \quad ,$$

where H is the number of units of housing services and X represents the non-racial characteristics of housing. Plugging Equation (22) into a utility function yields

$$(23) \quad U_w = U_w(Z_w, H_w) \quad .$$

This modest reformulation of Equation (21) allows us to specify several different forms for the interaction between X and r(u)--via Equation (22)--and still make use of simple separable forms for the utility function of a prejudiced household.

One straightforward form for the function H_w is

$$H_w = H_w r(u)^{-d_w} \quad .$$

This form is not acceptable, however, because it implies that when r(u)=0 (that is, when only whites live at u), H_w is equal to infinity. Prejudice is, to be sure, a powerful feeling, but I doubt that "whiteness" is infinitely valued by prejudiced whites.

Another possible form is

$$H_w = X_w(1-r(u))^{d_w} ,$$

but this form goes to another extreme: it implies that the number of units of housing services received by a white would approach zero as r(u) approached one.

A functional form that avoids these problems is

$$(24) \quad H_w = H_w \exp[-d_w r(u)] \quad .$$

In this case, H_w equals X_w in an all-white neighborhood and approaches $[X_w/\exp(d_w)]$ as r(u) approaches one. This form also implies that the change in housing services will increase with the quantity of housing services in the dwelling and decrease with the size of the neighborhood.[10] In other words, a black neighbor will have a greater impact on housing services (and hence on utility) for the owner of a fancy house (that is, one that contains a large quantity of housing services) than for the owner of a plain house, and a smaller impact in a large neighborhood than in a small one.

Plugging Equation (24) into a Cobb-Douglas utility function yields

$$(25) \quad U_w = c_1 \log(Z_w) + c_2 \log(X_w e^{-d_w r(u)})$$

$$= c_1 \log(Z_w) + c_2 \log(X_w) - c_w r(u)$$

where $c_w = d_w c_2$. Thus a white household's maximization problem is to

$$(26) \quad \begin{array}{ll} \text{Maximize} & U_w \\ \text{Subject to} & Y = P_z Z_w + P_w(u) X_w + tu. \end{array}$$

It is important to note that $r(u)$ does not appear in the budget constraint of this problem. It is well known that in the long run the implicit price of a housing characteristic is equal to its marginal production cost. This conclusion applies to the physical characteristics of a house and to the neighborhood characteristics associated with that house. Furthermore, if neighborhoods with a certain characteristic can be reproduced in the long run, then, for houses built in such neighborhoods, there will not be any marginal cost associated with that characteristic.[11] Since neighborhoods with any given racial composition can be reproduced in the long run, the implicit price of $r(u)$ will be zero.

In the short run, when $r(u)$ has a non-zero implicit price, H_w replaces X_w in the budget constraint of problem (26); however, no matter what the form of the H_w-function (Equation (22)), $r(u)$ does not affect the locational equilibrium condition in the short run. This result is proved in Note 4 of the Mathematical Appendix.

The Lagrangian expression for problem (26) is

$$L = U_w + \lambda(Y - P_z Z_w - P_w(u)X_w - tu)$$

and the first-order conditions are

$$(27.1) \quad \partial L/\partial Z_w = c_1/Z_w - \lambda P_z = 0$$

$$(27.2) \quad \partial L/\partial X_w = c_2/X_w - \lambda P_w(u) = 0$$

$$(27.3) \quad \partial L/\partial u = -c_w r'(u) - \lambda\ (P_w'(u)X_w + t) = 0$$

$$(27.4) \quad \partial L/\partial \lambda = Y - P_z Z_w - P_w(u)X_w - tu = 0 \quad .$$

Since the introduction of prejudice has only affected the locational equilibrium condition (27.3), the demand function that is derived from conditions (27.1), (27.2), and (27.4) is the same as the function derived without considering prejudice (Equation (16)):[12]

$$(28) \qquad X_w = k_2(Y-tu)/P_w(u)$$

where

$$k_2 = c_2/(c_1+c_2) \quad .$$

The substitution of condition (27.2) and the demand function (28) into the locational equilibrium condition (27.3) yields

$$c_2 P_w'(u)/P_w(u) = (c_1+c_2)\,(-t)/(Y-tu) - c_w r'(u) \quad .$$

Integrating and taking the exponential of this equation, we find that

$$(30) \qquad P_w(u) = [e^{-c_w r(u)}\,(Y-tu)^{c_1+c_2}/K_w]^{1/c_2}$$

where K_w is a constant of integration. The rent-distance function corresponding to Equation (29) is found, using Equations (6) and (7), to be

$$(31) \qquad R_w(u) = \bar{R}\; e^{d_w(r(\bar{u})-r(u))}\,[(Y-tu)/(Y-t\bar{u})]^{1/ak_2} \quad .$$

Equations (30) and (31) describe, respectively, the price- and rent-distance functions that, for a given racial distribution r(u), would make prejudiced whites indifferent to their location; in addition to declining with distance from the CBD, the equilibrium rent-distance function for prejudiced whites must also be lower at locations with higher concentrations of blacks.

Prejudiced blacks also choose how much housing to buy and where to live. A plausible H-function for blacks is

$$(32) \quad H_b = X_b \exp[d_b(r(u)-1)] \quad .$$

This function indicates that in an all-black neighborhood H_b equals X_b, and as a neighborhood approaches "whiteness," H_b approaches $[X_b/\exp(d_b)]$.

The utility function for blacks is thus

$$(33) \quad U_b = c_1 \log(Z_b) + c_2 \log(X_b e^{-d_b(r(u)-1)})$$

$$= c_1 \log(Z_b) + c_2 \log(X_b) + c_b(r(u)-1)$$

where $c_b = c_2 d_b$. Since adding a constant to a utility function is a monotonic transformation, we can rewrite (33) as

$$(34) \quad U_b = c_1 \log(Z_b) + c_2 \log(X_b) + c_b r(u) \quad .$$

Black households maximize this utility function subject to a budget constraint that, except for the subscript "w," is the same as that faced by whites. Furthermore, the only difference between the black and white utility functions [Equations (25) and (34), respectively] is that r(u) enters the former with a coefficient of c_b and the latter with a coefficient of $(-c_w)$; thus it can easily be seen that the locational equilibrium condition for blacks that is analogous to Equation (31) for whites is

$$(35) \quad R_b(u) = \bar{R}\, e^{-d_b(r(\bar{u})-r(u))} [(Y-tu)/(Y-t\bar{u})]^{1/ak_2} \quad .$$

For any given r(u), prejudiced blacks will be indifferent to their location if Equation (35) is satisfied.

4. RACIAL EQUILIBRIUM

In order for both prejudiced blacks and prejudiced whites to be in locational equilibrium, Equations (31) and (35) must be satisfied simultaneously; in this section, we will derive an r(u) function that makes such a result possible. If blacks and whites with a given income have the same tastes, aside from their prejudice, then the two rent-distance functions will both be satisfied only if

$$R_w(u)e^{-d_w(r(\bar{u})-r(u))} = \bar{R}[(Y-tu)/(Y-t\bar{u})]^{1/ak_2}$$
$$= R_b(u)e^{d_b(r(\bar{u})-r(u))} \quad .$$

Thus it must also be true that

$$R_b(u)/R_w(u) = e^{(d_w+d_b)(r(u)-r(\bar{u}))}$$

and

$$(36) \qquad r(u) = [\log(R_b(u)/R_w(u))]/(d_w+d_b) + r(\bar{u}) \quad .$$

This equation describes the function r(u) that will keep both blacks and whites in locational equilibrium, given the rent-distance functions (31) and (35). When Equation (36) holds we will say that an urban area is in racial equilibrium.

The key to Equation (36) is the term R_b/R_w. Under perfect competition, a factor that can be freely transferred from one use to another will earn the same return in both uses. In the short run, there is undoubtedly some cost to transferring land from the production of housing in white neighborhoods to the production of housing in black neighborhoods (that is, changing the racial composition of the neighborhood around a given unit of land[13]), but in the long run--and this is a long-run model--these transfer costs will disappear. Thus if both races live at u, R_b/R_w equals unity, $\log(R_b/R_w)$ equals zero, and

$$(37) \quad r(u) = r(\bar{u})$$

Equation (37) indicates that, given our assumptions about prejudice, the only continuous function r(u) that insures that both blacks and whites will be in locational equilibrium is one in which r is constant for all values of u. Since a complete urban model would include conditions guaranteeing that all blacks and all whites be supplied with housing, this result is equivalent to the statement that, at all values of u, r(u) must be equal to the ratio of the total number of blacks to the total population of the urban area. Note that if r(u) is a constant, the equilibrium price-distance function reflects, as it does when prejudice is not considered, the higher transportation costs at higher values of u, and the constant value for r(u) guarantees that no household can gain utility by moving away from the race against which it is prejudiced.

Although Equation (37) describes the only continuous racial equilibrium, it is by no means the only racial equilibrium when there is prejudice. In fact, in this model any completely segregated solution--any solution in which only blacks or only whites live at each distance--will have the same price-distance function as the model without prejudice and will be an equilibrium. Furthermore, such segregated solutions clearly represent a gain in utility for both blacks and whites; the trade-off between housing costs and transportation costs is the same for the integrated solution as for any such segregated solution, but in the case of segregated solutions no household has any disutility from living with members of the race against which it is prejudiced. In other words, if both groups are prejudiced, complete segregation is Pareto-superior to integration.

The logic of the racial equilibrium condition also tells us something about the dynamics of neighborhood change in this model. Starting from an integrated equilibrium, a small increase in the proportion of the population that is black at a given distance will give blacks an incentive to move to that location and whites an incentive to move away from it. Such moves will change the racial composition of other locations and, in turn, stimulate more moving. This process will continue until some completely segregated solution is reached. The model does not indicate, however, what the resulting segregated solution will look like. Therefore, unless everyone expects integration to be enforced by, say, the government, the integrated equilibrium is highly unstable; in the long run, prejudice of the form we have described is almost certain to lead to complete segregation.

It is also interesting to note that Equation (37) is the appropriate condition for racial equilibrium in the case of reverse prejudice--when either blacks or whites (or both) <u>prefer</u> to live with members of the other race. According to our formulation, reverse prejudice simply involves a change in the sign of the coefficient of $r(u)$ in the utility function of the group or groups with reverse prejudice; the derivation of Equation (36) is therefore still appropriate. As long as d_w is not equal to $(-d_b)$, the first term of Equation (36) will equal zero and perfect integration will be the only continuous racial equilibrium.

The equality of d_w and $(-d_b)$ represents the unlikely situation in which whites and blacks have identical tastes for racial composition. In this case any r(u) is consistent with racial equilibrium. Note also that reverse prejudice eliminates the possibility of a segregated equilibrium, since households with reverse prejudice have an incentive to move into areas inhabited by the other race. In short, the only racial equilibrium when there is reverse prejudice is the unstable equilibrium of perfect integration.

The results of recent surveys of the attitudes of urban blacks indicate that blacks differ on the neighborhood racial composition they prefer. Many blacks prefer racially mixed neighborhoods; others want to live in all-black neighborhoods.[14] These surveys are summarized by Pettigrew (1973). Thus it is appropriate to include groups of blacks with different tastes in our model and to add a third category--preference for a racially mixed neighborhood--to the two extreme categories of prejudice and reverse prejudice.[15] To be specific, if two groups of blacks, one with prejudice and one with a preference for a racially mixed neighborhood, are included in the preceding analysis, it is clear that the perfectly integrated solution is still an unstable equilibrium. Furthermore, no combination of segregated and integrated regions in an urban area will be a stable equilibrium. Prejudiced blacks will be in equilibrium when they are segregated from whites, but blacks who prefer integrated neighborhoods will not be in equilibrium unless they are living with whites. If some integration does take place, however, a small decrease in r(u) in one of the integrated neighborhoods would give whites an incentive to move to that location. This would cause changes in $r(u)$ at other locations, thereby causing other moves, and so on. Thus the combination of prejudice and either reverse prejudice or a preference for racially mixed areas is an unstable combination: no race-distance function will prove a stable locational equilibrium for every group.

The addition of more than one income group does not significantly change these results. Each income group will live in that range of values of u where its rent-distance function is higher than that of any other group. Within each income group, prejudice (or reverse prejudice) will affect location in the manner we have described for one income group. The perfectly segregated solution will involve a different proportion of blacks for different income classes, but a constant proportion of blacks throughout the distance occupied by any given class. The list of segregated solutions will include any combination of all-black and all-white locations that does not involve the mixing of income classes.

It will prove instructive to conclude this discussion of racial equilibrium by examining another possible type of racial equilibrium: one in which Equation (31) holds in some locations and Equation (35) holds in other locations. For example, take the case in which locations with a white majority are located in the outer part of the city and have a rent-distance function given by (30), whereas the centralized black locations have the rent-distance function (35). In this situation, competition would insure that at the border between the black and white areas, rent would be the same when calculated by either function. Thus the initial conditions for the rent-distance functions would be:

$$\text{for } R_w(u): \quad R_w(\bar{u}) = \bar{R} \quad ;$$

$$\text{for } R_b(u): \quad R_b(u^*) = R_w(u^*) \quad ,$$

where u^* is the border between the two areas.

This case can be illustrated in a diagram as follows:

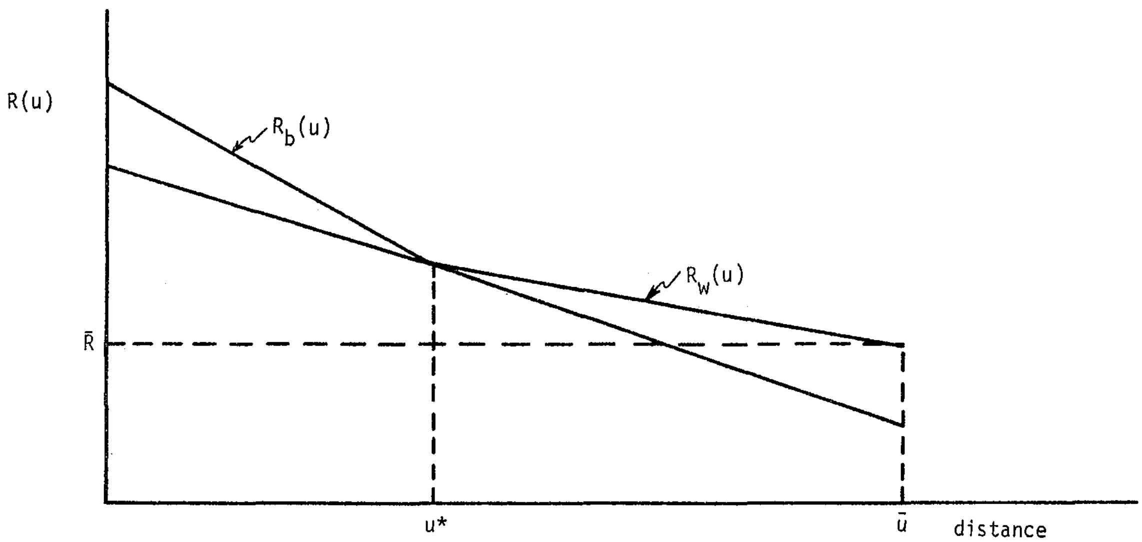

Figure 1.

An analysis of this diagram reveals that it cannot represent an equilibrium. If everyone is prejudiced, then no one will be willing to pay the land rent in the area where a majority of the residents are of the other race; complete segregation will inevitably result. And we have already shown that if some blacks want to live in racially mixed areas, those blacks and whites will both be in equilibrium only if the blacks are evenly distributed throughout the white area. In either case, the racial term will drop out of the rent-distance function. Note that these results will hold for any combination of Equations (30) and (35), not just for the example presented here.

In summary, the analysis in this section results in four main conclusions about racial equilibrium when prejudice takes the form we have postulated:

1. If there is complete segregation or perfect integration, racial composition will not affect the rent-distance function.
2. Complete segregation is a stable racial equilibrium only in the case of prejudice on the part of all blacks and all whites.
3. Perfect integration is an unstable racial equilibrium in the case of prejudice, reverse prejudice, or the desire to live in a racially mixed area.
4. If any group of blacks or whites has reverse prejudice or the desire to live in a racially mixed area, then there exists no stable racial equilibrium.

5. AN ALTERNATIVE SPECIFICATION OF PREJUDICE

In the preceding section a multiplicative form was used for the H-function (Equation (22)) in order to derive results about locational equilibrium; in this section we will show that the same results an be obtained using an additive form. An additive specification of the

H-function for white consumers can be written

$$(38) \qquad H_w = X_w - a_w r(u)$$

so that H_w equals X_w in an all-white neighborhood and approaches $(X_w - a_w)$ as a neighborhood becomes all-black. Equation (38) implies that the effect of a black neighbor on H_w decreases with neighborhood size, but, unlike the multiplicative form, it also implies that the effect of an additional black neighbor does not depend on the level of H_w.

When Equation (38) replaces Equation (24) in the maximization problem (26), one can derive, as shown in Note 5 of the Mathematical Appendix, the following locational equilibrium condition for whites:[16]

$$(39) \qquad a_w[P(u)^{k_1} r(u) - \bar{P}^{k_1}\bar{r}] - [(Y-tu)P(u)^{-k_2} - (Y-t\bar{u})\bar{P}^{-k_2}] = 0 \quad .$$

An additive H-function for black consumers takes the form

$$(40) \qquad H_b = X_b - a_b(1-r(u))$$

so that H_b equals X_b in an all-black neighborhood and approaches $(X_b - a_b)$ as a neighborhood becomes all-white. By plugging (40) into a maximization problem for a black consumer analogous to problem (26) for whites, one obtains, as shown in Note 5, the locational equilibrium condition:

$$(41) \qquad a_b[P(u)^{k_1}(1-r(u)) - \bar{P}^{k_1}(1-\bar{r})] - [(Y-tu)P(u)^{-k_2} - (Y-tu)\bar{P}^{-k_2}] = 0 \quad .$$

The racial equilibrium condition, which is derived in Note 5 by equating (39) and (41), is

$$(42) \qquad r(u) = (\bar{P}/P(u))^{k_2}(\bar{r} - A_b) + A_b$$

where

$$A_b = a_b/(a_b+a_w) \quad .$$

Although this condition is somewhat difficult to interpret, it is shown in Note 5 that Equation (42) is consistent with locational equilibrium for both blacks and whites if and only if

(43) $r(u) = \text{constant.}$

Equation (43) implies that with an additive H-function perfect integration represents a racial equilibrium. Furthermore, inspection of Equations (39) and (41) reveals that if Equation (43) holds, then r(u) drops out of the locational equilibirum condition. The analysis of the multiplicative case in the preceding section can also be used in the additive case to show that complete segregation is a stable racial equilibrium when all blacks and all whites are prejudiced and that there exists no stable racial equilibrium when some group has reverse prejudice or a preference for integration. In short, all four of the conclusions on page 25 are valid for both multiplicative and additive H-functions.

6. PREJUDICE AND DISCRIMINATION

Although discrimination against blacks has not been considered in the derivation of racial equilibrium conditions, the analysis of those conditions provides two important insights into the phenomenon of discrimination.

First, we have shown that as long as some blacks want to live in racially mixed areas, there is no stable locational equilibrium in areas inhabited entirely or partly by whites: if the price-distance function in those areas reflects white prejudice, then blacks who prefer mixed neighborhoods cannot be in equilibrium; and if the price-distance function does not reflect white prejudice, then whites will want to move to those areas with the fewest blacks. In either case, whites will be uncertain about the future racial composition of their neighborhoods. To the degree that this type of uncertainty involves disutility for whites--and I suspect that it involves considerable disutility--whites will have an incentive to discriminate against blacks by restricting them to certain areas. If such restrictions are possible, then an equilibrium can be attained when r(u) is determined by discrimination against blacks and the rent-distance function is given by Equation (31). It is appropriate, therefore, to restate the fourth conclusion from Section 4 as follows:

> 4'. If any group of blacks or whites has reverse prejudice or the desire to live in a racially mixed area, then <u>there exists no stable racial equilibrium without discrimination</u>. If discrimination against one group is possible, then an equilibrium can be obtained when r(u) is determined by discrimination and the price-distance function is the one derived above for the discriminating group.

The second insight provided by our analysis is that r(u) drops out of the price-distance function for every equilibrium that does not involve discrimination; therefore, if r(u) is found to have a significant coefficient in an empirically determined price-distance function,[17] it follows that either

a. the area is not in locational equilibrium, or

b. there is discrimination.

If one has reason to believe that the area under study is close to locational equilibrium, then one can infer something about the nature of the discrimination that is taking place. To be specific, a price-distance function that takes the form given by Equation (30) implies that $r(u)$ is determined by discrimination against blacks and that the price-distance function keeps whites in locational equilibrium. (Similarly, an empirically determined price-distance function that takes the form given by (35) implies that there is discrimination against whites.)

We have shown that the only way to obtain a stable pattern of racial composition in an urban area in the long run is by discrimination. Thus, to the extent that stability is valued by the white community, whites will have an incentive to discriminate against blacks. Another way of stating this result is that stability is a public good for the white community that can be purchased with discrimination. A discussion of the institutions that have developed for the purpose of purchasing this public good is beyond the scope of this paper; suffice it to say that the preponderance of stable white suburban communities testifies to the success of those institutions.

MATHEMATICAL APPENDIX

Note 1

In this note we will prove that

$$\partial^2 R/\partial u^2 > 0$$

where

$$\text{(A1)} \quad s_2 A[R(u)^{ak_1} - \bar{\bar{R}}^{ak_1}] - (Y^*-tu)R(u)^{-ak_2} + (Y^*-tu)\bar{\bar{R}}^{-ak_2} = 0$$

and

$$Y^* = Y - P_z s_1 \quad .$$

Proof. Taking the derivative of (A1) with respect to u, we find that

$$ak_1 s_2 AR(u)^{ak_1-1} R'(u) + ak_2(Y^*-tu)R(u)^{-ak_2-1} R'(u) + tR(u)^{-ak_2} = 0$$

or, since $k_1 = 1-k_2$,

$$R'(u)[ak_1 s_2 AR(u)^{a-1} + ak_2(Y^*-tu)R(u)^{-1}] = -t \quad .$$

This equation can be rewritten as

$$\text{(A2)} \quad R'(u) = -t/D$$

where

$$D = ak_1 s_2 AR(u)^{a-1} + ak_2(Y^*-tu)R(u)^{-1} \quad .$$

Now since R(u)--the price of land--is always positive and since all of a consumer's income is not spent on transportation so that (Y*-tu) is positive, D will always be greater than zero; therefore, by (A2),

$$R'(u) < 0 \quad .$$

Taking the derivative of (A2) with respect to u, we have

(A3) $R''(u) = t(\partial D/\partial u)/(D^2)$

where

(A4) $$\partial D/\partial u = (a-1)ak_1 s_2 AR(u)^{a-2}R'(u)$$

$$- ak_2(Y^*-tu)R(u)^{-2}R'(u) - tak_2 R(u)^{-1} \quad .$$

Thus R''(u) will be positive whenever (A4) is positive and (A4) will be positive if

$$R'(u)[(a-1)s_2 AR(u)^{a-1} - ak_2(Y^*-tu)R(u)^{-1}] > tak_2 \quad ;$$

that is, if

(A5) $-R'(u)[E] > tak_2$

where

$$E = (1-a)ak_1 s_2 AR(u)^{a-1} - ak_2(Y^*-tu)R(u)^{-1} \quad .$$

To determine when (A5) will hold, note from (A2) that

$$-R'(u)[D] = t$$

or

$$-R'(u)[D(1-a)] = t(1-a) \quad .$$

Now, by definition,

$$E = (1-a)K_1 - K_2$$

and

$$D(1-a) = (1-a)K_1 + (1-a)K_2$$

where

$$K_1 = ak_1 s_2 AR(u)^{1-a}$$

$$K_2 = ak_2(Y^*-tu)R(u)^{-1} \quad ;$$

therefore, since both a and k_2 are positive,

$$E < D(1-a)$$

and

$$\text{(A6)} \quad R'(u)E > R'(u)[D(1-a)] - t(1-a) \quad .$$

Furthermore, since

$$a \cong .2 \text{ (see Mills, 1972, p. 80)}$$

$$K_2 = c_2/(c_1+c_2) < 1 \quad ,$$

then

$$\text{(A7)} \quad tak_2 < t(1-a)$$

and

$$\text{(A8)} \quad R'(u)E > t(1-a) > tak_2 \quad .$$

Thus condition (A5) is fulfilled and $R''(u) > 0$. Q.E.D.

Note that a sufficient condition for (A7)—and hence for (A5)—to hold is that a be less than or equal to 0.5.

Note 2

Our task in this note is to derive a rent-distance function using the demand function

$$(18) \quad X = k(Y-tu)^{\theta_1}P(u)^{\theta_2} \quad .$$

The other relevant equations are

$$(6) \quad P(u) = AR(u)^a$$

$$(13) \quad P'(u) = aAR(u)^{a-1}R'(u)$$

$$(4) \quad P'(u)X + t = 0$$

$$(7) \quad R(\bar{u}) = \bar{R} \quad .$$

Plugging (18), (6), and (13) into (4), we obtain

$$(A10) \quad aAR(u)^{a-1}R'(u)k(Y-tu)^{\theta_1}[AR(u)^a]^{\theta_2} + t = 0$$

or

$$aA^{1+\theta_2}kR'(u)R(u)^{B-1} = -t/[(Y-tu)^{\theta_1}]$$

where

$$B = a(1+\theta_2) \quad .$$

Rearranging, this equation becomes

$$(A11) \quad R(u)^{B-1}R'(u) = E(-t)(Y-tu)^{-\theta_1}$$

where

$$E = (aA^{1+\theta_2}k)^{-1} \quad .$$

Now if $B \neq 0$ (that is, if $\theta_2 \neq -1$), integrate both sides of (A11) to find that

$$R(u)^B/B + C_1 = E(Y-tu)^{1-\theta_1}/(1-\theta_1) + C_2$$

or

$$\text{(A12)} \quad R(u) = [BE(Y-tu)^{1-\theta_1}/(1-\theta_1) + BC]^{1/B}$$

where $C = C_2 - C_1$ is a constant of integration. Now using the initial condition (7) to solve (A12) for C, we have

$$R(\bar{u}) = \bar{R} = [BE(Y-t\bar{u})^{1-\theta_1}/(1-\theta_1) + BC]^{1/B}$$

or

$$\text{(A13)} \quad C = (\bar{R}^B/B) - E(Y-t\bar{u})^{1-\theta_1}/(1-\theta_1) \quad .$$

Plugging (A13) back into (A12) yields

$$\text{(A14)} \quad R(u) = \bar{R}^B + BE[(Y-tu)^{1-\theta_1} - (Y-t\bar{u})^{1-\theta_1}]/(1-\theta_1)^{1/B} \quad .$$

Mills's result (1972, p. 83, eq. 5-14a), which uses Y instead of (Y-tu) in (18), is

$$R(u) = [\bar{R}^B + BtE(\bar{u}-u)]^{1/B} \quad .$$

If $B = 0$ (that is, if $\theta_2 = -1$), then (A11) becomes

$$\text{(A15)} \quad R'(u)/R(u) = E(-t)(Y-tu)^{-\theta_1} \quad .$$

Integrating, we find that

$$\log[R(u)] + C_1 = E(Y-tu)^{1-\theta_1}/(1-\theta_1) + C_2$$

or

$$\text{(A16)} \quad R(u) = C \exp[E(Y-tu)^{1-\theta_1}/(1-\theta_1)]$$

where $C = \exp(C_2-C_1)$ is a constant of integration.

Solving for C using (7), we find that

$$R(\bar{u}) = \bar{R} = C \exp[E(Y-t\bar{u})^{1-\theta_1}/(1-\theta_1)]$$

or

$$\text{(A17)} \quad C = \bar{R} \exp[-E(Y-t\bar{u})^{1-\theta_1}/(1-\theta_1)] \quad .$$

Thus

$$\text{(A18)} \quad R(u) = \bar{R} \exp \left\{ E[(Y-tu)^{1-\theta_1} - (Y-t\bar{u})^{1-\theta_1}]/(1-\theta_1) \right\} \quad .$$

This can be compared with Mills's result (1972, p. 83, eq. 5-14b):

$$R(u) = \bar{R} \exp[tE(\bar{u}-u)] \quad .$$

Note 3

In this note we will show (a) that the second order conditions of problem (1) in the text require that $P''(u)$ be positive, and (b) that if $R''(u)$ is positive, $P''(u)$ will also be positive.

By totally differentiating the first-order conditions (2), one obtains the following bordered Hessian for problem (1):

$$\begin{vmatrix} U_{ZZ} & U_{ZX} & 0 & -P_z \\ U_{XZ} & U_{XX} & -\lambda P' & -P \\ 0 & -\lambda P' & -\lambda XP'' & -(P'X+t) \\ -P_z & -P & -(P'X+t) & 0 \end{vmatrix} = |H|$$

Since, for a maximum, the principal minors of this Hessian must be alternately positive and negative starting with $|H_2|$, and since, by (2.3), $P'X+t = 0$, we know that a maximum requires that

$$|H_2| = \begin{vmatrix} U_{XX} & -\lambda P' & -P \\ -\lambda P' & -\lambda XP'' & 0 \\ -P & 0 & 0 \end{vmatrix} > 0$$

or

$$P^2\lambda XP'' > 0 \; .$$

Since λ and X are positive, this condition is equivalent to

$$P'' > 0 \; .$$

Now since $P = AR^a$,

$$P' = aAR^{a-1}R'$$

and

$$P'' = a(a-1)AR^{a-2}R' + aAR^{a-1}R''$$

$$= aAR^{a-1}[(a-1)R'/R + R''] \quad .$$

Furthermore, since $0 < a < 1$; $R' < 0$; and $A, R > 0$; P'' will clearly always be positive if R'' is positive.

Note 4

In the short run, a white household attempts to

Maximize $\quad U(Z,H) = c_1 \log(Z) + c_2 \log(H_w)$

Subject to $\quad Y = P_z Z + P(u)H_w + tu$

where

$$H_w = H_w(X_w, r(u)) \quad .$$

In this note we will show that the short-run locational equilibrium condition derived from this problem does not contain racial composition as an argument.

The Lagrangian for the above problem is

$$L = c_1 \log(Z) + c_2 \log(H_w) + \lambda[Y - P_z Z - P(u)H_w - tu]$$

and the first-order conditions are

$$\partial L/\partial Z = c_1/Z - \lambda P_z = 0$$

$$\partial L/\partial X = (c_2/H_w)(\partial H_w/\partial X) - \lambda P(u)(\partial H_w/\partial X) = 0$$

$$\partial L/\partial u = c_2(\partial H_w/\partial r)r^1/H_w - \lambda P'H_w - \lambda P(\partial H_w/\partial r)r^1 - \lambda t = 0$$

$$\partial L/\partial \lambda = Y - P_z Z - P(u)H_w - tu = 0 \quad .$$

The first two conditions can be used to eliminate λ as follows:

$$c_1/(P_z Z) = c_2/PH_w$$

or

$$Z = c_1 PH_w/(c_2 P_z) \quad .$$

Substituting for Z in the fourth condition, we obtain the demand function:

$$Y - c_1 PH_w/c_2 - PH_w - tu = 0$$

or

$$\text{(A19)} \quad H_w = c_2(y-tu)/[P(c_1+c_2)] \quad .$$

Now by eliminating λ from the locational equilibrium condition (the third first-order condition above), we find that

$$c_2(\partial H_w/\partial r)r^1/H_w - (c_2/PH_w)(P'H_w) - (c_2/PH_w)[P(\partial H_w/\partial r)r^1]$$
$$- (c_2/PH_w)t = 0$$

or

$$c_2(\partial H_w/\partial r)r^1/H_w - c_2P'/P - c_2(\partial H_w/\partial r)r^1/H_w - c_2t/PH_w = 0$$

or

$$\text{(A20)} \quad c_2P'/P - c_2t/PH_w = 0$$

Plugging in the demand function, the locational equilibrium condition becomes

$$c_2P'/P - c_2tP(c_1+c_2)/c_2P(y-tu) = 0 \quad .$$

or

$$c_2P'/P - t(c_1+c_2)/(Y-tu) \quad .$$

Thus r(u) drops out: racial composition does not appear in the short-run locational equilibrium condition.

Note 5

In this note a long-run racial equilibrium condition is derived for the case of an additive H-function.

The white consumer's problem is to

$$\text{Maximize } U(Z_w, H_w)$$

$$= c_1 \log(Z_w) + c_2 \log(H_w)$$

$$\text{Subject to } H_w = X_w - a_w r(u)$$

$$Y = P_z Z_w + P(u)X_w + tu \quad .$$

The Lagrangian for this problem is

$$L = c_1 \log(Z_w) + c_2 \log(X_w - a_w r(u))$$

$$+ \lambda \; (Y - P_z Z_w + P(u)X_w + tu)$$

and the first-order conditions are

(A21) $\partial L/\partial Z_w = c_1/Z_w - \lambda P_z = 0$

(A22) $\partial L/\partial X_w = c_2/H_w - \lambda P(u) = 0$

(A23) $\partial L/\partial u = (c_2/H_w)(-a_w r') - \lambda(P'X_w + t) = 0$

(A24) $\partial L/\partial \lambda = Y - P_z Z_w - P(u)X_w - tu = 0$

The demand function, which is derived by using (A21) and (A22) to eliminate λ and Z_w from (A24), is

(A25) $X_w = k_2(Y-tu)/P(u) + k_1 a_w r(u)$

where

$$k_1 = c_1/(c_1+c_2)$$

$$k_2 = c_2/(c_1+c_2) \quad .$$

The locational equilibrium condition is then derived by substituting (A22) and (A25) into (A23) to obtain

$$a_w r' + (P'/P(u))[k_2(Y-tu)/P(u) + k_1 a_w r(u)] + t/P(u) = 0$$

or

(A26) $dr[a_w P(u)] + dP[k_2(Y-tu)/P(u) + k_1 a_w r(u)] + du[t] = 0$

where d indicates a differential.

Using the integrating factor

$$P(u)^{-k_2} \quad ,$$

the solution to this total differential equation is found to be

$$\text{(A27)} \quad a_w P(u)^{1-k_2} r(u) - (Y-tu)P(u)^{-k_2} = K_w$$

where K_w is a constant of integration. The initial condition for (A27) is

$$\text{(A28)} \quad P(\bar{u}) = \bar{P}$$

so that the market locational equilibrium condition is

$$\text{(A29)} \quad a_w[P(u)^{k_1} r(u) - \bar{P}^{k_1}\bar{r}] - [(Y-tu)\ P(u)^{-k_2} - (Y-t\bar{u})\bar{P}^{-k_2}] = 0 \ .$$

The black consumer's problem is to

$$\text{Maximize} \quad U(Z_b, H_b) = c_1 \log(Z_b) + c_2 \log(H_b)$$

$$\text{Subject to} \quad H_b = X_b - a_b(1-r(u))$$

$$Y = P_z Z_b + P(u)X_b + tu \ .$$

The Lagrangian for this problem is

$$L = c_1 \log(Z_b) + c_2 \log[X_b - a_b(1-r(u))]$$

$$+ \lambda(Y - P_z Z_b - P(u)X_b - tu)$$

and the first-order conditions are

$$\text{(A30)} \quad \partial L/\partial Z_b = c_1/Z_b - \lambda P_2 = 0$$

$$\text{(A31)} \quad \partial L/\partial X_b = c_2/H_b - \lambda P(u) = 0$$

$$\text{(A32)} \quad \partial L/\partial u = (c_2/H_b)a_b r' - \lambda(P'X_b+t) = 0$$

$$\text{(A33)} \quad \partial L/\partial \lambda = Y-P_z Z_b - P(u)X_b - tu = 0 \ .$$

Following the same steps as with the white consumer's problem, one can derive a demand function

$$(A34)\quad X_b = k_2(Y-tu)/P(u) + a_b k_1(1-r(u))$$

and a locational equilibrium condition

$$a_b r' - (P'/P(u))[k_2(Y-tu)/P(u) + a_b k_1(1-r(u))] - t/P(u) = 0$$

or

$$(A35)\quad dr[a_b P(u)] - dP[k_2\ (Y-tu)/P(u) + k_1 a_b(1-r(u))] - du[t] = 0 \quad .$$

Using the integrating factor, $P(u)^{-k_2}$, the solution to (A34) is found to be

$$(A36)\quad a_b P(u)^{1-k_2} r(u) + (Y-tu)P(u)^{-k_2} - a_b P(u)^{1-k_2} = K_b$$

where K_b is a constant of integration.

Using the initial condition (A28), this becomes

$$(A37)\quad a_b[P(u)^{k_1}(1-r(u)) - \bar{P}^{k_1}(1-\bar{r})] - [(Y-tu)P(u)^{-k_2} - (Y-t\bar{u})\bar{P}^{-k_2}] = 0 \quad .$$

The racial equilibrium condition is found by equating (A29) and (A37) and solving for r(u). Thus,

$$a_w[r(u)P(u)^{k_1} - \bar{r}\ \bar{P}^{k_1}] = a_b[P(u)^{k_1}(1-r(u)) - \bar{P}^{k_1}(1-\bar{r})]$$

or

$$(A38)\quad r(u) = (\bar{P}/P(u))^{k_1}(\bar{r}-A_b) + A_b$$

where

$$A_b = a_b/(a_w + a_b) \quad .$$

Finally, we will prove that (A38) implies that there will be racial equilibrium if and only if r(u) is constant.

If r(u) is constant, there will clearly be racial equilibrium because in that case r(u) drops out of the market locational equilibrium conditions (A29) and (A37).

The "only if" part of the proof is more complicated. We will proceed by showing that a non-constant P(u) function leads to a contradiction. If there is to be racial equilibrium, then the individual locational equilibrium conditions for whites and blacks, (A26) and (A35), must both be satisfied, that is, it must be true that

$$\text{(A39)} \quad r'a_w P(u) + P'k_1\, a_w r(u) = -P'k_2(Y-tu)/P(u) - t =$$

$$- r'a_b P(u) + P'k_1\, a_b[1-r(u)] \quad .$$

Substituting (A38) and its derivative with respect to u into (A39), we find that

$$\text{(A40)} \quad (\bar{P}/P(u))^{k_2}(\bar{r}-A_b)(k_1-k_2)P'(a_w+a_b)$$

$$- k_1 P'(a_b(1-A_b)-a_w A_b) = 0 \quad .$$

But since

$$a_b(1-A_b) - a_w A_b = a_b aw/(a_b+a_w) - a_w ab/(a_b+aw) = 0 \quad ,$$

the second term in (A40) drops out and one can substitute (A38) into (A40) to obtain

$$(r(u)-A_b)(k_1-k_2)P'(a_w+a_b) = 0$$

or

$$\text{(A41)} \quad r(u) = A_b \quad .$$

Since A_b is a constant, (A41) contradicts our assumption that $r(u)$ is not constant. Thus racial equilibrium is possible if and only if $r(u)$ is constant.

FOOTNOTES

[1]For a more complete discussion of these terms, see Simpson and Yinger, 1972, ch. 1.

[2]Thurow, for example, lists seven types of discrimination of interest to economists (1969, pp. 117-118).

[3]For one proof that higher-income groups live farther from the CBD, see Mills (1972, pp. 85-88). See also the derivation of Equation (14) in Muth (1969, p. 30).

[4]In performing this maximization problem, we are implicitly assuming that the consumer has at least enough income to purchase the survival quantities of Z and X.

[5]In checking this result it is helpful to note that $k_1 + k_2 = 1$.

[6]A proof is given in Note 1 of the Mathematical Appendix. Note 2 shows that the second-order conditions require a positive curvature for P(u) and that if R"(u) is positive, this condition will be satisfied.

[7]Cf. Mills (1967, p. 121, eq. 22); Mills (1972, p. 83, eqs. 5-14a and 5-14b); and Muth (1969, p. 72, eq. 3).

[8]These results are derived in Note 3 of the Mathematical Appendix. The corresponding results from Mills (1972) are also presented for comparison.

[9]For a discussion of this conceptualization of the housing market, see Muth (1960) or Olsen (1969).

[10]To obtain these results, write r(u) = B(u)/N(u) where B(u) is the black population at u and N is the total population. Now assume that N is constant (so that the addition of a black neighbor implies the loss of a white neighbor), and differentiate (24) with respect to B to find that

$$DH_w/DB = X_w \exp(-d_w B/N)\ (-d_w/N) = -d_w H_w/N \quad ,$$

where, to avoid confusion, D denotes a derivative.

[11]For a more complete discussion of this result, see Hamilton (1972), and Yinger (1974, sec. II.I).

[12]Note that in order to simplify the notation, the survival quantities have been left out of this analysis; therefore, the first-order conditions (26) should be compared to the conditions (11) with s_1 and s_2 equal to zero. Similarly, the demand function (28) should be compared to (16) when s_1 equals zero. Alternatively, the survival quantity for Z can be included in the following analysis simply by reinterpreting Y to be $(Y - P_z s_1)$.

[13]See Yinger, (1974, sec. II.I).

[14]Preference for a racially mixed neighborhood reflects many different attitudes, including racial prejudice and the desire for high-quality schools and other local public services. Thus a preference by blacks for integrated neighborhoods could exist despite strong black prejudice against whites. In this paper we will make no attempt to disentangle the effects of these various attitudes.

[15]The preference for a racially mixed neighborhood might correspond to an H-function of the form

$$H_g = X_g \exp[-d_g(r^*_g - r(u))^2]$$

for any group g, where r* is the most desirable racial composition. Although the derivation is somewhat more complicated, Equation (37) can be derived for any two groups with H-functions of this form.

[16]In order to simplify the derivations of the locational equilibrium conditions in this section, the transformation into rent was not performed, and the initial condition (7) was replaced by

$$P(\bar{u}) = \bar{P} \quad .$$

Furthermore, a single P(u) function was used in both the white and the black consumer maximization problems--that is, the equality of $P_w(u)$ and $P_b(u)$ was assumed. See pages 19-20.

[17]There have not been, to my knowledge, any attempts in the literature to estimate price-distance functions in forms determined by urban models. One possible estimating procedure (along with some illustrative regressions) is presented in Yinger (1974, sec. I.7).

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