246-74

INSTITUTE FOR RESEARCH ON POVERTY DISCUSSION PAPERS

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Jonathan R. Kesselman

UNIVERSITY OF WISCONSIN - MADISON

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December 1974

This research was supported by a University of British Columbia Research Grant and by the funds granted to the Institute for Research on Poverty at the University of Wisconsin by the Office of Economic Opportunity pursuant to the provisions of the Economic Opportunity Act of 1964. This paper supercedes a University of British Columbia discussion paper, "Tax Instruments and Economic Equality." The author gratefully acknowledges comments on the version I economy by Peter Diamond, Robert Solow, members of the Applied Welfare Economics Seminar, University of Wisconsin (Madison), and of the Applied Microeconomics Workshop, University of British Columbia; and imaginative programming of the version II economy by Daniel Waldo. Full responsibility for views and errors resides with the author.

ABSTRACT

Previous studies of optimal income taxes on earnings have shown them to be very limited in their egalitarian potential. A graded earnings tax is a policy that discriminates between wage rate and hours worked in its treatment of earnings. This paper compares the optimal income tax and optimal graded earnings tax under two sets of assumptions about a general-equilibrium economy. One version of the economy follows the pioneering work of Mirrlees; the other extends this to a more realistic utility function and labor-supply behavior, as well as a distribution of property incomes. Primary interest focuses on the degree of income equality and welfare equality under the two policies, for a given social-welfare trade-off between equality and economic efficiency. The newly proposed tax policy is found to be much more egalitarian than the optimal income tax. Part of the analysis is repeated with the assumption that status differentials are attached to skill differences in the population.

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1. Introduction

Severe limitations on the egalitarian potential of labor income taxes have emerged in recent studies. Optimal tax schedules have been calculated including an income-transfer component at zero earnings. Mirrlees (1971) stimulated much of the interest in this subject with his estimates of optimal nonlinear income-tax schedules. Most of his marginal tax rates did not exceed the 30 percents and showed little if any progressivity.[1] Fair (1971) derived top marginal rates in the low 40-percent range. Atkinson (1973) and Itsumi (1974) found comparably low marginal rates in a linear income-tax schedule with realistic skill dispersions. Only with extreme assumptions about the importance of equality relative to efficiency and about the inelasticity of labor supply was Feldstein (1973) able to achieve marginal rates in the 60 percents. The positive income guarantee made average tax rates fall below the marginal rates reported here. Mirrlees concluded that "The income tax is a much less effective tool for reducing inequalities than has often been thought..." (p. 208). The successor studies do little to alter his statement unless economic efficiency receives a very low policy weight. This is a disturbing conclusion for the theory of taxation in its presumedly most equalizing policy instrument.

Previous research on the optimal taxation of earnings has dealt with a tax schedule based on gross earnings. This conventional income tax does not discriminate between a worker's wage rate and his hours worked. Elsewhere the optimal <u>structure</u> for a tax on earnings has been investigated

(Kesselman 1974b). As proposed there, a graded earnings tax (GET) would treat the two components of gross earnings separately. Theoretical analysis of the GET proved its social-welfare superiority over an optimal income tax. However, analytical conclusions on the degree of economic equality under the two alternative tax forms were very limited. The present paper provides evidence on this question for two sets of assumptions about the nature of the economy. Our results confirm the strong egalitarian potential of distinguishing between wage rates and hours worked in a tax on earned income. We shall also extend the model to consider status differentials related to worker productivity or wage rates. This new element affects the degree of income equality and welfare equality under the optimal tax.

2. Version I of the Economy

Our first set of assumptions about the economy follows closely the numerical examples of Mirrlees. Labor is a homogeneous service in the hypothesized economy. Individuals differ only in a one-dimensional skill parameter, n, reflecting their labor productivities. The value of n is the worker's gross or market wage rate.[2] For each individual, the product of his time worked (y_n) and his skill level yields his total quantity of effective labor (ny_n), denoted z_n. This product also equals his gross earnings. The distribution of skills is assumed to be lognormal, with standard deviation of 0.39 and mean log n of -1.[3] Individuals have identical additive logarithmic utility functions:

$$u_n = \log x_n + \log (1-y_n) , \tag{1}$$

where x_n is consumption and $(1-y_n)$ is leisure time of a person in skill class n. The individual's budget constraint depends on the fiscal policy assumed to be in force:

$$x_n = f(n, y_n, \xi) , \tag{2}$$

where ξ is a vector of tax parameters.

Social welfare of the economy is individualistic and evaluated under the criterion

$$W_B = \begin{cases} \int_0^\infty u_n f(n)\,dn & \text{for } B = 0 , \\ -\frac{1}{B}\int_0^\infty e^{-Bu_n} f(n)\,dn & \text{for } B > 0 . \end{cases} \tag{3}$$

The social welfare function possesses a parameter B. With W_0 the welfare standard is utilitarianism. The larger is B, the greater is the concern with economic equality vis-à-vis economic efficiency. The probability density function of the skill distribution is f(n), taken from a differentiable cumulative distribution function, F(n). Note that the social welfare function is additively separable in the individual utilities. Thus it does not allow for the possibility of envy between workers at different skill levels.

The economy has a constant-returns-to-scale linear production constraint, with a single variable input

$$X = Z + a . \tag{4}$$

Here X and Z are the economy-wide aggregates of the respective individual variables:[4]

$$X = \int_0^\infty x_n f(n)dn \ , \tag{5}$$

$$Z = \int_0^\infty ny_n f(n)dn \ . \tag{6}$$

Term a in (4) reflects the presence of public consumption, which requires the collection of tax revenues for nontransfer expenditures. Aggregate output of the economy is Z, as there are no property incomes. A certain proportion (p) of output goes toward private consumption:

$$p = X/Z = X/(X - a) \ . \tag{7}$$

The residual portion of output goes toward general revenues for nontransfer expenditures:

$$R = (1-p)Z = -a \ . \tag{8}$$

Since a equals minus the revenue required to finance public consumption, equation (4) has an implicit normalization. Namely, the price of the consumption good equals the wage rate for a standardized unit of effective labor.

Version I of the economy will compare Mirrlees's results on the optimal nonlinear income tax with a linear form of graded earnings tax. The latter device taxes each worker an amount depending negatively on his work time and positively on his wage rate:

$$x_n = \gamma^* + (g + \sigma n)y_n \ . \tag{9}$$

The GET policy offers a guaranteed wage rate at zero skill level, g. Parameter σ is the fraction of a worker's gross wage rate that he may keep in addition to g.[6] A so-called breakeven wage rate (n') arises at the skill level where gross and net wage rates are equal:

$$n' = g/(1-\sigma) . \tag{10}$$

For workers with $n < n'$ the marginal tax rate on working is negative. For them, work is subsidized and hence leisure is a taxed activity.[7] Whereas the income tax operates on the worker's gross earnings z_n ($= ny_n$), the GET operates separately on the components of gross earnings.

The assumed utility function (1) requires the presence of lump-sum income for labor supply not to be perfectly inelastic. Invariant labor supply would eliminate the interest of the problem. Therefore, we shall provide an identical lump-sum transfer γ to each individual. The GET value of γ will be set equal to its optimum value γ^* under the corresponding income-tax solution. This approach conforms with the analytical approach of Kesselman (1974b). Note that γ^* is not necessarily the optimal lump-sum transfer for the GET; it merely facilitiates comparisons with the optimal income tax.

We now generate results useful in simulating the economy. Maximization of utility function (1) subject to GET budget (9) yields the worker's supply and demand schedules:

$$y_n = \begin{cases} 0.5-0.5\gamma^*/(g+\sigma n) & \text{for } \gamma^* < g+\sigma n \\ 0 & \text{for } \gamma^* \geq g+\sigma n \end{cases} \tag{11}$$

$$x_n = \begin{cases} 0.5(\gamma^*+g+\sigma n) & \text{for } y_n > 0 \\ \gamma^* & \text{for } y_n = 0 . \end{cases} \tag{12}$$

The GET raises the following net revenue from an n-skill worker:

$$R_n = -\gamma^* - [g + (\sigma-1)n]y_n , \tag{13}$$

which aggregates to

$$\begin{aligned} R &= \int_0^\infty R_n f(n)dn \\ &= -\gamma^* - gY + (1-\sigma)Z . \end{aligned} \tag{14}$$

Similar to the earlier aggregates, we have

$$Y = \int_0^\infty y_n f(n)dn . \tag{15}$$

Substitution of result (14) into (8) yields

$$\sigma = p - (gY + \gamma^*)/Z . \tag{16}$$

If the value of p is given, this result is useful in achieving the economy's production frontier.

3. Version II of the Economy

Perhaps the weakest aspect of the Mirrlees-economy assumptions is the utility function (1). This weakness has been noted by other researchers but never fundamentally improved upon. One problem is that the resulting labor supply has unrealistically large wage elasticity.[8] The other problem is that labor-supply function (11) cannot portray a backward bend like the one empirical estimates show for prime-age male heads of households. Our version II of the economy overcomes these problems with a utility function from a generalized linear expenditure system:

$$u_n = [b(1 - y_n - y^*)^c + (x_n - x^*)^c]^{1/c} . \tag{17}$$

The parameters of the function are restricted:

$$b > 0,\ (1 - y_n - y^*) > 0,\ (x_n - x^*) > 0,\ c < 1\ . \tag{18}$$

Wales (1973) has employed this function in an empirical study of labor supply. One other advance in version II of the economy is the inclusion of property income systematically related to skill class. This further enhances the realism of the model. Other features are similar to those of version I of the economy, except that the lognormal skill distribution of version II is restricted to 0.39 standard deviation.

The complex computational procedure developed by Mirrlees (p. 188) for simulation of an optimal nonlinear income-tax schedule requires that $\partial^2 u/\partial x \partial y$ be zero. This is satisfied by the additive utility function (1) but not by the generalized function (17). Consequently, version II of the economy will confine its attention to the linear income tax:

$$x_n = \gamma + i_n + \tau n y_n\ . \tag{19}$$

The new policy parameter τ is one minus the marginal tax rate. Term i_n is per-capita property income in skill class n. Results for the optimal linear income tax will be compared with results for the optimal linear GET. The budget for the latter is equation (9) supplemented by i_n on the right-hand side. It is assumed that neither policy taxes property incomes; this issue and its interrelations with earnings taxation are entire research areas in themselves. Let w_n be the net marginal wage rate under a policy--τn for the income tax, $(g + \sigma n)$ for the GET. Then maximizing utility (17) subject to the budget yields the labor-supply function

$$y_n = \max\left[0, \frac{b^d(1-y^*) - w_n^d(\gamma+i_n-x^*)}{b^d+w_n^{d+1}}\right], \tag{20}$$

where $d = 1/(c-1)$. Consumption demand can be obtained by inserting equation (20) into the respective budget.

Labor supply of any given worker depends upon his property income as well as his skill class and the tax parameters. A result of Greenberg and Kosters (1973) on asset holdings of male family heads has been adapted to the form [9]

$$i_n = -0.000846 + 0.02546n - 0.00494n^2 . \tag{21}$$

The other empirical findings used to fit the utility function parameters were also taken for prime-age male family heads. These represented a rough characterization of results of several studies (Cain and Watts, 1973; Cohen, Rea, and Lerman, 1970). The labor-supply function has been constrained to pass through two points in wage-hours space with given elasticities at each point. One is the point where the supply schedule is exactly vertical: $n = 0.243$ (for \$2.00), $y = 0.25$, elasticity = 0, and $i_n = 0.0050$. The other is the point of population mean: $n = 0.397$ (for \$3.27), $y = 0.24$, elasticity = -0.10, and $i_n = 0.0085$.[10] The dollar figures apply for the United States in 1966. Most of the sample, and all of the higher-skill members, fall on the backward-bending portion of the schedule. The fitted parameters were $b = 2.4153$, $y^* = 0.33429$, $x^* = -0.15965$, and $c = -2.710$. With lump-sum income of 0.0050, they implied a zero-hours-worked intercept at net wage rate of 0.0133.

The version II economy has an aggregate production-consumption constraint:

$$X = Z + I + a , \tag{22}$$

where

$$I = \int_{o}^{\infty} i_n f(n)dn . \tag{23}$$

Aggregate output of the economy is now Z+I and includes a return to owners of capital. As before, a proportion of output (p) goes to private consumption:

$$p = X/(Z+I) = X/(X-a) . \tag{24}$$

It is readily confirmed that relation (8) still holds. We can aggregate private consumption under an income tax:

$$\begin{aligned} X &= \int_{o}^{\infty} x_n f(n)dn \\ &= \int_{o}^{\infty} [\gamma + i_n + \tau n y_n] f(n)dn \\ &= \gamma + I + \tau Z . \end{aligned} \tag{25}$$

Substitution of this result into (22) and rearrangement produce

$$\tau = 1 + (a-\gamma)/Z . \tag{26}$$

Similarly, we can work through the GET budget aggregation to get [11]

$$\sigma = 1 + (a-\gamma-gY)/Z . \tag{27}$$

4. Simulation Techniques

Discrete rather than continuous skill distributions have been used to implement simulations of the economy. Optimal tax parameters were explored with the population disaggregated into 100 skill classes. The normal distribution was generated by a highly accurate approximation formula.[12]

Classes were taken at 80 intervals of 0.04 standardized normal deviations between -1.60 and 1.60. Ten additional intervals of 0.15 standardized normal deviations each were taken at each end, constituting a total inclusive range of $\pm$ 3.10 standardized normal deviations. The omitted tails of the distribution contain less than 0.002 of the entire population. Frequency densities were calculated for each class, and values for the skill parameter n of each class were calculated from the midpoint of the respective interval. In both versions of the economy skill parameter n ranged from 0.113 to 1.197. For case 6 in the version I economy only, the more dispersed n ranged from 0.018 to 7.577.

For the version I economy, optimal nonlinear income-tax estimates have already been calculated by Mirrlees. The optimal linear GET in each case was determined by simulating the economy for the social-welfare-maximizing tax parameters. Lump-sum component γ was set in each case to γ^* in the corresponding optimal income tax. Trial values of g were taken at 0.001 intervals. For each trial value of g, result (16) was used to find the σ that achieved the production constraint. Because changes in σ affect Y and Z, several iterations of σ were required. The positive slope of the labor-supply schedule assured convergence of σ. The termination test criterion was a change in σ of less than 1×10^{-5}. Only for such pairs (g, σ) were values of social welfare W_B compared to find the optimal policy parameters.

For the version II economy, we estimated the optimal linear income tax and GET. The general procedure was similar to that for the version I economy, with one significant added complication. Intervals of 0.001 were employed for trial γ in the income tax. Owing to computational expense, intervals of only 0.01 were used for trial g in the GET. In one case an additional significant place was allowed for optimizing

g. For all cases $(a-\gamma)$ was negative, owing to positive public consumption and non-negative lump-sum transfers. Most of the population was on the negatively sloped portion of the labor-supply schedule. Thus $\partial Z/\partial\tau$ and $\partial Z/\partial\sigma$ are negative except for very low τ and low $(g + \sigma n)$. Together these conditions imply that iterations of relation (26) or (27) with the rest of the economy will not converge on the correct values of τ or σ. The technique adopted, which is illustrated below, reveals properties of the tax instruments in the hypothesized economy.

For the linear income tax we can write the two sides of the production-consumption constraint (22) as $X(\tau,\gamma)$ and

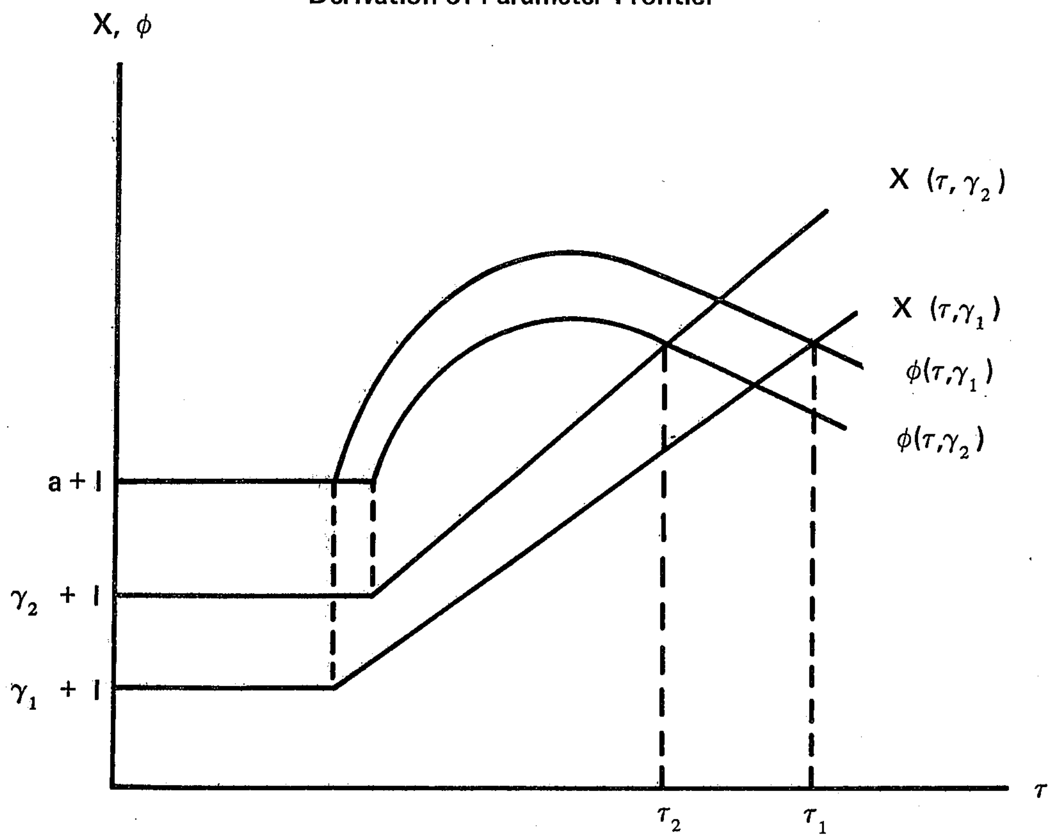
$$\phi(\tau,\gamma) = Z(\tau,\gamma) + I + a \; . \tag{28}$$

First, let us explore the properties of the two functions for a given trial γ value. There is a minimum value $\underline{\tau}$ at which and below which no individual works. For values of τ up to $\underline{\tau}$, $\phi(\tau,\gamma) = I + a$. Because no earned income arises in this range, $X(\tau,\gamma) = I + \gamma$. For values of τ above $\underline{\tau}$, $X(\tau,\gamma)$ is monotonically increasing as long as consumption is a normal good. For values of τ just above $\underline{\tau}$, the most skilled workers are drawn into the labor force on the positively sloped branch of the supply schedule. For sufficiently high τ, the majority of aggregate output will be supplied by workers on the backward bend of the supply schedule. Thus, above $\underline{\tau}$, $\phi(\tau,\gamma)$ will first rise and then fall, but never fall as low as $I + a$.

Figure 1 summarizes the preceding findings for two different γ values, with $\gamma_2 > \gamma_1$. The effects of a change in the trial γ value are easily derived and appear in the figure. For given γ_i the τ achieving production-consumption balance appears at the intersection of X and ϕ curves

Figure 1.

Derivation of Parameter Frontier



as τ_i. Note that the figure assumes $a > \gamma_i$, which cannot satisfy aggregate balance in the economy if no labor is supplied. With added messiness, the figure could as well have assumed $a < \gamma_i$. This would have yielded dual or multiple solutions to τ_i. In the case of multiple solutions τ_i, the only candidate for a social-welfare-maximizing solution is the highest value. Our simulation technique handled this problem quite simply. A numerical method was initialized at $\tau = 1$ and iterated downward until finding the <u>first</u> solution to $X(\tau,\gamma_i) = \phi(\tau,\gamma_i)$. The test criterion for halting the iterations of τ was a finding of the difference between X and ϕ less than 1×10^{-6} in absolute value. The economy possesses a "parameter frontier" of (τ,γ) pairs satisfying aggregate balance. In (τ,γ) space this frontier is everywhere negatively sloped except for $\tau \leq \tau$ where it is flat. The policy problem can be viewed as the choice of the social-welfare-maximizing point on the parameter frontier.

A similar approach can be implemented for the linear GET. Taking γ^* from the optimal income-tax parameter, we define $X(\sigma, g, \gamma^*)$ and $\phi(\sigma, g, \gamma^*)$. The value of γ^* is a constant and g is set at different trial values. The two functions can be graphed with σ on the horizontal axis. The higher the given value of g, the lower are the σ values at which $d\phi/d\sigma$ first becomes positive and at which ϕ attains its maximum. However, the shift in the intersection of X and ϕ with a change in g is ambiguous. This means that a positive slope in the parameter frontier (σ, g) is not ruled out.[13] While this may be analytically resolvable, there is no need to resolve it here. It suffices to note that a social-welfare maximizing choice of (σ, g) could not lie internally on a positively sloped stretch of the parameter frontier.

With individualistic social welfare function W_B, feasible increases in both σ and g improve welfare.[14] Any problems of multiple intersections between X and ϕ for given g can be treated by taking the highest σ solution--or the first solution if iterating downward for σ. The test criterion for stopping the iterations is identical to that employed for the income tax.

5. Economic Equality in the Version I Economy

Tables 1-6 report the estimated optimal income-tax and GET parameters for the version I economy. These correspond to the values of B and p investigated by Mirrlees. They include his findings on optimal nonlinear income taxes and an extension of his findings on general lump-sum transfers. The tables summarize the ranges of Mirrlees' marginal income-tax rates (m). Social-welfare values are reported for the optimum of each regime. Welfare values for the income-tax optimum and for the general optimum had to be estimated from Mirrlees's reported results, as he did not report them (see Appendix).

The tables present values of the individual's x, y, z, and U at cumulative distribution points of n of 0.01, 0.10, 0.50, 0.90, and 0.99. Tabulated values for utility (U) are the exponential of the individual utility function (1), or x(1-y). Instead of 0.01, Mirrlees chose the cumulative distribution point of zero, which applies to all variables subscripted "i" in the rows of F(n) = 0.01. Differences for individuals at F(n) = 0.01 and those at F(n) = 0 are trivial under the GET. Otherwise, tabulated results are fully comparable as between the two taxes on earnings in each case.

Mean values of x and z appear in the tables. Observe that mean x_i and mean x_g are very nearly identical in each case except case 6. Again excepting case 6, the mean z_i are almost equal to the corresponding mean z_g. This suggests that the two programs have similar efficiency costs. Mirrlees's optimal income-tax schedules happened to be fairly linear, despite the fully general form allowed them in optimization. The GET investigated here was confined to a linear form. The apparent similarity in aggregate efficiency costs of the two optimal programs may be related to this linearity. However, we do not yet know the deeper reason for this finding, if there is one.

The optimal marginal income-tax rates never get very high in the first five cases. They do not exceed 39 percent, with a typical value in the 20-percent range. In "unrealistic" case 6, the top marginal tax rate reaches 60 percent. Marginal tax rates on higher wage rates under the optimal GET are $(1-\sigma)$, or 89, 82, 81, 88, 91, and 84 percent in the six cases. These rates directly affect dynamic incentives. The net tax on marginal work effort can be determined only in conjunction with parameter g and the worker's skill level. This more conventional marginal tax rate on work effort is calculable as $[-g/n + (1-\sigma)]$. Thus the previous marginal tax rates are approached only for workers in the extreme upper tail of the skill distribution. For workers at the 0.01 and 0.99 cumulative distributions of skill, the six cases yield percentage tax rates of (-88, 61), (-86, 55), (-68, 57), (-60, 64), (-57, 67), and (-503, 79). As is characteristic of a wage subsidy, negative marginal tax rates obtain for workers below the breakeven wage rate (n'). The tables show the point in the skill distribution where the marginal tax rate is zero by F(n').

We next inquire into the relative effectiveness of the income tax and the graded earnings tax in promoting economic equality. The tabulated results

TABLE 1. Version I Case 1 Estimates

F(n)	x_o	x_i	x_g	z_o	z_i	z_g	y_o	y_i	y_g	U_o	U_i	U_g
0.01	.190	.03	.155	0	0	.066	0	0	.446	.190	.03	.086
0.10	.190	.10	.158	.033	.09	.100	.149	.42	.448	.162	.05	.087
0.50	.190	.16	.165	.178	.17	.166	.484	.45	.450	.098	.08	.091
0.90	.190	.25	.176	.416	.29	.275	.687	.48	.453	.060	.13	.096
0.99	.190	.38	.191	.722	.45	.417	.792	.49	.457	.040	.19	.103
Mean	.190	.17	.166	.207	.18	.178						

Social welfare function W_o; private consumption share p = 0.93.

Full optimum by general lump-sum transfers (subscript "o") has W = -2.324.

Income-tax optimum (subscript "i") has $\gamma = 0.03$; $0.16 \leq m \leq 0.26$; W = -2.439.

GET optimum (subscript "g") has g = 0.265 and $\sigma = 0.095$; F(n') = 0.28; W = -2.389.

TABLE 2. Version I Case 2 Estimates

F(n)	x_o	x_i	x_g	z_o	z_i	z_g	y_o	y_i	y_g	U_o	U_i	U_g
0.01	.210	.05	.163	0	0	.061	0	0	.409	.210	.05	.096
0.10	.210	.11	.169	.013	.08	.092	.059	.36	.413	.198	.07	.099
0.50	.210	.17	.182	.157	.15	.155	.429	.42	.420	.120	.10	.106
0.90	.210	.27	.203	.396	.28	.261	.654	.45	.430	.073	.15	.116
0.99	.210	.40	.230	.702	.43	.400	.770	.47	.439	.048	.21	.129
Mean	.210	.18	.184	.188	.17	.168						

Social welfare function W_o; private consumption share p = 1.10.

Full optimum by general lump-sum transfers (subscript "o") has W = -2.130.

Income-tax optimum (subscript "i") has $\gamma = 0.05$; $0.15 \leq m \leq 0.21$; W = -2.273.

GET optimum (subscript "g") has g = 0.250 and $\sigma = 0.175$; F(n') = 0.31; W = -2.235.

TABLE 3. Version I Case 3 Estimates

F(n)	x_o	x_i	x_g	z_o	z_i	z_g	y_o	y_i	y_g	U_o	U_i	U_g
0.01	.160	.07	.160	0	0	.053	0	0	.360	.160	.07	.102
0.10	.179	.12	.167	.044	.07	.082	.199	.28	.367	.143	.08	.106
0.50	.211	.17	.180	.157	.14	.140	.426	.37	.380	.121	.11	.112
0.90	.249	.26	.203	.357	.26	.240	.589	.43	.396	.103	.15	.122
0.99	.286	.39	.231	.626	.42	.374	.687	.46	.411	.090	.21	.136
Mean	.213	.18	.183	.183	.15	.152						

Social welfare function W_1; private consumption share p = 1.20.

Full optimum by general lump-sum transfers (subscript "o") has W = -8.305.

Income-tax optimum (subscript "i") has $\gamma = 0.07$; $0.19 \leq m \leq 0.28$; W = -9.345.

GET optimum (subscript "g") has g = 0.222 and $\sigma = 0.187$; F(n') = 0.22; W = -8.852.

TABLE 4. Version I Case 4 Estimates

$F(n)$	x_o	x_i	x_g	z_o	z_i	z_g	y_o	y_i	y_g	U_o	U_i	U_g
0.01	.143	.05	.144	.006	0	.059	.038	0	.395	.137	.05	.087
0.10	.164	.10	.148	.060	.08	.089	.267	.33	.399	.120	.07	.089
0.50	.193	.15	.157	.175	.15	.149	.475	.41	.405	.101	.09	.093
0.90	.228	.24	.171	.378	.28	.251	.624	.46	.414	.086	.13	.100
0.99	.261	.37	.189	.650	.44	.386	.713	.48	.424	.075	.19	.109
Mean	.194	.16	.158	.201	.17	.162						

Social welfare function W_1; private consumption share $p = 0.98$.

Full optimum by general lump-sum transfers (subscript "o") has $W = -9.921$.

Income-tax optimum (subscript "i") has $\gamma = 0.05$; $0.20 \leq m \leq 0.34$; $W = -11.212$.

GET optimum (subscript "g") has $g = 0.220$ and $\sigma = 0.119$; $F(n') = 0.16$; $W = -10.628$.

TABLE 5. Version I Case 5 Estimates

F(n)	x_o	x_i	x_g	z_o	z_i	z_g	y_o	y_i	y_g	U_o	U_i	U_g
0.01	.136	.04	.137	.013	0	.061	.085	0	.414	.124	.04	.080
0.10	.156	.09	.140	.068	.08	.093	.302	.36	.417	.109	.06	.082
0.50	.184	.14	.146	.184	.16	.155	.500	.43	.421	.092	.08	.085
0.90	.217	.23	.157	.389	.29	.259	.642	.48	.427	.078	.12	.090
0.99	.249	.36	.170	.663	.45	.395	.727	.50	.433	.068	.18	.096
Mean	.185	.15	.147	.211	.17	.167						

Social welfare function W_1; private consumption share p = 0.88.

Full optimum by general lump-sum transfers (subscript "o") has W = -10.950.

Income-tax optimum (subscript "i") has $\gamma = 0.04$; $0.21 \leq m \leq 0.39$; W = -12.619.

GET optimum (subscript "g") has g = 0.220 and $\sigma = 0.088$; F(n') = 0.14; W = 011.720.

TABLE 6. Version I Case 6 Estimates

F(n)	x_o	x_i	x_g	z_o	z_i	z_g	y_o	y_i	y_g	U_o	U_i	U_g
0.01	.250	.10	.158	0	0	.010	0	0	.269	.250	.10	.116
0.10	.250	.10	.164	0	0	.029	0	0	.280	.250	.10	.118
0.50	.284	.14	.185	.084	.06	.116	.227	.15	.315	.220	.11	.127
0.90	.436	.32	.261	.889	.54	.506	.671	.41	.382	.143	.19	.161
0.99	.618	.90	.456	3.151	.184	1.652	.836	.49	.438	.101	.46	.256
Mean	.316	.18	.203	.317	.20	.218						

Social welfare function W_1; private consumpt on share p = 0.93.

Full optimum by general lump-sum transfers (subscript "o") has W = -5.057.

Income-tax optimum (subscript "i") has $\gamma = 0.10$; $0.49 \leq m \leq 0.60$; W = -8.373.

GET optimum (subscript "g") has g = 0.211 and $\sigma = 0.159$; F(n') = 0.35; W = -7.561.

tell a very clear story. The GET creates by far the greater equality of welfare among individuals. Income equality can be measured by net incomes or by the consumption variable x. The GET is dramatically more income-egalitarian than the income tax. Interestingly, with social welfare function W_1, the GET achieves even greater income equality than does the full optimum. In all cases except 6, the GET achieves greater welfare equality than the full optimum. An inverse ranking of individual utility vis-a-vis skill class arises in the full social optimum. In short, the income tax has relatively weak egalitarian effects compared to the alternative GET. This is true even for a fixed view of the desirable trade-off between equality and efficiency.

We have observed the GET and income tax to have approximately identical efficiency costs (except in case 6). The greater welfare equality under the GET as compared with the income tax has also been observed. Thus, a larger social welfare value attainable under the GET is not surprising. This reinforces the analytical result that a linear GET yields higher social welfare than a linear income tax (Kesselman, 1974b). In the six cases of the version I economy, the social welfare values of the full optimum, GET, and income tax are (-2.324, -2.389, -2.439), (-2.130, -2.235, -2.273), (-8.305, -8.852, -9.345), (-9.921, -10.628, -11.212), (-10.950, -11.720, -12.619), and (-5.057, -7.561, -8.373). The GET closes from a quarter to a half of the gap in social welfare between the general optimum and the income-tax optimum.

6. Effects of Status Differentials

Social scientists outside of economics have long recognized the existence of status differentials related to workers' occupations. Such status differentials obviously affect workers' feelings of well-being. Yet, they have been ignored by economists generally and in particular relating to the optimal taxation of earnings. We shall adapt the version I economy to this problem by a simple change in utility function (1). In a homogeneous labor model, we can represent status by an increasing function of worker productivity. Let us examine the following specification:

$$u_n^s = \log x_n + \log (1-y_n) + 0.1 \log n \, . \tag{29}$$

This formulation allows status to affect utility levels in a moderate fashion. The rest of the version I economy remains unchanged. This represents a preliminary way of describing status differentials; another approach might take z_n as the indicator of worker status. Heterogeneous labor models, with occupational differences and perhaps occupational choice, open much richer possibilities.

The version I economy is simulated again only for the GET instrument. Table 7 presents the results of entering status differentials into the model alongside some of our earlier results for cases 1 and 5. We note first that in cases 1 and 2 optimal GET parameters g and σ are unchanged with the introduction of status. This follows directly from the utilitarian social-welfare function W_o and the additive form of individual utility (29). In these cases income equality is unaffected by the introduction of status. The GET parameters in the remaining four cases are altered with the introduction of status, namely by g rising and σ falling. In case 5, g rises

TABLE 7. Status Differentials

	Case 1		Case 5			
F(n)	U_w	U_w^s	x_w	x_w^s	U_w	U_w^s
0.01	.086	.078	.137	.140	.080	.074
0.10	.087	.082	.140	.142	.082	.078
0.50	.091	.090	.146	.146	.085	.084
0.90	.096	.100	.157	.153	.090	.092
0.99	.103	.112	.170	.162	.096	.100

TABLE 8. Version II Case 1 Estimates

F(n)	x_i	x_g	z_i	z_g	y_i	y_g	u_i	u_g
0.01	.038	.095	.035	.036	.233	.243	.180	.2141
0.10	.056	.095	.054	.054	.242	.241	.191	.2143
0.50	.088	.094	.088	.088	.239	.238	.211	.2146
0.90	.134	.093	.135	.142	.224	.234	.237	.2149
0.99	.183	.091	.186	.208	.204	.228	.263	.2149
Mean	.092	.094	.092	.094				

Social welfare function W_o; production function $X = Z + I - 0.008$.

Income-tax optimum (subscript "i") has $\gamma = 0.005$, $\tau = 0.859$; $p = 0.920$; $W = 0.2127$.

GET optimum (subscript "g") has $g = 0.37$, $\sigma = -0.077$; $p = 0.921$; $W = 0.2141$.

TABLE 9. Version II Case 2 Estimates

F(n)	x_i	x_g	z_i	z_g	y_i	y_g	u_i	u_g
0.01	.028	.077	.035	.037	.235	.251	.173	.203
0.10	.044	.078	.055	.056	.246	.250	.183	.204
0.50	.074	.079	.091	.090	.246	.246	.202	.205
0.90	.115	.080	.141	.146	.233	.240	.226	.207
0.99	.160	.082	.196	.213	.215	.234	.251	.208
Mean	.077	.079	.095	.096				

Social welfare function W_o; production function $X = Z + I - 0.026$.

Income-tax optimum (subscript "i") has $\gamma = 0$, $\tau = 0.726$, $p = 0.748$, $W = 0.2032$.

GET optimum (subscript "g") has $g = 0.30$, $\sigma = -0.031$; $p = 0.752$; $W = 0.2045$.

TABLE 10. Version II Case 3 Estimates

F(n)	x_i	x_g	z_i	z_g	y_i	y_g	u_i	u_g
0.01	.042	.083	.028	.034	.189	.227	.187	.210
0.10	.056	.084	.047	.050	.209	.224	.195	.211
0.50	.082	.087	.081	.081	.219	.220	.211	.213
0.90	.120	.090	.130	.130	.215	.214	.232	.216
0.99	.161	.093	.184	.189	.202	.208	.254	.219
Mean	.086	.087	.085	.086				

Social welfare function W_{25}; production function $X = Z + I - 0.008$.

Income-tax optimum (subscript "i") has $\gamma = 0.021$, $\tau = 0.660$; $p = 0.914$; $W = -0.0002085$.

GET optimum (subscript "g") has $g = 0.263$, $\sigma = -0.004$; $p = 0.915$; $W = -0.0001922$.

TABLE 11. Version II Case 4 Estimates

F(n)	x_i	x_g	z_i	z_g	y_i	y_g	u_i	u_g
0.01	.031	.073	.031	.036	.211	.243	.177	.202
0.10	.045	.074	.051	.054	.229	.240	.185	.203
0.50	.070	.075	.087	.087	.236	.236	.201	.204
0.90	.108	.077	.139	.139	.229	.230	.224	.206
0.99	.149	.078	.195	.203	.214	.222	.246	.208
Mean	.074	.075	.091	.093				

Social welfare function W_{25}; production function $X = Z + I - 0.026$.

Income-tax optimum (subscript "i") has $\gamma = 0.008$, $\tau = 0.628$; $p = 0.739$; $W = -0.0002653$.

GET optimum (subscript "g") has $g = 0.26$, $\sigma = -0.028$; $p = 0.742$; $W = -0.0002428$.

from 0.220 to 0.232, while σ falls from 0.088 to 0.057. This follows from the declining marginal social-welfare significance of utils for any worker under W_1. In such cases, consideration of status will increase income equality. The distributions of utility levels in the optimal GET with status (U_w^s) and without status (U_w) are reported in Table 7.[15] In all cases, the introduction of status yields a GET optimum with less welfare equality than would arise in the absence of status.

7. Economic Equality in the Version II Economy

Tables 8-11 report the estimated optimal income-tax and GET parameters for the version II economy. These correspond to values of B = 0, 25 and $\underline{a}$ = -0.008, -0.026. Because of space limitations, results for B = 1, 5 and $\underline{a}$ = -0.044, 0.010, as well as all combinations with reported B and $\underline{a}$ values, have been omitted. The arrangement of these tables is similar to that of Tables 1-6. As in the version I economy, mean x_i and mean x_g are typically close in each case, as are mean z_i and z_g. The GET mean values of these variables are always 0.001 to 0.002 higher than the respective income-tax values. The GET promotes slightly greater aggregate output and aggregate consumption than does the income tax.

Tabulated values for utility are the values taken by utility function (17). Even under laissez-faire (a = 0 and p = 1), the spread in individual utilities is quite small. At F(n) = 0.01, x = 0.039, y = 0.246, u = 0.180, and at F(n) = 0.99, x = 0.201, y = 0.200, u = 0.270. Thus, the marginal utility of income is rapidly diminishing to the individual under the assumed utility function. The nature of the problem requires that we place cardinal significance upon individual utility. Few observers would agree that a person with five times the consumption of another person, along with more leisure time, is only 50 percent better off. One solution to this difficulty would be to raise utility function (17) to some power greater than unity; as a monotonic transformation, this would not alter the utility-maximizing leisure-and-consumption choice of the individual. Instead we have opted to employ higher B values in the social-welfare function. Atkinson (1973) has shown how transformations of the utility function make the optimal tax structure more progressive. By taking a more egalitarian social welfare function, we are effectively creating a more satisfactory cardinalization of the individual utility function. Thus, given the original utility function, W_{25} may reflect only a modest egalitarian sentiment.

Let us examine the optimal tax parameters. The optimal marginal income-tax rates ($1-\tau$) are 14, 27, 34, and 37 percent in the four cases. These rates conform to the ranges familiar in the optimal-income-tax literature. Note in Table 9 that case 2 offers no income subsidy in the optimum ($\gamma=0$). This reflects the relatively high public consumption in the economy, plus the relatively inegalitarian welfare function and the fact that the lowest-skilled member of the society still has positive productivity and a bit of property income. The optimal income-tax γ for case 2 would have been negative. However, negative γs were not explored owing to the infeasibility of head taxes. Marginal tax rates on higher wage rates under the optimal GET are ($1-\sigma$), or 108, 103, 100, and 103 percent in the four cases. Consequently, no incentive for upgrading skill levels remains for individuals. The GET net tax rates on marginal work effort can be calculated as in the version I results. For workers at the

0.01 and 0.99 cumulative distributions of skill, the four cases yield percentage tax rates of (-142, 67), (-99, 70), (-77, 72), and (-72, 74).

In terms of either measure of economic equality, the optimal GET again strongly dominates the optimal linear income tax. In case 1, the GET produces a mildly reversed ranking of consumption vis-à-vis skill level. In all tabulated cases, individual utility rises with skill level under the optimal GET. However, we noted before that σ is negative in the reported cases. Therefore, the positive relation between skill level and property income is helping to maintain the utility ranking across the skill distribution. For the untabulated cases with $\underline{a} = -0.044$, the optimal income tax had $\gamma = 0$ and $\tau = 0.537$ for all values of B. The corresponding optimal GET had g between 0.18 and 0.21 and σ positive but less than 0.09. The private consumption share in output (p) was about 0.58, reflecting heavy public consumption. For the untabulated cases with B = 1, 5, the results were roughly intermediate between those for B = 0 and B = 25. In fact, results for B = 0 and B = 1 were almost indistinguishable. In all cases, the optimal GET raised social welfare above that attainable under the optimal income tax.

8. Qualifications and Extensions

Our results reveal the strong egalitarian potential of taxes on earnings that discriminate between wage rates and work hours. They confirm the relatively weak equalizing effects found in previous studies of labor income taxes. The graded earnings tax also makes possible a higher level of social welfare than does an optimal linear or nonlinear income tax. As in most simulation experiments, these results may be sensitive to variations in the model's assumptions. Nonetheless, our findings demonstrate robustness under variations in the model's key features: the social welfare function, the proportion of gross output devoted to public consumption, the dispersion of skill distribution, and the individual utility function. For the first time in the optimal tax literature, a complex utility function that produces realistic labor-supply behavior has been estimated and utilized. Variants of the functional form of the skill distribution, other than lognormal, have not been investigated. We have also not optimized for the lump-sum transfer (γ) in the GET nor explored a nonlinear form of the device.

The model underlying our analysis assumes that workers at all skill levels possess identical utility functions. This assumption is common to all of the analytical optimal tax theory. The special case where worker preferences for leisure are negatively correlated with skill level has been simulated by Blinder (1974, ch. 6). Within the framework of a flat-rate income tax, he compares the equalizing effects of an income subsidy with those of a conventional wage subsidy. The latter differs from our GET in not operating above the breakeven wage rate (n'). Blinder's results, both with and without correlation in tastes, strongly support the egalitarian superiority of wage subsidies over income subsidies. Assuming differential worker tastes <u>not</u> systematically related to skill levels opens further possibilities. Two workers, for example, may have the same gross flow of earned income but different wage rates. The income tax judges the two workers equal in "ability to pay" and assesses each the same tax liability. The GET judges the worker with the higher wage rate as having a greater "ability to pay." His equal gross income merely reflects his choice of shorter working hours.

The graded earnings tax warrants further study. In particular, non-linear forms of the GET with optimal lump-sum transfers need to be explored. Of especial concern for the GET are the large dynamic disincentives posed by the instrument. In the cases examined here, marginal tax rates on increases in the worker's wage rate ranged from 81 to 108 percent. The implications for incentives to undertake education, vocational training, or on-the-job training are worrisome. Following the optimal income-tax literature, the model has assumed the distribution of market wage rates to be given and fixed. This area needs further development, perhaps in a model combining static and dynamic incentive effects.[16] Finally, the basic administrative issues for a graded earnings tax are unanswered. Unlike a tax on innate ability, the GET can readily measure wage rates for most of the working population. Self-employed and unsupervised salaried workers are two difficult exceptions. Other problems arise in the accountability of employers and in the treatment of families, overtime pay, and multiple jobs. These programmatic problems should prove as intriguing as any encountered in implementing an income tax.

Appendix: Estimation of Mirrlees's W

One important question was how the optimal GET compared with the optimal income tax and the full optimum in social welfare. Mirrlees's failure to report his W values induced us to perform an unusual exercise. Rather than re-estimating his optimal tax schedules, we chose a less time-consuming approach. We first estimated the optimal income-tax function resulting from his simulation. This could then be used to forecast distributions of x_n and y_n, from which the respective W could be calculated.

For each case we estimated a quadratic tax function of gross income:

$$t = \alpha_o + \alpha_1 z + \alpha_2 z^2$$

by least-squares regression. The observations were for z = 0, 0.05, 0.10, 0.20, 0.30, 0.40, and 0.50 in cases 1-5; and for z = 0, 0.1, 0.25, 0.50, 1.00, 1.50, 2.00, and 3.00 in case 6. With the average tax rate (s) reported in percent, we have calculated

$$t = sz/100$$

to obtain the requisite data.[17] The regression fits were very tight. Table 12 shows the parameter estimates. Despite the low degrees of freedom, all coefficients were significant at better than the 0.005 level of confidence. The implicit marginal tax rates (= $\alpha_1 + 2\alpha_2 z$) in these estimated functions are almost always within one percentage point of Mirrlees' reported values. The exceptions arise in the extreme tails of the z distribution, containing less than 0.001 of the total population, and in one other instance in case 6.

TABLE 12. Income-Tax Schedule Estimates

case	α_o	α_1	α_2	R^2
1	-.02971	.25613	-.09602	.974
2	-.05042	.21661	-.07211	.964
3	-.06978	.28012	-.08364	.978
4	-.04995	.34410	-.14876	.992
5	-.04074	.40165	-.21517	.979
6	-.10251	.60578	-.02071	1.000

The second part of the exercise required forecasts from the model with the optimal income-tax functions. For a person with skill level n, the tax function is

$$t_n = \alpha_o + \alpha_1 n y_n + \alpha_2 n^2 y_n^2 .$$

Under an income tax, his consumption is

$$x_n = n y_n - t_n = -\alpha_o + (n - \alpha_1 n)\, y_n - \alpha_2 n^2 y_n^2 .$$

Maximization of the utility function (1) yields a labor-supply function:

$$(3\alpha_2 n^2) y_n^2 + 2(\alpha_1 n - n - \alpha_2 n^2) y_n + (\alpha_o + n - \alpha_1 n) = 0 .$$

If multiple roots appear when this quadratic function of y_n is solved, solutions outside the interval [0, 1) must be replaced by values within the interval. Then a comparison of utility level u(y,x(y)) for the two y-values must be made in order to find the worker's labor-supply choice. In each case, this must be done individually for the each of the 100 values of n. The W values are then readily calculated. As a final check on our procedure, we printed the resulting values of x, y, z, and x(1-y) at F(n) = 0, 0.10, 0.50, 0.90, and 0.99 in each case. These proved always to lie within 0.01 of Mirrlees's tabulated values.

Values of social welfare, as well as y, z, and U distributions, were obtained for the full social optimum. The procedure followed results of Mirrlees (p. 201) along with his published values of x° for each case. The latter variable denotes consumption for persons who are out of the labor force.

NOTES

[1]Most of the successor studies have employed minor variations of the Mirrlees model. Atkinson (1972, 1973) has demonstrated the model's sensitivity to several of its components: the form of individual utility, the social welfare function, and the distribution of ability.

[2]The linear nature of production technology in the hypothesized economy allows n to be associated with the real wage rate. Feldstein (1973) investigates a heterogeneous-labor model in which occupational wage rates are endogenous.

[3]These distribution parameters apply to the first five of Mirrlees's six cases. In the sixth case, the distribution of skills is assumed to be more dispersed, with standard deviation 1.00. For cases 1-5 the mean n value is 0.397; for the sixth case it is 0.610. See Aitchison and Brown (1963).

[4]Because of the population relation:

$$1 = \int_0^\infty f(n)dn ,$$

the aggregates X, Z, and Y (defined later) can also be interpreted as population mean values.

[5]The GET is an extension of the wage subsidy developed by Kesselman (1969). The GET merely extends the operation of the instrument to market wage rates above the breakeven wage rate.

[6]Wagstaff (1972) demonstrates that a nonlinear GET has a relation between net and market wage rates with slope sign identical to that of the ordinary labor-supply slope. Kesselman (1974b) examines the linear form as well. An optimal linear GET _may_ have negative σ if most of the output comes from workers on the backward bend of the labor-supply curve.

[7]If a proportional sales tax is also imposed on all consumption goods, there may be a skill class that bears no deadweight loss. This unusual result arises because all goods and activities (including leisure) are taxed at the same rate. See Little (1951).

[8]Feldstein (1973) has experimented with alternative wage elasticities of labor supply in a C.E.S. utility function.

[9]The result employed is their "basic male family head sample," evaluated at the mean age of the sample and imputing a 0.07 rate of return to assets. The coefficients in equation (21) are obtained after division by the number of hours in a year (8760) and then multiplication by the ratio of mean n (0.397) to mean sample wage rate (3.27). Thus property income is in the appropriate units for lognormal skill distribution with standard deviation 0.39 (Mirrlees's cases 1-5). This is the only distribution considered in version II of the economy; the larger standard deviation 1.00 led to forecasts of negative property incomes in very high and very low skill classes.

[10]Attempts to fit the function to these data with wage elasticity = -0.15 at the mean wage rate yielded imaginary roots for some parameters.

[11]Note the similarity of (27) to (16); whereas the latter holds p constant, the former holds <u>a</u> constant. Given the nature of the problem, the choice is not important.

[12]The formula, with a maximum error of less than 3×10^{-7}, appears in Hastings (1955, p. 187).

[13]The absence of ambiguity for the income-tax case results from the normality of leisure, so that increases in γ depress labor supply of <u>all</u> persons working. The present ambiguity stems from the fact that increases in g evoke increases and decreases for different parts of the population.

[14]An increase in σ <u>and</u> g increases the net wage rate of all persons. For all such persons who are working, the indirect utility function further implies an increase in everyone's utility level. See Kesselman (1974b).

[15]Tabulations of the utility distribution take the exponent of u_n^s at F(n):

$$U_w^s = x_n(1 - y_n)(n/\bar{n})^{0.1} .$$

Division of n by its mean value, $\bar{n}$ (0.397 in cases 1-5 and 0.610 in case 6), makes the tabulations of U_w and U_w^s readily comparable. Without this correction, the series U_w^s would lie uniformly above U_w in each case.

[16] Kesselman (1974a) has examined off-the-job training and job-search incentives under several tax and transfer forms. If Θ is the slope sign of the ordinary labor-supply curve, we can reach the following effects of a linear GET vis-à-vis laissez faire. Dynamic incentives are negative for $n < n'$ and $\Theta < 0$; negative for $n > n'$ and $\Theta > 0$; and ambiguous for the other sign combinations. These results require σ positive, which condition has been violated in some of our cases.

[17] Mirrlees's case 4 has an error for $z = 0.10$, which should be $s = -17.5$ (not the reported -34). This can be verified by working with the reported marginal tax rates.

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