229-74

INSTITUTE FOR RESEARCH ON POVERTY DISCUSSION PAPERS

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November 1974

The research reported here was supported by funds granted to the Institute for Research on Poverty at the University of Wisconsin-Madison by the Office of Economic Opportunity pursuant to the Economic Opportunity Act of 1964. The opinions expressed are those of the author.

ABSTRACT

Several alternative suggestions for methods of measuring segregation have appeared in the literature. This paper is an examination, both theoretical and empirical, of three measures of segregation, with the empirical focus on school segregation. The first measure is based on the absolute deviation of the racial composition of a school from that of the school district, the second is based on the square of that deviation, and the third is derived from information theory. The purpose of this paper is to examine and compare the properties of these three measures in terms of how useful they are both as descriptive devices and as indicators of appropriate policy actions. Separate discussions of the theoretical nature of each index are accompanied by summaries of their calculated values based on a sample of school districts.

Several arguments are given for preferring the information theory measure: it incorporates the notion of diminishing marginal payoff to desegregation; it depends on the entire distribution of students by race across schools; it may be interpreted as a measure of association between race and school assignment; it can be meaningfully aggregated; and, once aggregated, it can be decomposed into "between" and "within" components. Its main drawbacks are that it is somewhat more complicated to calculate and that its interpretation is not as easily grasped intuitively.

The use of any of the three indexes presented here as a policy aid would be substantially better than subjective judgment. Moreover, if the costs of implementation and of gaining acceptance are not too great, then the information theory index appears to be the most appropriate measure of school segregation.

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INTRODUCTION

Several alternative suggestions for methods for measuring desegregation have appeared in the literature. Excellent reviews of most of this literature appear in Taeuber and Taeuber (Appendix A) and in Duncan and Duncan. This paper is an examination, both theoretical and empirical, of three measures of desegregation, with the empirical focus on school desegregation. The first measure examined, the dissimilarly index, is discussed in the two sources cited above and is based on the absolute deviation of the racial composition of a school from that of the school district. The second measure is referred to here as the segregation index and is based on the squared deviation. The third measure investigated derives from information theory and has been suggested for this use by Theil and Finizza. The major purpose of this paper is to examine and compare the properties of these three measures in terms of how useful they are as both descriptive devices and indicators of appropriate policy actions.

Part I of this paper contains a separate discussion of the theoretical nature of each index and includes empirical calculations. The data used for these calculations are a subset of the information collected by DHEW from public elementary and secondary schools and school districts in the fall of 1972.[1] The sample was chosen in order to eliminate those school districts for which the issue of school desegregation is not meaningful. It includes all school districts surveyed in 1972 for which each of the following were true in that year:

(1) Either the district contained more than 6 school campuses or at least one grade was taught at more than one campus.

(2) At least 5 percent of the student population was minority.

(3) At least 5 percent of the student population was nonminority.[2]

Since the original DHEW survey was based on a random sample of school districts, with different sampling rates for different size strata, the universe projections that are possible using the entire survey are not reasonable based on our sample.[3] This selection resulted in a set of 2,393 districts, approximately 20 percent of all school districts in the country. Almost half of these were in the 17 southern and border states,[4] since minority students are relatively overrepresented in those states. While these 2,393 districts contain only 55 percent of the total national public school enrollment, they include more than 88 percent of all enrolled minority students.[5]

Part II of this paper contains a comparative discussion of the three indexes and Part III presents conclusions and additional comments.

I. ALTERNATIVE MEASURES OF SCHOOL SEGREGATION

A. Dissimilarity Index (D)

The first index we will consider was originally developed for the purpose of describing residential segregation.[6] It has since been applied to the study of school segregation as well as to other topics.[7] The numerator of the dissimilarity index, which we shall call D_n, is defined as simply the sum of the absolute deviations of the racial composition of the schools from the overall racial composition of the school district:

$$D_n = \sum_i T_i \left| p_i - p \right| , \tag{1}$$

where T_i and p_i are, respectively, the total enrollment and percent minority of the ith school, and where p is the percent minority of the district. An implicit rationale for this measure is that the contribution of the ith school to the "badness" of segregation is proportional to the absolute difference between p_i and p.

The index of dissimilarity (D) is then derived by dividing the value of D_n by its maximum. This maximum will occur in a totally segregated system and is given by[8]

$$\begin{aligned} \hat{D}_n &= \sum_{p_i < p} T_i (p - 0) + \sum_{p_i \geq p} T_i (1 - p) \\ &= p(\# \text{ of nonminority students}) \\ &\quad + (1 - p)(\# \text{ of nonminority students}) \\ &= p(1 - p)T + (1 - p)pT \\ &= 2Tp(1 - p) . \end{aligned} \tag{2}$$

Dividing D_n by $\hat{D}_n$ therefore gives an index that ranges from 0 to 1 for any given school district:[9]

$$D = \frac{\sum_i T_i \left| p_i - p \right|}{2Tp(1 - p)} . \tag{3}$$

An important characteristic of D is that its value is not dependent on the overall distribution of students by race but only on the numbers of students in schools with less than and those with greater than the district-wide proportion of minority students. This can be seen by decomposing D_n as follows:

$$D_n = \sum_{p_i<p} T_i(p - p_i) + \sum_{p_i \geq p} T_i(p_i - p)$$

$$= p\left[\sum_{p_i<p} T_i - \sum_{p_i \geq p} T_i\right] + \left[\sum_{p_i \geq p} T_i p_i - \sum_{p_i<p} T_i p_i\right] . \quad (4)$$

The first bracketed term on the right-hand-side of (4) is simply the difference between the numbers of students in the two groups of schools, while the second bracketed term is the difference between the numbers of minority students in the two groups of schools.[10] The value of D is unaffected by transferring students between any schools within each group; only by transferring them across the two groups will D change. Thus, D is independent of assignment among those schools for which $p_i<p$ or among those for which $p_i>p$ and is completely determined by the total numbers of minority and nonminority students in each of the two groups of schools. Alternatively, one can say that the payoff criterion implicit in D is linear (as opposed to the quadratic payoff criterion implicit in the second index to be discussed below). An important effect of this linearity is that the payoff (measured by changes in the value of D) is the same for bringing a particular school x percentage points closer to the overall racial composition of the district, regardless of how far away from that composition the school was originally. Since it is often assumed that achieving a given "amount" of desegregation is "easier" the more segregated are the schools to begin with, the use of D as a policy variable may not provide the desegregation incentives desired: if this assumption is valid, then the payoff should be nonlinear in the sense that a given "amount" of desegregation is rewarded more for initially more segregated districts.

As a measure of segregation, the dissimilarity index has two very appealing features. First, it is the easiest to compute of all indexes discussed here. This characteristic derives from the fact that the only disaggregated information required is the numbers of minority and non-minority students in the two groups of schools identified above. Second, D has a straightforward intuitive interpretation since it equals the proportion of minority (or non-minority) students who would have to be transferred in order to achieve the same racial composition in all schools. Furthermore, a method of decomposing the value of D on the basis of other attributes is available.[11] In addition, a convenient interpretation may be attached to the weighted sum of absolute deviations given by (1) taken as a percent of total student enrollment, or

$$\frac{D_n}{T} = \frac{1}{T} \Sigma_i T_i \left| p_i - p \right| .$$

This quantity is the minimum percent of the total student body who would have to be involved in two-way minority-nonminority trades between schools in order to achieve racial balance and has been called the replacement index.[12]

Table 1 displays the distribution of values of D across districts. The data sample used is the one described above in the Introduction and results are presented separately for southern school districts. Looking at the distributions of school districts across values of D, one sees little difference in the degree of segregation between the two regions. This is surprising, since most indications are that more school desegregation has occurred in recent years in the South than elsewhere. However,

TABLE 1

Percent Distribution of D by Region

Regions	Value of D 0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.7	0.7-0.8	0.8-0.9	0.9-1.0	Total Numbers
South*											
Districts	16.9	29.1	21.8	13.6	9.7	3.9	2.3	1.8	0.8	0.2	1,168
Schools	7.0	18.7	21.2	15.5	11.0	8.1	7.6	6.0	4.8	0.1	18,652
Students	6.3	17.2	20.4	15.1	10.3	9.2	8.4	7.2	5.8	0.1	11,210,761
Minority Students	6.2	16.0	16.0	12.8	8.3	10.6	8.6	10.7	10.7	0.2	3,970,169
Non-South											
Districts	10.5	30.5	24.4	17.5	8.7	4.4	2.5	1.1	0.2	0.0	1,225
Schools	3.3	17.4	17.6	14.4	12.5	8.2	14.8	10.6	1.2	0.0	21,093
Students	2.6	16.0	15.9	12.9	11.8	7.9	19.2	12.3	1.5	0.0	13,483,458
Minority Students	2.2	9.2	9.9	7.7	9.0	7.4	29.8	22.1	2.7	0.0	4,580,079

Source: U.S. Department of Health, Education, and Welfare, Office for Civil Rights, Directory of Public Elementary and Secondary Schools in Selected Districts: Enrollment and Staff by Racial/Ethnic Group, Fall, 1972 (Washington, D.C.: U.S. Government Printing Office, 1973).

Note: Rows may not sum to 100 due to rounding errors.

*Alabama, Arkansas, Delaware, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, Missouri, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, West Virginia.

a different picture emerges if one compares the distributions of students across values of D for the two regions: substantially large percentages of students, especially of minority students, are in relatively segregated school systems outside the South. The main reason that different conclusions are reached by looking at the two distributions stems mainly from the fact that the nonsouthern districts include more large school districts that are relatively segregated than do the southern districts. This is illustrated by the data in Table 2, which is taken from the table presented in the Appendix.[13] While more than 96 percent of the students (and 98 percent of the minority students) in the nine largest nonsouthern districts were enrolled in school systems with values of D greater than 2/3, this was true of only 30 percent of the students (and 66 percent of the minority students) in the eleven largest southern districts listed. (Note also that more than 73 percent of the minority students but only 40 percent of the nonminority students in these largest twenty districts were outside the South.)

B. <u>Segregation Index (S)</u>

An interesting feature of the segregation index (S) is that it was developed separately and independently by two groups each using different rationales, one statistical and the other in terms of policy goals.[14] Three conceptual bases for S will be discussed here in order to shed additional light on its interpretation.

1. <u>S as a Policy-Goal Measure</u>

Assume that the goal of school desegregation is to avoid racial isolation and that this goal is achieved for each student in proportion to the percent of students belonging to the other racial group in the

TABLE 2

Total Enrollment, Minority Enrollment, and Values of D for the 20 Largest Districts in the Sample

District Name	Total Enrollment	Minority Enrollment	D
South:			
Broward Co., Fla.	128,889	31,640	.31
Dade Co., Fla.	241,809	124,870	.52
Duval Co., Fla.	113,644	37,100	.33
Hillsborough Co., Fla.	106,294	27,196	.18
Baltimore City, Md.	186,600	129,250	.82
Montgomery Co., Md.	126,912	12,799	.29
Prince Georges Co., Md.	161,961	42,935	.61
St. Louis City, Mo.	105,617	72,985	.90
Memphis City, Tenn.	138,714	80,403	.86
Dallas, Texas	154,580	76,366	.70
Houston, Texas	225,410	127,128	.73
Totals	2,744,136	735,476	
Non-South:			
Los Angeles, Cal.	620,707	327,278	.69
San Diego, Cal.	124,604	32,790	.53
Chicago, Ill.	557,141	384,149	.80
Detroit, Mich.	276,655	192,259	.74
New York City, N.Y.	1,125,449	724,954	.67
Cleveland, Ohio	145,196	87,007	.88
Columbus, Ohio	106,676	31,825	.70
Philadelphia, Pa.	282,965	183,424	.78
Milwaukee, Wis.	128,734	43,665	.76
Totals	3,368,127	2,007,351	
Grand Totals	6,112,263	2,742,827	

same school. In other words, the contribution of each minority child towards this goal equals the proportion of nonminority children attending the same school. Averaging this criterion over all minority children then yields

$$DI_n = \frac{\sum_i T_i p_i (1 - p_i)}{\sum_i T_i p_i} = \frac{\sum_i T_i p_i (1 - p_i)}{Tp}, \tag{5}$$

where $T_i p_i$ equals the number of minority students in the ith school and $(1 - p_i)$ is the proportion of nonminority students attending that school. This quantity will be maximized when $p_i = p$ for all i, i.e., when all schools have the same racial composition. This maximum value equals $(1 - p)$, the district-wide percent nonminority.[15] We therefore define the segregation index to be one minus the value of (5) taken as a percent of its maximum possible value, or

$$S = 1 - \frac{DI_n}{p} = 1 - \frac{\sum_i T_i p_i (1 - p_i)}{Tp(1 - p)} . \tag{6}$$

In this context, the value of S may be interpreted as the amount of "exposure" between minority and nonminority students that has not been achieved within the schools relative to the maximum amount possible.

2. S as a Mean-Square-Deviation Measure

Assume that the goal of school desegregation is to avoid deviations from the mean racial composition and that the "costs" of such deviations increase with the square of the deviation.[16] The mean-square-deviation (MSD), averaged over all schools and weighted by school enrollments, is then

$$MSD = \sum_i T_i (p_i - p)^2 , \tag{7}$$

which can also be written as

$$MSD = Tp(1 - p) - \sum_i T_i p_i (1 - p_i) \ . \tag{8}$$

The maximum value of the expression in (8) occurs when schools are totally segregated and is given by

$$Tp(1 - p)^2 + T(1 - p)p^2 = Tp(1 - p) \ . \tag{9}$$

The first term on the left-hand-side of (9) is simply the number of minority students in the district (Tp) times the contribution of the all-minority schools to MSD, since $p_i = 1$ for each of these schools. The second term is likewise the number of nonminority students [T(1 - p)] multiplied by the contribution of their schools (p^2) in which $p_i = 0$. Thus we define the index as MSD (7) divided by its maximum value (9), or

$$\frac{\sum_i T_i (p_i - p)^2}{Tp(1 - p)} \ ,$$

which can also be written as:

$$1 - \frac{\sum_i T_i p_i (1 - p_i)}{Tp(1 - p)} = S \ . \tag{10}$$

Thus, minimizing the value of the MSD index is exactly equivalent to minimizing the value of S.

3. S as a Variance-Accountability Measure

Consider a binomial race variable R_{ij} that equals 1 if the jth student in the ith school is minority and 0 otherwise. Then the appropriate hypothesis test for equality of racial composition across schools can be derived from analysis of variance. The expected value of R_{ij} is p and its variance can be decomposed as follows:

$$\sum_{ij}\sum (R_{ij} - p)^2 = \sum_{ij}\sum (R_{ij} - p_i)^2 + \sum_i T_i(p_i - p)^2 . \qquad (11)$$

The first term on the right-hand-side of (11) is the "within samples" variation and can be interpreted as the variance "attributable to desegregation" since it measures the mean-square-deviation of R_{ij} within schools. The second term can be interpreted as the variance "attributable to segregation" since it measures the mean-square-deviation of R_{ij} between schools. In terms of S, the decomposition of (11) can be rewritten as

$$\sum_{ij}\sum (R_{ij} - p)^2 = Tp(1 - p)(1 - S) + Tp(1 - p)S . \qquad (12)$$

Since $Tp(1 - p)$ is the total variance in the system, S can be interpreted as the percent of the total variance attributable to segregation.[17]

To clarify this interpretation, consider the following measure of association between the binomial color variable and the school to which a student is assigned:

$$\phi = \sqrt{\frac{\chi^2}{T(L - 1)}} , \qquad (13)$$

where χ^2 is the Pearson chi-square computed from a 2xK contingency table (K is the number of schools in the district) and L is the smaller of the number of rows and columns in that table. ϕ is often called Cramer's statistic and should not be confused with the contingency coefficient. The value of ϕ must lie between 0 (complete independence) and 1 (perfect association).[18] Since we are constraining the number of racial/ethnic groups to be 2 (minority and nonminority), and since it is only meaningful to discuss desegregation when there is more than one school, L must always equal 2. Therefore:

$$\phi = \sqrt{\frac{\chi^2}{T}} .$$

However, for our purposes, we can write χ^2 as follows:[19]

$$\chi^2 = \sum_i \left[\frac{(p_i T - p_i T_i)^2}{pT_i} + \frac{[(1 - p)T_i - (1 - p_i)T_i]^2}{(1 - p)T_i} \right] .$$

Rearranging and combining terms yields

$$\chi^2 = \frac{\sum_i T_i(p_i - p)^2}{p(1 - p)} ,$$

which, using (10) above, reduces to

$$\chi^2 = TS .$$

Thus,

$$\phi^2 = S . \qquad (14)$$

Although ϕ^2 may not be conveniently interpreted as the proportion of the variance in one variable explained by the other, it does provide us with a measure of association between race and school assignment that can be compared across different school districts. As with MSD, minimizing ϕ^2 is equivalent to minimizing S, so that the two amount to the same desegregation criterion.

Table 3 shows the distribution of districts, schools, and students in the sample by values of S and by region. The notable difference between Table 3 and Table 1 (values of D) is that districts tend to be more heavily clustered under lower values of S than they were for D. This is not terribly

TABLE 3

Percent Distribution of S by Region

Regions	Value of S 0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.7	0.7-0.8	0.8-0.9	0.9-1.0	Total Numbers
South*											
Districts	70.1	15.1	6.6	3.1	2.1	1.5	0.9	0.4	0.1	0.2	1,168
Schools	48.5	19.1	7.5	6.4	5.7	6.1	3.8	1.9	1.0	0.1	18,652
Students	45.3	18.5	7.0	7.4	6.8	7.3	4.4	2.4	0.9	0.1	11,210,761
Minority Students	36.2	16.3	8.2	8.3	7.1	10.7	7.6	3.7	1.8	0.2	3,970,169
Non-South											
Districts	72.7	13.0	6.9	3.3	1.9	1.6	0.6	0.1	0.0	0.0	1,225
Schools	43.6	13.0	9.8	6.7	9.1	9.6	7.3	0.9	0.0	0.0	21,093
Students	39.1	11.9	8.9	6.9	11.8	10.4	10.0	1.1	0.0	0.0	13,483,458
Minority Students	21.0	8.9	7.8	6.8	20.9	13.9	18.9	1.9	0.0	0.0	4,580,079

Source: U.S. Department of Health, Education, and Welfare, Office for Civil Rights, Directory of Public Elementary and Secondary Schools in Selected Districts: Enrollment and Staff by Racial/Ethnic Group, Fall, 1972 (Washington, D.C.: U.S. Government Printing Office, 1973).

Note: Rows may not sum to 100 due to rounding errors.

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surprising, since using the mean-square-deviation should more heavily weight divergences (and, therefore, segregation) than using the average absolute deviation. It is important to note that our conclusions with respect to South/non-South comparisons are exactly the same as above: namely, although the distribution of districts tends to indicate about the same amount of segregation in the two regions, the distribution of students clearly shows more segregation outside the South.

C. Information Theory Index (H)

Information theory provides us with a technique for measuring the degree of association between two qualitative or categorical variables.[20] Consider the joint probability distribution given by P(A,B) where A and B are nonquantifiable events. The marginal and conditional distributions are given by P(A), P(B), P(A|B), and P(B|A). For our purposes, we define A as the school that an individual student attends and B as the minority/nonminority status of the student. Information theory then defines the average joint uncertainty of A and B as[21]

$$H(A,B) \quad = \quad - \sum_{ij}\sum P(A_i,B_j) \log P(A_i,B_j) \ .$$

Letting A_i represent assignment to school i (i = 1, ..., K), B_1 represent minority status, and B_2 represent nonminority status, we can write

$$H(A,B) \quad = \quad - \sum_i \left[\frac{p_iT_i}{T} \log \frac{p_iT_i}{T} + \frac{(1-p_i)T_i}{T} \log \frac{(1-p_i)T_i}{T}\right] . \quad (15)$$

The average marginal and conditional uncertainties are similarly defined and expressed as follows:

$$H(A) = -\sum_i P(A_i) \log P(A_i) = -\sum_i \frac{T_i}{T} \log \frac{T_i}{T} ; \tag{16}$$

$$H(B) = -\sum_j P(B_j) \log P(B_j) = p \log \frac{1}{p} + (1 - p) \log \frac{1}{(1 - p)} ; \tag{17}$$

$$\begin{aligned} H(A|B) &= -\sum_{ji}\sum P(A_i,B_j) \log P(A_i|B_j) \\ &= -\sum_i \frac{T_i}{T} \left[p_i \log \frac{p_i T_i}{pT} + (1 - p_i) \log \frac{(1 - p_i)T_i}{(1 - p)T} \right] ; \end{aligned} \tag{18}$$

$$\begin{aligned} H(B|A) &= -\sum_{ij}\sum P(A_i,B_j) \log P(B_j|A_i) \\ &= -\sum_i \frac{T_i}{T} \left[p_i \log \frac{1}{p_i} + (1 - p_i) \log \frac{1}{(1 - p_i)} \right] . \end{aligned} \tag{19}$$

The marginal uncertainty H(B) is the average prior amount of uncertainty about B over all possible cases, while the conditional uncertainty H(B|A) is the average amount of uncertainty concerning event B given knowledge of event A. The average relative reduction in uncertainty about B resulting from knowing A can then be written as

$$H = \frac{H(B) - H(B|A)}{H(B)} . \tag{20}$$

Certainly H(B) must be no less than H(B,A), since our uncertainty about B is reduced if we have knowledge of A so long as there is any relation at all between the two events. Thus, $H \leq 1$, with equality holding only when A and B are independent. H can therefore be interpreted as the relative reduction in uncertainty about the racial status of a particular student given that we know which school that student attends. The greater the value of H, the more certain we would be in predicting the race of any student in a particular school. H is therefore a measure of segregation: the larger its value for a particular school district, the more racially segregated are the schools of that district.

We have not yet justified defining our measure as the relative reduction in uncertainty about B given A rather than the relative reduction in uncertainty about A given B. Consider the following symmetric measure of association between A and B, again from information theory:[22]

$$\gamma^2 = \frac{H(A) + H(B) - H(A,B)}{\min[H(A), H(B)]} . \tag{21}$$

The numerator of γ^2 is called the "expected mutual information" and can be shown to be nonnegative.[23] In fact, it is true in general[24]

$$H(A) + H(B) - H(A,B) = H(B) - H(B|A) . \tag{22}$$

Furthermore, so long as the number of schools (K) is greater than 1 and no single school contains more than one-half the students, then, taking logarithms to the base 2 (as suggested by Theil and Finizza), we have the result that[25]

$$H(A) \geq 1 \geq H(B) .$$

The denominator of γ^2 is simply H(B), which, together with (22), implies that $\gamma^2 = H$. Thus H can be interpreted not only as a measure of the relative reduction in uncertainty but also as a measure of association highly analogous to a squared coefficient of correlation (ρ^2). The analogy is particularly strong in that both γ^2 and ρ^2 indicate how much of a reduction in uncertainty/variation in one particular variable can be achieved by knowing another.[26]

The relevant school segregation index is therefore

$$H = \frac{p \log \frac{1}{p} + (1 - p) \log \frac{1}{(1 - p)} - \sum_i \frac{T_i}{T} \left[p_i \log \frac{1}{p_i} + (1 - p_i) \log \frac{1}{(1 - p_i)}\right]}{p \log \frac{1}{p} + (1 - p) \log \frac{1}{(1 - p)}} . \tag{23}$$

Theil and Finizza have directly derived this index as a measure of school desegregation. They offer the interpretation of

$$p_i \log \frac{1}{p_i} + (1 - p_i) \log \frac{1}{(1 - p_i)}$$

as the "racial entropy" of the student body of the ith school. Analogously, then, H(B) can be termed the racial entropy of the district and H(B|A) the average school racial entropy.

Theil and Finizza also demonstrate that this type of index can be easily aggregated over large units. Switching to their terminology and notation for the moment for ease of presentation, we consider a set of G school districts (such as a city) and define the following "entropies" using the subscript g to denote values for the gth district:

$$\text{School:} \quad E_i = p_i \log \frac{1}{p_i} + (1 - p_i) \log \frac{1}{(1 - p_i)} .$$

$$\text{District:} \quad E_g = p_g \log \frac{1}{p_g} + (1 - p_g) \log \frac{1}{(1 - p_g)} .$$

$$\text{City:} \quad E = p \log \frac{1}{p} + (1 - p) \log \frac{1}{(1 - p)} . \qquad (24)$$

$$\text{Average district:} \quad \overline{E}_g = \sum_{i \varepsilon g} \frac{T_i}{T_g} E_i .$$

$$\text{Average city:} \quad \overline{E} = \sum_i \frac{T_i}{T} E_i = \sum_g \frac{T_g}{T} \overline{E}_g .$$

Unsubscripted values of p and T are now calculated over the entire set of G school districts.[27] Note that, for the gth school district, E_g is the same as H(B) and $\overline{E}_g$ is equal to H(B|A) as defined above in (17) and (19). The aggregation over the set of districts is straightforward: to obtain the value of H(B|A) for the city ($\overline{E}$), one simply takes a weighted average of the values of $H(B|A)$ for each district $(\overline{E}_g)$ with weights corresponding to the proportion of the city's enrollment in each district.

As well as providing a convenient method of aggregation, this formulation also allows us to make an interesting decomposition of $\overline{E}$. Theil and Finizza show that

$$\overline{E} = \sum_g \frac{T_g}{T} E_g - \sum_g \frac{T_g}{T} \overline{I}_g , \tag{25}$$

where

$$\overline{I}_g = \sum_{i\varepsilon g} \frac{T_i}{T_g} \left[p_i \log \frac{p_i}{p_g} + (1 - p_i) \log \frac{(1 - p_i)}{(1 - p_g)} \right] . \tag{26}$$

The quantity $\overline{I}_g$ is known in information theory as the average "expected information of the message that transforms the proportions $(p_g, 1 - p_g)$ to a second set of proportions $(p_i, 1 - p_i)$."[28] In other words, if we already know the percent minority of the gth district's student body (p_g), then $\overline{I}_g$ defines the expected information content, on the average, of a message that tells us the percent minority of the ith school in that district (p_i).

Since $\overline{I}_g$ is a measure of the extent to which the racial composition of the gth district differs from that of one of its schools, then the second term on the right-hand-side of (25) may be interpreted as a weighted average of the degree of racial segregation in each district. The first term in (25) is a weighted average of each district's total "entropy" and may be interpreted as a measure of the racial composition of each district relative to that of the city as a whole. Thus, (25) represents a decomposition of the city's average "entropy" into a component representing "between district" segregation and one representing "within district" segregation. This clearly provides a potentially fruitful method for investigating the currently controversial issue of cross-district school desegregation. Using the decomposition of (25), we can determine not only how segregated a set of school districts is, but also to what extent differences in the racial compositions of the districts contribute to the segregation of the overall system.

The distributions of districts, schools, and students across different values of H is given by region in Table 4. The notable point about these distributions is their remarkable similarity to the distributions across S of Table 3. Virtually any conclusion one would draw from the data of Table 3 would be identical if Table 4 were used. Some of the reasons for this similarity will be discussed in the next section.

II. COMPARISON OF MEASURES

There are some important qualifications that must be kept in mind when interpreting actual values of these indexes as measures of the extent of school desegregation. Each index is computed here on the basis of the entire student body across the district. This means that two implicit and erroneous assumptions must be recognized: (1) that students can be transferred between grade levels as well as between schools, since no account is taken of the grade span offered at each school; (2) that a particular student can be transferred to any school in the district just as "easily" as to any other. Assumption (1) is necessary even if one is only considering how much desegregation has been achieved within a particular district relative to what that district could accomplish. However, it is likely to create serious problems of interpretation in only two instances: if the district contains only a few schools, or if either the racial composition or actual degree of

TABLE 4

Percent Distribution of H by Region

Regions	Value of H: 0-0.1	0.1-0.2	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.7	0.7-0.8	0.8-0.9	0.9-1.0	Total Numbers
South*											
Districts	66.8	18.2	7.3	3.5	1.5	1.5	0.7	0.3	0.0	0.2	1,168
Schools	46.3	20.3	9.3	8.2	4.0	5.9	3.6	2.4	0.0	0.1	18,652
Students	43.5	19.6	8.6	9.6	4.5	7.1	4.3	2.8	0.0	0.1	11,210,761
Minority Students	36.5	16.9	8.4	11.1	4.3	10.3	7.6	4.8	0.0	0.2	3,970,169
Non-South											
Districts	69.1	16.5	7.6	3.5	1.6	0.9	0.5	0.1	0.0	0.2	1,225
Schools	40.8	15.6	11.7	6.9	9.8	8.5	5.8	0.9	0.0	0.1	21,093
Students	37.0	13.7	11.1	6.7	12.4	10.0	8.0	1.1	0.0	0.1	13,483,458
Minority Students	21.9	8.5	9.5	7.0	21.1	15.4	14.7	1.9	0.0	0.0	4,580,079

Source: U.S. Department of Health, Education, and Welfare, Office for Civil Rights, Directory of Public Elementary and Secondary Schools in Selected Districts: Enrollment and Staff by Racial/Ethnic Group, Fall, 1972 (Washington, D.C.: U.S. Government Printing Office, 1973).

Note: Rows may not sum to 100 due to rounding errors.

*Alabama, Arkansas, Delaware, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, Missouri, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, West Virginia.

desegregation differs substantially between sets of schools offering different grade spans (e.g., between elementary and secondary schools). Because we have excluded the very small districts from our sample, the effect of the first problem has been somewhat alleviated. Although we have not dealt explicitly with the second case, there is no particular reason to believe that it causes much of a problem except, perhaps, as a result of different dropout rates for older students.

Assumption (2) is not necessary unless one wishes to compare index values across districts as measures of relative desegregation efforts. In that case, account must be taken of the factors influencing relative costs of desegregation in different districts. Some of these factors are racial residential segregation, location of and distances between schools, and school capacities relative to population densities. No notion of these cost factors is included in the definition of any of the indexes. The closest we come to dealing with these problems here is in recognizing that the incremental cost of desegregation rises with the absolute level of desegregation. This implies that our choice of a measure for policy purposes should be one whose marginal payoff is a decreasing function of the level of desegregation. As we shall see below, both S and H exhibit this characteristic, while D does not.

The three indexes discussed here have several characteristics in common. First, they are all perfectly symmetrical with respect to the two racial/ethnic groups. Second, they are all nonconcave functions of the racial mix in each school. This insures that optimization on any one of the indexes will yield the most homogeneous possible racial composition of the schools.[29] The linearity of D, however, distinguishes it from the other two indexes, since the incremental payoff per student in terms of D is the same for a particular school once it is known whether that school's p_i is less than or greater than p. Figure 1 shows the marginal payoff per student in a particular school for H and S over different values of p_i.[30] The two values have been plotted using different scales (since their maximum values are not the same) to show that the shapes of the two payoff functions are very similar.[31] This is not surprising, since both of these indexes are measures of association between student racial affiliation and school assignment. For this reason, we would also expect any set of calculated values of S and H to be highly correlated, as, in fact, they turn out to be.[32]

There are other possible applications for these types of indexes within the context of school segregation. They can and have been used to examine issues of school faculty segregation by race as well as racial segregation of students between classrooms within grade level. Both of these issues have been very important in the South, first, because faculty desegregation has been interpreted by the courts as a necessary step in eliminating dual school systems and, second, because instances have been uncovered of southern systems that, after having desegregated their schools, effectively resegregate students by classroom. Table 5 displays simple correlation coefficients between the three indexes computed, for the sample of districts described above, on the bases of faculty desegregation and classroom desegregation for grades 3, 6, 9, and 12. The means and standard deviations of the index values are also presented.

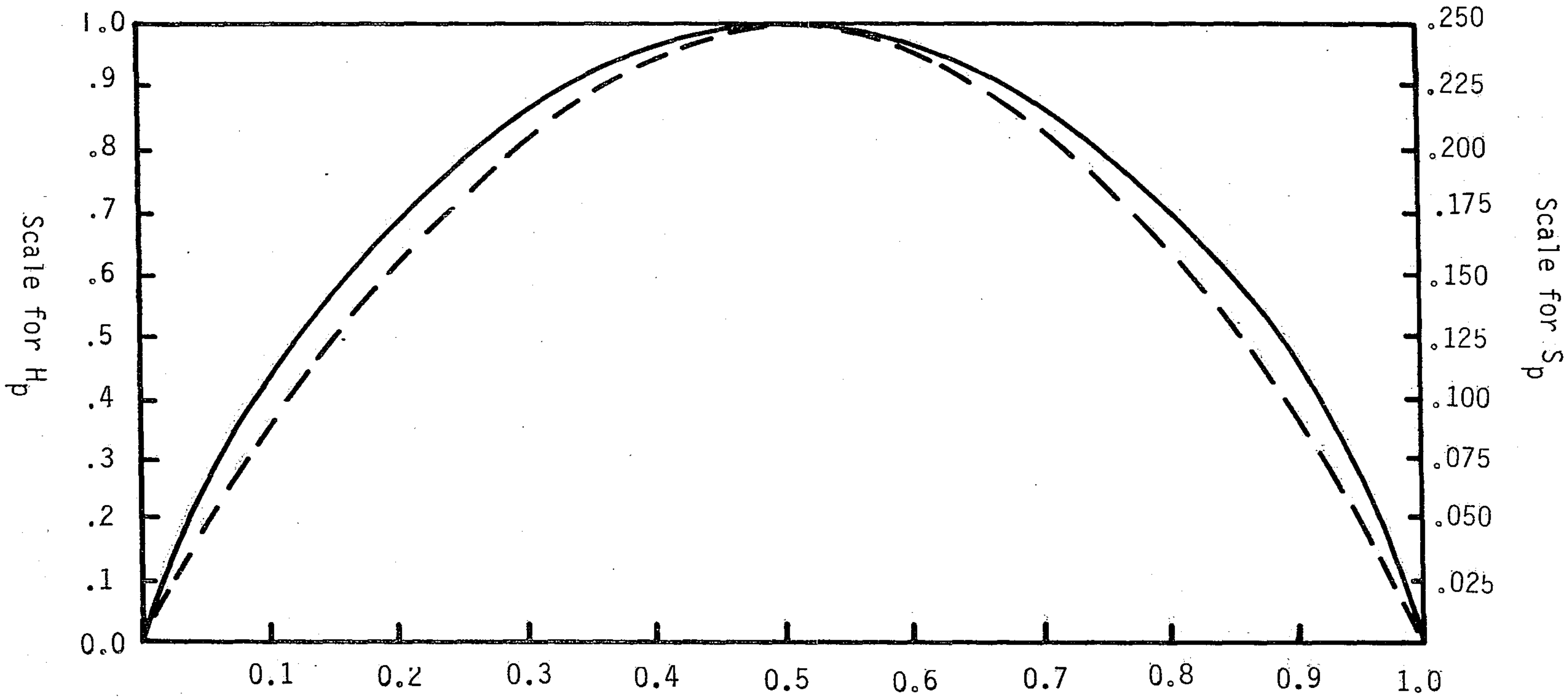
The indexes D_F, S_F, and H_F are straightforward extensions of D, S, and H, with the focus now on the numbers and racial composition of faculty

Figure 1

Segregation Index and Information Theory Index: Payoff per Student for Increasing the Desegregation Level of the _ith_ School as a Function of p_i

$$H_p = -[p_i \log_2 p_i + (1 - p_i) \log_2 (1 - p_i)]$$

$$S_p = p_i(1-p_i)$$



P_i = percent minority in the _ith_ school

TABLE 5

Simple Correlation Coefficients Between Different Measures of School, Faculty, and Classroom Segregation for 2,393 School Districts

Indexes	D	S	H	D_F	S_F	H_F	S_3	H_3	S_6	H_6	S_9	H_9	S_{12}	H_{12}
S	.87													
H	.93	.98												
D_F	.18	.07	.12											
S_F	.04	.02	.04	-.24										
H_F	.04	-.04	.00	.05	.94									
S_3	.47	.54	.52	-.04	-.04	-.09								
H_3	.48	.50	.50	.01	-.01	-.05	.98							
S_6	.45	.52	.50	-.08	-.05	-.11	.86	.83						
H_6	.47	.48	.49	-.02	-.03	-.07	.84	.84	.98					
S_9	.08	.13	.10	.03	.06	.05	.04	.03	.06	.04				
H_9	.08	.10	.09	.08	.08	.10	.02	.02	.03	.03	.99			
S_{12}	.05	.09	.07	.05	.08	.09	.01	-.00	.02	.02	.83	.83		
H_{12}	.06	.06	.05	.09	.11	.14	-.01	-.00	-.01	.00	.81	.83	.98	
Mean	.260	.097	.099	.317	.108	.161	.198	.209	.191	.202	.247	.259	.267	.288
Standard Deviation	.163	.131	.122	.211	.226	.227	.239	.230	.235	.225	.308	.299	.320	.308

Note: D_F, S_F, H_F: Measures of faculty segregation between schools.
S_3, H_3: Measures of classroom segregation in third grade.
S_6, H_6: Measures of classroom segregation in sixth grade.
S_9, H_9: Measures of classroom segregation in ninth grade.
S_{12}, H_{12}: Measures of classroom segregation in twelfth grade.

members in different schools. Extending the indexes to measure classroom segregation is slightly more complicated because of aggregation problems. As noted above, aggregation is no problem at all when using the information theory index. However, with both D and S, the issue arises as to which racial composition each classroom should be compared against--that of the school or that of the entire district. Using the former creates the problem of how to aggregate over all the schools in the district, while using the latter implies that students can be transferred between any two classrooms (of their grade level) in the district regardless of which school that classroom is in. We present here results for only one classroom segregation index other than H: S_3, S_6, S_9, and S_{12} are computed analogously to S using the district-wide percent minority for the appropriate grade level. The correlation coefficients between the S and H measures of classroom segregation reconfirm our theoretical claim that these two indexes tend to measure the same thing.

The high correlations between D and S and between D and H are somewhat surprising, since the distribution in Table 1 seems to be very different from those in Tables 3 and 4. However, if we note the fact that the variances of S and H appear to be somewhat smaller than that of D and that their ranges are lower, it is reasonable to assert that the three indexes do, indeed, move together linearly across districts. This can be confirmed by scanning the listing of index values for large districts in Table A.2 of the Appendix. Not only is the value of D always higher than those of S and H, but, while D never falls below .1 for this set of districts, S and H frequently do. Note also that D_F is not at all highly correlated with either S_F or H_F.

III. CONCLUSION AND COMMENTS

The evidence we have shown here leads us to conclude that, among the three segregation indexes discussed, the one derived from information theory (H) is the most useful. To appropriately qualify this statement, we will now consider the reasons for this choice.

Three reasons can be stated for preferring either S or H as a measure of segregation over D. First, both S and H incorporate the notion of diminishing marginal payoff to desegregation. This is useful in both a descriptive and a policy sense since there is good reason to believe that the cost of additional desegregation rises with the level. It is also relevant to incorporate this notion as a policy incentive since more weight is thereby given to desegregation efforts by the most segregated districts. Second, we have seen that S and H both depend on the entire distribution of students across schools rather than, as D does, on the numbers of students in schools with less than and those with more than the district-wide percent minority. Finally, although the dissimilarity index has a convenient and appealing interpretation, so do S and H. There seems to be no particular reason for preferring one of these interpretations to another. The ease of calculating D is an additional point in its favor and certainly relevant, although computers can just as easily handle one index as another.

Why, then, do we prefer H to S? Again, three arguments are put forth. First, we have seen that H can be conveniently and meaningfully aggregated, whereas the proper aggregation procedure for S is somewhat ambiguous. Although this point is not relevant when considering simply the level of school segregation within a district (or the level of classroom segregation within a school), it becomes very important in issues such as cross-district desegregation or state-by-state comparisons of desegregation efforts. Second, we have also seen that, for certain issues, a convenient decomposition of H is available, while this is not true of S. Finally, although both S and H are measures of association between racial/ethnic affiliation and school assignment, the interpretation of H is a bit more precise because of its analogy with the squared correlation coefficient. This last reason is a rather marginal one, since both S and H can be interpreted as the percent of one thing "attributable to" another. However, it should be noted that, unlike S, the definition of the information theory measure H would allow us to extend it to the case of more than two racial/ethnic categories.[33] Although this has not yet been applied to the issue of school segregation, it is potentially useful in areas with more than one predominant minority group, such as Blacks and Chicanos in the Southwest.

In addition to the usefulness of indexes as descriptive devices, they can have important applications as policy tools. Some examples relevant to the issue of school segregation are worth mentioning. Segregation indexes can be an informative aid in enforcing civil rights legislation. Indexes can be used to identify where problems exist as well as where progress has been made. In addition, appropriate indexes can be used as funding criteria for certain expenditure programs. The Emergency School Aid Act of 1972 is a case in point. This legislation was developed to provide financial assistance to desegregating school districts, and one of the explicit funding criterion was the extent to which minority isolation of students was reduced. Unfortunately, minority group isolation was defined by the bill to refer to any school whose enrollment was greater than 50 percent minority. This ruled out the use of a general segregation index as a funding criterion, although it would prevent an incentive for resegregation in districts which were, overall, more than 50 percent minority. Nevertheless, it provides a good example of the type of policy uses to which such indexes can be put.

The uses of indexes similar to the ones presented here are not, of course, limited to the issue of school segregation. The concepts embodied in this paper are directly transferable to the issue of residential segregation and, indeed, to any issue involving the distribution of a two-category (binomial) variable across some specified units, such as the distribution by race and by sex across occupations.

Finally, it is important to note that the two characteristics of the dissimilarity index that have made it so appealing--its ease of computation and its convenient interpretation--should not be dismissed lightly, especially given the realities of federal policy making. It is this writer's experience that even slightly complex analytic techniques are very slow to gain acceptance within the government bureaucracy. Nevertheless, if the effort is to be made, it should be towards a useful and meaningful end. In conclusion, then, the use of any of the three indexes presented here as a policy aid would be substantially better than a seat-of-the-pants type of judgment. However, if the costs of implementation and of gaining acceptance are not too great, we would opt for the information theory index as the most appropriate measure of segregation.

FOOTNOTES

[1]These data are published in U.S. Department of Health, Education, and Welfare, Office for Civil Rights, Directory of Public Elementary and Secondary Schools in Selected Districts: Enrollment and Staff by Racial/Ethnic Group (Washington, D.C.: U.S. Government Printing Office, 1973).

[2]The term minority is used throughout this paper to refer to all persons who were classified in the DHEW survey as American Indian, Negro, Oriental, or Spanish-Surnamed American. All other persons were reported in a single category and are referred to as nonminority.

[3]The sampling procedure used by DHEW resulted in all districts containing at least 3,000 students being surveyed while none of those with an enrollment of 300 were included.

[4]Alabama, Arkansas, Delaware, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, Missouri, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia, and West Virginia.

[5]Almost all of the excluded districts were omitted because they were either too small (2,424 districts) or greater than 95 percent non-minority (3,211 districts). In only 28 of the surveyed school districts was the student body greater than 95 percent minority, and the only large district in this category was the District of Columbia.

[6]Karl E. Taeuber, and Alma F. Taeuber, Negroes in Cities (Chicago: Aldine Publishing Company, 1967), Appendix A.

[7]See Farley and A. Taeuber, and Leslau for its use as a measure of school segregation. Farley and A. Taeuber also compare school with residential segregation using this index. Among other things, the dissimilarity index has been used to measure occupational segregation by sex. See the Council of Economic Advisors 1973 Report, Supplement to Chapter 4.

[8]Schools for which $p_i = p$ may arbitrarily be placed in either summation group.

[9]Note that D can equivalently be expressed as

$$D = \frac{1}{2} \Sigma_i \left| \frac{M_i}{M} - \frac{W_i}{W} \right| ,$$

where M and W refer to numbers of minority and nonminority students respectively. Note also that D is perfectly symmetrical with respect to minority and nonminority students since its value would be unchanged if p_i and p were defined instead as the proportions of nonminority students.

[10]Since D is symmetrical with respect to the two racial groups, it can also be written as

$$2D = (1 - p)\left[\sum_{p_i<p} T_i - \sum_{p_i\geq p} T_i\right] + \left[\sum_{p_i\geq p} T_i(1 - p_i) - \sum_{p_i<p} T_i(1 - p_i)\right]$$

[11]See Halliman H, Winsborough, "A Note on the Decomposition of Indexes of Dissimilarity," Institute for Research on Poverty (University of Wisconsin-Madison) Discussion Paper No. 201-74 (Madison, Wisconsin: 1974), for the derivation and discussion. The sample he uses compares racial residential segregation with between- and within-group income distributions.

[12]Reynolds Farley and Karl E. Taeuber, "Population Trends and Residential Segregation Since 1960," Science, Vol. 159, No. 3818 (March 1, 1968): 956.

[13]Three large districts do not appear in Table 2 because they were excluded from the sample: the District of Columbia, which is greater than 95 percent minority, and Baltimore County, Md., and Fairfax County, Va., both of which are less than 5 percent minority.

[14]See Ira H. Cisin, "Statistical Indices of School Integration," Technical Memorandum 70-1, Social Research Group, George Washington University, for the first and George Pugh, "Criteria for Measurement of Integration Level," Paper 65, Lambda Corporation, Arlington, Virginia, for the second. The Pugh paper also contains a helpful discussion of several alternative measures, including the one described by Cisin.

[15]S (like D) is perfectly symmetrical between the two racial groups and can be derived by averaging the percent minority in each school over all non-minority students. This average would then be

$$\sum_i \frac{T_i p_i (1 - p_i)}{T(1 - p)}$$

and its maximum value would be p.

[16]A similar concept can be used to derived the dissimilarity index (D) using absolute deviations as the criterion.

[17]One could perform a standard F test on the null hypothesis that $p_i = p$ for all i using

$$F = \frac{S/(K - 1)}{(1 - S)/(T - K)},$$

where K is the number of schools in the district. In practice, however, this is a somewhat misleading test to perform, particularly for policy purposes, since T is almost always very much larger than K. Thus, very slight deviations from racial balance will result in a rejection of the null hypothesis.

[18]See William L. Hays, Statistics (New York: Holt, Rinehart, and Winston, 1963), pp. 604-606, for a fuller discussion of the ϕ coefficient.

[19]This is the standard Pearson chi-square statistic computed from a 2xK contingency table using $p_i T_i$ and $(1 - p_i)T_i$ as the expected number of minority and nonminority students, respectively, in the ith school.

[20]See Hays, op. cit., pp. 610-612, and Henri Theil, Economics and Information Theory (Amsterdam: North-Holland Publishing Company, 1965), Chapters 1-3, for fuller discussions of this approach.

[21]This formula amounts to the expected value of the quantity $-\log P(A_i, B_j)$ over all i and j. When the logarithm is taken to the base 2 (as we choose below to do), then this quantity equals the minimum number of "yes-no" questions one would have to ask in order to determine the school and racial/ethnic affiliation of any particular student. In the language of information theory, it is the "information content" of the message containing both of these pieces of information about the student. For a univariate application of this concept to measures of industrial concentraction, see Theil, op. cit., Chapter 8. An additional justification for using the logarithmic function is its additive properties. See Theil, op. cit., Chapter 4.

[22]See Hays, op. cit., p. 611.

[23]For a proof, see Theil, op. cit., pp. 34-35.

[24]See Theil, op. cit., pp. 49-50, for the proof. In his words, this result can be described as follows: "The expected mutual information is equal to the unconditional entropy [i.e., expected information content], given the messages sent." In equation (22), the left-hand-side represents the expected mutual information, H(B), the unconditional entropy, and $H(B|A)$ the entropy conditioned on knowledge of A.

[25]If the proportion of students attending school k (T_k/T) is no greater than 1/2 for all k, then

$$-(T_k/T)\log_2 (T_k/T) \geq 1/2 \text{ for all } k$$

and

$$H(A) = -\sum_k (T_k/T)\log_2 (T_k/T) \geq K/2.$$

But $(K/2) \geq 1$ so long as there is more than one school. Therefore, $H(A) \geq 1$. The value of H(B) is solely determined by p and, taking logs to the base 2, has a maximum value of 1 and a minimum of 0.

[26]Measures of association between more than two categorical variables can also be derived from information theory. See Theil, op. cit., pp. 55-59.

[27]Thus, $T = \sum_g T_g$, and $p = \sum_g (T_g/T)p_g$.

[28]See Henri Theil and Anthony Finizza, "A Note on the Measurement of Racial Integration of Schools by Means of Informational Concepts," Journal of Mathematical Sociology 1971, Vol. 1, p. 191. Note that the value of I_g for the gth district is defined as $(E_g - \overline{E}_g)$ and is the same as $H(B) - H(\overline{B}|A)$ as defined above.

[29]Since each index is defined here as a measure of segregation minimization of the indexes will result in racially balanced schools. Any of the three indexes could be redefined as one minus its current value without loss of its properties, in which case maximization would be the appropriate goal.

[30]A graph like Figure 1 cannot be drawn for D independently of the value of p. Such a graph would simply be two straight lines, one rising from 0 at $p_i = 0$ to its maximum where $p_i = p$ and the other falling to zero at $p_i = 1$.

[31]The same change of scale in Figure 1 could have been accomplished by taking logarithms to the base 16 for H_p, thereby making its maximum value also equal to .25.

[32]See Table 5 below.

[33]Applications of an information theory measure using more than two categories include measurements of the inequality of income and of industrial concentration. See Ann R. Horowitz, "Trends in the Distribution of Family Income Within and Between Racial Groups," in George M. von Furstenberg, et al., editors, Patterns of Racial Discrimination, Volume II: Employment and Income (Lexington, Mass.: D.C. Heath and Company, 1974), for the former and George J. Stigler, The Organization of Industry (Homewood, Ill.: Richard D. Irwin, Inc., 1968), pp. 32-35 for the latter. Horowitz makes interesting use of the decomposition properties to compare black and white income distributions. Theil also suggests a wide range of applications.

APPENDIX

TABLE A.1: School Districts and Enrollment in the 1972 Sample by Region and Size of District

TABLE A.2: Segregation Indexes for Districts in the 1972 Sample Enrolling 25,000 or More Students

TABLE A.1

School Districts and Enrollment in the 1972 Sample
by Region and Size of District

	Districts with Less Than 25,000			Districts with 25,000 +		
Region	# Districts	# Students	# Minority Students	# Districts	# Students	# Minority Students
South	1,086	7,007,060	1,531,277	82	5,103,729	1,932,025
Non-South	1,148	6,107,032	2,038,144	77	6,476,398	3,048,802
Totals	2,234	13,114,092	3,569,421	159	11,580,127	4,980,827

TABLE A.2

Segregation Indexes for Districts in the 1972 Sample
Enrolling 25,000 or More Students

District Name	K # Schools	T Total Enrollments	p Percent Minority	Percent Bused*	National Size Rank**	D	S	H
Alabama:								
Birmingham City	92	57,729	59.51	0.07	63	.7613	.6519	.6151
Huntsville City	40	35,407	16.08	1.56	110	.4407	.2272	.2297
Jefferson Co.	74	55,191	24.15	60.96	69	.4811	.3798	.3510
Mobile City Co.	82	66,263	45.76	38.82	50	.5252	.3910	.3473
Montgomery City Co.	55	36,949	46.35	31.37	105	.5788	.4413	.3852
Alaska:								
Anchorage	54	33,852	10.50	52.63	123	.2918	.0745	.0870
Arizona:								
Phoenix Union High	11	28,773	26.16	0.00	158	.5644	.3476	.2987
Tucson Elementary	83	43,323	34.97	14.25	92	.6165	.4690	.3949
Arkansas:								
Pulaski Co. Special	40	27,211	18.83	62.51	168	.4253	.2354	.2316
California:								
Anaheim Union High	26	37,340	11.32	9.52	103	.1955	.0292	.0394
Fremont USD	47	32,857	11.79	18.66	129	.1969	.0275	.0371
Fresno USD	76	55,002	31.45	11.60	70	.4635	.2974	.2467
Garden Grove USD	68	51,373	14.38	16.17	76	.3262	.0917	.0959
Hacienda-La Puente USD	43	30,439	35.31	11.81	145	.2983	.1100	.0869
Hayward USD	48	25,139	29.36	8.48	184	.1819	.0445	.0357
Long Beach USD	79	63,838	20.28	4.77	55	.4957	.2502	.2218
Los Angeles USD	618	620,707	52.73	5.66	2	.6879	.5602	.5029
Mt. Diablo USD	61	46,457	5.73	20.41	85	.2273	.0440	.0624
Norwalk-La Miranda USD	38	27,742	28.09	9.52	164	.3106	.1303	.1082
Oakland USD	100	65,505	75.30	1.84	52	.5845	.3659	.3428
Orange USD	39	28,471	10.38	15.21	159	.2990	.0829	.0930
Pasadena USD	41	26,225	52.24	46.90	177	.1035	.0187	.0140
Richmond USD	62	39,968	39.89	9.88	97	.4299	.2775	.2386

TABLE A.2 (cont.)

District Name	K # Schools	T Total Enrollments	p Percent Minority	Percent Bused*	National Size Rank**	D	S	H
California (cont.):								
Riverside USD	35	25,555	24.62	18.29	180	.1547	.0307	.0265
Sacramento City USD	75	48,774	37.98	4.08	78	.2892	.1211	.0965
San Bernadino City USD	61	34,459	37.34	13.07	118	.3605	.2091	.1706
San Diego USD	162	124,604	26.32	2.77	19	.5264	.3792	.3246
San Francisco USD	171	81,970	68.20	26.14	36	.2264	.0755	.0644
San Jose USD	47	37,125	29.22	25.23	104	.5857	.3643	.3019
Santa Ana USD	35	27,014	47.34	8.59	169	.3591	.1956	.1550
Stockton USD	42	31,406	42.78	12.96	138	.4918	.2899	.2276
Torrance USD	40	31,433	11.71	6.84	137	.2909	.0646	.0796
Colorado:								
Colorado Springs	48	35,853	16.74	10.97	108	.3603	.1992	.1730
Denver	119	91,616	41.69	16.34	29	.4690	.3076	.2558
Pueblo City	42	26,947	42.61	5.95	170	.3501	.1818	.1420
Connecticut:								
Hartford	40	28,069	71.04	13.39	162	.6433	.4505	.4129
Florida:								
Brevard Co.	69	62,285	12.14	37.82	59	.3771	.1354	.1525
Broward Co.	141	128,889	24.55	33.05	17	.3080	.1420	.1309
Dade Co.	239	241,809	51.64	18.10	6	.5217	.3544	.3002
Duval Co.	139	113,644	32.64	46.97	20	.3273	.1807	.1524
Escambia Co.	71	47,952	29.11	57.64	81	.5189	.3197	.2831
Hillsborough Co.	132	106,294	25.58	53.42	21	.1810	.0399	.0309
Okaloosa Co.	36	26,892	9.69	50.65	171	.2686	.0436	.0651
Orange Co.	101	86,093	20.41	39.03	33	.6334	.4474	.4184
Palm Beach Co.	83	67,030	32.75	48.08	49	.3229	.1726	.1361
Pinellas Co.	115	90,177	16.51	44.04	30	.2430	.0565	.0612
Polk Co.	88	57,006	22.93	45.26	65	.4463	.2353	.2288
Seminole Co.	32	26,458	19.76	62.90	174	.4709	.2051	.2058
Volusia Co.	53	34,578	22.24	46.09	114	.2500	.0833	.0834

TABLE A.2 (cont.)

District Name	K # Schools	T Total Enrollments	p Percent Minority	Percent Bused*	National Size Rank**	D	S	H
Georgia:								
Atlanta City	153	96,006	77.41	2.18	27	.8023	.6176	.6072
Bibb Co.	58	30,817	50.36	30.10	142	.5154	.3982	.3592
Chatham Co.	63	34,998	51.73	54.96	112	.2014	.0833	.0714
Clayton Co.	32	29,483	5.17	68.75	151	.4704	.0545	.1428
Dekalb Co.	115	86,144	10.07	49.21	32	.6504	.4170	.4298
Fulton Co.	76	34,584	10.79	37.84	113	.6450	.3960	.4064
Muscogee Co.	64	38,349	34.84	31.31	100	.1871	.0747	.0577
Richmond Co.	57	32,501	43.85	50.65	131	.3860	.2376	.2012
Illinois:								
Chicago	656	557,141	68.95	1.48	3	.7987	.6894	.6586
Elgin	43	25,470	9.86	33.10	181	.3708	.1169	.1366
Rockford	72	41,364	15.07	8.61	95	.6357	.3776	.3805
Indiana:								
Evansville-Vandeburgh	39	31,937	9.77	35.90	135	.2659	.0483	.0697
Fort Wayne	62	43,245	18.03	23.14	93	.5055	.3674	.3393
Gary	50	44,830	77.89	7.38	88	.8596	.6885	.6534
Indianapolis	125	98,076	39.76	9.06	25	.6705	.5630	.5146
South Bend	46	34,361	19.81	27.10	120	.5838	.3857	.3742
Iowa:								
Des Moines	80	43,226	10.25	9.84	94	.5785	.3167	.3239
Kansas:								
Kansas City	58	32,945	38.33	30.01	128	.6260	.5186	.4595
Wichita	109	57,222	19.74	29.10	64	.1688	.0344	.0311
Kentucky:								
Fayette Co.	47	35,648	18.37	58.77	109	.2971	.0903	.0991
Louisville	67	49,133	51.13	0.56	77	.8030	.7099	.6519

TABLE A.2 (cont.)

District Name	K # Schools	T Total Enrollments	p Percent Minority	Percent Bused*	National Size Rank**	D	S	H
Louisiana:								
Caddo Parish	80	52,336	50.24	27.86	73	.6756	.5571	.5116
Calcasieu Parish	67	38,520	26.92	45.09	99	.6793	.5638	.5007
East Baton Rouge Parish	106	67,242	39.55	48.33	48	.7355	.6546	.6103
Jefferson Parish	78	65,656	23.37	77.69	51	.2549	.0639	.0569
Lafayette Parish	39	29,144	25.22	79.84	154	.2619	.1117	.0966
Orleans Parish	141	103,839	76.37	8.83	24	.7662	.5805	.5718
Rapides Parish	49	28,118	34.86	60.54	161	.6640	.5762	.5194
Maryland:								
Anne Arundel Co.	97	76,756	13.30	66.29	40	.4549	.1694	.1890
Baltimore City	218	186,600	69.27	31.02	8	.8218	.6943	.6690
Harford Co.	39	32,418	10.35	77.96	133	.3570	.0648	.0956
Montgomery Co.	197	126,912	10.08	43.66	18	.2928	.0577	.0749
Prince Georges Co.	235	161,961	26.51	48.35	9	.6077	.4383	.3853
Massachusetts:								
Boston	202	96,239	40.35	4.26	26	.7082	.5832	.5210
Springfield	53	30,497	32.35	32.03	143	.4555	.2864	.2366
Worcester	64	29,426	5.97	22.22	152	.4530	.0953	.1641
Michigan:								
Detroit	325	276,655	69.49	5.23	5	.7448	.6037	.5658
Flint City	59	46,115	46.63	1.22	86	.5992	.4434	.3831
Grand Rapids	72	33,890	29.07	19.24	122	.5313	.4307	.3823
Lansing	58	31,404	22.47	14.17	139	.2244	.0670	.0614
Minnesota:								
Minneapolis	119	61,565	15.83	7.48	61	.5059	.2465	.2427
St. Paul	85	48,151	11.59	7.29	80	.4960	.2516	.2540
Mississippi:								
Jackson	54	29,861	66.03	24.89	149	.3871	.1805	.1580

TABLE A.2 (cont.)

District Name	K # Schools	T Total Enrollments	p Percent Minority	Percent Bused*	National Size Rank**	D	S	H
Missouri:								
Kansas City	99	65,414	54.39	19.05	53	.8643	.7946	.7529
St. Louis City	181	105,617	69.10	9.59	23	.8983	.8318	.7952
Nebraska:								
Omaha	98	63,125	21.77	1.67	56	.6710	.5135	.4727
Nevada:								
Clark Co.	95	75,207	18.01	27.56	42	.2130	.0418	.0451
Washoe Co.	51	29,705	7.27	26.35	150	.3660	.1156	.1381
New Jersey:								
Jersey City	37	38,616	63.97	4.04	98	.5973	.4333	.3811
Newark	95	78,492	87.72	6.61	38	.7360	.4396	.4797
Paterson	32	27,548	72.58	4.65	166	.4862	.2846	.2594
New Mexico:								
Albuquerque	111	86,650	42.57	35.17	31	.4907	.2964	.2388
New York:								
Buffalo	99	64,262	45.10	21.00	54	.6299	.5020	.4452
New York City	1,205	1,125,449	64.41	29.13	1	.6715	.4890	.4409
Rochester	60	43,340	44.00	20.88	91	.5005	.3451	.2917
Syracuse	45	27,603	28.94	11.60	165	.4723	.2802	.2403
Yonkers	45	29,444	22.48	1.75	153	.5643	.3292	.3006
North Carolina:								
Cumberland Co.	51	32,999	27.73	65.66	127	.2026	.0527	.0471
Forsyth Co.-Winston Salem	67	46,675	30.45	63.05	84	.1536	.0505	.0413
Gaston Co.	55	33,173	15.46	37.58	125	.2418	.0503	.0625
Greensboro	46	28,321	37.42	66.47	160	.1407	.0277	.0214
Mecklenburg Co.-Charlotte	107	79,812	32.81	59.45	37	.1387	.0331	.0270
Wake Co.	42	29,878	25.12	63.92	148	.3331	.1091	.0967

TABLE A.2 (cont.)

District Name	K # Schools	T Total Enrollment	p Percent Minority	Percent Bused*	National Size Rank**	D	S	H
Ohio:								
Akron	67	53,997	29.01	4.13	71	.6398	.4779	.4480
Cincinnati	108	77,880	47.66	6.29	39	.6942	.5721	.5230
Cleveland	191	145,196	59.92	2.62	11	.8804	.7971	.7537
Columbus	171	106,676	29.83	8.86	22	.7003	.5709	.5274
Dayton	68	52,162	44.98	7.83	75	.7809	.6818	.6240
Toledo	78	61,694	30.67	36.83	60	.6572	.5235	.4660
Oklahoma:								
Oklahoma City	109	60,275	29.95	33.38	62	.2682	.0914	.0748
Tulsa City	108	71,190	20.13	15.31	45	.5966	.3772	.3507
Oregon:								
Portland	124	68,613	13.97	9.32	47	.4359	.2194	.2057
Pennsylvania:								
Philadelphia	277	282,965	64.82	5.03	4	.7805	.6769	.6328
Pittsburgh	152	70,050	42.25	14.26	46	.6553	.5341	.4788
South Carolina:								
Charleston Co.	84	55,562	49.17	48.26	68	.6233	.4855	.4333
Greenville Co.	91	56,636	22.24	41.35	66	.1399	.0132	.0134
Richland Co.	63	36,074	56.86	56.27	107	.2806	.1273	.1103
Tennessee:								
Knoxville City	65	34,524	18.31	22.43	116	.7467	.5720	.5505
Memphis City	163	138,714	57.96	1.63	13	.8551	.7874	.7551
Nashville-Davidson Co.	137	85,406	28.11	51.18	34	.3779	.1683	.1636
Texas:								
Aldine ISD	25	28,909	29.84	47.94	157	.5223	.4256	.3736
Amarillo ISD	48	27,355	16.32	4.74	167	.3539	.1347	.1334
Austin ISD	75	55,861	36.96	13.25	67	.6152	.4943	.4273
Corpus Christi ISD	64	45,567	58.75	2.35	87	.6178	.4404	.3832

TABLE A.2 (cont.)

District Name	K # Schools	T Total Enrollment	p Percent Minority	Percent Bused*	National Size Rank**	D	S	H
Texas (cont.):								
Dallas ISD	189	154,580	49.40	9.08	10	.7040	.5787	.5233
El Paso ISD	62	62,403	60.66	9.68	58	.6137	.4074	.3547
Fort Worth ISD	114	82,268	40.63	13.65	35	.6371	.5098	.4550
Houston ISD	232	225,410	56.40	7.71	7	.7266	.5955	.5334
Lubbock ISD	53	32,830	38.12	2.48	130	.7760	.6472	.5763
North East ISD	31	29,142	13.51	25.51	155	.1939	.0296	.0352
Northside ISD	31	25,407	29.52	23.65	182	.3539	.1918	.1556
Pasadena ISD	37	35,018	10.48	21.93	111	.2869	.0775	.0879
San Antonia ISD	99	72,315	80.40	6.60	44	.5531	.2722	.2781
Ysleta ISD	35	36,736	66.95	6.44	106	.5748	.3763	.3177
Utah:								
Davis Co.	51	34,541	5.20	25.36	115	.4376	.0464	.1141
Salt Lake City	50	31,322	12.08	13.01	140	.4047	.1320	.1553
Virginia:								
Chesapeake City	32	25,005	29.73	80.09	185	.2283	.0647	.0562
Hampton City	39	32,165	31.72	24.28	134	.2312	.0640	.0522
Henrico Co.	45	33,167	9.92	66.40	126	.4548	.1110	.1640
Newport News City	38	30,195	37.56	78.78	146	.2401	.0846	.0650
Norfolk City	68	48,701	50.67	59.37	79	.1371	.0338	.0252
Prince William Co.	43	34,466	7.31	69.50	117	.2224	.0282	.0494
Richmond City	83	43,825	70.56	51.22	90	.2894	.0968	.0822
Virginia Beach City	48	47,919	11.59	91.08	80	.2399	.0452	.0587
Washington:								
Seattle	127	75,239	22.88	12.74	41	.5409	.3202	.2792
Spokane	55	33,281	5.06	20.60	124	.3167	.0459	.0808
Tacoma	63	34,453	15.29	28.06	119	.2657	.0665	.0705

TABLE A.2 (cont.)

District Name	K # Schools	T Total Enrollment	p Percent Minority	Percent Bused*	National Size Rank**	D	S	H
West Virginia:								
Kanawha	122	52,289	6.72	54.71	74	.5551	.1740	.2575
Wisconsin:								
Milwaukee	161	128,734	33.92	6.42	16	.7575	.6719	.6112
Racine	47	31,309	18.28	27.02	141	.4716	.2711	.2549

*This figure equals the number of students reported to have been "transported at public expense" to school as a percent of the total district enrollment.

**The size rank is based on all 185 districts in the country that enrolled 25,000 or more students in 1972, only 159 of which are listed here.

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