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THE EQUILIBRIUM LEVEL OF UNEMPLOYMENT:
A SIMULATION

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ABSTRACT

This paper describes a dynamic mathematical model of the labor market which can simulate its equilibrium under a variety of circumstances. The model is neoclassical in origin, but frictions have been built into it in a variety of ways in an attempt to replicate the effect of information costs, uncertainty, and capital market imperfections. The study is designed to explore the general behavior of a nonhomogeneous labor market, but the simulations can shed light on many specific questions in that context.

One thousand laborers are specified to differ from each other by a normally distributed characteristic--called talent--which affects their productivity. The market is divided into ten skill groupings, which differ from each other in their talent requirements, and laborers attempt to get into the highest skill class in which they can find work. Ten firms offer employment in each of these skill classes to those workers whose talent is sufficient to make them productive in that class.

With hiring, firing, quit, and production decisions being made endogenously, the model determines an equilibrium level of frictional unemployment that depends on the various frictional parameters and the nature of the shocks to which the model is exposed. Several experiments are reported to examine the effect of cycles, random shocks, and demand changes on that equilibrium.

THE EQUILIBRIUM LEVEL OF UNEMPLOYMENT: A SIMULATION

I. INTRODUCTION

Is there an efficient level of unemployment? Is the level of unemployment that would exist in a perfectly competitive labor market efficient? What are the characteristics of the perfectly competitive rate of unemployment? How can aggregate policy be used to offset non-competitive aspects of the labor market? These are but a few of the many questions that must be answered before we can begin to talk about optimal macroeconomic policies. Unfortunately, we have answers to none of these questions, and even worse, we are not sure of what apparatus to use in deriving those answers. The problem is not that labor market theorists are backward, or even that economic theory in general is backward; it is simply that the theory is not well suited to answering questions of this kind.

Solow says "The art of successful theorising is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive."[1] Unfortunately, for a variety of labor market problems, the results seem to be exceptionally sensitive to the kinds of assumptions economists usually make. Solow defines a crucial assumption as "one on which the conclusions do depend sensitively," and he notes that crucial assumptions should be realistic. Several of the assumptions most frequently made by economists in other contexts--presumably where they are not crucial--seem to be crucial in labor market analysis. Thus, at present, labor market economists are finding great difficulty in choosing a set of assumptions which will reduce their problems to a mathematically tractable level without affecting significantly the conclusions that can be drawn about several important phenomena. Specifications that are simple enough to yield results do not seem to be able to lend insight into many questions of great importance.

Part of this is due to the historical development of economics as we know it. Our greatest accomplishments have been our concise characterizations of competitive markets along with our imaginative manipulations of those characterizations. With monopoly, however, we have done much worse. While it is true that we have a precise theory of how a single monopolist should behave when he finds himself in a world otherwise characterized by perfect competition, or how a single laborer should carry out a policy of optimal search for employment in a variety of given environments, we have no good way of aggregating those monopolies or workers into an economy which simultaneously determines the environments within which each of those agents behaves. This is true not only of the most abstract, mathematical models of general equilibrium, but of practical, applied models of the labor market as well. Our theory is most powerful when it can be applied to problems that can be conveniently represented by competitive markets for homogeneous goods and factors.

For many problems, this shortcoming of conventional theory is no more than an annoyance; it comprises but one of the many awkward but realistic factors that a streamlined theory does well to ignore. Even in the labor market, there are problems that can be conveniently handled in the conventional framework; the explanation of why wage rates differ in different contries, for example, is most easily accomplished using supply and demand. But for other problems, and in particular, for the explanation of unemployment, the use of supply and demand, or any other apparatus that treats laborers identically, merely sidesteps the problem. In this case, the heterogeneity of labor lies at the very heart of the problem, and no matter how awkward it is to model that factor, some account of it must be taken if the model is to have any predictive power. Unfortunately, we cannot at this date model heterogeneity in a very tractable way.

It is difficult enough to generate a static model of equilibrium for a heterogeneous commodity, but in labor market analysis, equilibrium is even more difficult to specify because unemployment is dynamic in nature. It is a state through which workers pass, not a permanent state for a particular group of workers. While we expect a certain percentage of houses to be vacant at any moment in time, for example, it is a rare house for which we predict vacancy in the long run. A static model of housing or unemployment will eventually clear with no vacancies or unemployment unless further shocks are encountered.

For these reasons, economists' models of unemployment equilibrium are very awkward at the moment. Part of the awkwardness may be circumvented in the future by theoretical breakthroughs--by what Solow calls the art of successful theorizing. It is idle to speculate here about the nature of these breakthroughs, but it should be clear that a great contribution will have been made by the person who finds a way to represent with great economy the interactions among the many complex search, informational, and subjective factors that exist in markets for heterogeneous commodities like land, labor, structures, or used equipment.

Another part of the awkwardness is simply computational. Even an economical theory of how a heterogeneous market functions may require a great amount of computational work if it is ever to be applied. In this vein, it is interesting to ask how much of our present inability to grapple with some important labor market problems is a result of the weakness of our theory, and how much is simply due to the computational problems that arise when attempting to aggregate the behavior of diverse individuals into that of a market as a whole.

The simulation model I report on in this paper is intended to shed light on this question. This model uses a very simple specification of hiring, firing, quit, and wage decisions that are applied to 1000 workers of differing abilities. These decisions constitute a labor market that has certain realistic characteristics not seen in previous models. The computational complexity that necessitates the use of simulation as opposed to analytical methods results simply from the fact that the productive power of each worker is assumed to be different. While this makes the numerical detail of the problem enormous, the economic behavior can be kept to a minimum so that the effect on the equilibrium of a change in the level of one parameter or another can be easily determined.

While I cannot reach, in this way, general answers to the grand questions posed at the outset of the paper, I can approach those problems in a way very different from the ways they have been considered to date. Answers to questions of a more limited nature can be reached, and these questions are far broader than those which can be addressed with conventional analytical means. Specifically, in this paper I can answer the following questions: (1) Is there a non-zero rate of unemployment which maximizes steady state GNP? (2) Does an economy with a cyclically fluctuating unemployment rate have a higher or lower GNP on average than one with the same average, but constant, unemployment rate? (3) How does the equilibrium unemployment rate depend on the structure of the labor market?

It is true, of course, that the answers to these questions that are generated by any model whether analytic or simulative, will be of direct policy applicability only if the model is a careful representation of the economy in question. The model I will describe is a pilot, or laboratory, model whose purpose is to explore in the abstract the relations between certain economic concepts which can be related only with great difficulty using analytic techniques. Therefore, the quantitative results I derive are of little interest; it is the qualitative dependencies that I wish to isolate.

Since the questions cover rather broad areas of labor economics and macroeconomics, some of which are not usually thought of in conjunction with each other, the antecedents of this model can be found in a diverse literature that I can only briefly mention here.

The macroeconomic issues were described in the volume by Phelps et al. (1970) which explored the link between inflation and unemployment. Various factors must be considered when describing that link, and a large literature now exists on each. Holt and David (1966), in a seminal paper, had first described the links between turnover and unemployment, thereby giving an empirical foundation to the dynamic concept of frictional unemployment. Recently, estimates of many aspects of these complex hypotheses have appeared in several issues of the Brookings Papers, most notably the work of Hall (1970, 1972), Gordon (1971, 1973), and Perry (1970, 1972). The emphasis in this literature has been macroeconomic in the sense that its purpose has been to improve our understanding of the effect of macroeconomic policy on inflation and unemployment.

Various institutional theories have also grown up to explain other labor market phenomena not easily described by neoclassical theory. These include the work of Thurow and Lucas (1972), Piore and Doeringer (1971), and the more radical market segmentation theories of Reich, Gordon and Edwards (1973). Feldstein (1973) also examined the effect of a group of institutional forces on the equilibrium unemployment rate.

The reader of this literature cannot fail to be impressed with the difficulty of the problems being considered, and the inadequacy of existing theory to give concise, satisfactory answers to the important questions being asked.

II. AN OVERVIEW OF THE MODEL

In section III below, I describe the basic functional forms and parameter values which are to be used in the subsequent simulations. Here I describe broadly the model's structure.

There are 1000 workers in the simulation who differ from each other by a single normally distributed characteristic called <u>talent</u>. There are ten firms, each of which produces <u>output</u> according to a production function that uses ten different labor <u>skills</u> as inputs. Skills differ from each other only in the level of talent that they require. Different workers will be able to contribute different amounts of each skill with high talent workers being able to outproduce low talent workers in all skills. The functional dependency is nonlinear so that high talent individuals have a comparative advantage at high skill jobs. There is no skill-specific training that workers must have.

Thus there are 100 different jobs (firm-skill combinations) that a worker might acquire, each with its own wage rate. Workers attempt to get the best jobs they can--those with the highest wage rates--while firms attempt to hire the best workers they can--those with the most talent. The heart of the simulation is the set of rules which governs the search processes that are carried out in each time period in order to match workers and jobs. An outline of that process follows.

Taken as given is some allocation of workers to jobs (or to unemployment), and a wage rate for each job. For the first time period, the wage rates are determined exogenously, while the allocation of workers to jobs is done randomly. For subsequent periods, those data are carried over from the preceding period.

Each worker, if employed, decides whether or not to quit. He makes this decision after considering the unemployment rate, the wages available on his present job and elsewhere, and his own talent relative to that of his co-workers. If he quits, he determines an asking wage and becomes unemployed. He seeks work in the skill classification above the one he left. If unemployed at the outset of the period, he lowers his asking wage by five percent, and decides whether or not to lower his skill-classification and search for less desirable jobs.

Firms examine their employees and fire those workers whose production is less than eighty percent of the wage being paid at that level. I will use the term marginal product to denote that production, though the heterogeneity of the labor force implies that each worker will have a different marginal product at each job. It is marginal in the sense that the production is calculated taking as given the allocation of the other workers to their jobs within that firm. Eighty percent is an arbitrary number and its purpose is to signify the uncertainty that firms have about the productiveness of their workers.

Firms then search the unemployed for workers who have a level of talent that is high enough to make them productive at the job in question. The search is carried out for each job in order according to the wage rate being offered, with the high wage jobs getting the first crack. The unemployed are classified by skill, and only those classed one above, at, or one below the skill listing of the jobs are searched. An offer is made to any unemployed worker discovered through this process whose talent is sufficiently large that his marginal product will exceed the wage at the job in question. A worker accepts the first job offer which has a wage in excess of his asking wage. These hire, fire, and quit decisions determine a new allocation of workers to jobs which is maintained until the next period. The only behavior of importance that is not contained in this sequence is that which determines wage rates. Firms determine wage offers in a rather complex manner that is described more fully below. Here, we need only note that wages are increased when they are less than the marginal product of the worst worker on the job, and decreased when they are greater.

Aggregate demand can be simulated by changing the demand for the output of all firms. This demand is an important element of the demand for labor. These output demand functions can also be subjected to random shocks in order to create the need for labor turnover. This is the only stochastic force which I have used in this paper.

III. SPECIFIC FUNCTIONS AND PARAMETER VALUES

The 1000 workers in the model are numbered consecutively from one and are indexed by the letter K. The single characteristic, TALENT, which distinguishes workers from each other is normally distributed with mean of 1.0 and standard deviation of .15.[2] Specifically, each worker is assigned a level of talent according to the following implicit function.

$$\text{(1)} \qquad \frac{K-1/2}{1000} = \int_{-\infty}^{\text{TALENT (K)}} \frac{1}{.15\sqrt{2\Pi}} e^{-1/2\left(\frac{x-1}{.15}\right)^2} dx$$

Thus worker #100 has that talent level which is greater than 9.95 percent of all talent levels while worker #500 has the level which is greater than 49.95 percent of all other levels. For these workers, talent levels of .803 and 1.000 respectively are assigned.

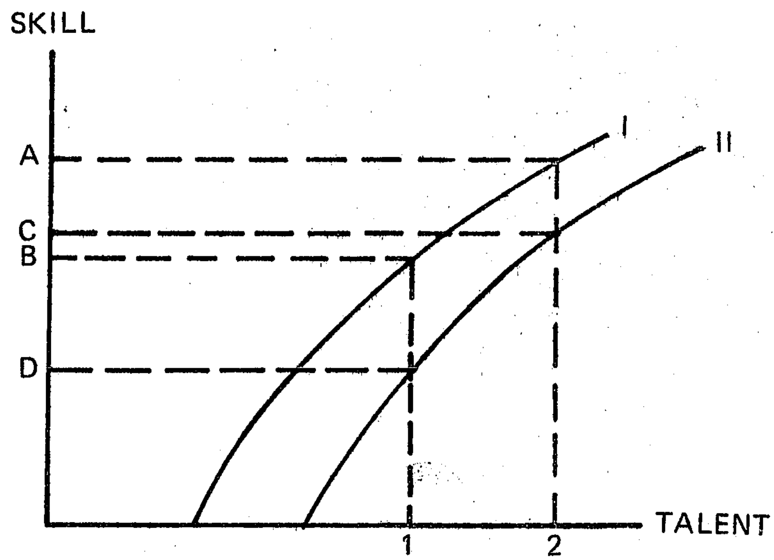
TALENT is transformed into the various labor SKILLS by a set of nonlinear functions. SKILLS are indexed by the letter I. The quantity of the I-th SKILL input that the K-th worker can produce is determined by (2).

$$\text{(2)} \qquad \text{SKILL(I,K)} = \text{LOG [TALENT(K)} + .68 - .08*\text{I]}$$

Thus in the most demanding skill class, (I = 10), it takes a level of talent greater than 1.12 for a worker to be productive while in the least demanding skill class, a level of talent greater than .4 is required. Even worker #1 has a talent level of .5, however, so the possibility for productive employment of that worker exists. The numbers .68 and .08 are arbitrary, of course, and are chosen relative to the distribution of talent so as to exert strong pressure for certain men to gravitate to certain jobs without completely dominating that allocation. The sensitivity of the results to this arbitrary choice will be examined.

Equation (2) guarantees diminishing returns to talent in any skill classification, and it guarantees high talent individuals a comparative

FIGURE 1



advantage at high talent jobs. Since the functions for each skill classification differ from each other by a constant, it will be true that the ratio of the output of a specific skill of a high talent individual to that of a low talent individual will be higher the higher the skill classification. In the diagram, this means that A/B will always be less than C/D. Since C and D represent the levels of skill II of individuals with talents 1 and 2, while A and B represent their skill levels at less difficult job I, it is easily seen why the logarithmic form of these equations guarantees that high talent individuals will have a comparative advantage at high skill jobs. This should guarantee the existence of a unique optimal allocation of men to jobs in the absence of stochastic disturbances and market frictions.

Each of the ten different firms in the economy faces a separate demand curve for its output and must produce that output using a Cobb-Douglas production function defined over the ten labor skill classes. Firms are indexed by the letter J. Thus the 100 different jobs that can be acquired are distinguished by I, J subscripts. $K \varepsilon$ Job (I, J) will denote the fact that worker K holds job I, J. The skill of the I-th class that is used as an input by the J-th firm is simply the sum of the effective levels of skill of all workers employed by that firm at that skill level.

(3) $$\text{SKILLS}(I,J) = \sum_{K \varepsilon \text{ JOB}(I,J)} \text{SKILL}(I,K)$$

These skill aggregates are used to produce the firm's output.[3]

(4) $$\text{OUTPUT}(J) = \prod_{I=1}^{10} \text{SKILLS}(I,J)^{.1}$$

Thus there are 100 different jobs that a worker might acquire. He can also be unemployed, and when he is, he remains attached to one of the ten different skill classifications. Because the production functions are Cobb-Douglas, each firm has a strong incentive to hire some labor at each skill classification.

Demand curves are assumed to be rectangular hyperbolae.

(5) OUTPUT(J) = B(J)/PRICE(J)

When random shocks are used, they enter in the form of changes in the constants B(J). Changes in aggregate demand are simulated by increasing all the B(J) simultaneously. For the experiments reported below, B(J) = 10.0 unless otherwise noted.

These five equations complete the environment within which decisions are to be made. The environment is technically very simple, yet it leads to difficult decisions because of the problems introduced by heterogeneous labor. Next, we examine the decisions that must be made in order to allocate the workers to the correct jobs.

While the number of decisions to be made in this model are small, the environment within which these decisions are to be made is complex. Since each worker is different and since each firm has at any point in time a work force of differing composition, the marginal product of a particular worker may vary dramatically from firm to firm even at the same skill classification. A very lengthy search procedure for both workers and firms would be necessary if a state of perfect knowledge were to be characterized and which guaranteed each worker that job at which his productivity was highest. The procedure followed here does not replicate a state of perfect knowledge. Instead, a few simple rules of behavior are followed which it is felt are generally consistent with profit and utility maximization in the long run.

Each of the 100 firm-skill job classifications has an individually determined wage rate. Workers examine these rates and determine whether they feel they can improve their income by quitting their present job and looking for a different one. This calculation is made by comparing two numbers, one to represent the costs of search and possible unemployment, the other to represent the expected income gain to be attained once the job switch has been completed. Since all hiring is done from the pool of unemployed workers, it is necessary for a worker first to quit before he can attain a better job. However, it is possible for a worker to accept employment in the same time period in which he had quit. Thus he need not be unemployed for any finite time since all production takes place at the end of the period. There are no internal promotions in the model.

The costs of unemployment are assumed by the worker to be his present wage rate, WAGE(I,J), multiplied by the present unemployment rate for workers in the skill class in question with a constant added to the unemployment rate and another constant multiplying the whole expression. These constants are to be varied to determine their effect on the labor market's adjustment to equilibrium. The expression denotes the cost of being unemployed for one time period, (the present wage rate) multiplied by terms which represent the probability of being unemployed and the expected duration of that unemployment. In equation (6), the latter constant has been substituted out. The intention of the numbers shown in (6) is to have the worker estimate the cost of quitting as exactly equal to his present wage when the unemployment rate is ten percent. This is the same cost as would result from knowing with certainty that he would be unemployed one time period.

The benefits to be gained from switching jobs are estimated to be equal to the difference in wage rates between the present job and that paid on average at the next higher classification multiplied times the difference in talent between the worker in question and the average of his co-workers multiplied by a constant. The worker is assumed to feel underpaid only if he feels he's better than his co-workers. The constant in this expression serves two purposes. It converts talent into man-time periods, and it multiplies the resulting expression by the number of time periods the new job is expected to be held. The effect of the constant is to determine the talent differential necessary to make quitting profitable. The number 12 which appears results from a division of the number 60, which is used to express the expected benefits of the job switch, by the number 5 which comes from the right hand side of the expression. The sense of these numbers is seen in the following example: With a talent differential of one standard deviation, a quit is just profitable if the expected wage differential is one ninth and the unemployment rate is ten percent. If the unemployment rate is eight percent, the same quit is made if the wage differential exceeds ten percent.

(6) $$\text{WORKER}(K') \text{ QUITS JOB}(I',J') \text{ IF } 12 \cdot [\text{TALENT}(K') - \sum_{K \in JOB(I',J')} \text{TALENT}(K)/N(I',J')] \geq [\sum_{J} \text{WAGE}(I'+1,J)/10 - \text{WAGE}(I',J')] \; \text{WAGE}(I',J') \; [.1 + \text{UNEMP}(I')/\text{LABFORCE}(I')].$$

N(I,J) denotes the number of workers on job I, J; UNEMP(I) denotes the number of workers seeking work at skill level I; and LABFORCE(I) is the total number of workers in all firms at skill level I plus UNEMP(I).

Once the worker quits, he seeks work in the job classification immediately above the one he just left. His asking wage is set equal to a weighted average of his old wage and the average wage paid at the new classification.

$$\text{(7)} \qquad \text{ASK(K)} = .8*\text{WAGE(I}',\text{J}') + .2 \sum_{J} \text{WAGE(I}' + 1,\text{U)}/10$$

Fired workers must also determine an asking wage. It is a fixed percentage of the wage on the job they just left. Fired workers seek work in the job classification below the one they just left.

$$\text{(8)} \qquad \text{ASK(K)} = .9\ \text{WAGE(I}',\text{J}')$$

Each period, all those unemployed who do not find work lower their asking wages by five percent. When the asking wage falls to be equal to the average wage paid in the next lower skill classification, the worker drops to that classification. There is no other worker behavior.

The behavior of firms is a bit less simple. The firms must determine employment and the wage rates at each skill classification. Each worker at a given firm in a given skill class earns the same wage. Thus the firm must determine how much to pay a diverse group of employees, and it must take account of several factors when making this decision.

- The firm realizes that workers' talents differ and that it can generally hire better workers by paying higher wage rates.

- The firm realizes that its best present workers will quit if wages are too low.

- The firm knows that it is easier to hire and retain workers when unemployment rates are high.

- For a given labor force, the firm obviously makes higher profits the lower are the wages it pays.

- The firm wishes to hire anyone whose marginal product exceeds the real wage.

These factors make the problem sufficiently difficult that I confess to have little prior idea of how wages must be set if profits are to be maximized in the long run. I do constrain the problem somewhat by requiring that the firm behave competitively; that is, it moves in the general direction of having real wages equal to marginal products. I attempt to have the firm act as a price taker, but there are no natural functions to use to generate marginal revenue products or prices for workers in particular classes. Each worker has his own level of talent and his own wage demand. I can arbitrarily align these workers in order according to talent and/or wage demand, but the resulting alignment is not a labor supply curve to the firm; it is still an ordered shopping list of individual workers. Economists have not yet, to my knowledge, derived any general results concerning the optimal behavior of firms in such an environment, whether or not they assume the firms to be price takers.

At present, I determine wages in the following fashion. The worst worker employed in a particular skill class is the marginal worker. He is denoted $\underline{K}(I,J)$. His marginal product is attributed to the skill class.

(9) MPL(I,J) = .1*PRICE(J)*OUTPUT(J)*SKILL[I,$\underline{K}$(I,J)]/SKILLS(I,J)

Note that this marginal product is defined per worker while output is a function of skill units. Thus a term appears in (9) in addition to the marginal product of an additional unit of skill; this term converts skill units into workers. It is equal to the number of units of skill possessed by the worst worker in the relevant skill class currently employed by the firm. Note also that the use of the Cobb-Douglas production function and the unitary elastic output demand curve simplify this formula a great deal. If the marginal product exceeds the wage being paid, the firm attempts to expand employment in that skill class while it attempts to contract in the opposite case. When the firm is expanding, wages are determined according to (10a).

(10a) WAGE(I,J) = (.2 + .6*U(I,J))*WAGE(I,J)
+ (.8 - .6*U(I,J))*MPL(I,J)

where U(I,J) is a measure of unemployment or the availability of labor relative to the size of the firm.

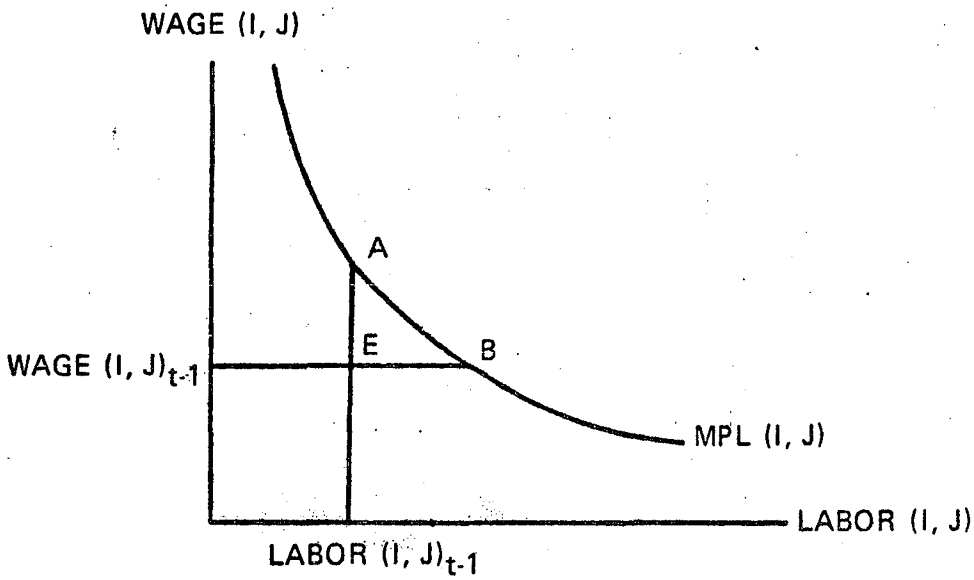
U(I,J) = UNEMP(I)/[UNEMP (I) + N(I,J)]

There are some further constraints on the rate at which wages can go up which are merely designed to prevent awkward results during unusual periods of turmoil (such as the period of adjustment to the initial random allocation of workers). These constraints prevent real wage rates from going up more than 25 percent per period unless the firm's wage would still be below the average asking wage of the unemployed in that class.

For contraction, the wage equation is (10b) which merely reverses the weights used in (10a).

(10b) WAGE(I,J) = (.2 + .6*U(I,J))*MPL(I,J)
+ (.8 - .6*U(I,J))*WAGE(I,J)

FIGURE 2



The sense of these functions can be understood with reference to the accompanying figures. In Figure 2, the downward sloping marginal product function acts as a demand curve for labor of this class. If E is taken to be a point of historical, long-run, stable equilibrium, then the demand curve drawn indicates an expansion in demand has occurred. The firm must select some point between A and B as its target for the coming time period. If it raises wages to be exactly equal to the present marginal product, the solution will be at point A and the firm need do no hiring or firing in the coming time period. If it keeps wages fixed, the solution will be at point B and the firm must hire EB laborers in the coming time period. Its choice depends on the unemployment rate of the relevant workers. If unemployment is high, it will choose a point near B, while if unemployment is low, it will choose a point near A.

I have described why there is no supply curve to the firm. However, it is possible to trace out loci of the points just mentioned by varying demand with unemployment held constant. Figure 3 shows these equilibria for high rates of unemployment while Figure 4 shows them for low unemployment. Note that both functions are kinked at the prevailing equilibrium.

Once the firm has chosen a set of wage rates, the rest of its behavior is simple. Firms hire those unemployed workers whose marginal products exceed the real wage of the relevant labor classification, who are looking for work at that classification, and whose asking wage is less than the firm's offer. They search for these workers in the pool of unemployed, and, generally, offer work to the most talented workers first. As each worker is hired, he reduces the marginal product of a unit of labor at that classification.

A worker is fired if his marginal product is less than 80 percent of the real wage he is to be paid. This requirement is checked immediately before and after the firm searches for new workers in the given skill class.

Behavior is simulated in the following manner. At the beginning, workers are assigned job classifications according to a pseudo-random process.[4] The initial wage offers are supplied exogenously and various behavioral parameters are assigned.

The program then enters the basic loop which determines a complete time period of behavior. First, some housekeeping calculations are performed to generate various aggregates which are needed as inputs to the behavioral decisions. These include the calculations of average skill levels by job and wage rates by skill class. Various rankings are performed which affect the order in which certain behavior occurs later.

Then come the behavioral decisions which form the heart of the model. First, workers decide whether or not to quit, and once they become unemployed, they determine an asking wage. In subsequent time periods, those already unemployed determine whether or not to drop to a lower skill class. There now exists a pool of unemployed with fixed wage demands in each skill class, and a set of firms with a stock of employees and fixed wage offers. At this point, market clearing behavior occurs.

The 100 jobs, each denoting a firm and skill classification, are considered in order according to wage offers, highest first. The firm fires workers whose marginal products are less than 80 percent of the wage rate. These workers immediately join the unemployed of the next lowest skill class. The unemployed in the relevant skill classes are then searched to see if job offers should be made. These classes include the ones immediately above and below that of the job in question, as well as its

FIGURE 3

LOCUS OF EQUILIBRIA WHEN UNEMPLOYMENT IS HIGH

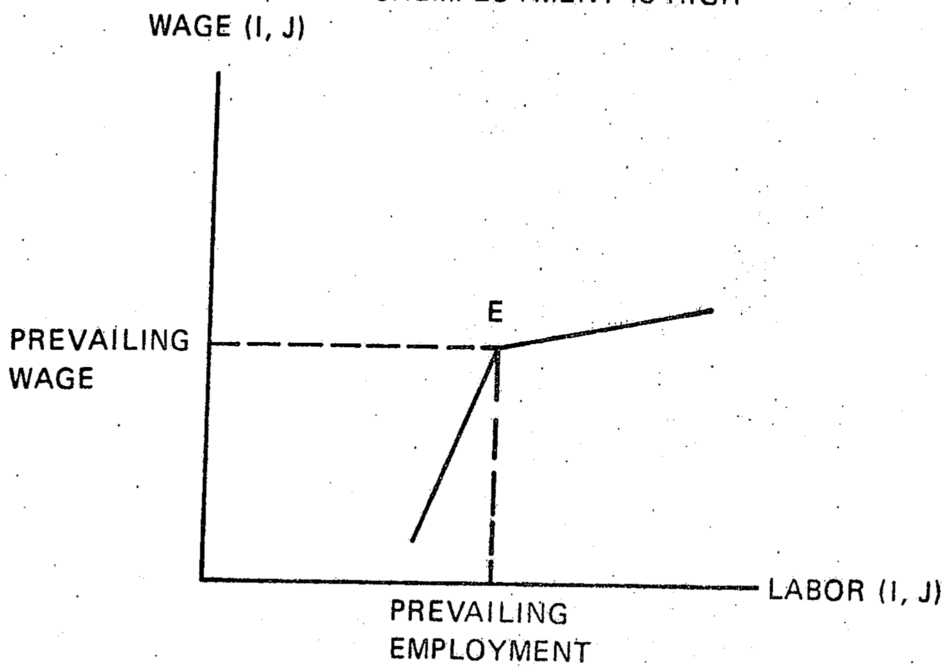
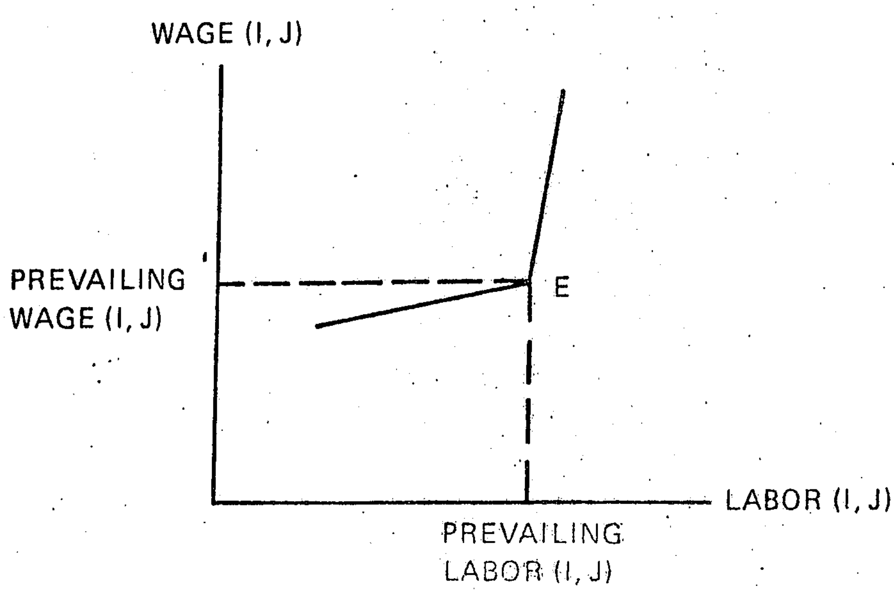


FIGURE 4

LOCUS OF EQUILIBRIA WHEN UNEMPLOYMENT IS LOW



own class. When an unemployed worker is found to have a marginal product in excess of the real wage, he is offered a job. He accepts if the wage offer equals or exceeds his asking wage. If any hiring is done, it reduces the marginal product of labor in the class in question, and it is again necessary to determine that the workers' marginal products are at least 80 percent of the real wage.

After the market clearing behavior is completed, the program proceeds to calculate output and prices for each firm. Real GNP is calculated; aggregate unemployment rates, wage rates and prices are determined and recorded. In the final time period, the program exits at this point.

For all other time periods, the housekeeping calculations are performed again, beginning this time with a calculation of new wage offers for each job. This done, the program repeats the calculations within the main loop until the required number of time periods has been reached.

IV. BEHAVIOR OF THE MODEL: A PRELIMINARY CHECK

The problems one faces when reporting the results of a complex simulation are a bit different from those usually encountered when reporting other empirical results, or when reporting theoretical results. When presenting a theoretical model, one lists the assumptions that have been made in order to simplify the complex real world. These assumptions usually restrict the breadth of the model sufficiently that a complete description of its behavior is then possible. The difficult choices that must be made are those concerning the assumptions, and these are made prior to the analysis.

The simulations I am about to describe, however, have yielded as output an extensive and rich set of data that are too numerous to report completely. In order to make any sense from this complex collection of numbers, it is necessary to use at this stage some of the conventions and assumptions that are usually made at the outset as part of the ordinary routine of the art of theorizing.

Often, theorists build models using a variable called the unemployment rate. This aggregative rate ignores all differences between workers and obscures much of importance that goes on in the labor market. Yet, the convenience that is gained from treating the unemployed as identical is apparently thought to be worth the information that is lost when this is done. The model I have used keeps track of several characteristics of the unemployed workers. They are not treated as a homogeneous aggregate. Yet, when it comes time to report the results of the simulations, I must find some way of summarizing those characteristics since a mere listing of them would be too lengthy to serve any purpose. Not surprisingly, I choose as one of the summaries, the unemployment rate--one of the very aggregates the model is designed to do without. The reader must remember that this aggregate is a sum of many unique hire, fire and quit decisions, and that it is not a part of the basic structure of the model. My use of it when reporting results should not obscure the fact that it was generated from a rich set of microanalytic decisions.

The first behavior I report on is not really an experiment. It concerns the adjustment of the model to the initial random allocation of workers in the absence of any additional shocks. Can the model take this random allocation and sort them into a new allocation at which no turnover exists? How close is this final allocation to the optimum? By examining the behavior of the model when confronted with this problem, a rough idea of the efficiency of the search procedure and an indication of the stability of the equilibrium can be gained. This problem was also the one used while debugging the program.

For this problem, the 1000 workers were distributed randomly across the 100 jobs, no one being unemployed at the outset. The randomization was done using the random number routine RANUN described in UWCC [1969]. The distribution of the initial numbers of workers assigned to each job is shown in Figure 5. The turnover that resulted from the initial allocation can be seen in Table 1, which lists the quits, hires, fires and unemployment for the first 40 periods. At the outset, turnover is very high as workers with high levels of talent quit the low skill jobs and workers with low levels of talent are fired from the high skill jobs. Fires in the first three periods alone are almost half the labor force, while quits are about one-fourth.

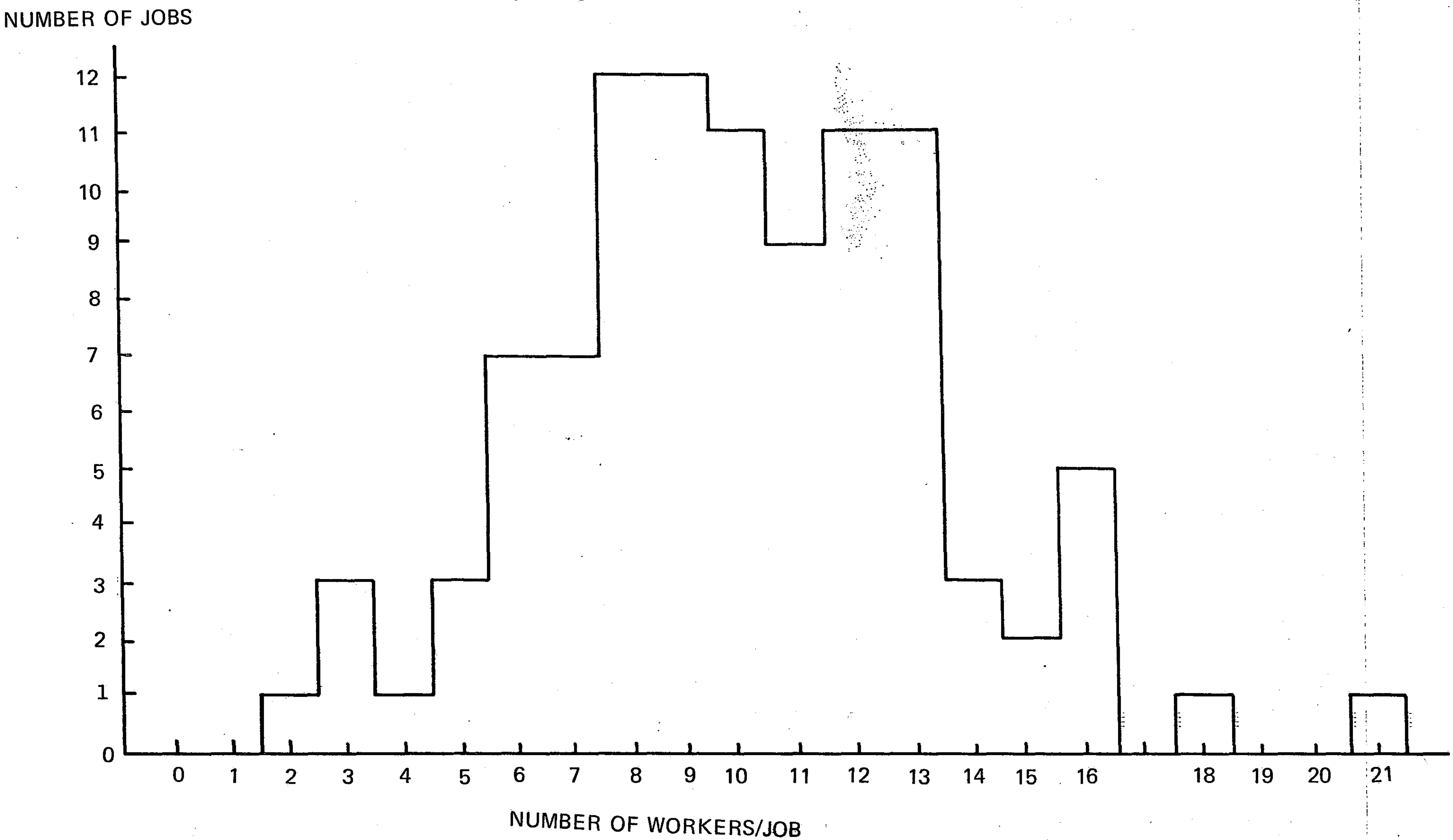
Unemployment averages more than a fourth of the labor force for the first nine time periods. But gradually, as the Table shows, it declines steadily until it becomes less than one percent in the 37th period.

The allocation of workers that results is far more efficient than the random one that appeared at the outset. Real GNP is 13,016 in period 1 with one quarter of the labor force unemployed. In period 39, GNP is 20,892, which represents an increase of 21 percent per employed worker and 61 percent overall. This understates the increase since GNP even in period 1 has benefited from a great deal of turnover. The initial allocation would have produced a much lower level of GNP even with zero unemployment. This is due to the fact that many of the low talent workers have a negative effect on output when they are assigned to high skill jobs.

This experiment does indicate that the model will approach an equilibrium even when it begins with a badly allocated labor force. The

FIGURE 5

FREQUENCY DISTRIBUTION OF THE NUMBER OF WORKERS IN EACH JOB AT THE INITIAL RANDOM ALLOCATION



stability, however, is not very robust in the near neighborhood of the equilibruum. Even after 199 periods with no shocks, there is still some turnover in the model though it is very minor. In the 199th period, which was the last one run, there is one quit and two hires. Unemployment is eight with seven of these being in the lowest skill class. Six of those seven are workers that have numbers one through six, which means that they are the least talented of all workers. They are apt to stay unemployed forever. Workers one and two were never employed throughout the entire 199 periods, while the other low talent workers had minimal employment at best. Since the talent of these workers is three standard deviations below the mean, it is not surprising that they remain unemployed. If, for example, we thought of talent as being synonymous with I.Q., these workers would have I.Q.s of about 55 and would be institutionalized in any real world.

After 199 periods, real GNP has reached 21.031 which is only 0.6 percent higher than its level in time period 39. Thus all of the subsequent turnover has little impact on real output. The labor force is better allocated in period 199 than it is in 39, but not much better. Maximum possible output, it should be noted, is 23.674. This was established in a deterministic solution of the model in which laborers were assigned the skill class in which they have a comparative advantage. Initially, the best 100 workers were placed in skill class ten, for example, but the lowest of these (worker #901) was transferred to skill class nine if the transfer would increase total output. Then #902 would be transferred if that would increase output. Similarly, worker #900 would be promoted if the move would increase output. A solution was reached at which no further switches of this kind could increase output. Output is 23.674 at

TABLE 1

Period	Quits	Hires	Fires	Unemployment	Real GNP
1	170	254	331	247	13.016
2	55	143	107	266	14.126
3	24	60	58	288	14.410
4	13	40	13	274	14.697
5	11	20	6	271	15.164
6	18	40	4	253	15.792
7	5	24	3	237	16.119
8	2	25	7	221	16.430
9	1	21	2	203	16.764
10	0	22	3	184	17.470
11	26	51	0	159	18.094
12	18	53	14	138	18.061
13	6	41	7	110	18.785
14	8	19	8	107	18.952
15	39	61	5	90	19.200
16	23	62	10	61	19.741
17	31	44	11	59	19.744
18	17	30	12	58	19.751
19	34	56	3	39	20.022
20					
21	35	63	14	25	20.472
22	45	64	14	20	20.473
23	17	25	7	19	20.551
24	8	10	2	19	20.557
25	7	11	0	15	20.678
26	10	13	0	12	20.762
27	17	19	1	11	20.801
28	22	17	2	18	20.530
29	11	20	2	11	20.792
30	14	18	4	11	20.802
31	16	13	2	16	20.699
32	12	20	2	10	20.748
33	3	7	5	11	20.778
34	0	3	3	11	20.792
35	4	5	0	10	20.809
36	3	5	2	10	20.889
37	2	6	3	9	20.924
38	2	5	2	8	20.928
39	8	11	0	5	20.892

that equilibrium. Thus the simulation can bring about an allocation in which output is 87 percent of its maximum value.

V. THE EFFECT OF RANDOM SHOCKS ON UNEMPLOYMENT

For all subsequent experiments, it was decided to use as the initial allocation, the one that resulted after 19 periods in the previous experiment. Much of the churning that occurs in those early periods gets replicated exactly for all other experiments, and it is so large that it swamps turnover due to other sources. Thus there is little to be learned from running those periods again and again. Accordingly, for the remaining experiments, time period 21 is the first to be reported while 39 is the last.

The previous simulation showed that in the absence of disturbances, the model will settle down to an equilibrium at which turnover is minimal. In order to generate turnover, it was decided to subject the model to a continuing stream of shocks. These were applied to the firm's output demand functions according to equation (11). Remember that the output demand functions are rectangular hyperbolae, and that the initial value of the constant level of sales is 10.0.

This constant, B(J), is multiplied each time period by a shock which shifts the demand function permanently. Thus the constant at any point in time is ten times the product of all past shocks. The random number routine RANNM [described in UWCC(1969)] yields normally distributed disturbances with mean of zero and standard deviation of one. These are used as powers of the exponential function to yield a lognormally distributed variable. Since the mean of the logs is not the log of the mean, however, half the variance must be subtracted from the random term to correct for this bias if the mean of the log normally distributed shocks is to be one. The shocks are multiplied by a constant, DEV, before exponentiation, and this constant can be varied to examine the effect of the shocks on the equilibrium.

$$B(J)_t = B(J)_{t-1} * EXPF(RANNM * DEV - DEV^2/2) \tag{11}$$

The random number generator produces the same sequence of shocks when the same constant is fed in to start it. Thus I can examine the effect of a given series of shocks which differ only by the multiplicative constant DEV, or I can hold DEV constant and use many different series of shocks. Both approaches were tried.

As expected, turnover (hirings) increases as the shocks get larger. The next to last column in Table 2 shows this. The reason for this is simply that the shocks cause firms to grow and contract which necessitates hiring and firing. Unemployment increases with turnover for most sequences of shocks, but one was found in which the unemployment rate varied in a narrow range as DEV went from .1 to .25. The sequence which yielded Table 2 was more typical, however.

Real GNP generally reaches a maximum for a value of DEV between .1 and .15. It is not surprising that some level of shocks would improve the allocation of workers to jobs; as firms are forced to hire and fire, the worst misallocations get corrected since underplaced workers are the first to quit and overplaced workers are the first to get fired. Thus a moderate amount of turnover improves the job-worker match. On the other hand, it may take a few time periods for the model to adjust completely to a major shock. With a continuing stream of large shocks, the market is perpetually in the process of adjusting, and the short run misallocations that result eventually exceed in importance the long-run benefits gained

TABLE 2

The Effect of Random Shocks of Different Size on Unemployment Equilibrium

Size of Shock (DEV)	Real GNP	Quits	Fires	Hires	Unemployment
0	414.7	243	65	339	259
.05	419.4	453	146	622	390
.10	421.8	482	163	664	446
.11	419.9	474	154	649	498
.12	420.0	547	202	772	523
.13	422.4	560	224	814	441
.14	419.8	544	230	799	510
.15	419.8	579	252	863	482
.16	418.3	588	247	863	467
.17	421.2	579	266	866	507
.18	419.5	595	301	921	593
.19	419.7	565	337	928	601
.20	417.9	597	362	978	577
.25	417.3	608	544	1166	877

The figures in Columns 2 through 6 represent sums of the values of the variables for the entire 19 periods of each simulation.

by stirring up the dead wood. Thus there appears to be an optimum size to the shocks in this model though it is difficult to be precise about its level since it varies with the particular sequence of random shocks used.

An interesting relationship between unemployment and output emerges here. For small levels of shocks, both variables increase with the size of the shocks. This is a confirmation of the importance of turnover for efficiency. While one of the costs of turnover is a high unemployment rate, one of the benefits is an efficient allocation of labor. Over some ranges, the benefits can outweigh the costs so that real GNP can be larger despite the higher unemployment rate if turnover is high.

VI. THE EFFECT OF A STEADY GROWTH IN DEMAND ON OUTPUT AND EMPLOYMENT

The demand experiments were satisfying in one respect: they revealed the existence of a macroeconomic phenomenon that has been difficult to establish analytically. That is, when unemployment rates are held below the long-run equilibrium levels associated with a specific level of shocks, steady state GNP is not increased. In fact, long-run GNP declines with the long-run unemployment rate over a substantial range.

In this experiment, aggregate demand was allowed to increase at a constant percentage rate. The demand changes were programmed by shifting all firms' demand curves by the same percentage. With the demand for their output growing, the marginal products of the workers grow. But wage rates increase in response to changes in marginal products, albeit with a slight lag. If the wage rates increase at the same rate as the marginal products, there is no incentive to hire. But given the lag in wages rates, there should be some effect on employment from a change in aggregate demand. Note that the ability to affect the long-run rate of unemployment by changes in aggregate demand results from the fact that firms and workers do not make forecasts of the rate of inflation when making wage and price decisions. If such forecasts were used, and if those forecasts were realized, then any constant rate of growth of demand would yield the same allocation of men to jobs as any other. That is, there would exist a natural rate of unemployment in this model, and that rate would be independent of the rate of growth of prices.[6] As the model is specified, however, the absence of price forecasts means that changes in the level of aggregate money demand can change output and employment as well as prices.

This nonneutral effect is described as a Phillips curve in Figure 6. The rate of unemployment that corresponds to a zero rate of inflation is the natural rate of unemployment since zero is the forecast rate of inflation. Before the recent surge of interest in the microeconomics of unemployment equilibrium, curves like Figure 6 were presented as loci of points from which a selection could be made by a policy maker. Inflation and unemployment were both to be viewed as evils, but unfortunately, to have less of one, an economy had to put up with more of the other. In discussions of the relative merits of lessening one or the other evil, those who argued for less unemployment have always had a very powerful argument in that decreases in unemployment correspond to increases in output. Since more output is better than less output, inflation was argued to be the lesser of two evils.

One advantage of the simulation model being described is that we can calculate the level of output for each growth path and see to what extent reductions in the unemployment rate correspond to increases in output. The results are shown in Table 3, for a random deviation in demand of .13.

TABLE 3

Variation in the Rate of Growth of Demand. DEV = .13

% Growth in Demand	Total GNP	Total Unemployment	Total Quittings	Total Firings	Total Hirings
-5	407.4	1870	229	531	632
-4	414.1	1294	280	426	632
-3	419.3	963	310	389	665

FIGURE 6

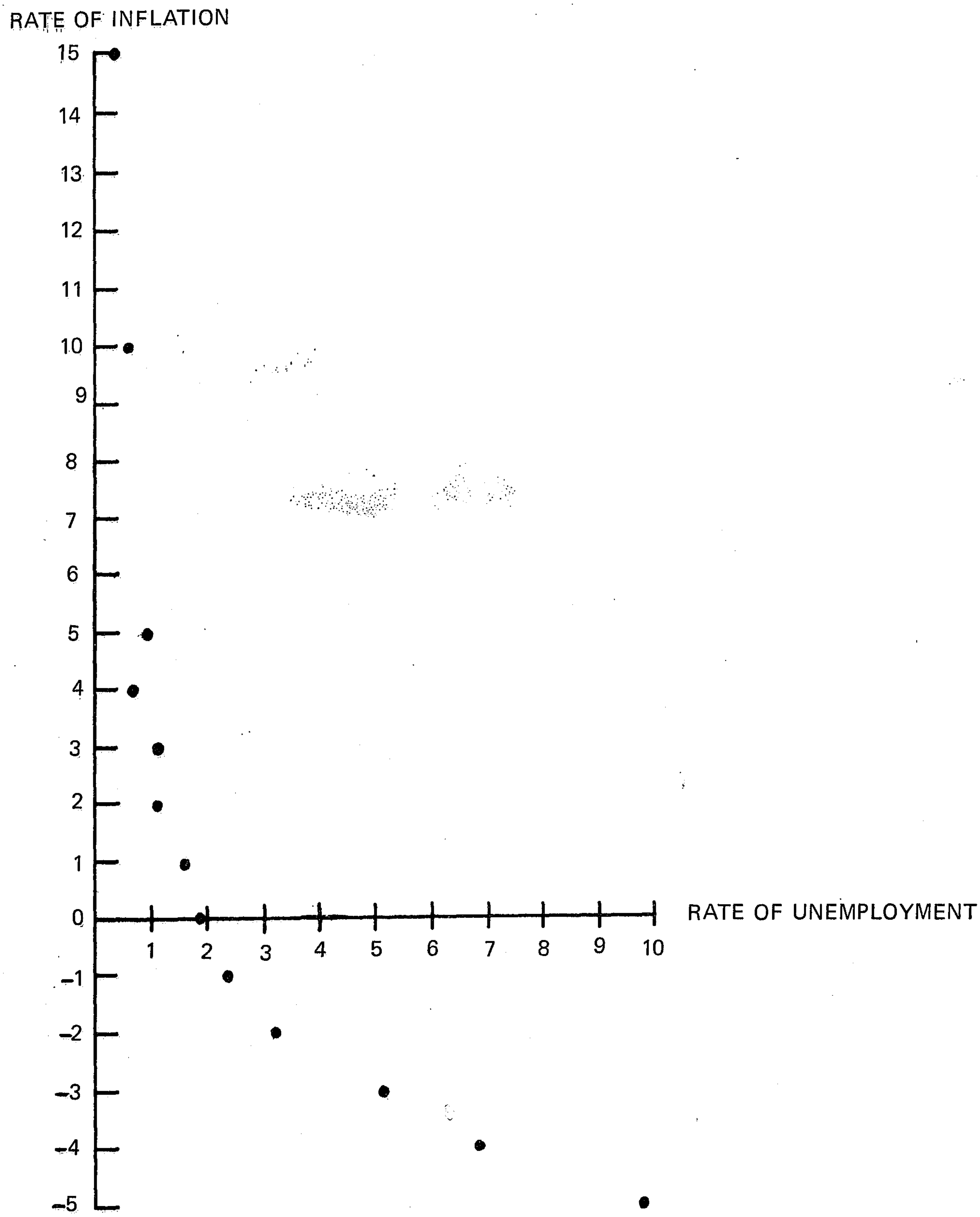

RATE OF INFLATION
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
–1
–2
–3
–4
–5
1
2
3
4
5
6
7
8
9
10
RATE OF UNEMPLOYMENT

TABLE 2 (cont.)

& Growth in Demand	Total GNP	Total Unemployment	Total Quittings	Total Firings	Total Hirings
-2	421.2	588	381	296	668
-1	420.3	432	479	252	739
0	423.7	373	507	251	769
1	417.3	341	562	194	779
2	416.1	237	581	180	792
3	416.9	241	612	148	793
4	410.1	153	593	105	734
5	410.9	204	633	113	780
10	401.5	135	664	55	756
15	386.6	102	618	36	690

Output is at a maximum when there is no growth in demand. Even though unemployment can be reduced below the 2.0 percent value it takes on then, output can not be increased further. This same result occurs regardless of the size or series of the random shocks used. Zero growth in demand is efficient in this model, in the sense that aggregate output is at a maximum for the rate of growth of demand. Two percent is the natural rate of unemployment, we noted, and in this model, the natural rate is also the efficient rate. From the table, one can also see where the inefficiency comes from. The rate of firings falls off rapidly as the growth in demand increases. Some workers must be remaining in jobs that are beyond their abilities. While a higher rate of firing would probably increase the unemployment rate, it would apparently improve the allocation of workers to jobs by enough to offset the effect of unemployment on the level of output. Not surprisingly, if unemployed workers are viewed as an inventory of people awaiting jobs, there exists an efficient level of that inventory. The costs associated with an overheated economy, then, are not just those of inflation, but also those associated with a badly allocated labor force.

VII. CYCLES

A deterministic cycle can generate the same need for turnover as do random shocks. To examine the cyclical behavior of the model, 12 period cycles were programmed, each cycle consisting of a sequence of identical changes in the demand for the output of all firms. The sequence consists of two periods of increases in demand, two of no change, four decreases, two of no change, and two increases respectively. Runs of 36 periods (three cycles) were carried out, and within each run, the percentage changes in demand were the same (up and down) in each period in which demand changed.

The results can be seen in Table 4. Where there are no other shocks to the firms demand functions, moderate sized cycles do increase the average level of GNP over the cycle. Beyond a certain point, however,

TABLE 4

Cyclical Behavior; 36 Periods, 3 Complete Cycles

% Change in Demand during Boom/Slump	Total GNP	Total Unemployment	Quittings	Turnover Firings	Hirings
	A. Without Random Shocks, DEV = 0				
0	749.6	336	265	78	378
5	761.4	807	474	131	625
10	765.7	775	429	173	635
15	760.4	2154	489	485	962
20	739.7	3838	477	759	1157
	B. With Random Shocks, DEV = .10				
0	766.3	836	640	268	920
5	765.3	1115	614	298	920
10	767.9	1603	602	480	1071
15	749.2	3023	587	754	1254
20	727.1	4621	573	969	1378

TABLE 4 (cont.)

% Change in Demand during Boom/Slump	Total GNP	Total Unemployment	Quittings	Turnover Firings	Hirings
C. With Random Shocks, DEV = .15					
0	763.8	827	702	427	1138
5	762.7	1013	725	528	1244
10	752.4	2471	637	727	1277
15	733.9	3954	653	942	1445
20	707.2	5663	619	1146	1529

further increases in amplitude cause the average level of output to decline.

When other shocks exist, the benefits from the cycle are reduced. The Table shows that for DEV equal to .10 or .15, the cycle adds less to output than it does for DEV equal to zero. Thus the cycle seems to be a reasonable substitute for shocks as a way of inducing turnover.

The average rate of unemployment increases with the cycle. Again, output does not fall as much as one would predict from the unemployment rates alone, and in some cases, it increases with unemployment in the manner noted above.

A final, interesting calculation can be performed using the cyclical data. Since output and unemployment vary cyclically as expected, it is possible to calculate an aggregate production function from the time series data. I was curious to find out whether the model would exhibit the peculiar characteristic of the U.S. economy noted by Okun, namely that the elasticity of output with respect to employment is greater than one.

I have only performed the calculation for one simulation. When DEV equals .1 and a cycle of demand changes of .1 occurs, the unemployment rate varies cyclically between two and nine percent. For this simulation, output grows by just .67 percent when the unemployment rate falls by a percentage point; Okun's Law is not confirmed. An r^2 of .47 was obtained, however, indicating that the relationship is not very tight. I have not attempted to discover which model changes would be necessary to get the cited elasticity above one.

VIII. CONCLUSION

The results just presented are but a few of many that can be derived from the model. Some will be presented in subsequent papers. In particular, questions of microeconomic labor market policy will be examined with an eye on their effect on the distribution of income. Minimum wages, wage subsidies and unemployment compensation have, at this date, been programmed and a variety of results, which are too lengthy to enumerate here, have been derived.

What can we learn from results of this kind? Certainly, the model is not a replication of the U.S. economy, and therefore, the quantitative results must be ignored. But I find the qualitative results to be very interesting. (1) Even in a model with arbitrarily chosen parameter values for quit, hire, fire and wage decisions, there appears to be a cost to violating individuals' expectations about the rate of inflation. (2) There exists, for every alternative specification examined, a positive level of unemployment associated with the maximum level of output. That is, there exists a rate of frictional unemployment which is efficient in the macroeconomic sense. (3) A business cycle of modest size can improve the allocation of labor resources even after taking account of the fact that it increases the average rate of unemployment. The ability of the cycle to improve that allocation is less when there already exists substantial labor turnover than it is otherwise.

Much remains to be done to guarantee that these results are not due to some unknown peculiar specification. My intention when building the model was to choose those functional forms which were least likely to generate problems of nonexistence, nonuniqueness, or instability of equilibrium and to keep the number of equations to a minimum. Cobb-Douglas production functions, for example, probably behave more "regularly" than real world functions do. It is hoped, then, that the possibilities for strange results have been kept to a minimum. There is no way to guarantee that, of course, but the use of a small number of well behaved inputs have usually yielded well behaved outputs in other contexts.

The most important contribution of results of this kind may be the stimulus they provide for analytic research. The timetable for the discovery of analytic methods for handling problems of this kind may be accelerated once researchers have concrete goals to shoot for. Some of the more surprising results noted above provide goals of this kind since researchers may be interested in designing analytic models to replicate one or another of them in order to get at the root of the surprise. The hunt remains difficult, but simulations can give us useful knowledge about the quarry.

FOOTNOTES

1. Solow [1956] p. 65.

2. Initially, I intended to use the variable I.Q. instead of talent. I.Q.s initially had a mean of 100 and a standard deviation of 15. While my colleagues cautioned me that I.Q. and productive ability are not identical and therefore a different variable name was in order, I kept the original dimensions and occasionally made references to I.Q.

3. The constant .01 has been added to each level of skills for computational convenience only. Occasionally, a job may have no employees and in such cases, the constant allows some output to be produced despite the nature of the Cobb-Douglas production function.

4. Pseudo-random numbers are numbers that appear to be random for statistical purposes, but are in fact generated by a deterministic process. Since the process can be replicated, it is possible to use the same set of random numbers for successive experiments.

5. For example, see Nichols [1970].

6. See Friedman [1968] for a definition of the natural rate of unemployment.

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