ALTERNATIVE DEFINITIONS OF INCOME REDISTRIBUTION

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ABSTRACT

Four alternative general equilibrium definitions of income redistribution are offered which differ only because they assume different counterfactuals. If the objective is to show the extent to which vertical equity is achieved through government, two of the definitions taken together provide an analytical framework consistent with prevailing theory that can serve empirical research. The definitions considered do not, however, exhaust the logical possibilities.
Interest in measuring the effects of government, at all levels, on the size distribution of income has increased in recent years. Much has been written about the technical difficulties inherent in such measurements, but how income redistribution ought be defined—the most basic of all technical difficulties—has received little detailed consideration. Perhaps this is because the theoretical literature seems to offer little direct guidance.

Four alternative general equilibrium definitions of redistribution will be presented here. These definitions differ because they assume different counterfactuals. The most commonly assumed counterfactual in the literature is an economy in which there are neither government expenditures nor taxes. This concept of zero government, a much criticized counterfactual, leads to including in redistribution all the effects of government fiscal activities on the size distribution. In general, it is true that this

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2 A. R. Prest, "Statistical Calculations of Tax Burdens," *Economica*, 22 (August 1955), 234-245. Perhaps it should be noted that the preponderance of "redistributive" studies are of a particular program or program change. These studies generally take as their counterfactual the status quo ante. One more general study evaluating a marginal change against the status quo ante is, Social Development and Manpower Policy Division, Department of Finance, Canada, *The Impact of Tax, Transfer and Expenditure Policies of Government on the Personal Distribution of Incomes in Canada*, mimeo., August 1972.
counterfactual is inconsistent with the theory of the public sector, if the redistributive effects of government are to be evaluated on a principle of vertical equity. Three additional counterfactuals are described here, each of which has certain appealing characteristics. Two of the three have a somewhat better claim to being consistent with prevailing theory, and these two, taken together, characterize an analytical framework which empirical researchers will probably value, even though they may retreat from it because of difficult implementation problems. However, because only rarely has the question "What should income redistribution studies measure?" been confronted directly, the degree to which current practice has retreated from any norm is unclear. We provide here a basis for evaluating the ongoing research in this area, not only by examining the implications of the most common definition of redistribution and offering several alternatives, but, more generally, by making explicit the nature of the normative choice among alternative definitions.

In section I, the nature of the problem of defining redistribution and the limited scope of the definitions to be considered will be indicated. The four definitions will be presented in section II, and will be compared and evaluated in section III. An algebraic comparison is in the Appendix.

I. The Concept of Redistribution

In general, income redistribution due to government is simply the change which comes about in the income of individuals when the parameters describing governmental behavior change. The specific definition of income,

\[ \text{But see footnote 6 for a qualification.} \]
the changes in parameters examined, and the way in which the redistributive effects on individuals are aggregated must all depend upon the objectives of the inquiry. Studies of the benefits and burdens of government by income class are usually concerned with answering questions about vertical equity, and it is this particular question which motivates us here.

Given an interest in vertical equity, the primary income distribution, by which is meant the vector of individual incomes which arises from the assumed counterfactual, must be an appropriate index for ranking individuals, i.e., for determining who are equals and who are unequals. Once the primary distribution is determined, the redistributive effect of government on each individual is the difference between his primary income and the income he receives under existing governmental parameters, which is called here his final income.  

4 Of course, specifying an initial set of parameters is not sufficient for a complete definition of redistribution. It is also necessary to decide whether to group individuals into families or consumer units and, given this unit of analysis, to specify an appropriate concept of income. These issues have received considerable attention in the literature and will not be addressed here. See, e.g., G. A. Bishop, "Income Redistribution in the Framework of the National Income Accounts," National Tax Journal, 19, (Dec. 1966), 378-390. In the definitions presented in section II, we will for convenience use individuals as the unit and specify a general definition of income in order to focus on the problem of choosing a proper set of initial parameters, an issue which has been relatively neglected.

The theoretical literature in public finance has often been vague on this point. For example, horizontal and vertical equity are usually discussed as though only a proper definition of income is necessary to define equals and unequals, given the unit of analysis. The comparative-static dimension of the problem, i.e., the necessity of choosing a counterfactual, is generally left in the background. For example, see Richard A. Musgrave, The Theory of Public Finance (New York: McGraw-Hill, 1959), Ch. 8. A.C. Pigou explicitly adopts the zero-government counterfactual but he offers no justification for this choice. See his, A Study in Public Finance (London: Macmillan, 1928), p. 58.
Redistribution can then be defined as a vector which shows the difference between primary and final income for each individual and which can be evaluated on criteria of both vertical and horizontal equity.

The zero-government counterfactual used in most studies of redistribution has as its rationale the value judgment that individuals should be ranked by their private-market income, that is, the income they would receive if there were no government (given individual preference functions, existing technology, and individual resource endowments). However, problems arise when the zero-budget variant of this counterfactual is used along with the assumption that other policy parameters remain unaltered. If current policies permit discrimination, the exercise of monopoly power, etc., such effects

5 It may be necessary, particularly for empirical work, to group individuals by their primary income and define redistribution as a vector of average effects by income class. However, such a definition can always be derived from any definition specified in disaggregated terms.

6 The definition of redistribution need not be confined to changes in government parameters when the objective is to measure vertical equity. For example, one assumption governing the counterfactual could be that there are no private transfers. However, it may still be said that the resulting definition of redistribution is redistribution by government. If the counterfactual defines equals and unequals, and government is charged with the responsibility of achieving vertical and horizontal equity, then the effects of private transfers can be attributed to government on the grounds that it can take into account the predicted level of private transfers in designing redistributive policies. Only changes in government parameters are involved in the definitions of redistribution discussed in this paper. (See footnote 27, however, for a possible modification of one of the definitions to include private transfers.)
will not be removed under a zero-budget counterfactual. To fully specify a state which appropriately defines equals and unequals, it is necessary to decide not only which fiscal parameters but also what other rules of the game should prevail. Even if the rules are to be those of a perfectly competitive private market, questions still remain: Is a taste for discrimination, for example, to be allowed to be a factor in determining the ranking of individuals?7

The problem of defining the nonfiscal policies appropriate to a desirable primary distribution is a thorny one which, nevertheless, cannot be ignored if individuals are not to be ranked according to a partly arbitrary definition of primary income. A complete definition of redistribution requires judgments about nonfiscal policies, but it seems possible to leave this issue in abeyance while analyzing the choice of fiscal parameters to include in a counterfactual. Accordingly, we will assume (1) that the private market is perfectly competitive, (2) that the nonfiscal policies prevailing in the counterfactual are those appropriate for defining equals and unequals, and (3) that the same nonfiscal policies govern the final distribution.

Similarly, we will eliminate consideration of the redistributive effects of stabilization policy by assuming a neo-classical system, balanced budgets and unchanging aggregate output, employment, and price level. As in the case

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7These issues have been raised by several authors, including: Stephan Michelson, "The Economics of Real Income Redistribution," Review of Radical Political Economy, 2 (Spring 1970), 75-86; George Rottier and Jean Francois Albert, "The Social Services and Income Redistribution in France,"; and Alan T. Peacock, "Introduction," in Alan T. Peacock, ed., Income Redistribution and Social Policy (London: Cape, 1954).
of nonfiscal policies, however, a complete definition of redistribution must consider whether the counterfactual is to include the maintenance of full employment and, if so, what instruments are to be used to achieve it. 8

One final qualification of our definitions should be mentioned here. In what follows, the term "government" will refer to all levels of government combined, on the assumption that vertical and horizontal equity are national objectives. If, on the other hand, individual localities pursue their own social welfare functions then a separate definition of redistribution might be appropriate for each level of government. 9 However, since sub-national political jurisdictions may be unable to achieve independent vertical equity goals because persons will migrate in response to unequal fiscal residua, it may be that the only practical view of vertical equity is as a national objective. 10

II. Alternative Definitions 11

In all cases, the same definition of the final distribution of income is assumed. It is the distribution of after-tax factor income plus transfers and

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8 Note that the incorporation of stabilization policy in the counterfactual forces consideration of monetary, debt management, and other nonfiscal policies.

9 A. T. Eapen, who advocates allowing states to make equity decisions, has suggested that the federal government should rank persons by their income net of the effects (both burdens and benefits) of state governments. See his "Federalism and Fiscal Equity Reconsidered," National Tax Journal, 19 (Sept. 1966), 325-29.


11 A more precise description is in the algebraic appendix.
benefits of all final government services. The benefits of general government services (which include any taxpayer benefits accruing from transfer programs) and recipient benefits of in-kind transfers are all valued at their marginal valuation to each individual as given by his demand curve for each service.\textsuperscript{12} The final distribution is that which arises from actual governmental parameters, \textit{i.e.}, those which prevail in the time period for which redistribution is to be measured, and these parameters are assumed to result in a balanced budget. The different definitions of redistribution which follow, therefore, result from alternative \textit{a priori} views of the appropriate definition of the primary distribution.

\textbf{Case I.} One conception of the primary distribution is the zero-government counterfactual (which hereafter refers to a zero budget since nonfiscal policies are ignored). The private sector distribution of factor incomes is taken to satisfactorily define equals and unequals. Any and all changes in this private-sector distribution resulting from any and all expenditures of government is then viewed as its income redistributational consequences. We will refer to this definition as Gillespie redistribution, since it is roughly consistent with his recent approach.\textsuperscript{13}

\textbf{Case II.} A second approach is to define the primary distribution of income as that arising from the private sector plus the allocative activities

\textsuperscript{12}This treatment of government benefits is consistent with the valuation of private goods.

\textsuperscript{13}\textit{Op. cit.} The main differences between Gillespie's conception and ours is that our final distribution includes taxpayer benefits from transfers and explicitly values benefits at their marginal value to recipients.
The primary distribution differs from that of the public sector. One such distribution is that which would obtain if the government used only marginal benefit taxation. The primary distribution which defines equals and unequals is then that distribution which arises from a Lindahl equilibrium. This primary distribution differs from that which would prevail in the Gillespie case because, although benefits received by each individual are balanced by taxes paid, factor incomes may have been altered in going from zero government to the Lindahl equilibrium. What we will call Lindahl redistribution, therefore, excludes the effects of factor income which arise incidentally from the undertaking of allocative activities by government.

Any Lindahl redistribution which does take place, comes about primarily in three ways. First, the taxes actually levied to finance the purely allocative expenditures of government may deviate from benefits received.

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14 A Lindahl equilibrium combines the provision of private goods through the perfectly competitive private market with the provision of public goods according to a Lindahl solution, given a distribution of individual endowments. See Duncan Foley, "Lindahl's Solution and the Core of an Economy with Public Goods," *Econometrica*, 38 (Jan. 1970), 66-72.

15 This change in factor income is often ignored, implying that the provision of public goods by a Lindahl solution involves no alteration in individual incomes. See Henry Aaron and Martin McGuire, "Public Goods Income Distribution," *Econometrica*, 38 (Nov. 1970), 909.

16 The label given to this definition refers only to the use of the Lindahl solution to specify the primary distribution. Lindahl himself would probably define redistribution as the alteration in initial endowments before the allocation of resources to the public sector, when the latter are provided according to a Lindahl solution. See Erik Lindahl, "Just Taxation--A Positive Solution," in R. A. Musgrave and A. T. Peacock, eds., *Classics in the Theory of Public Finance* (London: Macmillan, 1958).

17 A fourth source of redistribution is unintended inefficiencies in the provision of government services. These are not formally different, however, from inefficiencies which are deliberately created to redistribute factor income and will not be referred to separately in the analysis.
Second, the government may undertake exhaustive expenditures not fully justified on efficiency grounds to provide factor income to certain subgroups of the population. Third, the government may make transfers, either in cash or in-kind, to certain groups in the population at the expense of other groups. We will refer to these three methods of redistributing income as the "redistributive policies" of government. 18

In defining Lindahl redistribution, we have found it necessary to make a distinction between the allocative and the redistributive policies of government. The Pareto optimal redistribution literature makes that distinction ambiguous. To the extent that taxpayers benefit from redistributive policies, the provision of these programs may be viewed as collective consumption on the part of the taxpayers. 19 Redistribution resulting from taxpayer demand, therefore, is essentially an allocative activity of government: its provision would be necessary to the attainment of an efficient allocation of resources even if redistribution were not a separate governmental objective. An element of this type of redistribution exists whenever a redistributive policy generates donor benefits.

Viewing efficient transfers as an allocative activity suggests that their effects should be assigned to the primary distribution, under the assumption that donors can be taxed according to the marginal benefits they receive from

18 Although the Lindahl definition excludes the effects of allocative activities as an independent source of redistribution, the redistributive effects of changes in allocative expenditures induced by redistributive policies are included. 19 Redistribution may be necessary to achieve a Pareto optimum if the utility of some individuals, or their consumption of particular commodities, enter the utility functions of others, or if the income distribution itself enters individual utility functions. See, Harold M. Hochman and James D. Rodgers, "Pareto Optimal Redistribution," American Economic Review, 59 (Sept. 1969), 542-57; and Lester C. Thurow, "The Income Distribution as a Pure Public Good," Quarterly Journal of Economics, 85 (May 1971), 327-36.
the programs in question and that benefits to recipients (i.e., those excess of their marginal valuations of in-kind transfers) are not taxed. It is important to note that this procedure involves the inclusion of recipient benefits as well as donor benefits in the primary distribution. Efficient transfers, therefore, are not considered redistribution under this procedure.

An alternative treatment of efficient redistribution is suggested by a different line of reasoning. We previously described the primary distribution as that which would prevail if only benefit taxation were used. The provision of efficient transfers, however, violates this principle of taxation. Although donors are taxed according to their marginal benefits, the recipients receive benefits for which they are not taxed. This consideration suggests removing the recipient benefits from the primary distribution and including them in the redistributive effects of government. There are, therefore, two variants of Lindahl redistribution, which we will call Lindahl I and II, each involving

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20 It is possible that all transfers are efficient in this sense and that the part of exhaustive expenditures which are not otherwise justifiable on allocative grounds are also efficient redistribution while the taxes collected for these transfers and other expenditures are in fact equal to what would be assessed in a regime of marginal benefit taxation. Were all these conditions to hold simultaneously, the primary distribution would be identical with the final distribution, and hence it would follow that no redistribution takes place through government.

21 This treatment of recipient benefits is consistent with Lampman's view of redistribution as the receipt by individuals of "consumer-power income" which does not correspond to their "producer-contribution." See his "Transfer and Redistribution as Social Process," in Shirley Jenkins, ed., Social Security in International Perspective (New York: Columbia University Press, 1969).
the same counterfactual but a different specification of which components
of income are appropriate for defining equals and unequals.\textsuperscript{22}

Case III. A third definition of redistribution, which we call Smolensky-
Behrens redistribution, has as its essential characteristic that the primary
distribution is an optimal one. In the counterfactual which defines the primary
distribution, the government both engages in allocative activities and carries
out redistributive policies (as defined above) on the basis of some ability-to-
pay criterion thereby achieving an optimum.\textsuperscript{23} The Smolensky-Behrens (S-B)
definition requires first that equals and unequals be identified, and in this it
does not differ from the other two cases. In addition, however, S-B necessitates
making an explicit judgment about vertical equity by requiring that an ability-to-
pay criterion be specified. The definition of equals and unequals may be based
on the Gillespie or Lindahl or any other counterfactual, which is then altered
on the basis of the ability-to-pay criterion to produce the S-B counterfactual.

As in the other cases, S-B redistribution is simply the difference between
the final and primary distributions. Given a definition of equals and unequals,
S-B redistribution is the difference between the actual fiscal treatment individ-
uals receive and the way they would be treated if the optimum distribution were
achieved.

\textsuperscript{22}Note that while Lindahl II redistribution differs from Gillespie only in
excluding the effect of governmental allocative activities on factor incomes,
Lindahl I has the additional difference of excluding recipient benefits in Pareto
optimal transfers.

\textsuperscript{23}The term ability-to-pay as we will use it differs from the usual meaning
in that we are assuming a principle which takes into account all benefits of
government expenditures as well as taxes. Such a principle may require that
the net benefits of government to an individual (his fiscal residuum) be either
positive or negative, depending upon his primary income.
III. Evaluation of the Definitions

The key differences among these concepts of redistribution lie along two dimensions. One important difference, that which distinguishes Gillespie from both variants of Lindahl, is the conception of the allocative activities of government. The other major difference, that which differentiates S-B from Lindahl, is the tax principle invoked to define the primary distribution.

The common acceptance of the private market distribution for defining equals and unequals (e.g., the Gillespie counterfactual) seems to rest on two principal rationales. One is normative: persons should be ranked by their marginal products. The second is taxonomic: all effects of government activities should be included in any overall measure of redistribution.24 Both rationales ignore the role of government in providing goods which cannot be provided efficiently by the private market. If there are externalities in production or consumption, factors would not be paid their marginal social products in the zero-government counterfactual and the first rationale is inoperative. Thus, the objective of ranking people by their marginal products is better served by a counterfactual which includes the allocative activities of both the private and public sectors. The existence of a public-sector analogue to the private price mechanism, the Lindahl solution, is a way to specify such a counterfactual.

The second rationale, that all effects of government should be included in a concept of redistribution, is also suspect, for the private market then determines

24 Gillespie seems to reason this way to arrive at his choice of the zero-government counterfactual (op. cit., pp. 123-124).
the appropriate ranking of individuals by virtue of an a priori judgment about the nature of governmental redistribution made without reference to the objective the definition is to serve. Although such a definition of redistribution may seem intuitively correct at first, it becomes less so when it is shown to lead to an asymmetric treatment of the allocative activities of the private and public sectors. For example, if the steel industry were to disappear, the incomes of some factors might be lowered and those of others might rise. However, the consequences of steel production for the distribution of income are not included in Gillespie redistribution since they simply contribute to the formation of the private distribution (assuming, for the sake of argument, that all demand for steel is private). On the other hand, Gillespie redistribution does include the redistributive consequences of the production of nuclear submarines, simply because the demand for this commodity emanates from the public rather than the private sector.

This asymmetry between the treatment of the two sectors does not arise in Lindahl, since all consequences of the pursuit of allocative efficiency are excluded from the definition of redistribution. If the allocative function of government is viewed only as a use of income by individuals to buy collectively-consumed goods and services, which are paid for at a "market" price, there seems to be no compelling reason to include in a measure of the fiscal treatment of persons the distributional effects of merely allocating resources to the public sector, while excluding the effects of allocating resources to particular uses in the private sector. These arguments suggest that Lindahl
Redistribution is preferable to the Gillespie concept if individuals are to be ranked by their market incomes. 25

Since most theoretical and empirical studies of redistribution do not give an explicit reason for using the zero-government counterfactual, it should be mentioned that there is another possible rationale for its use. This is the value judgment that persons should be ranked by their private market incomes, whether or not these are equal to marginal products. One cannot argue with this type of specific value judgment, of course, but one can point out that it involves an asymmetric treatment of the allocative activities of the private and public sectors. 26

25 That there are at least two definitions of redistribution (Lindahl I and II) based on market counterfactual which exclude the effects of governmental allocative activities, may also resolve an issue which has arisen in the incidence literature.

Musgrave has argued that it is not very meaningful to measure the distributional consequences of the budget. Such a measure includes the effects of expenditures which he asserts are largely unintended. Musgrave would like to remove these effects, but doing so and measuring only the effects of taxes would force the analysis to examine an unbalanced budget change. We have shown, however, that the redistributive effects of allocative expenditures can be meaningfully removed, in a balance-budget way, from the measure of redistribution by assigning these expenditures to the counterfactual on the assumption that they are financed by marginal benefit taxes of the same amount. For Musgrave's discussion, see his text: Op. cit., pp. 213-15.

26 At least one study of redistribution seems to be based on the idea that persons should be ranked by their private incomes, \textit{per se}. Tibor Barna (Redistribution of Incomes in 1937 (Oxford: Claredon Press, 1945)) defines redistribution as a deviation from a system of "neutral finance," i.e., "from a state in which the distribution of incomes is left unaltered by government" (p. 11). His definition derives from an earlier discussion by F. C. Benham ("Notes on the Pure Theory of Public Finance," Economica, 1 (1934), 436-58) who apparently saw neutral finance as a means of fulfilling the social compact. Benham's definition of neutral finance was that system which individuals would voluntarily choose (not necessarily equivalent to Barna's definition), and he viewed all deviations from that system as inequitable. Once it is admitted, however, that deviations from "neutral finance" may be desirable, and Barna clearly thinks they are, the philosophy of government as a social compact has been rejected. There appears to be no reason why government should, on the basis of this philosophy, rank individuals by their private-market incomes, if it may violate the social compact in achieving vertical equity. The significance of the social contract has recently been revived: John Rawls, A Theory of Justice (Cambridge, Mass.: The Belknap Press of Harvard University Press, 1971).
Of the two variants of Lindahl redistribution, Lindahl I is consistent with a strict ranking of individuals by their marginal products. Since Pareto optimal transfers yield benefits to taxpayers, the receipt of such transfers can be called part of the recipient's marginal product. The fact that the recipient is not taxed for the benefits he receives from Pareto optimal transfers is not then relevant, because in receiving these transfers he is not simply receiving the benefits of government expenditures but engaging in a productive activity.

There seems to be no logical reason for excluding the payments for this particular activity from the primary distribution. An individual may incur a cost in receiving transfers (e.g., any associated stigma), as in other productive activities. As with the case of the zero-government counterfactual, therefore, theoretical analysis subverts an intuitively acceptable notion, in this case the idea that all effects of transfers should be included in a measure of redistribution. One may, of course, simply choose to exclude, for the purpose of ranking individuals, that part of a person's marginal product which is earned by virtue of the fact that his income or consumption of certain commodities enters the utility functions of others. While such a value judgment may be widely accepted, its arbitrary nature leads to our preference for Lindahl I over Lindahl II.  

26 It should be noted also that Variant 2 of Lindahl redistribution involves an asymmetric treatment of the public and private sectors in one respect. When Pareto optimal redistribution is carried out in the public sector, the benefits to recipients are considered part of redistribution. When the private sector provides such "redistribution," however, recipient benefits are excluded. Lampman, op. cit., has implied that the concept of redistribution should be extended to private sector transactions including transactions in the insurance sector. If the Lindahl II definition is used it might be preferable to modify it by including recipient benefits of private transfers.
The differences among the Gillespie and the two Lindahl counterfactuals turn on the alternative views of the appropriate ranking of individuals. By contrast, the Smolensky-Behrens counterfactual takes as given a definition of equals and unequals and asks what measure of redistribution will best characterize the degree to which vertical equity is achieved.

A measure of redistribution based on the Gillespie or Lindahl counterfactuals will show whether the relative positions of unequals has narrowed, widened, or remained unchanged. The usual interpretation of such results is that progressivity is always a movement towards an optimum. Perfect equality of incomes is thus used as an implicit standard even while it is explicitly rejected as the ultimate equity objective. Even if perfect equality were the objective of redistribution, however, progressivity carried far enough could lead to a reversal of economic positions and is not unambiguously desirable.

The S-B counterfactual has an advantage over the others in that it yields a definition of redistribution which can be readily interpreted: any redistribution represents deviations from a specified optimum and can always be interpreted as detrimental. There is additional semantic advantage to the S-B definition. Progressivity in the fiscal system commonly has the connotation of taking from the rich and giving to the poor. If, however, existing progressivity is judged to be insufficient to produce an optimum, the S-B definition will yield the result that the government, in failing to achieve an equity goal, has redistributed income from the poor to the rich.

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28 The idea of measuring redistribution as deviations from an optimum is not entirely new. F. C. Benham, op. cit., suggested that a topic of economic inquiry should be the deviations of the existing distribution from that distribution which would be produced by a "neutral" budget (cf. footnote 26). He considered the latter distribution an optimum.
A disadvantage of the S-B concept is that its use requires an explicit conception of vertical equity. Traditionally, there has been a reluctance to make normative judgments about the optimal distribution of income; however, analysts are coming to believe increasingly that this reluctance places an unnecessary constraint on research and that the question of what is an optimal distribution is a proper subject of economic analysis. Furthermore, as was pointed out earlier, in section I, a complete counterfactual (including nonfiscal parameters) which defines equals and unequals requires many judgments. A specific definition of vertical equity is, therefore, only somewhat more value-laden than the concept of horizontal equity, since it requires but one more value judgment. Finally, the S-B definition merely makes explicit the normative judgment used by the researcher to evaluate what he has measured and labeled redistribution.

The comparison of the four alternative counterfactuals has pointed to the advantage of the Lindahl I concept in providing a correct ranking of individuals on the basis of marginal products and of the S-B formulation in yielding an unambiguous measure of redistribution. These two attributes can be combined in an S-B counterfactual which ranks individuals by their incomes under a Lindahl counterfactual as a basis for specifying an optimal distribution. For example, an optimal distribution might be specified which is less skewed, has a lower

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29 One proponent of this view is Lester Thurow. See his "Toward a Definition of Economic Justice," The Public Interest, 31 (Spring 1973), 56-80.

30 Indeed, by the time an analyst finished specifying all the rules-of-the-game appropriate to defining equals and unequals, he might well find that he had arrived at an optimum. Thurow, for example, argues that what constitutes a fair economic game is not intuitively obvious and ultimately is determined by one's view of the equitable distribution of prizes (ibid., pp. 58-61). If this is true, then it would not be possible to use the idea of a fair economic process to derive a primary distribution which determines the ranking of individuals but which is not an optimum.
relative variance, and is more peaked than the current one, and which, in addition, maintains the rank order of individuals as given by the Lindahl I counterfactual.

Conclusion

We have considered four alternative definitions of income redistribution that might be used in empirical studies designed to show the extent to which vertical equity is achieved through government. All of the definitions are in terms of comparisons of long-run equilibria arising from actual government policies and some specific counterfactual. Given measuring vertical equity as the objective, the distribution of income arising from the counterfactual chosen must appropriately rank individuals as equals or unequals. The zero-government counterfactual, the basis for Gillespie redistribution, ignores the role of government in achieving allocative efficiency, and therefore, does not rank individuals by their marginal products. The two variants of what we call Lindahl redistribution include the allocative activities of government in the counterfactual, although they differ in their treatment of Pareto optimal redistribution. Finally, the Smolensky-Beihrens concept, which defines redistribution as deviations of the actual distribution from an optimal one, has the advantage of an unambiguous interpretation: all redistribution is a movement away from the optimum. It also is consistent with the theory of public sector if the optimal distribution maintains the rank order of individuals as given by the Lindahl I counterfactual.

31 Eugene Smolensky and J. Douglas Comery have indicated their preference for these changes in the shape of the size distribution. See their, "The Urban Problems as an Exercise in the Theory of Efficient Transfer," Regional Science Perspectives, 2 (1972), 172-88; reprinted as Reprint # 97, Institute for Research on Poverty, University of Wisconsin, Madison.
The definitions presented in this paper by no means exhaust the possibilities. Indeed, they serve to illustrate the wide range of interpretations which may be given to the concept of redistribution. However, we feel that the Lindahl I and the combined Lindahl I--Smolensky-Behrens concepts are both reasonable and appealing. They also demonstrate that the appropriate definition of redistribution is not intuitively obvious, but rather, must be tailored to the objective the definition is to serve.
Appendix: Algebraic Comparison of the Definitions

The differences and similarities among the alternative definitions of redistribution can be clarified by presenting the components of each in algebraic form. Let $P_i$ be the income received by individuals $i$ in the primary distribution. Similarly, let $F_i$ be $i$'s final income. The effect of governmental redistribution on individual $i$ is the difference between his final and primary income:

$$R_i = F_i - P_i \quad i = 1, \ldots, N$$

where $N$ is the number of individuals in the economy. The primary distribution, the final distribution, and the redistributive effects of government are defined as vectors of the $P_i$'s, the $F_i$'s and the $R_i$'s, respectively, and are related by the vector identity,

$$R = F - P .$$

Each definition of redistribution can be described by general equations which identify the elements of $R$.

Subscripts on income types: The subscripts on $P$ and $R$ will refer to the alternative concepts of redistribution which define their components.

The subscripts are:

- $G$ = Gillespie redistribution
- $LI$ = Lindahl redistribution, Variant I
- $LII$ = Lindahl redistribution, Variant II
- $S$ = Smolensky-Behrens redistribution
No subscript is needed for \( F \), since final income is defined identically for all concepts of redistribution.

**Income components:**

\[
Y = \text{pretax factor income}
\]

\[
B = \text{benefits of general government expenditures (i.e., expenditures other than transfers)}
\]

\[
TD = \text{donor benefits of transfers}
\]

\[
TRE = \text{recipient benefits from efficient transfers}
\]

\[
TRO = \text{recipient benefits of transfers other than efficient transfers}
\]

\[
TR = \text{recipient benefits from all transfers, equal to } TRE + TRO
\]

\[
X = \text{taxes paid}
\]

\[
XM = \text{taxes on a marginal benefit basis for benefits of general government expenditures and donor benefits of transfers}
\]

\[
XA = \text{taxes on an ability-to-pay basis}
\]

Benefits of general government expenditures \((B)\) and donor benefits of transfers \((TD)\) are valued at their marginal values to individuals. The benefits to recipients of in-kind transfers (a portion of \(TRE\), \(TRO\), and \(TR\)) are valued at their marginal values net of any charges to the recipient for the goods and services included in these transfers. Recipient charges, therefore, are excluded from the tax components. This method of valuing recipient benefits of in-kind transfers allows us to exclude the marginal benefit taxes for these benefits from \(XM\), which is necessary to obtain the simplified forms of the definitions in Table 2 below.

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32 The value of all components are assumed to be adjusted for any differences in relative prices between counterfactuals, i.e., the components are real values.

33 TRO consists of the recipient benefits attributed to deviations in transfers from the level justified by efficiency considerations alone. TRO may be positive or negative, but since for all practical purposes receipt of transfers is voluntary \( TRO + TRE = TR > 0 \).
Subscripts on income components: The subscripts on the income components refer to the assumptions about governmental parameters which give rise to these components. The subscripts denoting the four alternative assumptions relevant to the definitions of redistribution are:

- \( n \): government does not exist (i.e., taxes and government expenditures equal zero);
- \( m \): government exists, makes efficient allocative expenditures, and taxes according to marginal benefits received;
- \( a \): government exists, makes allocative expenditures and carries out redistributive policies according to an ability-to-pay principle to achieve an optimal distribution;
- \( f \): government exists, and pursues actual policies.

For example, \( Y_n \) denotes the factor income an individual would receive in long-run equilibrium, if there were no government, given existing technology, individual resource endowments, and individual preference functions.

To derive the components of \( R \) for each concept of redistribution, note first that the definition of final income is the same for all of the redistributive concepts under consideration. In our notation, final income is given by line (1) of Table 1. Final income, then, consists of factor payments, plus all benefits of government expenditures (including recipient and donor benefits of all transfers), less taxes paid. Appropriately, all of the components of final income have an \( f \) subscript, because they represent the actual values of the components in the time period in which redistribution is measured. 34

The components of primary income vary with the alternative concepts of redistribution, and are indicated in lines (2)-(5) of Table 1. In the case of Gillespie redistribution, primary income is simply the factor income an individual would receive if there were no government as indicated in line (2).

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34 Of course, this does not imply that any published data accurately reports the appropriate values of these components.
TABLE 1
Final Income, Primary Income and Redistribution Specified

<table>
<thead>
<tr>
<th>Final Income (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) For all definitions</td>
</tr>
<tr>
<td>of redistribution</td>
</tr>
<tr>
<td>$Y_f + TR_f + TD_f + B_f - X_f$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Primary Income (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) Gillespie ($P_G$)</td>
</tr>
<tr>
<td>$Y_n$</td>
</tr>
<tr>
<td>(3) Lindahl I ($P_{LI}$)</td>
</tr>
<tr>
<td>$Y_m + TRE_m + TD_m + B_m - XM_m$</td>
</tr>
<tr>
<td>(4) Lindahl II ($P_{LII}$)</td>
</tr>
<tr>
<td>$Y_m + TD_m + B_m - XM_m$</td>
</tr>
<tr>
<td>(5) S-B ($P_S$)</td>
</tr>
<tr>
<td>$Y_a + TR_a + TD_a + B_a - (XM_a + XA_a)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Redistribution (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6) Gillespie</td>
</tr>
<tr>
<td>$(Y_f - Y_n) + TR_f + TD_f + B_f - X_f$</td>
</tr>
<tr>
<td>(7) Lindahl I</td>
</tr>
<tr>
<td>$(Y_f - Y_m) + TRO_f + (TRE_f - TRE_m) + (TD_f - TD_m) + (B_f - B_m) - (X_f - XM_m)$</td>
</tr>
<tr>
<td>(8) Lindahl II</td>
</tr>
<tr>
<td>$(Y_f - Y_m) + TR_f + (TD_f - TD_m) + (B_f - B_m) - (X_f - XM_m)$</td>
</tr>
<tr>
<td>(9) S-B</td>
</tr>
<tr>
<td>$(Y_f - Y_a) + (TR_f - TR_a) + (TD_f - TD_a) + (B_f - B_a) - [X_f - (XM_a + XA_a)]$</td>
</tr>
</tbody>
</table>
Under the first variant of the Lindahl definition, primary income is given by line (3). Primary income in this case consists of factor payments, all benefits of allocative activities of government (including both recipient and donor benefits of efficient transfers), less marginal benefit taxes for general government expenditures and donor benefits of transfers. (In the case of the Lindahl equilibrium, recipient charges for in-kind transfers are equal to marginal benefits received. Marginal benefit taxes for recipient benefits of these transfers, therefore, are already netted out of TRE.) The \( m \) subscript indicates that the values of all components are the equilibrium values which would prevail if the government engaged only in allocative activities and employed marginal benefit taxation.

The definition of primary income under Variant II of Lindahl redistribution differs from Variant I in that it excludes recipient benefits of efficient transfers, which are considered part of redistribution. The resulting expression for primary income is indicated on line (4).

The S-B primary distribution is simply an expression for any primary, but it is assumed that the primary is the optimal distribution and is constructed to maintain the rank order of individuals as given by a counterfactual which defines equals and unequals. Primary income becomes line (5). Two kinds of taxation have been distinguished: ability-to-pay taxes are defined as deviations from marginal benefit taxation. \(^{35}\)

\(^{35}\) Note that if \( P_E \) is a vector of individual incomes under a counterfactual (labeled \( E \)) which defines equals and unequals, the optimal distribution (the S-B primary distribution) is given by:

\[
P_S = P_E + (Y_a - P_E) + TR_a + TD_a + B_a - (XM_a + XA_a)
\]

\[
= P_E + (Y_a - P_E) + TR_a - XA_a
\]

Therefore, the fiscal treatment persons receive under the optimal distribution includes the deviations of their factor income from their income under the \( E \) counterfactual as well as transfers received and deviations of taxes from
By subtracting each expression for primary income from final income, expressions for the alternative definitions of redistribution are obtained. These are listed in the final section of Table 1. \(^36\)

The expressions in Table 1 can be simplified by noting that, by the definition of XM:

\[ XM = B + TD \]

for all counterfactuals. These terms, therefore, drop out of both P and R for the Lindahl and S-B cases, yielding the simplified expressions in Table 2.

In comparing the four redistributive concepts, as shown in Table 2, the simplest comparison is between Gillespie and Lindahl (lines (6) and (8)). These differ only in the treatment of factor income. The similarity of the two definitions in other respects can best be explained by reference to the definitions of primary income associated with these concepts. In going from the counterfactual of no government (n) to that of marginal benefit taxation (m) all that has happened is that persons have used income to "buy" collectively consumed goods. The benefits of these goods to any individual has been offset by taxes (since recipient benefits of efficient transfers are excluded from primary income in the Lindahl II case). The only change in any individual's income which occurs by the shift in counterfactuals is a

\(^{35}\) con't. benefits. It is the combination of these effects, not just the effect of XA, which conforms to the ability-to-pay principle.

The advantage of specifying the optimal income distribution in terms of its separate components is that it forces consideration of how the optimum is to be achieved and, thus, its feasibility.

\(^{36}\) It may be useful to point out that the tax and transfer terms in all expressions for F, P, or R are equal to real taxes and transfers actually paid or received. Any shifting which occurs will be reflected in either the terms for factor income, if shifting affects nominal factor payments, or in any of the components, if prices are affected.
TABLE 2
Redistribution When $XM = B + TD$

<table>
<thead>
<tr>
<th>Final Income ($F$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) For all definitions</td>
</tr>
<tr>
<td>of redistribution</td>
</tr>
<tr>
<td>$Y_f + TR_f + TD_f + B_f - X_f$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Primary Income ($P$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2) Gillespie</td>
</tr>
<tr>
<td>$Y_n$</td>
</tr>
<tr>
<td>(3) Lindahl I</td>
</tr>
<tr>
<td>$Y_m + TRE_m$</td>
</tr>
<tr>
<td>(4) Lindahl II</td>
</tr>
<tr>
<td>$Y_m$</td>
</tr>
<tr>
<td>(5) S-B</td>
</tr>
<tr>
<td>$Y_a + TR_a - XA_a$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Redistribution ($R$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6) Gillespie</td>
</tr>
<tr>
<td>$(Y_f - Y_n) + TR_f$</td>
</tr>
<tr>
<td>$+ TD_f + B_f - X_f$</td>
</tr>
<tr>
<td>(7) Lindahl I</td>
</tr>
<tr>
<td>$(Y_f - Y_m) + TRO_f + (TRE_f - TRE_m) + TD_f + B_f - X_f$</td>
</tr>
<tr>
<td>(8) Lindahl II</td>
</tr>
<tr>
<td>$(Y_f - Y_m) + TR_f$</td>
</tr>
<tr>
<td>$+ TD_f + B_f - X_f$</td>
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<tr>
<td>(9) S-B</td>
</tr>
<tr>
<td>$(Y_f - Y_a) + (TR_f - TR_a)$</td>
</tr>
<tr>
<td>$+ TD_f + B_f - (X_f - XA_a)$</td>
</tr>
</tbody>
</table>
possible change in his factor income due to the new allocation of resources. Therefore, only the exclusion of this effect of government from redistribution in the Lindahl II case distinguishes it from Gillespie.37

In contrast to Lindahl II, the Lindahl I concept (line 7) excludes all effects of allocative activities of government from redistribution. Lindahl I excludes recipient benefits of efficient transfers as well as the change in factor income induced by allocative expenditures. The expressions for redistribution in Table 2 show the different treatment of transfers between the two variants of Lindahl redistribution. All recipient benefits of transfers not justified on efficiency grounds are included in both variants. However, the only recipient benefits of efficient transfers included in Lindahl I result from a possible change in demand for efficient transfers induced by other redistributive policies.

37 The similarity of Gillespie and Lindahl II redistribution raises the possibility that the two will not greatly differ empirically. The two counterfactuals are equivalent when the reallocation of resources which occurs in going from zero-government to the Lindahl equilibrium leaves the distribution of factor incomes unaffected. This will be the case when two conditions hold simultaneously. First, the factors which are reallocated must be employed in industries using the same overall factor intensities as the industries in which they were previously employed. This will insure that the nominal income of these factors will not change. Second, any changes in relative prices between counterfactuals must be such as to leave real factor incomes unchanged.

The significance of this observation turns on whether the two counterfactuals differ very much, because there must be some difference between them. The introduction of collective consumption goods in going from Gillespie to Lindahl II will raise real incomes and hence alter the primary distribution. It may be that the ranking of individuals remain unchanged, however, if price indices do not differ very greatly among individuals.
S-B redistribution is just the difference between the final distribution and the optimal distribution. It consists of (1) the differences in factor income which individuals currently receive and that which they would receive under the optimal distribution (the first term), (2) differences in transfers received between the final and optimal distributions (the second term), and (3) the remaining deviations of current taxes from current benefits received after removing the deviations which would exist under the optimal distribution (the last three terms).

The expressions in Table 2 indicate the imposing data requirements and analytical problems involved in measuring redistribution as given by any of the definitions we have considered. In no case can the redistributive effects of government be calculated by looking at either the tax or expenditure side alone. All of the definitions require an analysis of how factor incomes might

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38 Data is not readily available even for current income. Distributions of income that arise naturally from survey and tax data do not conform to the final distribution which forms the basis of each of these concepts of redistribution. In particular they generally fail to include donor benefits of transfers and also they fail to value benefits to recipients of in-kind transfers at their cash equivalence to recipients. Both of these omissions bias published distributions towards progressivity since donor benefits accrue to higher income families while in-kind transfers, which accrue by and large to families at the low end of the distribution, are over-valued. The benefits of general government expenditures are also absent from usual data series, largely because of the difficulties involved in measuring these benefits.
differ between the final distribution and the other counterfactuals, and there is reason to expect that these differences will not be negligible. In addition, the S-B concept requires both a determination of individual incomes under a counterfactual which defines equals and unequals and an explicit choice of an ability-to-pay criterion. Finally, in the case of Lindahl II, the level of efficient transfers under the m counterfactual must be determined.

39 If factor incomes remained unaltered among the n, m and f counterfactuals, Gillespie and Lindahl II redistribution would be equivalent (see footnote 37) and could be measured using current data on taxes, transfers, and benefits of government services. (The conditions under which factor incomes would not change between two counterfactuals were given in footnote 37.) However, differences in factor incomes going from either the zero-government or the Lindahl counterfactual to the current situation are likely to arise from various redistributive policies employed by government. If these policies cause inefficiencies, real incomes will be reduced with consequent redistributive effects. In addition, these policies may affect relative prices of commodities with different consequences for different individuals.