

FILE COPY  
DO NOT REMOVE

103-71

**INSTITUTE FOR  
RESEARCH ON  
POVERTY**

MAKING INFERENCES FROM CONTROLLED INCOME

MAINTENANCE EXPERIMENTS

Charles E. Metcalf

**DISCUSSION PAPERS**

THE UNIVERSITY OF WISCONSIN, MADISON, WISCONSIN

MAKING INFERENCES FROM CONTROLLED INCOME MAINTENANCE EXPERIMENTS

Charles E. Metcalf

The research reported here was supported by funds granted to the Institute for Research on Poverty at the University of Wisconsin by the Office of Economic Opportunity pursuant to the provisions of the Economic Opportunity Act of 1964. The author is a staff member of the Institute and an assistant professor of economics. The author wishes to thank Glen Cain, Robert Haveman, and Walter Nicholson for their comments on an earlier draft of this paper. Valuable comments were also received from participants in seminars sponsored by the Institute for Research on Poverty and the Social Systems Research Institute. The conclusions are the sole responsibility of the author.

September 1971

## ABSTRACT

There are a wide variety of issues related to how controlled income maintenance experiments should be interpreted. This paper addresses a single question: If an experiment of limited duration is conducted under "ideal" conditions, what can be inferred from the experimental results about individual behavior in a world where a negative income tax is adopted permanently?

Part I of the paper summarizes a conventional analysis of the effects of a negative income tax, given a one period, static model of behavior. Part II introduces a simplified multiperiod model to compare the effects of a temporarily and a permanently adopted negative income tax. Part III considers how one might make inferences about permanent behavior based upon the results of a temporary experiment and summarizes how the results derived from the presented two good model can be extended to apply to the multigood case. Part IV briefly explores the implications of the results outside the context of income maintenance experimentation and outlines some mitigating complications which would alter the results presented in this paper.

# MAKING INFERENCES FROM CONTROLLED INCOME MAINTENANCE EXPERIMENTS

## Introduction

Since 1968 substantial resources have been devoted to the design and execution of controlled income maintenance experiments.<sup>1</sup> In the typical experiment, a sample of low income households is placed on a variety of negative income tax plans for a temporary period of time (usually three years). The primary objective of these experiments has been to determine the impact of a negative income tax on work incentives and the effect of this impact on the monetary cost of adopting such a welfare scheme nationally.

While there are a wide variety of issues related to how the results of such experiments are to be interpreted, this paper addresses a single question. If an experiment of limited duration is conducted under "ideal" conditions, what can be inferred from the experimental results about individual behavior in a world where a negative income tax is adopted permanently?

By assuming "ideal" conditions, we mean (a) that individuals (or households)<sup>2</sup> in the experiment are utility maximizers in the conventional sense, given the structure of the experiment, and (b) that the experiment is sufficiently well designed to measure the differential response of individuals to the experiment.<sup>3</sup> The discussion is limited to a partial equilibrium analysis of an individual's supply of labor, or, equivalently, his demand for leisure time.

Part I of the paper summarizes a conventional analysis of the effects of a negative income tax given a one period, static model of behavior.

Part II introduces a multiperiod model to compare the effects of a temporarily and a permanently adopted negative income tax. Part III considers how one might make inferences about permanent behavior based upon the results of a temporary experiment, and summarizes how the results derived from the presented two good model can be extended to apply to the multigood case. Part IV briefly explores the implications of the results outside the context of income maintenance experimentation and outlines some mitigating complications which would alter the results presented in this paper.

## I

Consider a one period model in which a well-behaved function  $U(C,L)$  may be used to represent the maximum utility an individual may obtain from  $C$  dollars of goods purchases and  $L$  hours of leisure consumption. Given fixed goods prices, a nonwage income of  $G$  dollars, and a fixed quantity of time  $\bar{L}$ , the individual (by assumption) may allocate his consumption between goods and leisure by varying his quantity of labor supplied at a fixed net wage rate  $W$ . He is assumed to maximize

$$(1) \quad U(C,L) \text{ subject to } [G + W(\bar{L} - L) - C] \geq 0.$$

With a binding budget constraint and the absence of corner solutions the conditions for a utility optimum become

$$(2) \quad U_C - \lambda = 0$$

$$U_L - \lambda W = 0$$

$$[G + W(\bar{L} - L) - C] = 0 \text{ and}$$

$$(3) \quad |H| > 0,$$

where  $|H|$  is the determinant of the Hessian matrix of second order conditions.

A negative income tax (N.I.T.) ordinarily provides a household with a fixed income guarantee and simultaneously taxes other income sources of the household at a positive rate up to the "break-even" point where the amount of the guarantee is exhausted. In this model, the N.I.T. is assumed to increase nonwage income ( $dG > 0$ ) and to decrease the net wage rate ( $dW < 0$ ).<sup>4</sup> The household is assumed to be below the break-even point, such that

$$(4) \quad dG + (\bar{L}-L)dW > 0.$$

The effect of a N.I.T. on the demand for leisure time can be derived by totally differentiating (2).

$$(5) \quad \frac{\partial L}{\partial G} = \frac{-(WU_{CC} - U_{LC})}{|H|}$$

$$(6) \quad -\frac{\partial L}{\partial W} = -(\bar{L}-L)\frac{\partial L}{\partial G} + \frac{\lambda}{|H|}$$

If leisure is a normal good, an increase in the guarantee raises the demand for leisure time. A decline in the net wage rate (through an increase in the tax rate) decreases the demand for leisure through the income effect (again if leisure is normal good) and increases the demand for leisure through the substitution effect. From condition (4), the income effect of the guarantee dominates the income effect of the reduction in the wage rate. Thus, if leisure is a normal good, the N.I.T. unambiguously increases the demand for leisure, compared to a situation where no such program is in effect.

## II

In order to compare the effects of a temporary and a permanent N.I.T., the above analysis must be extended to a multiperiod framework. Consider an intertemporally additive utility function of the form

$$(7) \quad V = \sum_{i=1}^N (1+d)^{1-i} U^i(C_i, L_i),$$

where  $d$  is the individual's subjective discount rate and where the form of  $U^i(\cdot)$  may vary across time periods.<sup>5</sup> The individual maximizes (7) subject to the income constraint

$$(8) \quad \sum_{i=1}^N (1+r)^{1-i} [G_i + W_i(\bar{L} - L_i) - C_i] \geq 0,$$

where  $r$  is the real period market rate of interest (assumed to be constant).

With a binding budget constraint and the absence of corner solutions, the conditions for a regular multiperiod utility optimum become

$$(9) \quad \left(\frac{1+d}{1+r}\right)^{1-i} \cdot UC_i^i - \lambda = 0 \quad i = 1, \dots, N,$$

$$\left(\frac{1+d}{1+r}\right)^{1-i} \cdot U_{L_i}^i - \lambda W_i = 0 \quad i = 1, \dots, N,$$

$$\sum_{i=1}^N (1+r)^{1-i} [G_i + W_i(\bar{L} - L_i) - C_i] = 0, \text{ and}$$

$$(10) \quad |H^*| > 0$$

where  $|H^*|$  is the determinant of the relevant Hessian matrix. With the added assumptions that  $d = r$ , that  $U^i(\cdot)$  is identical for all time periods, and that relative prices are fixed through all time periods, the optimization of (7) would imply a uniform consumption stream throughout the life of the individual. We shall proceed without imposing these added assumptions, but shall note their implications for the derived results as they are presented.

A temporary N.I.T., effective during the first period of the individual's lifetime, can now be compared with a permanent N.I.T., in force throughout the individual's life.<sup>6</sup> In comparing these two situations, we assume that the individual knows with certainty the duration of the N.I.T. in each instance, as well as all other relevant information about the future.

#### A. Temporary Negative Income Tax

The effects of a temporary N.I.T. can be derived by changing the values of  $G_1$  and  $W_1$ . By differentiating (9) we obtain the effect of a temporary N.I.T. upon the first period demand for leisure time.

$$(11) \quad \frac{\partial L_1}{\partial W_1} = \frac{- [W_1 U_{CC}^1 - U_{LC}^1] \cdot \left( \prod_j^{2,N} |D_j| \right)}{|H^*|} \quad \text{and}$$

$$(12) \quad - \frac{\partial L_1}{\partial W_1} = - (\bar{L} - L_1) \frac{\partial L_1}{\partial G_1} - \frac{\lambda}{|H^*|} \left\{ [U_{CC}^1 \sum_i^{2,N} (|J_i| \prod_{j \neq i}^{2,N} |D_j|)] - \left[ \prod_j^{2,N} |D_j| \right] \right\}$$

where we define

$$|D_j| = \begin{vmatrix} \left(\frac{1+d}{1+r}\right)^{1-j} U_{CC}^j & \left(\frac{1+d}{1+r}\right)^{1-j} U_{CL}^j \\ \left(\frac{1+d}{1+r}\right)^{1-j} U_{LC}^j & \left(\frac{1+d}{1+r}\right)^{1-j} U_{LL}^j \end{vmatrix}, \quad j = 1, \dots, N; \text{ and}$$

$$|J_i| = \begin{vmatrix} \left(\frac{1+d}{1+r}\right)^{1-i} U_{CC}^i & \left(\frac{1+d}{1+r}\right)^{1-i} U_{CL}^i & -1 \\ \left(\frac{1+d}{1+r}\right)^{1-i} U_{LC}^i & \left(\frac{1+d}{1+r}\right)^{1-i} U_{LL}^i & -W_i \\ -(1+r)^{1-i} & -(1+r)^{1-i} W_i & 0 \end{vmatrix}, \quad i = 1, \dots, N^7$$

Equations (11) and (12) are qualitatively similar to equations (5) and (6) in Part I. What must be ascertained is whether or not the magnitudes

of  $\frac{\partial L_1}{\partial G_1}$  and  $-\frac{\partial L_1}{\partial W_1}$  are unbiased estimates of the magnitudes associated with a permanent change in G and W.

### B. Permanent Negative Income Tax

We now differentiate (9) with respect to G and W, where  $dG_i = dG$ , for all i, and  $dW_i = dW$ , for all i.

$$(13) \quad \frac{\partial L_1}{\partial G} = \sum_{i=1}^N (1+r)^{1-i} \frac{\partial L_1}{\partial G_1} = (1+R) \frac{\partial L_1}{\partial G_1}, \text{ where}$$

$$R = \left[ \frac{1 - (1+r)^{1-N}}{r} \right]$$

$$(14) \quad -\frac{\partial L_1}{\partial W} = -\sum_{i=1}^N (1+r)^{1-i} (\bar{L} - L_i) \frac{\partial L_1}{\partial G_1} - \frac{\lambda}{|H^*|} \left\{ [U_{CC}^1 \sum_i^{2,N} (|J_i| \prod_{j \neq i}^{2,N} |D_j|)] \right\}$$

$$- \left[ \prod_{j \neq i}^{2,N} |D_j| \right] \left\{ [U_{CC}^1 \sum_i^{2,N} (|J_i| \prod_{j \neq i}^{2,N} |D_j|)] \right\} - \frac{\lambda}{|H^*|} [W_1 U_{CC}^1 - U_{LC}^1] \sum_{i=2}^N \{ (1+d)^{1-i} [W_1 U_{CC}^i - U_{CL}^i] \prod_{j \neq i}^{2,N} |D_j| \}$$

$$(15) \quad -\frac{\partial L_1}{\partial W} = -\frac{\partial L_1}{\partial W_1} - \sum_{i=2}^N (1+r)^{1-i} (\bar{L} - L_i) \frac{\partial L_1}{\partial G_1}$$

$$- \frac{\lambda}{|H^*|} [W_1 U_{CC}^1 - U_{LC}^1] \sum_{i=2}^N \{ (1+d)^{1-i} [W_1 U_{CC}^i - U_{CL}^i] \prod_{j \neq i}^{2,N} |D_j| \}$$

Equation (13) reflects the conventional result that a transitory change in income will have a smaller effect on consumption than a permanent change. Evaluated locally, the income effect of a permanent N.I.T. is larger than the income effect of a temporary N.I.T. by the multiple (1+R). From (14) and (15) we observe that the difference in price effects has both income and substitution components. If leisure is a normal good, the estimated effect of a reduction in the net wage rate on the demand for leisure, based upon a temporary experiment, will typically overstate the

price effect of a permanent N.I.T. for two reasons. First, the negative influence of the income effect (for a normal good) is understated due to the downward bias in the income effect. Second, the difference between the permanent and transitory substitution effects, measured by the final term in equation (15), will typically assume a negative value.<sup>8</sup>

Two basic results can be stated concerning biases in estimates of the effect of a N.I.T. on the demand for leisure derived from a temporary experiment. First, the income effect is understated by the experiment. Second, both the gross and the compensated price effects are overstated by the experiment. It should be noted that a directional statement can be made concerning the bias in the gross price effect even though the price effect itself is ambiguous in sign.<sup>9</sup>

### III

The above results complicate the process of drawing inferences about permanent behavior from a temporary experiment. First, in order to make proper inferences the magnitude of the identified biases must be determined. Second, in order to sort out the biases, independent estimates of the income and substitution effects must be obtained. This implies the need to vary income guarantees and tax rates independently across sample points in a N.I.T. experiment, rather than compare a single N.I.T. scheme to a control group. Third, to the extent that policymakers are concerned with the efficiency implications of the N.I.T. rather than with the demand for leisure per se (this may not in fact be true), it should be noted that the estimates derived from the experiments place an outer bound on the substitution effect, upon which efficiency implications depend.

A further examination of equations (13) - (15) suggests procedures for measuring the biases in the estimated income and substitution effects.<sup>10</sup>

From (13), the value of  $R$  must be known in order to infer the permanent income effect from the estimated transitory measure;  $R$  depends upon the periodic interest rate facing the individual and upon his time horizon or length of life, stated as a multiple of the length of the experiment.

Table 1 reports calculated values of  $R$  as a function of the discount rate and of the time horizon. If the household has a real annual discount rate of ten percent and a time horizon of 30 years ( $N = 10$ ), the bias in the income effect would be 2.8 times the measured transitory effect.

TABLE 1  
EVALUATION OF  $R$

$$R = \frac{1 - (1+r)^{1-N}}{r}$$

$r$  = real discount rate (3-year base period)

$N$  = number of periods in lifetime (3-year periods)

		VALUE OF $R$					
Annual Discount Rate	$r$	$N=1$	$N=2$	$N=5$	$N=10$	$N=20$	$N=\infty$
.05	.1576	0	.864	2.812	4.645	5.952	6.435
.08	.2597	0	.794	2.321	3.369	3.803	3.851
.10	.3310	0	.751	2.059	2.791	3.008	3.021
.12	.3937	0	.718	1.867	2.412	2.535	2.540
.15	.5209	0	.658	1.561	1.876	1.913	1.920
.20	.7280	0	.579	1.220	1.364	1.374	1.374

Even if the time horizon is as short as two times the length of the experiment,<sup>11</sup> the bias would be 75 percent as large as the measured coefficient. It should be noted that these measures of bias are applicable when the measured income effect is negative as well as positive.

A number of alternatives are available for estimating the value of  $R$ . If sufficient assumptions are made to imply a uniform lifetime consumption stream for the individual, it can easily be demonstrated that  $\left(\frac{R}{1+R}\right)$  equals the transitory marginal propensity to save. If a portion of the N.I.T. sample were placed on an experiment of longer duration, it would be possible (in principle) to measure  $R$  independently by observing the difference in income responses of the groups on experiments of unequal duration. If we are unwilling to impose these additional assumptions and if the experiment is of uniform duration, independent evidence must be sought to verify the value of  $R$ .

The income effect component of the bias in the gross price effect [equation (15)] is also a multiple of the observed, temporary income effect, but the value of  $R$  is insufficient information to determine the size of the bias. What must be known is the present discounted quantity future of labor services provided by the individual; in addition to specifying an interest rate, we must be willing to assert a relationship between the present and future labor force behavior in order to estimate this component of the bias.

In order to quantify the bias in the substitution effect, we must be able to evaluate the expression

$$(16) \quad \left[ \left( \frac{\partial L_1}{\partial W_1} \Big|_U \right) - \left( \frac{\partial L_1}{\partial W_1} \Big|_U \right) \right] = - \frac{\lambda}{H^*} [W_1 U_{CC}^1 - U_{LC}^1] \sum_{i=2}^N \{ (1+d)^{1-i} [W_i U_{CC}^i - U_{CL}^i] \prod_{j \neq i} |D_j| \}^{2,N}$$

from equation (15). In the following discussion, two alternative methods of evaluating (16) are presented. The first method involves observing the substitution effects of a temporary change in the net wage rate on consumption of all goods during the experiment. The second method involves

placing restrictions on the form of the single period utility function ( $U^1$ ) and calculating (16) as a function of marginal budget shares expended on cash consumption and leisure.

In the single period optimization problem of part I we can state the conventional adding-up condition from neoclassical consumption theory that the price-weighted sum of substitution effects of a single price change across all goods equals zero:

$$(17) \quad \frac{\partial C}{\partial W} \Big|_{\bar{U}} + W \cdot \frac{\partial L}{\partial W} = 0$$

In the multiperiod model of part II we are concerned with a first period change in the wage rate resulting from the experiment. The adding-up condition comparable to (17) places a zero restriction on the price-weighted substitution effects summed over all time periods, but places no restriction on the substitution effects summed only over the period of the experiment. The deviation of this sum from zero is directly related to the substitution bias in (16).

After differentiating (9) to obtain compensated price effects, the weighted sum of  $\left(-\frac{\partial L_1}{\partial W_1} \Big|_{\bar{U}}\right)$  and  $\left(-\frac{\partial C_1}{\partial W_1} \Big|_{\bar{U}}\right)$  can be written as

$$(18) \quad \left\{ -W_1 \frac{\partial L_1}{\partial W_1} \Big|_{\bar{U}} - \frac{\partial C_1}{\partial W_1} \Big|_{\bar{U}} \right\} = - \frac{\lambda}{|H^*|} \left\{ [W_1 U_{CC}^1 - U_{CL}^1] \sum_i^{2,N} (|J_i|) \prod_{j \neq i}^{2,N} |D_j| \right\},$$

which is positive or negative depending upon whether leisure is a normal or an inferior good during the experimental period.

The importance of the present discounted quantity of future labor services in determining the income effect component of the bias in the gross price effects was mentioned above; similarly, the marginal share of leisure in future consumption is the remaining element needed to quantify the substitution bias. Again, differentiating (9), we obtain

$$(19) \quad \frac{\partial L_i}{\partial G_1} = \frac{-\left(\frac{1+d}{1+r}\right)^{1-i} [W_i U_{CC}^i - U_{LC}^i] \prod_{j \neq i}^{1,N} |D_j|}{|H^*|} \quad \text{and}$$

$$(20) \quad \left[ \frac{\partial C_i}{\partial G_1} + W_i \frac{\partial L_i}{\partial G_1} \right] = \frac{\left(\frac{1}{1+r}\right)^{1-i} [ |J_i| \prod_{j \neq i}^{1,N} |D_j| ]}{|H^*|}$$

If the discounted sum of the partial derivatives defined by (19) for periods  $\underline{2}$  through  $\underline{N}$  is divided by the discounted sum corresponding to (20), we obtain

$$(21) \quad \frac{\sum_i^{2,N} (1+r)^{1-i} \frac{\partial L_i}{\partial G_1}}{\sum_i^{2,N} (1+r)^{1-i} \left[ \frac{\partial C_i}{\partial G_1} + W_i \frac{\partial L_i}{\partial G_1} \right]} = \frac{\sum_i^{2,N} (1+d)^{1-i} [W_i U_{CC}^i - U_{LC}^i] \prod_{j \neq i}^{2,N} |D_j|}{\sum_i^{2,N} [ |J_i| \prod_{j \neq i}^{2,N} |D_j| ]}$$

An examination of the above results reveals that the substitution bias defined in (16) equals the product of equations (18) and (21). Thus we conclude

$$(22) \quad \left[ \left( \frac{\partial L_1}{\partial W_1} \Big|_{\bar{U}} \right) - \left( \frac{-\partial L_1}{\partial W_1} \Big|_{\bar{U}} \right) \right] = \left\{ -W_1 \frac{\partial L_1}{\partial W_1} \Big|_{\bar{U}} - \frac{\partial C_1}{\partial W_1} \Big|_{\bar{U}} \right\} \cdot \left\{ \frac{\sum_i^{2,N} (1+r)^{1-i} \frac{\partial L_i}{\partial G_1}}{\sum_i^{2,N} (1+r)^{1-i} \left[ \frac{\partial C_i}{\partial G_1} + W_i \frac{\partial L_i}{\partial G_1} \right]} \right\}$$

i.e. that the bias in the observed substitution effect equals the product of the two terms; the first is the weighted sum of substitution effects observed during the experimental period; the second is the ratio of the effect of an income change on the present discounted quantity of future leisure consumption to the effect of an income change on the present discounted value of future total consumption. Since the numerator of the ratio is in quantity rather than value terms, the expression differs from a marginal budget share.

Evaluation of the substitution bias requires some estimate of future leisure consumption. If we were to impose the additional restrictions cited in part II as being sufficient to guarantee a uniform life-time consumption stream [i.e.  $\underline{d} = \underline{r}$ ,  $U^i(\cdot)$  identical for all time periods, and fixed relative prices through time], the ratio required in (22) would equal the corresponding ratio observed during the experimental period. The same result would hold with  $\underline{d} \neq \underline{r}$ , if we were to add the requirement that  $U^i(\cdot)$  be homogeneous. It should be emphasized, however, that so long as the assumptions of our more general model are fulfilled, failure of the more stringent restrictions to hold affects our ability to estimate the substitution bias only insofar as it affects the accuracy of our estimate of the relative future consumption of leisure. A given error in the estimated value of the ratio in (22) translates into an equal percentage error in the estimated substitution bias. Failure to estimate the appropriate market interest rate or the time horizon of the individual plays precisely the same role. It should be noted that the fact that relative leisure consumption increases substantially after an individual reaches retirement age does not have a major impact on the appropriateness of using current leisure behavior as an approximation of discounted future behavior, since consumption patterns in the distant future are presumably discounted heavily.

While the expression in (22) is directly measurable, given assumptions about the relationship between present and future leisure consumption, it provides no intuitive assessment of the relative magnitude of the substitution bias. By again placing further

restrictions on the form of the utility function, we can state the ratio of the transitory and permanent substitution effects in terms of the marginal shares of expenditures on leisure and cash consumption.

Reevaluating the components of (22), the first element drawn from equation (18) can be written as

$$(23) \quad \left\{ -w_1 \frac{\partial L_1}{\partial w_1} \Big|_{\bar{U}} - \frac{\partial C_1}{\partial w_1} \Big|_{\bar{U}} \right\} = - \frac{\lambda(\text{MPS}) [w_1 U_{CC}^1 - U_{CL}^1]}{[U_{CC}^1 U_{LL}^1 - U_{CL}^1 U_{LC}^1]},$$

where MPS is the marginal propensity to save observed during the experimental period.<sup>12</sup> If, in addition, we impose the approximation that

$$(24) \quad \frac{\sum_i^{2,N} (1+r)^{1-i} \frac{\partial L_i}{\partial G_1}}{\sum_i^{2,N} (1+r)^{1-i} \left[ \frac{\partial C_i}{\partial G_1} + w_1 \frac{\partial L_i}{\partial G_1} \right]} \sim \frac{\frac{\partial L_1}{\partial G_1}}{\frac{\partial C_1}{\partial G_1} + w_1 \frac{\partial L_1}{\partial G_1}} = \frac{-[w_1 U_{CC}^1 - U_{LC}^1]}{|J_1|}$$

equation (22) can be rewritten as

$$(25) \quad \left[ \left( - \frac{\partial L_1}{\partial w} \Big|_{\bar{U}} \right) - \left( - \frac{\partial L_1}{\partial w_1} \Big|_{\bar{U}} \right) \right] \sim \frac{\lambda(\text{MPS}) [w_1 U_{CC}^1 - U_{LC}^1]^2}{|J_1| \cdot [U_{CC}^1 U_{LL}^1 - U_{CL}^1 U_{LC}^1]}$$

If we now impose the restrictions that  $\underline{d} = \underline{r}$ , that prices are uniform across time, and that  $U^i(\cdot)$  is identical in every time period and additive within as well as across time periods, we can utilize the results that  $- \frac{\partial L_1}{\partial w} \Big|_{\bar{U}} = \frac{\lambda}{|J_1|}$  and

$$(26) \quad \frac{[w_1 U_{CC}^1 - U_{LC}^1]^2}{[U_{CC}^1 U_{LL}^1 - U_{CL}^1 U_{LC}^1]} = \frac{w_1^2 U_{CC}^1}{U_{LL}^1} = \frac{w_1 \frac{\partial L_1}{\partial G_1}}{\frac{\partial C_1}{\partial G_1}}$$

to obtain

$$(27) \frac{\left[ \left( -\frac{\partial L_1}{\partial W} \Big|_U \right) - \left( -\frac{\partial L_1}{\partial W_1} \Big|_U \right) \right]}{\left( -\frac{\partial L_1}{\partial W} \Big|_U \right)} = (\text{MPS}) \cdot \begin{bmatrix} W_1 \frac{\partial L_1}{\partial G_1} \\ \frac{\partial C_1}{\partial G_1} \end{bmatrix}$$

Thus the substitution bias as a proportion of the permanent substitution effect equals the product of the observed marginal propensity to save and the ratio of marginal budget shares expended on leisure and cash consumption, given the restrictions stated.<sup>13</sup>

In section III we identified the bias in the income effect to be a function of  $R$ , which is a function of the interest rate and the time horizon of the household. The bias in the substitution effect can be identified by measuring the weighted sum of single period substitution effects, and by approximating future relative leisure consumption; under more stringent restrictions, the substitution bias depends upon the marginal propensity to save and marginal budget shares.

The results of this paper can easily be extended to a model with  $m$  goods. The reported results concerning income effects can be generalized directly to the multigood case. The biases in the substitution effects of a price change on any two goods can be shown to be proportional to the effects of an income change on consumption of the two goods.<sup>14</sup> If an approximation such as (24) holds as an equality for the good undergoing the price change, the price-weighted sum of substitution biases will equal the deviation of the observed sum of substitution effects from zero.

## IV

The above analysis concerns a situation in which there exists a desire to make inferences about permanent behavioral relationships from measurements of behavior generated from a temporary experiment. The converse problem exists in many discussions of the use of fiscal instruments, where policy makers may wish to predict the effect of temporary fiscal measures, based upon the observed empirical effects of earlier fiscal measures which were viewed to be permanent. For example, if one used evidence drawn from the 1964 reduction in U.S. Personal Income Tax rates (viewed to be permanent) to predict the effect on consumption of the 1968 Income Tax Surcharge (viewed to be temporary) one would expect the income effect of the 1968 Surcharge to have been substantially overestimated. The permanent income hypothesis has been widely cited in making this argument concerning the effects of temporary changes in income tax rates; the analysis of income effects in this paper corresponds to the implications of the permanent income hypothesis.

Furthermore, the analysis of this paper suggests that temporary fiscal measures which affect the prices faced by decisionmakers (such as investment credits, excise taxes on consumer durables, and general sales or expenditure taxes) will have a stronger effect on short-run behavior than would be predicted from empirical measures of responses to price changes viewed to be permanent. This paper suggests procedures for predicting the strength of such fiscal measures. The reader must be cautioned, however, that this paper, as a partial equilibrium analysis, stops far short of making predictions about the general equilibrium or macroeconomic magnitude of such measures.

At least three critical assumptions implied in the above analysis would, if relaxed, create mitigating complications in the reported results. First, it is assumed that the households in question face perfect capital and labor markets. The nature of borrowing and labor market constraints facing low income households, and the expected impact of these constraints on behavior, must be investigated before considering seriously either the results of current negative income tax experiments or the measures of bias in those results suggested in this paper. Second, it is assumed that the households in question adjust instantaneously and costlessly to changes in their economic environment. Again, the magnitude of adjustment costs and lags must be investigated. Finally, it is assumed that the households behave as if they know with certainty the duration of the experiment. Given the possibility that households may expect a negative income tax similar to the experimental tax to be permanently enacted prior to the end of the experiment, and given that the conductors of the experiment may have obscured its duration in the minds of the participating households, the effects of uncertainty and of different patterns of expectations on behavior must also be accounted for.

## NOTES

1. In addition to the New Jersey - Pennsylvania Graduated Work Incentive Experiment and the Rural Graduated Work Incentive Experiment (Iowa-North Carolina) funded through the Office of Economic Opportunity, similar experiments are operating or in the planning stages in Gary, Seattle, and Denver.
2. The terms "individual" and "household" are used interchangeably in this paper. The household is assumed to possess a single decision maker and to have a finite lifetime corresponding to that of the decision maker.
3. We are abstracting from all problems related to sample design and stochastic estimation. The analysis of this paper is done in a nonstochastic framework. For a discussion of sample design procedures in such experiments, see Orcutt and Orcutt [3], and Conlisk and Watts [1].
4. For expositional purposes, we now treat the income guarantee as the only source of nonwage income.
5. The individual is assumed to have no bequest motive; he derives utility only from his own consumption.
6. We can interpret the individual's "lifetime" as being that portion of his life commencing with the initiation of the negative income tax. Given the form of (7), behavior prior to "period 1" affects behavior during the period of analysis only through its effect on the budget constraint.
7. It should be noted that

$$(a) \quad |H^*| = \sum_{i=1}^N (|J_i| \prod_{j \neq i} |D_j|) > 0$$

Additional requirements implied by the second order conditions for optimization include

$$(b) \quad |J_i| > 0, \text{ all } i, \text{ and}$$

$$(c) \quad \{|J_i| \cdot |D_j| + |J_j| \cdot |D_i|\} > 0, \text{ all } i, j, i \neq j$$

Conditions (b) and (c) imply that  $|D_j|$  have a nonpositive value for at most one  $j$ . If  $|D_j|$  is positive for all  $j$ , an increase in the present value of the individual's income stream will increase total expenditures in every time period. If  $|D_j|$  is negative for one  $j$ , an increase in the present value of income will lead to an increase in expenditures in all other periods. (If  $|D_j| = 0$  for one  $j$ , expenditures would increase in period  $j$  and remain constant in all other periods). In this paper it is assumed that  $|D_j| > 0$  for all  $j$ ,

i.e. that the aggregate expenditures in each time period, viewed as a composite commodity, be a "normal good." Nothing in this assumption rules out the possibility of inferiority of individual goods in the consumption bundle. The above discussion of restrictions placed on the value of  $|D_j|$  parallels the treatment of a single period additive utility function in Slutsky [4].

8. Given that  $\lambda$  and  $|H^*|$  are positive, the sign of the expression depends upon the signs of all  $[W_i U_{CC}^i - U_{CL}^i]$  and  $|D_j|$ . From the

restrictions placed on  $|D_j|$  in footnote 7, all  $|D_j|$  are assumed to be positive. From equation (11) and from similar expressions

for other time periods,  $\frac{\partial L_i}{\partial G_i} \geq 0$  as  $[W_i U_{CC}^i - U_{LC}^i] \geq 0$ .

So long as the effect of an income change on current leisure consumption is of the same sign (positive or negative) as the effect of an income change on the discounted sum of future leisure consumption, the final term in equation (15) will be negative.

9. If leisure is an inferior good, the income effect would still be understated in absolute value. The statement in footnote 8 about the compensated price effect would stand, but the bias in the gross price effect would be ambiguous in sign.
10. The entire measurement discussion is in terms of the magnitude of locally evaluated partial derivatives. Additional measurement problems associated with the use of local approximations to evaluate the effects of finite parameter changes are not dealt with in this paper.
11. While many have argued that low income households have very short time horizons, it is not clear that evidence used to support these arguments in fact supports the existence of short time horizons as opposed to high discount rates.

$$12. \text{MPS} = \left\{ \sum_{i=2}^N (1+r)^{1-i} \left[ \frac{\partial C_i}{\partial G_1} + W_i \frac{\partial L_i}{\partial G_1} \right] \right\} = \frac{|D_1| \sum_i^{2,N} [ |J_i| \prod_{j \neq i}^{2,N} |D_j| ]}{|H^*|}$$

13. The additional restrictions guarantee that the approximation is an equality.
14. The close relationship between substitution and income effects has its origins in the assumption of intertemporal additivity. For a general discussion of additive preferences, see Houthakker [2].

## REFERENCES

- [1] J. Conlisk and H. Watts, "A Model for Optimizing Experimental Designs for Estimating Response Surfaces," 1969 Proceedings of the Social Statistics Section, American Statistical Association, pp. 150-156.
- [2] H.S. Houthakker, "Additive Preferences," Econometrica 28 (April 1960): pp. 244-256.
- [3] G.H. Orcutt and A.G. Orcutt, "Incentive and Disincentive Experimentation for Income Maintenance Policy Purposes," The American Economic Review 58 (September 1968): pp. 754-772.
- [4] Eugen E. Slutsky, "On the Theory of the Budget of the Consumer," Giornale degli Economisti, Vol. LI (1915), pp. 1-26, reprinted in American Economic Association, Readings in Price Theory, (Homewood, Illinois, 1952): pp. 27-56.