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Insufficient Human Capital Accumulation Resulting in a Dual Economy Caught in a Poverty Trap

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Abstract

This paper examines from a theoretical point of view the observation of a dual economy in many underdeveloped countries. The notion of duality arises from the coexistence of a large labor-intensive sector offering bare subsistence wages and a smaller capital-intensive sector offering better employment opportunities. The question is, then, why do those participating in the labor-intensive sector remain in poverty rather than move into the capital-intensive sector and better their lot? The results presented here suggest that one answer may lie in the absence of markets for human capital, the consequence being that parental income and wealth exercise an independent, constraining effect on children's education capable of explaining the perpetuation of a dual economy.

The analysis is carried out within a general equilibrium framework. The assumption that future labor earnings cannot serve as collateral on a loan results in a continuum of possible steady states. They are characterized by the decomposition of production between a labor-intensive and a skill-intensive good. The larger is the labor-intensive sector in a steady state, the lower is the economy's gross national product and the larger is the relative wage difference between educated and uneducated workers.

1. INTRODUCTION

A Chilean friend told me that when you park your car in Santiago, often a youngster will come up to you to offer his services. He will look after your car, and he will only pay the parking fee if the parking controller shows up. There are numerous examples of these low-paid, labor-intensive services in the developing world. At the same time, a few of these countries have an economic sector which is capital-intensive in terms of both real and human capital. It seems puzzling that these two layers of the economy can coexist when the allocation of resources takes place in competitive markets. How is it that there is no transfer of labor between sectors, given substantial relative wage differences?

The concept of a dual economy is far from new. Lewis [1954] constructed a model in which economic development results from a transfer of labor from a subsistence sector into a capitalist sector. The speed of the labor transfer is solely determined by the rate of capital accumulation in the capitalist sector, since the supply of labor from the subsistence sector is assumed to be perfectly elastic at a subsistence wage. In 1979 W. Arthur Lewis shared the Nobel Prize with Theodore W. Schultz, who stressed the importance of the quality of human agents for understanding the situation in low-income countries. In his Nobel Lecture, Schultz [1980] also said that "...poor people are no less concerned about improving their lot and that of their children than rich people are." In this paper we will try to show a possible link between the process of human capital accumulation and the existence of dual economies within a general equilibrium framework.[1]

In the human capital literature, whose early contributors included Becker [1962] and Mincer [1958], educational decisions are based on maximizing behavior.

A common assumption has also been that a perfect market for educational loans exists. This seems questionable, since the embodiment of human capital in people ought to affect the credit market for such investments. In particular, the prohibition of slavery makes it impossible to seize the capital from a borrower who does not honor his debt. Our way of modeling this is to adopt the assumption of Loury [1981], which is that future labor earnings cannot serve as collateral on a loan.

We formulate an overlapping-generations model with two goods. One good is skill-intensive, in the sense that only educated workers can be employed in its production. The other good can be produced with both educated and uneducated labor, so it is said to be labor-intensive. Each agent lives for two periods. In the first period of life, an agent is either educated or works in the labor-intensive industry, depending on a parental decision. As an adult in the second period, an agent seeks the best possible employment given his educational status. He derives utility from consuming the two goods and from the welfare of his exogenously given child. In addition to the educational decision, the offspring's utility can be affected through bequest.

The described model has many features in common with the work of Loury [1981], which is a good reason for comparing the two models' implications. But let us first point out a few differences in assumptions. In contrast to our model with no uncertainty, Loury assumes that the child's ability is stochastic and unknown at the time of the educational decision. In addition, Loury lets the educational-choice variable be continuous instead of discrete. On the other hand, there can be no bequest between generations in his model. Finally, Loury assumes that families live isolated from each other, consuming their own produced good.

Despite the somewhat similar structure of the two models, the results are dramatically different with respect to a stationary income distribution. Loury proves the existence of a unique equilibrium distribution, which is globally stable. However, in our model there is a continuum of stationary income distributions. They correspond to a continuum of steady states characterized by the decomposition of the labor force into educated and uneducated workers. The lower the ratio of educated workers in a steady state, the lower is the economy's gross national product and the larger is the relative wage difference between uneducated and educated labor. An agent's well-being will therefore depend not only on his own educational status but also on the production structure of the economy as a whole, which can be compared to Loury's model, where households live in autarky.

Lucas [1988] theorizes about the effects of externalities from human capital in the context of economic development. He shows that a neoclassical growth model with such a feature is consistent with the permanent maintenance of per capita income differentials *between* countries. Our focus on intergenerational transfers of human capital offers another explanation for this outcome, and relates it to income differentials *within* countries. This latter linkage brings us back to a question in political economy. Both Lewis [1954] and Schultz [1964] spoke about underinvestment in human capital because of some agents' vested interests in maintaining the status quo. Our model suggests how such a situation can arise, but the analysis stops short of theorizing with respect to agents' influence over the choice of institutional arrangements within their economy.

The organization of the paper is as follows. In section 2 we set up the model. Section 3 defines a perfect-foresight equilibrium. The concept of a steady state is then developed in section 4, where we also prove existence and nonuniqueness. The set of steady states is computed for a particular parameterization of the model in section 5. Section 6 establishes the existence of Pareto improvements for any stationary allocation except for one. This Pareto optimal allocation coincides with the unique steady state under a student loan program outlined in section 7. We offer a few conclusions in section 8.

2. MODEL

The economy has a population that is constant over time. In each period, a continuum of agents are born who live through two periods and behave competitively. Every member of the older generation is the parent of an agent in the current young generation. Let the families be indexed by the variable $i \in [0, 1]$. All agents are identical with respect to inherent capabilities and preferences. As a youth the agent is endowed with $f \geq 0$ labor units, and in the second period of life he has one labor unit. The preferences of an agent born at time t–1 in family i are given by

$$\text{<2.1>} \quad U\left[c_t^{\ell i}, c_t^{si}, u_{t+1}^i\right] = \log c_t^{\ell i} + \alpha \log c_t^{si} + \beta u_{t+1}^i ,$$

$$\alpha > 0 , \quad 0 < \beta < 1 ,$$

where c_t^{ji} is the agent's consumption of good $j \in \{\ell, s\}$ in his second period of life and u_{t+1}^i is his child's utility in adulthood. (This way of modeling altruism toward children can be found as early as Barro [1974].)

The two goods produced, ℓ and s, are labor-intensive and skill-intensive, respectively. Both production processes exhibit constant returns to scale and the only input is labor, but the technologies differ when it comes to what kind of labor can be utilized. To produce the labor-intensive good ℓ, workers of any educational status can be employed, and they all have the same productivity. In the case of the skill-intensive good s, only educated workers can be used. Let L_t^j be the number of labor units supplied by workers with an adequate educational status to industry $j \in \{\ell, s\}$ at time t. The produced quantities of the two goods in that period are then given by

$$\langle 2.2\rangle \quad q_t^{\ell} = q^{\ell}\left[L_t^{\ell}\right] = L_t^{\ell} \ ,$$

$$\langle 2.3\rangle \quad q_t^{s} = q^{s}\left[L_t^{s}\right] = \gamma L_t^{s} \ , \qquad \gamma > 0 \ .$$ [2]

Both goods can be consumed in the period in which they are produced. Only good ℓ can be stored between periods and is not subject to any depreciation. [3] Good s is also used in the education process. The education of a person requires a quantity e of good s as an input, where $0 < e < \beta \gamma$, and it precludes the person from working in that period. [4] Obviously, this technology gives rise to only two possible educational statuses – educated or uneducated.

It remains to spell out the agents' choice sets, and in particular the interaction between members of the same family. In the first period of life, an agent has no decisions to make; he simply submits to his parent's authority. The parent will either give him an education or let him work in the labor-intensive industry, which does not require any education. The parent gets control over any wage income and can decide to pass on a nonnegative inheritance to the child. Because of the described storage technology, any inheritance will be in form of good ℓ, which of course can be exchanged for the other good in future periods.

In the second period of life, the agent is a parent himself and maximizes the utility function in <2.1>. Since there is no disutility from working, the agent will clearly sell his labor unit in the labor market. If the agent is uneducated, he seeks employment in the labor-intensive industry. An educated worker chooses to work in the industry with the highest wage. The agent must also decide how to allocate his income and any wealth to his own material consumption and his child's future welfare. The child is educated and/or given an inheritance if the agent values the increase in the child's utility more than his own forgone consumption. In the case of education there are two costs: the direct cost of education, i.e., a quantity e of good s, and the indirect cost in form of lost wage income from the child's labor. Since there is no disutility from working, a child who is not being educated will work in the labor-intensive industry.

To set up the agent's maximization problem we need the following notation. Normalize the price of good ℓ to unity in every period, and let p_t be the price of good s in period t. In the same period, the wages in these two industries are denoted w_t^ℓ and w_t^s. The inheritance of an agent i born at time t from his parent is given by h_t^i, which he will gain access to in period t+1. That same agent is educated if $x_t^i = 1$, but he will remain uneducated if his parent sets this choice variable equal to zero. The objective function is obtained by successively substituting future generations' utility functions into <2.1>. We start by replacing the child's utility u_{t+1}^i by its utility function as a grown-up, which then allows us to do the same with the grandchild's utility u_{t+2}^i and so on;

$$\text{<2.4>} \quad \underset{\left\{c_t^{\ell i},\, c_t^{si},\, h_t^i,\, x_t^i\right\}_{t=1}^{\infty}}{\text{Max}} \quad \sum_{t=1}^{\infty} \beta^{t-1} \left[\log c_t^{\ell i} + \alpha \log c_t^{si} \right] ,$$

subject to

$$c_t^{\ell i} + p_t\, c_t^{si} + h_t^i + x_t^i\, p_t\, e$$

$$\leq x_{t-1}^i \left[\max\left\{ w_t^{\ell},\, w_t^{s} \right\} \right] + \left[1 - x_{t-1}^i \right] w_t^{\ell} + \left[1 - x_t^i \right] w_t^{\ell} f + h_{t-1}^i ,$$

$$c_t^{\ell i},\, c_t^{si},\, h_t^i \geq 0 , \qquad x_t^i \in \{0, 1\} , \qquad \forall\ t \geq 1 ,$$

$$\text{given } h_0^i,\, x_0^i,\, \left\{ p_t,\, w_t^{\ell},\, w_t^{s} \right\}_{t=1}^{\infty} .$$

The resulting maximization problem is time consistent, in the sense that each agent will choose an allocation which all ancestors would have agreed upon as optimal for that individual. This follows from the assumption that agents evaluate future consequences of any actions on the basis of future generations' preferences, and any actions in a period t only affect the state in that period and possibly future periods. (For example, the parent's inheritance decision influences the child's wealth as an adult.) Because of the nonnegativity restriction on inheritance, i.e., $h_t \geq 0$, the budget constraints cannot be collapsed into a single present-value constraint.

The logarithmic utility function gives rise to a constant relation over time between expenditures on the two consumption goods,

$$\langle 2.5\rangle \quad p_t\, c_t^{si} = \alpha\, c_t^{\ell i} .$$

From the first-order conditions for a maximum, it can also be seen that an optimal, intertemporal consumption allocation must satisfy

$$\langle 2.6\rangle \quad c_{t+1}^{\ell i} \geq \beta\, c_t^{\ell i} .$$

The assumption that agents behave competitively implies that a labor unit is paid the value of its marginal product in respective industries,

$$\langle 2.7\rangle \quad w_t^{\ell} = 1 \quad , \qquad \text{and} \qquad w_t^{s} = p_t\, \gamma .$$

Since the technologies exhibit constant returns to scale, another implication of perfect competition is that profits are zero and therefore the ownership of these industries has no bearing on the economy.

3. EQUILIBRIUM

Let N_t^e be the educated fraction of an adult generation in period t. These adults received their education as youths in period t–1, i.e., $N_t^e \equiv \int_0^1 x_{t-1}^i\, di$. The corresponding uneducated fraction is denoted $N_t^\eta \left[= 1-N_t^e\right]$. In any period t, there are two goods markets and two labor markets. The market equilibrium conditions are

$$\langle 3.1\rangle \quad \int_0^1 \left[c_t^{\ell i} + h_t^i - h_{t-1}^i\right] di = q^{\ell}\left[L_t^{\ell}\right] ,$$

$$\langle 3.2\rangle \quad \int_0^1 c_t^{si}\, di + e\, N_{t+1}^{e} = q^{s}\left[L_t^{s}\right] ,$$

$$\langle 3.3\rangle \quad L_t^{\ell} = N_t^{\eta} + f\, N_{t+1}^{\eta} + \phi_t\, N_t^{e} ,$$

$$\langle 3.4\rangle \quad L_t^{s} = \left[1 - \phi_t\right] N_t^{e} .$$

The market equilibrium condition <3.1> for good ℓ reflects the fact that the good is used for both consumption and storage, while condition <3.2> displays the consumption and education demand for good *s*. ϕ_t in <3.3> and <3.4> is the fraction of educated workers that are employed in the production of good ℓ at time t. An equilibrium for this economy can be defined as:

Definition 1:[5] Given initial conditions $h_0^i \geq 0$, $x_0^i \in \{0, 1\}$ for $i \in [0, 1]$ and $\int_0^1 x_0^i\, di \equiv N_1^{e} > 0$, a perfect foresight equilibrium is a set $\left\{ c_t^{\ell i}, c_t^{si}, h_t^i, x_t^i, \text{ for } i \in [0, 1] ;\ p_t, w_t^{\ell}, w_t^{s} \right\}_{t=1}^{\infty}$ satisfying

a. given prices, the private agents maximize utility by solving the optimization problem in <2.4>,

b. given prices, profits in the two industries are maximized, which implies <2.7>,

c. market-clearing conditions <3.1> – <3.4> hold.

The relative price of good *s* can be expressed in terms of other variables. After substituting <2.5> into <3.2>, we substitute the resulting expression into <3.1> along with <2.2>, <2.3>, <3.3> and <3.4>. This allows us to solve for p_t,

$$\text{<3.5>} \qquad p_t = \alpha \frac{N_t^{\eta} + f\, N_{t+1}^{\eta} + \phi_t\, N_t^{e} - \int_0^1 \left[h_t^i - h_{t-1}^i\right] di}{\gamma\left[1 - \phi_t\right] N_t^{e} - e\, N_{t+1}^{e}} .$$

Besides the composition and employment of the current labor force, <3.5> shows us that the equilibrium value of p_t depends on the older agents' decisions to affect the younger agents' future welfare by providing education and inheritance. With an iterative argument it follows that p_t depends on the infinite future. However, for our purposes it will be sufficient to study steady states.

4. STEADY STATE

A steady state is defined to consist of initial conditions and a corresponding equilibrium, in which agents facing stationary prices find it optimal to make decisions identical to those of their ancestors. A formal definition is:

<u>Definition 2:</u> A <u>steady state</u> is a set $\left\{c_t^{\ell i} = c^{\ell i},\ c_t^{si} = c^{si},\ h_0^i = h_t^i = h^i,\ x_0^i = x_t^i = x^i, \text{ for } i \in [0,\ 1];\ p_t = p,\ w_t^{\ell} = w^{\ell},\ w_t^{s} = w^{s}\right\}_{t=1}^{\infty}$, where $\int_0^1 x_0^i\, di \equiv N_1^{e} > 0$, satisfying the conditions for a perfect foresight equilibrium.

From agents' maximization behavior it is possible to further describe the stationary allocation. According to the following proposition, the economy can be thought of as being inhabited by two representative families: one educated, indexed by e, and one uneducated, indexed by η.

Proposition 1: In any steady state, bequests are zero and there are two groups of identical families;

a. if $x^i = 0$, then $c^{\ell i} = c^{\ell \eta}$, and $c^{si} = c^{s\eta}$,

b. if $x^i = 1$, then $c^{\ell i} = c^{\ell e}$, and $c^{si} = c^{se}$,

where $c^{\nu e} > c^{\nu \eta}$ for $\nu \in \{\ell, s\}$.

Proof: Bequests must clearly be zero in a steady state. Because a nonzero bequest implies that <2.6> holds at equality, which contradicts constant consumption. Knowing that $h^i = 0$ for all $i \in [0, 1]$ and that there is no mechanism other than education for intergenerational transfers, it follows immediately that families with the same educational status will end up with the same consumption allocation. A necessary condition for a steady state with educated workers is also that educated families have a higher utility level than uneducated families. If this were not true, an educated person would not choose to educate his child, since he could thus avoid bearing the cost of education without lowering the utility level of any descendants. By <2.5> the higher utility level of educated families implies a higher consumption of both goods.

Q.E.D.

Given zero bequests in a steady state, we can find the consumption allocation that solves optimization problem <2.4> for each representative agent,

$$c^{\ell j} = \frac{1}{1+\alpha} I^j , \qquad c^{sj} = \frac{\alpha}{1+\alpha} \frac{I^j}{p} , \qquad \text{for } j \in \{\eta, e\} ,$$

<4.1>

$$\text{where} \quad I^\eta = 1 + f , \qquad I^e = (\gamma - e)\, p .$$

So the representative agents' indirect utility functions are

$$\text{<4.2>} \quad V^\eta\left[p\right] = \frac{1}{1-\beta} \left\{(1+\alpha) \log\left[\frac{1+f}{1+\alpha}\right] + \alpha \log \alpha - \alpha \log p\right\} ,$$

$$\text{<4.3>} \quad V^e\left[p\right] = \frac{1}{1-\beta} \left\{(1+\alpha) \log\left[\frac{\gamma - e}{1+\alpha}\right] + \alpha \log \alpha + \log p\right\} .$$

According to Proposition 1, educated families have a higher consumption level than uneducated families. This can only be true if the wage is higher in the skill-intensive industry and all the educated workers are employed in that industry. Therefore, the steady-state version of <3.5> becomes

$$\text{<4.4>} \qquad p = \alpha \frac{(1+f)\left[1-N^e\right]}{(\gamma\text{-}e)\; N^e} \equiv p\left[N^e\right] .$$

The function $p\left[N^e\right]$ allows us to determine how a representative agent's welfare varies with the composition of the labor force. It can be seen that the utility of an uneducated agent increases with the ratio of educated individuals, while the opposite is true for an educated agent,

$$\frac{\delta\ V^{\eta}\left[p\left[N^{e}\right]\right]}{\delta\ N^{e}} = \frac{\alpha}{(1-\beta)\ \left[N^{e} - \left[N^{e}\right]^{2}\right]} > 0\ ,$$

$$\frac{\delta\ V^{e}\left[p\left[N^{e}\right]\right]}{\delta\ N^{e}} = -\frac{1}{(1-\beta)\ \left[N^{e} - \left[N^{e}\right]^{2}\right]} < 0\ .$$

The reason is that a shift of the labor force away from one industry reduces that industry's production, which in our model leads to an increase in both the relative price of the good and the relative wage in that industry.

In light of Proposition 1, there is then an upper bound on N^{e} which is a necessary condition for a steady state,

$$\text{<4.5>} \qquad V^{e}\left[p\left[N^{e}\right]\right] > V^{\eta}\left[p\left[N^{e}\right]\right] \quad \Leftrightarrow \quad N^{e} < \frac{\alpha}{1+\alpha}\ .$$

A corresponding sufficient condition is proved to exist in the following theorem, which therefore establishes the existence of a continuum of steady states.

Theorem 1: There exists a $\bar{N}^{e} > 0$ such that any $N^{e} \in \left[0, \bar{N}^{e}\right]$ is a steady state.

The proof is relegated to Appendix A, but it may be helpful for understanding the model to summarize how the proof is constructed. As a first step we show that a family will not find it optimal to change its educational status more than once when facing stationary prices. Thereafter we establish the existence of a $\left[\bar{N}^{e}\right]_{\eta}$

such that for all $N^e < \left[\bar{N}^e\right]_\eta$ the uneducated agents would not like to change their educational status. The intuitive interpretation is as follows. At these relatively high ratios of uneducated workers, the relative price of the labor-intensive good is low and, hence, the educational cost is substantial compared to the income of an uneducated worker. It turns out that the loss of utility from forgone consumption while saving for educational expenditures and the discounting of the future outweigh the higher welfare of the educated descendants. For the educated agents there is a similar upper bound $\left[\bar{N}^e\right]_e$ such that for all $N^e < \left[\bar{N}^e\right]_e$ it will not be optimal for them to change their educational status. At these compositions of the labor force, there is a significant relative wage difference favoring the educated workers. An educated agent will therefore willingly bear the educational cost in order to grant his child the same standard of living. Finally, the theorem is obviously true for $\bar{N}^e = \min\left\{\left[\bar{N}^e\right]_\eta, \left[\bar{N}^e\right]_e\right\}$.

In section 6 it is shown how different steady states can be ranked according to a Pareto criterion, and the following proposition compares these equilibria with respect to some common economic measurements.

Proposition 2: When comparing two steady states, the equilibrium with a higher ratio of educated workers has a larger gross national product in terms of good s and a smaller relative wage difference.

Proof: It is obvious that the production of good s increases with a higher ratio of educated workers, and by using <4.4> we can verify that the value of industry ℓ's production in terms of good s also increases,

$$\frac{\delta \dfrac{q^{\ell}\left[(1+f)\left[1-N^{e}\right]\right]}{p\left[N^{e}\right]}}{\delta \quad N^{e}} = \frac{\gamma - e}{\alpha} > 0 \ .$$

Similarly, it can be shown that the relative wage of the lower-paid uneducated workers increases with N^e,

$$\frac{\delta \dfrac{w^{\ell}}{w^{s}}}{\delta \ N^{e}} = \frac{\delta \dfrac{1}{p\left[N^{e}\right] \ \gamma}}{\delta \ N^{e}} = \frac{\gamma - e}{\alpha \ \gamma \ (1+f)\left[1-N^{e}\right]^{2}} > 0 \ .$$

Q.E.D.

5. A NUMERICAL EXAMPLE OF STEADY STATES

The model is fully parameterized by five parameters. Two of them describe the agents' preferences and the other three pertain to the production technologies. Their values in this numerical example are

$$\alpha = 1 \ , \quad \beta = .55 \ , \quad \gamma = 1 \ , \quad e = .1 \ , \quad f = .5 \ .$$

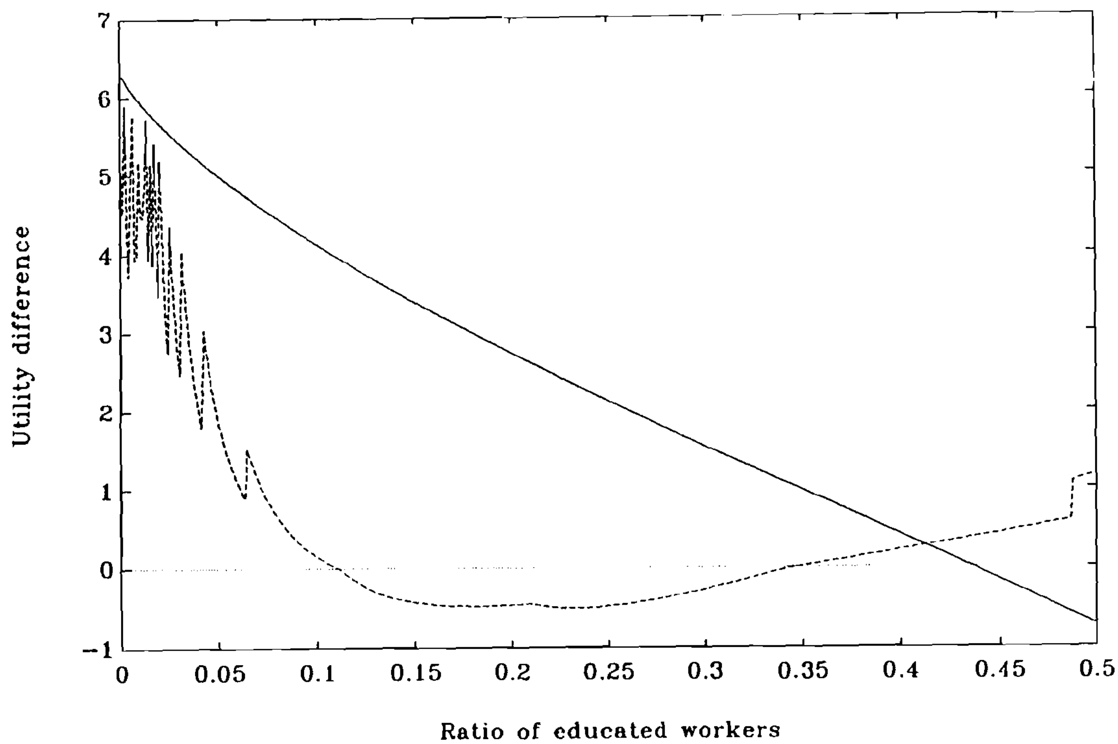
With $\alpha = 1$ the two types of goods have the same weight in the utility function. The discount factor $\beta = .55$ is comparable with a 3% annual interest rate during a 20-year period. By normalization the input-output coefficient is one in industry ℓ and $\gamma = 1$ assigns the same one to industry s. The educational cost of a child e =.1 will then constitute 10% of an educated worker's income. The productivity of a child f =.5 makes a child half as productive as an adult in the labor-intensive industry.

The set of steady states is computed by searching over N^e. According to <4.5>, the search can be confined to $N^e \in \left[0, \alpha/(1+\alpha)\right]$. For each picked value of N^e, we compute the difference in utility between retaining and changing the family's educational status.[6] This difference must clearly be nonnegative for both representative agents in order for N^e to constitute a steady state. The utility differences for the given parameterization are reported in Table 1 in Appendix B and in Figure 1. The figure displays two disjoint sets of possible steady states.[7] The subset with low ratios of educated workers contains the set of equilibria established in Theorem 1. As we pointed out above, these compositions of the labor force imply that the educational expenditures are high relative to the income of an uneducated worker. An uneducated agent will therefore deem the sacrifice associated with a change in the family's educational status to be too high, in spite of a significant increase in any educated descendants' well-being. But for the subset of steady states with higher ratios of educated workers, the explanation for an uneducated family's decision not to change its educational status is different. In such a steady state, the cost of education is low relative to the income of an uneducated worker, but so is the reward from education. On the other hand, if the ratio of educated workers is between the two sets of steady states, an uneducated agent would like to change his family's educational status at a stationary price given by <4.4>.

The two subsets of steady states have obviously very different welfare implications. The lower end of N^e is characterized by a considerable difference in

Figure 1: Difference in utility between retaining and changing the family's educational status. The differences for both representative agents must be positive in order for N^e to constitute a steady state (see footnotes 6, 7).

Educated agent	=	solid line
Uneducated agent	=	broken line



the standard of living of workers employed in the two different sectors. This is reflected in the relation between an uneducated and an educated family's income, (1+f) / $p\gamma$, which can be found in Table 2 in Appendix B. Another reported measure of the highly unequal income distribution is the Gini coefficient. In the same table, it can be seen that the other prediction of Proposition 2 becomes true, i.e., the value of the production of goods expressed in terms of good *s* increases with the ratio of educated workers. The growing gross national product associated with a more educated work force can be thought of as economic development. But instead of being driven by technological change, the development is due to a change in the economy's production structure. In the next section, we examine the existence of Pareto improvements when the economy is in a steady state.

6. PARETO IMPROVEMENTS

Let the social welfare function at time 1 be

$$\langle 6.1\rangle \qquad \Gamma\left[\left\{\left\{c_t^{\ell i}, c_t^{si}\right\}_{i\in[0,1]}\right\}_{t=1}^{\infty}\right]$$

$$= \int_0^1 \left[\omega^i \sum_{t=1}^{\infty} \beta^{t-1}\left[\log c_t^{\ell i} + \alpha \log c_t^{si}\right]\right] di \ ,$$

where $\omega^i > 0$ is the weight attached to individual i's utility. These weights can, without loss of generality, be normalized so that

$$\int_0^1 \omega^i \, di = 1 \ .$$

When allowing for lump-sum transfers, the maximization of <6.1> subject to feasibility conditions implies that family i receives a fraction ω^i of the aggregate consumption of the two goods in each period,

$$\langle 6.2\rangle \qquad c_t^{\nu i} = \omega^i \int_0^1 c_t^{\nu j}\, dj \ , \qquad \text{for } \nu \in \{\ell, s\} \ , \quad \forall\, i \in [0,1] \ , \quad \forall\, t \geq 1 \ .$$

To simplify the notation we define a few aggregate variables,

$$C_t^{\nu} \equiv \int_0^1 c_t^{\nu j}\, dj \ , \qquad \text{for } \nu \in \{\ell, s\} \ ,$$

$$C_1 \equiv \left\{C_t^{\ell}, C_t^{s}\right\}_{t=1}^{\infty} \ .$$

When substituting <6.2> into <6.1>, the result is

$$\langle 6.3\rangle \qquad \Gamma\left[\left\{\left\{c_t^{\ell i}, c_t^{si}\right\}_{i\in[0,1]}\right\}_{t=1}^{\infty}\right]$$

$$= \int_0^1 \frac{(1+\alpha)\ \omega^i \ \log\ \omega^i}{1-\beta}\, di \ + \ \sum_{t=1}^{\infty} \beta^{t-1}\left[\log C_t^{\ell} + \alpha \log C_t^{s}\right] \ .$$

This expression tells us that it is possible to rank different sequences of aggregate consumption in terms of potential social welfare without knowing the actual weights attached to each family's utility. The separation of the social planner's decisions with respect to production and distribution depends obviously on all agents having identical preferences and the marginal rate of substitution in consumption being only a function of the ratio between consumed quantities. It is therefore possible to study the existence of Pareto improvements by examining the criterion

$$<6.4> \quad W\left[C_1\right] \equiv \sum_{t=1}^{\infty} \beta^{t-1}\left[\log C_t^{\ell} + \alpha \log C_t^{s}\right] .$$

Definition 3: An allocation $\hat{C}_1$ is said to Pareto dominate C_1 if

$$W\left[\hat{C}_1\right] > W\left[C_1\right] .$$

The following theorem establishes the existence of Pareto improvements to all stationary allocations except for one.

Theorem 2 (Existence of Pareto improvements):

Given $N^e \in (0, 1]$ and a feasible, stationary allocation C_1, where

$$C_t^{\ell} = (1+f)\left[1 - N^e\right] \equiv C^{\ell} , \qquad \forall\ t \geq 1 ,$$

$$C_t^{s} = (\gamma - e)\, N^e \equiv C^{s} , \qquad \forall\ t \geq 1 ,$$

there exists another feasible allocation which Pareto dominates C_1

$$\text{if and only if} \qquad N^e \neq \frac{\alpha}{\alpha + \dfrac{(\gamma - e)(f+\beta)}{(\beta\gamma - e)(1+f)}} \equiv \hat{N}^e .$$

Proof: We will first show that $\hat{N}^e$ with its stationary consumption allocation is a

Pareto optimal allocation. A Pareto optimum requires that individuals who are being educated must be employed in the production of the skill-intensive good, since it would be wasteful to educate workers for the labor-intensive industry. Suppose also that good ℓ is not stored between periods, which must obviously be true for any stationary Pareto optimal allocation. The substitution of feasibility conditions into <6.4> will then give rise to a concave objective function in the employment of the initially given labor force of grown-ups and the remaining choice variable N_t^e (for t>1). Conjecturing an interior solution for N_t^e, a first-order condition for an optimum with respect to that variable is

$$-\beta^{t-2}\left(\frac{f}{1-N_{t-1}^e+f\left[1-N_t^e\right]}+\frac{\alpha\ e}{\gamma\ N_{t-1}^e-e\ N_t^e}\right)$$

$$+\beta^{t-1}\left(\frac{-1}{1-N_t^e+f\left[1-N_{t+1}^e\right]}+\frac{\alpha\ \gamma}{\gamma\ N_t^e-e\ N_{t+1}^e}\right)=0\ .$$

After replacing the ratio of educated workers in all periods by a constant fraction N^e, we can verify that N^e is equal to $\hat{N}^e$ as defined in the theorem. $\hat{N}^e$ is permissible, i.e., $0<\hat{N}^e<1$, because of our assumption that $0<e<\beta\,\gamma$. This concludes the proof that $\hat{N}^e$ with its stationary consumption allocation is a Pareto optimal allocation.

It remains to be shown that any other stationary ratio of educated workers can be improved upon. Let $\tilde{C}_1$ be an alternative consumption allocation associated with permanently changing the educated labor force in the first period by a permissible $\Delta N^e \in \{\, y \in \mathbb{R} : -N^e < y < 1-N^e,$ and if $y > 0$ then $(\gamma-e)N^e > ey \,\}$;

$$\tilde{C}_1^{\ell} = (1+f)\left[1-N^e\right] - f\,\Delta N^e ,$$

$$\tilde{C}_1^{s} = (\gamma - e)\,N^e - e\,\Delta N^e ,$$

$$\tilde{C}_t^{\ell} = (1+f)\left[1-N^e-\Delta N^e\right] \equiv \tilde{C}^{\ell} , \qquad \forall\ t \geq 2 ,$$

$$\tilde{C}_t^{s} = (\gamma - e)\left[N^e+\Delta N^e\right] \equiv \tilde{C}^{s} , \qquad \forall\ t \geq 2 .$$

The change in the educated labor force will constitute a Pareto improvement if $W\left[\tilde{C}_1\right] - W\left[C_1\right]$ is positive. This difference is zero for $\Delta N^e = 0$ and can be shown to be strictly concave in that variable. It is therefore both sufficient and necessary to examine the impact of an infinitesimal ΔN^e in order to determine whether there exists a Pareto superior $\tilde{C}_1$,

$$\frac{\delta}{\delta\ \Delta N^e}\left[W\left[\tilde{C}_1\right] - W\left[C_1\right]\right]\Bigg|_{\Delta N^e=0}$$

$$= \frac{1}{1-\beta}\left(\frac{\alpha\ (\beta\gamma - e)}{(\gamma - e)\ N^e} - \frac{f+\beta}{(1+f)\left[1-N^e\right]}\right)\begin{cases} >0\,, & \text{for } N^e \in \left[0\,,\ \hat{N}^e\right] \\ =0\,, & \text{for } N^e = \hat{N}^e \\ <0\,, & \text{for } N^e \in \left[\hat{N}^e,\ 1\right] \end{cases} ,$$

where we have made use of the assumption that $e < \beta\,\gamma$. This result establishes the existence of a Pareto dominating allocation to any stationary ratio of educated workers other than $\hat{N}^e$. In particular, welfare is improving with the closeness of the ratio of educated workers to $\hat{N}^e$.

Q.E.D.

In the next section, it will be shown that the Pareto optimal steady state is the only possible one under a particular student loan program.

7. STUDENT LOANS

Suppose that a student loan program can overcome the assumed imperfection in the market for human capital.[8] In particular, a family can borrow funds to cover the direct cost of education and the indirect cost in form of lost labor income while studying:

$$\text{<7.1>} \qquad \text{student loan at time t} \;\leq\; p_t\, e \;+\; w_t^{\ell} f \;.$$

The loans are financed through the issue of student bonds and there is no cost associated with the intermediation. The gross rate of return on these bonds in terms of good ℓ between period t and t+1 is R_{t+1}, which is market-determined and fully paid by the students. By also assuming that all contracts are enforceable, it follows that the budget constraint of the institution offering student loans will be satisfied in every period. In addition, with zero cost of intermediation there is no loss of generality to proceed from hereon as if all education is financed with student loans, i.e., <7.1> holds with equality.

Let b_t^i denote family i's holdings of student bonds between period t and t+1. The substitution of <7.1> at equality into the agents' maximization problem allows us to cancel several terms in the budget constraint. The optimization problem becomes

$$\text{<7.2>} \quad \underset{\left\{c_t^{\ell i},\, c_t^{si},\, h_t^i,\, x_t^i,\, b_t^i\right\}_{t=1}^{\infty}}{\text{Max}} \quad \sum_{t=1}^{\infty} \beta^{t-1} \left[\log c_t^{\ell i} + \alpha \log c_t^{si}\right] ,$$

subject to

$$c_t^{\ell i} + p_t\, c_t^{si} + h_t^i + b_t^i + x_{t-1}^i\, R_t \left[p_{t-1}\, e + w_{t-1}^{\ell}\, f\right]$$

$$\leq x_{t-1}^i \left[\max\left\{w_t^{\ell},\, w_t^s\right\}\right] + \left[1 - x_{t-1}^i\right] w_t^{\ell} + w_t^{\ell}\, f + h_{t-1}^i + R_t\, b_{t-1}^i ,$$

$$c_t^{\ell i},\ c_t^{si},\ h_t^i,\ b_t^i \geq 0 , \qquad x_t^i \in \{0, 1\} , \qquad \forall\ t \geq 1 ,$$

$$\text{given } h_0^i,\ x_0^i,\ b_0^i,\ \left\{p_t,\ w_t^{\ell},\ w_t^s,\ R_t\right\}_{t=1}^{\infty} .$$

The description of the new economy is completed with an equilibrium condition for the market in student loans and bonds,

$$\text{<7.3>} \quad \int_0^1 b_t^i \, di = N_{t+1}^e \left[p_t\, e + w_t^{\ell}\, f\right] .$$

It can be established that unity is a lower bound on the gross rate of return on student bonds in an equilibrium. The argument is the following. With logarithmic preferences both goods are always desired to be consumed in strictly positive quantities. Since good s is not storable, this implies that education takes place in every period. Therefore, student bonds cannot be dominated by the alternative of storing good ℓ at a gross rate of return of unity. Another constraint on the rate of return on student bonds can be derived from the existence of both educated and uneducated workers in an equilibrium. With student loans it can be seen from <7.2> that the educational decision does not affect the current period's budget constraint, i.e., the parent's budget set. So a parent will choose between educating or not educating his child on the basis of which alternative maximizes the child's net labor income (after repaying student loans) as an adult. For there to be both educated and uneducated workers, it must be true that no investment strategy in human capital dominates the other one,

$$\langle 7.4\rangle \qquad w_t^{\ell} = \max\left\{w_t^{\ell},\ w_t^{s}\right\} - R_t\left[f\, w_{t-1}^{\ell} + p_{t-1}\, e\right] .$$

The production of good s in an equilibrium requires that $w_t^s \geq w_t^{\ell}$, and after substituting <2.7> into the expression above we can solve for R_t,

$$\langle 7.5\rangle \qquad R_t = \frac{p_t\,\gamma - 1}{p_{t-1}\, e + f} .$$

Since student bonds are an additional vehicle for intergenerational transfers with a gross rate of return bounded below by unity, it can be seen from the first-order conditions to maximization problem <7.2> that the previous restriction on an optimal, intertemporal consumption allocation, <2.6>, is replaced by

$$\langle 7.6\rangle \qquad c_{t+1}^{\ell i} \geq R_{t+1}\,\beta\, c_t^{\ell i} .$$

The definitions of a perfect foresight equilibrium with student loans and a steady state with student loans are:

<u>Definition 4:</u> Given initial conditions $h_0^i \geq 0$, $x_0^i \in \{0, 1\}$, $b_0^i \geq 0$ for $i \in [0, 1]$

and $\int_0^1 x_0^i \, di \equiv N_1^e > 0$, a <u>perfect foresight equilibrium with student loans</u> is a set $\left\{c_t^{\ell i},\ c_t^{si},\ h_t^i,\ x_t^i,\ b_t^i,\ \text{for } i \in [0, 1]\ ;\ p_t,\ w_t^{\ell},\ w_t^{s},\ R_t \right\}_{t=1}^{\infty}$ satisfying

a. given prices, the private agents maximize utility by solving the optimization problem in <7.2>,

b. given prices, profits in the two industries are maximized, which implies <2.7>,

c. market-clearing conditions <3.1> – <3.4>, and <7.3> hold.

<u>Definition 5:</u> A <u>steady state with student loans</u> is a set $\left\{c_t^{\ell i} = c^{\ell i},\ c_t^{si} = c^{si},\ h_0^i = h_t^i = h^i,\ x_0^i = x_t^i = x^i,\ b_0^i = b_t^i = b^i,\ \text{for } i \in [0, 1]\ ;\ p_t = p,\ w_t^{\ell} = w^{\ell},\ w_t^{s} = w^{s},\ R_t = R \right\}_{t=1}^{\infty}$, where $\int_0^1 x_0^i \, di \equiv N_1^e > 0$, satisfying the conditions for a perfect-foresight equilibrium with student loans.

According to the following proposition, the welfare distribution in a steady state is independent of the families' educational statuses and depends only on their initial endowments of student bonds. The subsequent theorem proves the existence of a unique steady state. So the existence of student loans causes the multiplicity of steady states to go away.

<u>Proposition 3:</u> In any steady state with student loans, all bequests are in the form of student bonds and a family's welfare is determined by and increasing in its student bond holdings.

<u>Proof:</u> As argued above, education will take place in every period in an equilibrium. The implied positive amount of student bonds guarantees that <7.6> holds with equality. It then follows from constant consumption and $\beta \in (0, 1)$ that R exceeds unity,

$$\text{<7.7>} \qquad R = 1 / \beta .$$

All bequests will therefore be in the form of student bonds, since the storage of good ℓ is dominated in rates of return.

After substituting the constant holdings of student bonds and the equilibrium condition given by <7.4> into the budget constraint in <7.2>, we obtain an expression for an agent's consumption expenditures in a steady state,

$$c^{\ell i} + p\, c^{si} = (1 + f)\, w^{\ell} + (R - 1)\, b^{i} ,$$

where $R > 1$. This shows that a family's welfare is determined by and increasing in its student bond holdings.

Q.E.D.

<u>Theorem 3 (Student loans):</u> There exists a unique steady state with student loans in which N^e is equal to $\hat{N}^e$ as defined in Theorem 2.

<u>Proof:</u> The relative price of good s in a steady state is given by <4.4>, which can be substituted into <7.5> to obtain a function for the gross rate of return on student bonds in terms of N^e. By the proof of Proposition 3, this expression can be set equal to $1/\beta$. It is then found that $\hat{N}^e$ is the only ratio of educated workers that can satisfy the resulting equation. This establishes that a steady state must be unique. Existence is implied by all relationships being derived from agents'

maximization behavior and market clearing conditions.

Q.E.D.

The unique steady state under the proposed student loan program Pareto dominates all other stationary allocations according to Theorem 2. The allocation does actually coincide with a competitive equilibrium for the same economy without the imperfection in the market for human capital. As can be seen from <7.7>, the rate of return on an investment in human capital is equal to the marginal rate of intertemporal substitution. And by equilibrium condition <7.4>, the two investment strategies in human capital have the same present value. To understand how the student loan program works, it is therefore helpful to consider the corresponding investment project. An investor in human capital can be thought of as contacting a parent and paying him an amount equal to the value of a child's labor in industry ℓ. This is done in order to educate the child and then reap the benefits when the child becomes an adult. The investor is entitled to the child's future labor income minus the value of an adult's wage earnings in industry ℓ. This income retained by the "investment object" itself corresponds to an inherent earning capacity independent of any education.

8. CONCLUSIONS

In the introduction we motivated this paper with the observation of what appears to be a dual economy in some underdeveloped countries. Our simple model offers an explanation for the question of why a high percentage of uneducated workers can be a sustained, competitive market equilibrium. This is possible even though the country's citizens have the same preferences and have access to the same production technologies as the rest of the world. The model predicts that such an "unbalanced" economy will display a low gross national product and a large income difference between educated and uneducated workers. The two crucial assumptions for obtaining these results were incomplete credit markets for human capital and indivisibilities in education. The first assumption is based on the notion that human capital, being embodied in people, is less suited to serve as collateral on a loan. It follows that parental income and wealth may exercise an independent, constraining effect on children's education, which we conclude would be binding for a larger segment of the population in an underdeveloped than in a developed country. We also believe that indivisibilities in education are a plausible assumption. The common practice, for whatever reason, is to provide education in "packages" like high school and college degrees.

By focusing on human capital we will hopefully contribute to the understanding of controversies in economic development brought about by theories concerned with real capital accumulation. One argument is, or at least was, that large income inequalities may be necessary in poor countries to ensure high savings and investments. Our model shows that this kind of statement may need to be qualified; a highly unequal income distribution can be the sign of a severe impediment to economic development in the form of missing credit markets for human capital. If this is the case, the uneducated population and the economy as a whole will find itself in a poverty trap, since a correction of underinvestments in human capital can potentially increase the welfare of all living and future agents. Underinvestment in human capital is also an implication in models with a single composite good, perfect markets and positive externalities from human capital. To appreciate the differences in mechanism, we consider why an uneducated agent in an underdeveloped country would like to migrate to a developed country when the labor-intensive good is nontradable. In the latter models the agent would enhance his own productivity by working in close proximity to highly educated individuals, while in our model his productivity would stay constant but he would benefit from a higher relative wage within the developed country owing to a relatively smaller supply of uneducated labor.

In our model, economic development arising from a change in the economy's production structure may result in winners and losers. An economic policy aimed at increasing the ratio of educated workers will affect relative prices and therefore have redistributive effects. In particular, the welfare derived from the labor income of an educated agent will decrease due to a lower relative wage and a higher relative price of goods produced by uneducated workers. To achieve a welfare improvement for everyone, it may therefore be necessary to compensate the agents with an already higher standard of living. Such circumstances will inevitably lead us to seek a better understanding of the political economy and the role of institutions in economic development. Our model stresses also the importance of going beyond aggregate economic data. Per capita income statistics and measures of income distributions should spur our interest in a closer, comparative study of these economies. It may turn out that migration pressure between countries due to restrictions on immigration has its counterpart in "occupational migration" or social mobility within economies because of imperfections in the process of human capital accumulation.

APPENDIX A

PROOF OF THEOREM 1

In a steady state it must be true that no family finds it desirable to change its educational status over time. To establish whether a certain N^e is a steady state, it is therefore helpful to narrow down the set of educational alternatives that a utility-maximizing family would consider when facing stationary prices. This is done in the following lemma before turning to the proof of Theorem 1.

Lemma 1: Given stationary prices satisfying <4.5> (a necessary condition for a steady state) and no initial bequests, a family will not find it optimal to change its educational status more than once. In addition, an optimal change of the educational status will be such that after a well-defined time horizon all future descendants will be indistinguishable from the representative agent of that other educational status.

Proof: The nonoptimality of multiple changes of the educational status within the same family will be established last, since that argument will be better appreciated after having seen the proof to the second part of the lemma. But we will first derive a useful constraint on an optimal, intertemporal consumption allocation when the nonnegativity constraint on inheritance is not binding, let us say for $t \in [t_0, t_1]$. After adding the one-period budget constraints from <2.4> for these periods, we use <2.5> to express all consumption in terms of the labor-intensive good, which in turn can be written as functions of $c_{t_0}^{\ell i}$ through repeated substitution of <2.6> at equality. From the resulting expression we solve for $c^{\ell i}_{t_0}$,

$$\langle A.1\rangle \qquad c^{\ell i}_{t_0} = \frac{(1-\beta)\left[h^{i}_{t_0-1} - h^{i}_{t_1+1} + \sum_{t=t_0}^{t_1+1} I^{i}_{t}\right]}{(1+\alpha)\left[1-\beta^{t_1+2-t_0}\right]} ,$$

where $I^i_t \equiv x^i_{t-1}\, w^s_t + \left[1-x^i_{t-1}\right] w^\ell_t + \left[1-x^i_t\right] w^\ell_t f - x^i_t\, p_t\, e$.

Consider an uneducated agent named i, in a period labelled 1, who finds it optimal to change the educational status of his family once and for all in period T. By <2.6> and <A.1>, the optimal consumption allocation of good ℓ is

$$\langle A.2\rangle \qquad \begin{aligned} c^{\ell i}_t &= \beta^{t-1}\, c^{\ell\eta}_1(T) , && \text{for } 1 \le t \le T , \\ c^{\ell i}_t &= c^{\ell e} , && \text{for } t > T , \end{aligned}$$

$$\text{where} \quad c^{\ell\eta}_1(T) = \frac{(1-\beta)\left[(T\text{-}1)\ (1+f) + 1 - p\ e\right]}{(1+\alpha)\left[1-\beta^{T}\right]} .$$

Here we have made use of the initial condition that $h^i_0 = 0$ and the terminal condition of $h^i_T = 0$. This latter condition of no inheritance to the educated child must obviously be true since $c^{\ell i}_T < c^{\ell\eta} < c^{\ell e} = c^{\ell i}_{T+1}$.

Lemma 1.1: $c^{\ell\eta}_1(T)$ in <A.2> is strictly increasing in T and unbounded.

Proof: $c^{\ell\eta}_1(T)$ is defined over $T \in \mathbb{Z}_+$, but the proof of the function being strictly increasing in T is simplified by studying its derivative for $T \in \mathbb{R}_+$. A sufficient condition for this derivative to be strictly positive is

$$\frac{1}{\beta^T} - 1 + T \log \beta > 0 \quad .$$

The left-hand side of this expression is zero at the not permissible value of T=0, and its derivative is strictly positive. We therefore conclude that $c_1^{\ell\eta}(T)$ is strictly increasing in T.

The function $c_1^{\ell\eta}(T)$ is clearly unbounded, since the numerator goes to infinity while the denominator converges to a constant.

Q.E.D.

Feasibility of the allocation in <A.2> requires that the accumulated income up to T is at least equal to the educational cost, which imposes a lower bound on T;

<A.3> $\underline{T}^{\eta}$ is the smallest T satisfying $(T-1)(1+f) + 1 - p e \geq 0$.

Another feasibility condition is that the value of period one's consumption does not exceed that period's income, since negative inheritance is not allowed. In light of Lemma 1.1 and <2.5>, this gives rise to an upper bound on T;

<A.4> $\bar{T}^{\eta}$ is the largest T satisfying $(1+\alpha)\, c_1^{\ell\eta}(T) \leq 1 + f$.

We now turn to an educated agent named i, who finds it optimal for his family to become uneducated from a period labeled 1 and thereon. This agent must decide upon an optimal time horizon T for wealth deaccumulation. In a manner similar to <A.2>, a consumption allocation for good ℓ can be found,

$$c_t^{\ell i} = \beta^{t-1} c_1^{\ell e}(T) \ , \qquad \text{for } 1 \leq t \leq T \ ,$$

<A.5>
$$c_t^{\ell i} = c^{\ell \eta} \ , \qquad \text{for } t > T \ ,$$

$$\text{where} \quad c_1^{\ell e}(T) = \frac{(1-\beta)\left[p\,\gamma + f + (T-1)\,(1+f)\right]}{(1+\alpha)\left[1 - \beta^T\right]} \ .$$

Lemma 1.2: $\beta^{T-1} c_1^{\ell e}(T)$ is strictly decreasing in T and its limit is zero, where $c_1^{\ell e}(T)$ is defined in <A.5>.

Proof: As in the proof of Lemma 1.1, the variable T will be extended from $\mathbb{Z}_+$ to $\mathbb{R}_+$ when showing that $\beta^{T-1} c_1^{\ell e}(T)$ is decreasing in T. A sufficient condition for the derivative of this function to be strictly negative is

$$1 - \beta^T + T \log \beta < 0 \ .$$

The left-hand side of this expression is zero at the not permissible value of T=0, and its derivative is strictly negative. We therefore conclude that $\beta^{T-1} c_1^{\ell e}(T)$ is strictly decreasing in T.

The limiting value of $\beta^{T-1} c_1^{\ell e}(T)$ is zero, since β^{T-1} declines geometrically while the other factor grows linearly in the limit.

Q.E.D.

Lemma 1.3: The allocation in <A.5> that is both feasible and maximizes utility is given by a unique cutoff point for inheritance $T = T^e$, which is the only T satisfying

<A.6> $$c^{\ell\eta} < \beta^{T-1} c_1^{\ell e}(T) \leq c^{\ell\eta} / \beta .$$

Proof: We will first establish the existence of a unique time horizon satisfying <A.6>. By Lemma 1.2 there is a unique $T^e \geq 1$ such that

<A.7> $$\beta^{T-1} c_1^{\ell e}(T) \leq c^{\ell\eta} / \beta \qquad \text{if and only if} \qquad T \geq T^e .$$

To show that T^e also satisfies the strict inequality in <A.6>, we use <4.1> to rewrite this condition as

<A.8> $$\beta^{T^e-1} c_1^{\ell e}(T^e) > \frac{1 + f}{1 + \alpha} .$$

Given <4.5>, this can be seen to hold for $T^e = 1$. In the case of $T^e > 1$, suppose, to obtain a contradiction, that <A.8> does not hold. Together with <A.5>, this allows us to find an upper bound for the accumulated income between period 1 and T^e,

<A.9> $$\sum_{t=1}^{T^e} I_t^i = \frac{1 - \beta^{T^e}}{1 - \beta} (1+\alpha) c_1^{\ell e}(T^e) \leq \frac{1 - \beta^{T^e}}{1 - \beta} \frac{1 + f}{\beta^{T^e-1}} .$$

And according to <A.7>,

$$\beta^{T^e-2} c_1^{\ell e}(T^e-1) > c^{\ell\eta} / \beta = \frac{1 + f}{(1+\alpha) \beta} ,$$

which will similarly give rise to a lower bound for the accumulated income between period 1 and T^e-1,

<A.10> $$\sum_{t=1}^{T^e-1} I_t^i = \frac{1-\beta^{T^e-1}}{1-\beta}(1+\alpha)\, c_1^{\ell e}(T^e-1) > \frac{1-\beta^{T^e-1}}{1-\beta}\;\frac{1+f}{\beta^{T^e-1}} .$$

The difference between the two accumulated incomes is trivially 1+f, but from <A.9> and <A.10> we obtain

$$\sum_{t=1}^{T^e} I_t^i - \sum_{t=1}^{T^e-1} I_t^i < \left[\left[1-\beta^{T^e}\right]-\left[1-\beta^{T^e-1}\right]\right]\frac{1+f}{(1-\beta)\;\beta^{T^e-1}} = 1+f .$$

This contradiction establishes that T^e satisfies <A.8> and, by <A.7>; it therefore also satisfies <A.6>.

To prove that T^e is the only T satisfying <A.6>, it is sufficient by Lemma 1.2 to show that no two consecutive time horizons, T and T+1, can simultaneously satisfy <A.6>. To get a contradiction, suppose that they do. These constraints can then be used as before to find lower and upper bounds for accumulated incomes,

$$\sum_{t=1}^{T+1} I_t^i - \sum_{t=1}^{T} I_t^i > \frac{1+f}{(1-\beta)\,\beta^T}\left[\left[1-\beta^{T+1}\right]-\left[1-\beta^{T}\right]\right] = 1+f .$$

The inequality constitutes a contradiction, since the true difference in accumulated incomes is once again 1+f.

The allocation in <A.5> with $T=T^e$ is feasible because of the strict inequality in <A.6>, which ensures that there is no negative inheritance. By <2.6> it can also be seen that the intertemporal allocation maximizes utility given the educational decision. Since T^e is the only T satisfying <A.6>, it follows that any other cutoff point for inheritance would violate <2.6> and therefore be nonoptimal. This concludes the proof of Lemma 1.3.

Q.E.D.

It remains to be shown that multiple changes of the educational status are not optimal. An implication of <4.5> is that $p(w^s - e) > (1+f)w^\ell$, which means that being educated will increase the opportunity set of an agent more than just inheriting the combined value of a child's labor and the direct educational cost. It follows from utility maximization that the inheritance of an uneducated child must be strictly smaller than this value. In other words, a parent who educates his child will sacrifice more of his own consumption than any parent who leaves his child uneducated. Let us first consider a family which started out uneducated and has accumulated funds for educating a child in a particular period. The parent in that period will clearly consume less than the representative, uneducated agent owing to the restriction on an optimal, intertemporal consumption allocation given by <2.6>. It follows that this parent's resources will fall below the labor income of an educated agent. It would therefore be contradictory, if this parent with a relatively high marginal utility for own consumption will divert resources to expand the opportunity set of his child, while a future educated descendant with a lower marginal utility for own consumption will deem that sacrifice too high and leave his child uneducated. A similar contradiction with respect to a multiple change of the educational status is obtained for an initially educated family. That family will command the most resources when being educated. So if it is not optimal to educate the child during that period, it will certainly not be optimal to do that with fewer resources. We conclude that an agent will not find it optimal to change his family's educational status more than once. This result together with $\left\{\underline{T}^\eta, \bar{T}^\eta\right\}$ and T^e from <A.3>, <A.4> and Lemma 1.3 establish Lemma 1.

Q.E.D.

<u>Proof of Theorem 1</u>

The proof is divided into two parts. In Part 1 it will be shown that there will always exist a $\left[\bar{N}^e\right]_\eta$ such that for all $N^e < \left[\bar{N}^e\right]_\eta$, it will not be optimal for uneducated agents to change their educational status. Part 2 establishes a similar upper bound $\left[\bar{N}^e\right]_e$ for educated agents such that for all for $N^e < \left[\bar{N}^e\right]_e$, they would not like to change their educational status. Theorem 1 will then obviously be true for $\bar{N}^e = \min\left\{\left[\bar{N}^e\right]_\eta, \left[\bar{N}^e\right]_e\right\}$. Because of Lemma 1, we need only consider once-and-for-all changes in a family's educational status. <4.4> defining p as a strictly decreasing function of N^e permits us also to cast the proof in terms of the relative price p.

Part 1: Existence of an upper bound $\left[\bar{N}^e\right]_\eta$.

Suppose an uneducated, adult agent named i living in a period labeled 1 faces a price p and starts to accumulate savings in order to change his family's educational status in period $\hat{T}(p)$. An optimal savings plan gives rise to a consumption allocation for the transition period given by <A.2>, $\left\{c_t^{\ell i}, c_t^{si}\right\}_{t=1}^{\hat{T}(p)}$. We will now establish the existence of a $\bar{p}$ such that this educational decision is not desirable for any $p > \bar{p}$, i.e.,

$$\text{<A.11>} \quad \sum_{t=1}^{\hat{T}(p)} \beta^{t-1}\left[\log\frac{c_t^{\ell i}}{c^{\ell\eta}} + \alpha\log\frac{c_t^{si}}{c^{s\eta}}\right] + \beta^{\hat{T}(p)}\left[V^e(p) - V^\eta(p)\right] < 0\,,$$

$$\forall\, p > \bar{p}\,.$$

The method to prove this will be to show that the utility loss during the transition period given by the first term in <A.11> has a nonzero lower bound for $p > \bar{p}$, while the second term displaying the utility gain from education goes to zero in the limit when p goes to infinity.

By choosing $p > 1/e$, it follows that the uneducated agent i in period 1 will not even be able to pay the educational cost out of his income and therefore we have $\hat{T}(p) \geq 2$. This allows us to compute a bound for the first term in <A.11>,

$$\langle A.12\rangle \quad \sum_{t=1}^{\hat{T}(p)} \beta^{t-1} \left[\log \frac{c_t^{\ell i}}{c^{\ell \eta}} + \alpha \log \frac{c_t^{si}}{c^{s\eta}}\right] = \sum_{t=1}^{\hat{T}(p)} \beta^{t-1} (1+\alpha)(t-1) \log \beta$$

$$+ \frac{1 - \beta^{\hat{T}(p)}}{1 - \beta} (1+\alpha) \log \frac{c_1^{\ell i}}{c^{\ell \eta}} < \beta (1+\alpha) \log \beta .$$

In deriving the limit of the second term in <A.11>, we define a function

$$\tilde{T}(p) \equiv \frac{p\ e\ + f}{1 + f} \qquad \Longrightarrow \qquad \tilde{T}(p) \leq \hat{T}(p) ,$$

since any time horizon shorter than $\tilde{T}(p)$ would imply an accumulated income less than the educational cost. Using this fact and the indirect utility functions in <4.2> and <4.3>, we can find an upper bound for the second term in <A.11>,

$$\beta^{\hat{T}(p)} \left[V^e(p) - V^\eta(p)\right] < \beta^{\tilde{T}(p)} \frac{1 + \alpha}{1 - \beta} \left[\log \left[\frac{\gamma - e}{1 + f}\right] + \log p\right] .$$

After applying L'Hospital's Rule twice to the right-hand side of this expression, its limit is found to be zero. Together with <A.12>, it then follows that the existence of $\bar{p}$ in <A.11> has been established, and therefore also the existence of $\left[\bar{N}^e\right]_\eta$.

Part 2: Existence of an upper bound $\left[\bar{N}^{e}\right]_{e}$.

Suppose an educated, adult agent named i living in a period labeled 1 faces a price p and decides that all descendants will be left uneducated. The family's optimal, intertemporal consumption allocation can be found from <A.5> and Lemma 1.3. The associated utility level is given by a function $V^{i}(p)$. In addition, let the superscript 'Δ' denote a fictitious agent with the indirect utility $V^{\Delta}(p)$ derived from an income stream

$$\text{<A.13>} \qquad I_1^{\Delta} = p\,\gamma \ , \qquad I_t^{\Delta} = 0 \ , \qquad \text{for } t > 1 \ .$$

It must obviously be true that

$$V^{i}(p) - V^{\Delta}(p) > 0 \ , \qquad \forall\, p \ .$$

However, we show below in Part 2.b that this dominance in utility can be made arbitrarily small by increasing p, while Part 2.a establishes that the positive difference between $V^{e}(p)$ and $V^{\Delta}(p)$ is independent of p. It follows that there exists a $\bar{p}$ such that

$$V^{i}(p) - V^{e}(p) < 0 \ , \qquad \forall\, p > \bar{p} \ ,$$

which proves the existence of $\left[\bar{N}^{e}\right]_{e}$.

Part 2.a: There exists a $\epsilon > 0$ such that $V^{e}(p) - V^{\Delta}(p) = \epsilon, \quad \forall\, p$.

An optimal, intertemporal consumption allocation of good ℓ, given the income stream in <A.13>, is

$$\langle \text{A.14} \rangle \qquad c_1^{\ell\Delta} = \frac{(1-\beta)\ p\ \gamma}{1\ +\ \alpha}\ , \qquad c_t^{\ell\Delta} = \beta^{t-1}\, c_1^{\ell\Delta}\ , \qquad \text{for } t > 1\ .$$

(This can be seen from <A.1> when setting $t_0 = 1$ and $t_1 = \infty$.) The associated utility can be computed using <2.5>,

$$V^{\Delta}(p) \;=\; (1+\alpha) \log \beta \sum_{t=1}^{\infty} \beta^{t-1}\,(t-1)$$

$$+\ \frac{1}{1\ -\ \beta} \left\{ (1+\alpha) \left[\log (1-\beta) - \log (1+\alpha) + \log \gamma \right] + \log p + \alpha \log \alpha \right\}\ .$$

The following side calculation simplifies the first term in the previous expression,

$$\langle \text{A.15} \rangle \qquad \sum_{t=t_0}^{\infty} \beta^{t-1}\,(t-1) \;=\; (t_0 - 1) \sum_{t=t_0}^{\infty} \beta^{t-1} + \sum_{k=t_0}^{\infty} \sum_{t=k}^{\infty} \beta^{t}$$

$$=\ (t_0 - 1)\, \frac{\beta^{t_0-1}}{1\ -\ \beta} + \frac{\beta^{t_0}}{\left[1\ -\ \beta\right]^2}\ .$$

We can then find an expression for the difference in the utilities of one family which remains educated, i.e., $V^e(p)$ in <4.3>, and one family with the income stream in <A.13>,

$$V^e(p)\ -\ V^{\Delta}(p) \;=\; \frac{1\ +\ \alpha}{1\ -\ \beta} \left[\log \left[\frac{\gamma\ -\ e}{\gamma} \right] - \log \left[\beta^{\beta/(1-\beta)}\,(1-\beta) \right] \right]\ ,$$

which is obviously independent of p and strictly positive only if

$$\frac{e}{\gamma} < 1 - \beta^{\beta/(1-\beta)}\,(1-\beta).$$

By assumption $\beta > e/\gamma$, so it is sufficient to show that

$$0 < (1-\beta)\left[1 - \beta^{\beta/(1-\beta)}\right],$$

which is clearly true for $\beta \in (0, 1)$. Part 2.a is therefore established.

Part 2.b: For any $\partial > 0$, there exists a $\hat{p}$ such that for $p > \hat{p}$, $V^i(p) - V^\Delta(p) < \partial$.

Let T be the optimal cutoff point for inheritance in <A.5> given by Lemma 1.3. From <4.1>, <A.5>, <A.14> and <A.15>, the utility difference above can be written as

<A.16> $V^i(p) - V^\Delta(p)$

$$= \frac{1-\beta^T}{1-\beta}\,(1+\alpha)\log\left[\frac{p\,\gamma + f + (T-1)\,(1+f)}{p\,\gamma\,\left[1-\beta^T\right]}\right]$$

$$+ \frac{1+\alpha}{1-\beta}\,\beta^T\left[\log(1+f) - \log\gamma - \log p - \left[T + \frac{\beta}{1-\beta}\right]\log\beta - \log(1-\beta)\right].$$

To determine the limiting value of this expression, it will be necessary to understand the interrelationship between T and p. By Lemma 1.3, T can be written as a step function of p,

$$T(p) = \bar{T} \quad \text{for} \quad p \in \mathbb{P}(\bar{T}) \equiv \left\{x \in \mathbb{R}_+ : c^{\ell\eta} < \beta^{\bar{T}-1}\, c_1^{\ell e}(\bar{T}, x) \le c^{\ell\eta} / \beta\right\},$$

where we have included p as an argument of $c_1^{\ell e}$. The derivative of $c_1^{\ell e}$ with respect to p is constant within the continuous set $\mathbb{P}(\bar{T})$. The length of the interval $\mathbb{P}(\bar{T})$, denoted $\Delta p(\bar{T})$, can therefore be computed from

$$\left.\frac{d\ c_1^{\ell e}\left[T(p),\ p\right]}{d\ p}\right|_{p \in \mathbb{P}(\bar{T})} \Delta p(\bar{T}) \;=\; c_1^{\ell e}(\bar{T}, \max\{\ \mathbb{P}(\bar{T})\ \}) \;-\; c_1^{\ell e}(\bar{T}, \min\{\ \mathbb{P}(\bar{T})\ \})\ ,$$

from which we obtain

$$\Delta p(\bar{T}) \;=\; \frac{\left[1 - \beta^{\bar{T}}\right]\ (1+f)}{\beta^{\bar{T}}\ \gamma}\ .$$

Let $\tilde{p}(\bar{T})$ be the price for which $\beta^{\bar{T}-1}\ c_1^{\ell e}\left[\bar{T}, \tilde{p}(\bar{T})\right] = c^{\ell\eta} / \beta$. After noting that $c_1^{\ell e}(1, p) = c^{\ell\eta}$ for $p = 1 / \gamma$, $\tilde{p}(\bar{T})$ can be recovered as

$$\tilde{p}(\bar{T}) \;=\; \frac{1}{\gamma} + \sum_{T=1}^{\bar{T}} \Delta p(T) \;=\; \frac{1}{\gamma} + \frac{1+f}{\gamma}\,\beta^{-\bar{T}}\left[\frac{1-\beta^{\bar{T}}}{1-\beta} - \bar{T}\,\beta^{\bar{T}}\right]\ .$$

We can then compute the limiting value of the relative change in the price level associated with an agent's decision to increase the time horizon by one period,

$$\frac{\Delta p(T+1)}{\tilde{p}(T)} = \frac{\left[1 - \beta^{T+1}\right]\ (1+f)}{\beta\left[\beta^{T} + (1+f)\left[\dfrac{1-\beta^{T}}{1-\beta} - T\,\beta^{T}\right]\right]} \xrightarrow[T \to \infty]{} \frac{1-\beta}{\beta}\ .$$

Since $\tilde{p}(T)$ is growing in the limit by a factor of $1/\beta$, it follows that <A.16> goes to zero. Part 2.b is therefore established.

Q.E.D.

APPENDIX B

The two following tables contain statistics from a numerical example with the parameterization:

α	$= 1$	taste parameter,
β	$= .55$	discount factor,
γ	$= 1$	input-output coefficient in industry s,
f	$= .5$	productivity of a child in industry ℓ relative to the one of an adult,
e	$= .1$	educational cost.

Notation in headings

N^e	fraction of educated workers,
$V^i(p)$	utility of an agent whose family retains its educational status forever, $i = \eta$ for uneducated and $i = e$ for educated (defined by <4.2> and <4.3>),
$V^{i\Delta}(p)$	utility of an agent with educational status $i \in \{\eta, e\}$ who decides to change his family's educational status (see footnote 6),
$T^{\eta*}$	chosen time horizon by an uneducated agent after which all descendants will be educated (see footnote 6),
$\underline{T}^\eta$, $\bar{T}^\eta$	lower and upper bound for the time horizon that an uneducated agent can choose for changing his family's educational status (given by <A.3> and <A.4> in Appendix A),

T^e	optimal time horizon for an educated agent to disperse savings, from a change in the family's educational status, to descendants through inheritance (given by Lemma 1.3 in Appendix A),
$1/p$	relative price of good ℓ, given by the inverse of <4.4>,
Income	earnings of an uneducated family in relation to those of an educated family, $(1 + f) / (p\ \gamma)$,
Gini	Gini coefficient, calculated as $\gamma N^e / \left[(1+f)(1-N^e) / p + \gamma N^e\right] - N^e$,
GNP	gross national product, calculated as $(1+f)(1-N^e) / p + \gamma N^e$,
Equil.	indication whether a certain ratio of educated workers can constitute a steady state (Equil. = 1) or not (Equil. = 0).

TABLE 1

N^e	$V^\eta(p)-V^{\eta\Delta}(p)$	T^{η^*}	$\underline{T}^\eta$	$\bar{T}^\eta$	$V^e(p)-V^{e\Delta}(p)$	T^e	Equil.
.0100	4.4992	13.	12.	13.	5.9474	7.	1.
.0200	5.2010	7.	6.	7.	5.6653	6.	1.
.0300	2.4655	6.	4.	6.	5.4200	5.	1.
.0400	1.9313	5.	4.	5.	5.1984	5.	1.
.0500	1.8330	4.	3.	4.	4.9937	5.	1.
.0600	1.0491	4.	3.	4.	4.8007	4.	1.
.0700	1.1066	3.	2.	3.	4.6190	4.	1.
.0800	.6422	3.	2.	3.	4.4464	4.	1.
.0900	.3470	3.	2.	3.	4.2809	4.	1.
.1000	.1479	3.	2.	3.	4.1211	4.	1.
.1100	.0087	3.	2.	3.	3.9659	4.	1.
.1200	-.1654	2.	2.	3.	3.8152	3.	0.
.1300	-.2914	2.	2.	2.	3.6694	3.	0.
.1400	-.3747	2.	2.	2.	3.5277	3.	0.
.1500	-.4286	2.	1.	2.	3.3896	3.	0.
.1600	-.4613	2.	1.	2.	3.2544	3.	0.
.1700	-.4785	2.	1.	2.	3.1219	3.	0.
.1800	-.4839	2.	1.	2.	2.9916	3.	0.
.1900	-.4803	2.	1.	2.	2.8632	3.	0.
.2000	-.4697	2.	1.	2.	2.7365	3.	0.
.2100	-.4555	1.	1.	2.	2.6113	3.	0.
.2200	-.4953	1.	1.	2.	2.4873	3.	0.
.2300	-.5100	1.	1.	2.	2.3645	2.	0.
.2400	-.5061	1.	1.	2.	2.2432	2.	0.
.2500	-.4883	1.	1.	2.	2.1236	2.	0.
.2600	-.4596	1.	1.	2.	2.0053	2.	0.
.2700	-.4224	1.	1.	2.	1.8882	2.	0.
.2800	-.3785	1.	1.	2.	1.7721	2.	0.
.2900	-.3291	1.	1.	2.	1.6569	2.	0.
.3000	-.2753	1.	1.	2.	1.5424	2.	0.
.3100	-.2178	1.	1.	2.	1.4285	2.	0.
.3200	-.1572	1.	1.	2.	1.3151	2.	0.
.3300	-.0940	1.	1.	2.	1.2021	2.	0.
.3400	-.0295	2.	1.	2.	1.0893	2.	0.
.3500	.0101	2.	1.	2.	.9767	2.	1.
.3600	.0502	2.	1.	2.	.8641	2.	1.
.3700	.0906	2.	1.	2.	.7515	2.	1.
.3800	.1315	2.	1.	2.	.6388	2.	1.
.3900	.1726	2.	1.	2.	.5258	2.	1.
.4000	.2141	2.	1.	2.	.4126	2.	1.
.4100	.2560	2.	1.	2.	.2989	2.	1.
.4200	.2981	2.	1.	2.	.1849	2.	1.
.4300	.3405	2.	1.	2.	.0702	1.	1.
.4400	.3833	2.	1.	2.	-.0445	1.	0.
.4500	.4263	2.	1.	2.	-.1592	1.	0.
.4600	.4697	2.	1.	2.	-.2739	1.	0.
.4700	.5134	2.	1.	2.	-.3888	1.	0.
.4800	.5574	2.	1.	2.	-.5040	1.	0.
.4900	1.0942	1.	1.	1.	-.6195	1.	0.

TABLE 2

N^e	1/p	Income	Gini	GNP	Equil.
.0100	.0061	.0091	.5163	.0190	1.
.0200	.0122	.0184	.5063	.0380	1.
.0300	.0186	.0278	.4963	.0570	1.
.0400	.0250	.0375	.4863	.0760	1.
.0500	.0316	.0474	.4763	.0950	1.
.0600	.0383	.0574	.4663	.1140	1.
.0700	.0452	.0677	.4563	.1330	1.
.0800	.0522	.0783	.4463	.1520	1.
.0900	.0593	.0890	.4363	.1710	1.
.1000	.0667	.1000	.4263	.1900	1.
.1100	.0742	.1112	.4163	.2090	1.
.1200	.0818	.1227	.4063	.2280	0.
.1300	.0897	.1345	.3963	.2470	0.
.1400	.0977	.1465	.3863	.2660	0.
.1500	.1059	.1588	.3763	.2850	0.
.1600	.1143	.1714	.3663	.3040	0.
.1700	.1229	.1843	.3563	.3230	0.
.1800	.1317	.1976	.3463	.3420	0.
.1900	.1407	.2111	.3363	.3610	0.
.2000	.1500	.2250	.3263	.3800	0.
.2100	.1595	.2392	.3163	.3990	0.
.2200	.1692	.2538	.3063	.4180	0.
.2300	.1792	.2688	.2963	.4370	0.
.2400	.1895	.2842	.2863	.4560	0.
.2500	.2000	.3000	.2763	.4750	0.
.2600	.2108	.3162	.2663	.4940	0.
.2700	.2219	.3329	.2563	.5130	0.
.2800	.2333	.3500	.2463	.5320	0.
.2900	.2451	.3676	.2363	.5510	0.
.3000	.2571	.3857	.2263	.5700	0.
.3100	.2696	.4043	.2163	.5890	0.
.3200	.2824	.4235	.2063	.6080	0.
.3300	.2955	.4433	.1963	.6270	0.
.3400	.3091	.4636	.1863	.6460	0.
.3500	.3231	.4846	.1763	.6650	1.
.3600	.3375	.5063	.1663	.6840	1.
.3700	.3524	.5286	.1563	.7030	1.
.3800	.3677	.5516	.1463	.7220	1.
.3900	.3836	.5754	.1363	.7410	1.
.4000	.4000	.6000	.1263	.7600	1.
.4100	.4169	.6254	.1163	.7790	1.
.4200	.4345	.6517	.1063	.7980	1.
.4300	.4526	.6789	.0963	.8170	1.
.4400	.4714	.7071	.0863	.8360	0.
.4500	.4909	.7364	.0763	.8550	0.
.4600	.5111	.7667	.0663	.8740	0.
.4700	.5321	.7981	.0563	.8930	0.
.4800	.5538	.8308	.0463	.9120	0.
.4900	.5765	.8647	.0363	.9310	0.

FOOTNOTES

[1] It is doubtful whether the two economists referred to would endorse our intention. Concerning dual economies, Lewis [1954] questioned the application of neoclassical economic theory by saying that "The student of such economies has ... to work right back to the classical economists before he finds an analytical framework into which he can relevantly fit his problems." Schultz [1964], on the other hand, was very critical towards the class of models represented by Lewis's work, which he, among others, identified with the marginal product of labor being equal to zero.

[2] The unit coefficient on L_t^ℓ in <2.2> can be thought of as a normalization, where both <2.2> and <2.3> have been multiplied by the input–output coefficient in industry ℓ. Such a normalization will clearly not affect the preference ordering given by <2.1>.

[3] The assumption that good *s* cannot be stored is just for expositional simplicity. Its relaxation would not change our results, since they all pertain to steady states in which storage of goods turns out to be nonoptimal.

[4] The upper bound on e is sufficient for finding steady states, and necessary and sufficient for finding a stationary allocation that is Pareto optimal.

[5] The definition of an equilibrium excludes economies that start out without any educated workers. Because of the postulated technologies, such economies would never be able to produce good *s*. All agents' utilities would therefore be equal to minus infinity irrespective of their actions.

[6] The utility associated with retaining the family's educational status can be computed from <4.2>–<4.4>. To obtain the utility from changing the family's educational status, we use the findings in Lemma 1 in Appendix A. The educated

agent is supposed not to educate his child and instead choose time horizon T^e for an optimal inheritance scheme, where T^e is given by Lemma 1.3. The uneducated agent is constrained to maximize utility by picking a time horizon T belonging to the closed interval $[\underline{T}^\eta, \bar{T}^\eta]$ defined by <A.3> and <A.4>, from which all descendants will be educated.

[7] The apparent nonsmoothness of the uneducated agents' curve describing the difference in utility between retaining and changing their educational status is a consequence of the computation of the latter utility level. According to footnote 6, the uneducated family can be thought of as being forced to educate all descendants from a period $T \in [\underline{T}^\eta, \bar{T}^\eta]$. $\bar{T}^\eta$ will obviously be binding whenever a change in the educational status is not desirable, i.e. for positive utility differences. This can be seen in Table 1 in Appendix B where a higher $\bar{T}^\eta$ associated with a lower ratio of educated workers (a higher relative price of good s) will be chosen immediately, which gives rise to a dicontinuous jump in Figure 1. On the other hand, for negative utility differences the curve is smooth since the family chooses "interior" time horizons.

[8] The analysis of a student loan program is primarily meant to further our understanding of the effects of the missing market for human capital. Since that imperfection is not endogenously explained in the model, it is clear that any policy implications must be of a limited nature. This is also true for Loury [1981] who similarly assumes that private loan and insurance markets are absent for human capital, while taxes contingent on education and income are fully enforceable. An important research task remains to be the formulation of an explicit environment that gives rise to the described market imperfection. Such a model would allow us to carefully address the role for government intervention.

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