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The Effects of Grants-in-Aid on State and Local Expenditures: The Case of AFDC

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Abstract

One of the important policy issues facing the federal government concerns the best way to provide grants-in-aid to state and local governments. Given that the federal government wishes to provide such aid for a particular type of expenditure, several options are available: (1) block grants, under which the federal government simply provides a lump-sum amount of aid; (2) open-end grants, under which the federal government matches state or local expenditures at a constant matching rate, without limit; and (3) closed-end grants, under which state or local expenditures are matched only up to some maximum amount, beyond which no additional aid is given.

This paper discusses the econometric issues involved in estimating the effect of these and other such grant programs on state and local expenditures. It is shown that closed-end grants are a special case of a more general class of grants in which the subsidy rate is not constant, depending instead upon the level of state or local expenditures. Such grants create a "piecewise-linear" budget constraint for the recipient government, for the constraint consists of a series of linear segments--within each segment the subsidy rate is constant but the subsidy rate varies across segments. The segments of the constraint are joined by "kinks," points at which the subsidy rate changes.

The econometric methods are applied to the AFDC program, for the federal government subsidizes state expenditures for that program at varying rates. The results show that (1) the federal formula actually increases cross-state inequality of benefits; (2) state benefits would actually increase if some of the subsidy rates in the federal formula

were eliminated; and (3) the so-called "flypaper effect"--the notion that federal grants have a much bigger effect on state and local expenditures than do tax dollars from their own jurisdictions--no longer appears to hold.

The Effects of Grants-in-Aid on State and Local Expenditures: The Case of AFDC

As economists who have studied the public sector are well aware, actual government programs often differ significantly from those outlined in undergraduate and graduate texts. This is especially true in the area of grants-in-aid, for grant programs as actually legislated and then administered are frequently far different from the simple models of those programs with which economists generally work. The two most common models of the effect of grants-in-aid on state and local expenditures are those which depict either an open-end matching grant, under which expenditures by the receiving government are subsidized by the granting government at a constant rate, or a block grant, under which the receiving government is given a subsidy fixed in amount and independent of its own expenditures. These two models are simple to analyze because they alter the budget constraint of the receiving government in a simple fashion, by a change in slope in the first case and by a parallel outward shift in the second case, but they are far less common in practice than programs which alter the budget constraint in more complex ways.

A different type, closed-end grants, which outnumber all other types in the United States, create a single convex kink in the recipient budget constraint. Many other grant programs create a series of kinks, some convex and some non-convex, as the subsidy rate varies over different levels of expenditure in discrete brackets. If the resulting budget constraints are convex in all regions, the fundamental comparative statics of demand are not significantly altered by these programs--uncompensated price (i.e., subsidy-rate) effects are non-positive and income effects are non-negative on expenditures. However, the magnitude is changed--the respective effects may be zero rather than negative and positive. But if the budget constraint is non-convex, the certainty of these comparative statics is lost--uncompensated price effects may be positive and income effects negative. Therefore the incentive effects of existing programs and of changes in those programs may be opposite to those generally expected.

This paper addresses the analysis of such programs, with an emphasis on the econometric issues that arise when the effects of the programs are statistically examined. Difficult econometric issues are created because the demand functions created by piecewise-linear constraints are also non-linear. The functions are inherently non-linear in their parameters and, more importantly, non-linear in their error terms. As a result, simple techniques for estimating such functions are generally subject to serious biases. However, more sophisticated techniques have been developed to estimate demand functions in this circumstance (e.g., Burtless and Hausman, 1978) and these techniques are applied here.

The substantive application in the paper is to the AFDC (Aid to Families with Dependent Children) program. The federal government subsidizes state AFDC expenditures at varying matching rates, creating a budget constraint with both convex and non-convex regions. The econometric techniques for piecewise-linear budget constraints are applied to this problem and estimates for the state response to federal AFDC subsidies are obtained. The results are used to predict the effects of several policy changes of interest in this area, such as block grants for AFDC, changes in the progressivity of the matching-rates schedule, and a more simplified closed-end grant form.

Part I of the paper provides a discussion of piecewise-linear budget constraints in grants-in-aid applications and summarizes the econometric techniques for estimating state and local expenditure functions when the constraints are of that form. Part II applies the model to the AFDC case, discusses the issues specific to that program, and presents the results of the estimation. A conclusion and recommendations for future research follow.

I. GRANTS-IN-AID, PIECEWISE-LINEAR BUDGET CONSTRAINTS, AND ESTIMATION

The derivation of consumer-demand functions when budget constraints are piecewise-linear, and the econometric techniques necessary to estimate them have appeared in a number of labor-supply studies (Burtless and Hausman, 1978; Hanoch and Honig, 1978; Hausman, 1980, 1981; Moffitt, (forthcoming); Moffitt and Nicholson, 1982; Wales and Woodland, 1979; see Moffitt, 1982a, for a review). However these techniques have not been applied to the problem of grants-in-aid. In this section these techniques are summarized and their application to the grants-in-aid problem are discussed.

Figures 1 and 2 illustrate convex and non-convex sets, respectively, facing a state or local government that is offered non-linear grants-in-aid. A recipient government is assumed to have a utility function U(X,Y), where X is the good to be subsidized and Y is all other goods. It will be assumed throughout that an internally consistent function of this type exists for the local public sector; this is usually rationalized on the basis of the median-voter principle. The local public sector is assumed to have total own resources of M; hence the budget

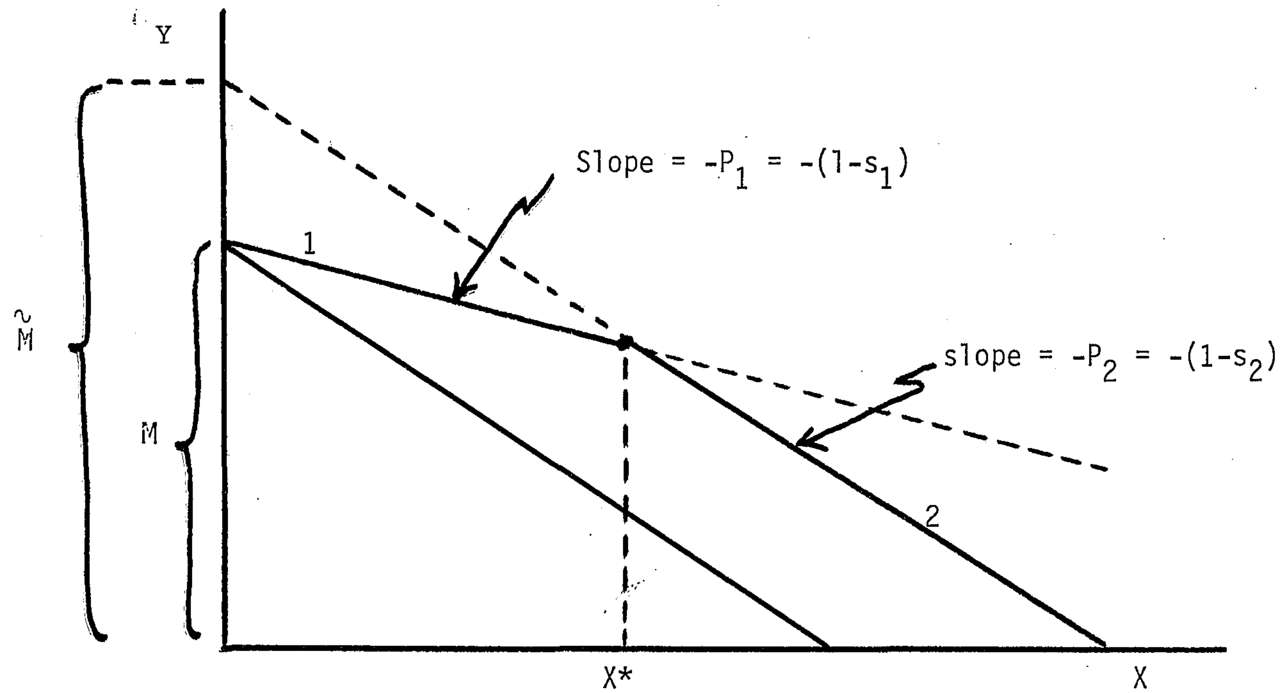

Figure 1. Convex Budget Set.

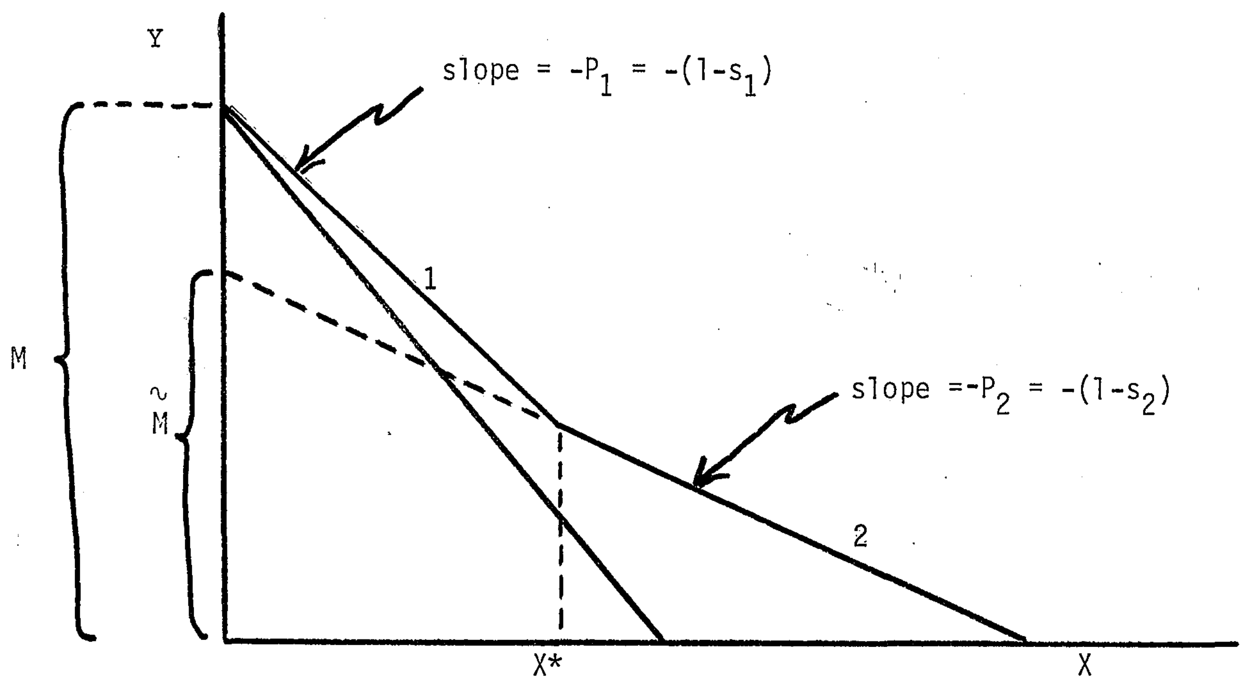

Figure 2. Non-convex Budget Set.

constraint in the absence of grants-in-aid is $M = X + Y$. In both diagrams the government is offered a subsidy rate of s_1 for expenditures greater than X^*.

In Figure 1 the subsidy rate is reduced for high values of X. A special case of such a grant is the closed-end grant, under which $s_2 = 0$. Another common case of such a grant is the take-it-or-leave-it "block" grant under which the state is offered a grant for a special category of expenditures (e.g., rat control), all of which must be spent on the category. In this case $s_1 = 1$ and $s_2 = 0$ (i.e., the first segment is horizontal), and many governments will be observed to take the grant but spend none of its own funds on the given category. In Figure 2 the subsidy rate rises with expenditures. Such programs are rarer than those in Figure 1, but do occur for AFDC as will be discussed in the next section.

The major difference between the two diagrams is that local maximums need not be global maximums in the non-convex case. As noted in the introduction, this has major implications for the comparative statics of these models. In the convex case a government will locate on one of the two segments or at the kink. An increase in M or a decrease in either or both prices (e.g., an increase in the subsidy rates) will either raise expenditure on X, or will leave it unchanged. The second will occur if the government "sticks" at the kink. However, in the non-convex case an increase in M or a decrease in the demand price may decrease X by inducing a change of segments, as the government "jumps" from segment 2 to segment 1. The segment upon which the global maximum occurs can change in such a fashion without any violation of the standard restrictions on the shapes of indifference maps.

There is also a "new" comparative static of interest here, which is the effect of a change in X*. In Figure 1, increasing the level of X* (e.g., increasing the ceiling amount on a closed-end grant) will have (a) no effect on a government already below X*, (b) a pure income effect on a government initially relatively far above X*, and (c) a combined substitution-income effect on a government initially spending X* or a bit above it.[1] In the last case, one possibility is that a government initially at X*, spending only up to the ceiling amount, will simply follow the kink and spend up to the new limit but no more. In Figure 2, an increase in X* will have (a) again no effect on a government spending less than X*, and (b) either a pure income effect on those initially spending more than X* or a large negative, combined substitution-income effect. In the second case the government retreats to segment 1; the increase in X* effectively postpones the point at which the more generous subsidy rate is obtained, making the second segment less attractive.

This brief discussion should be sufficient to demonstrate that changes in individual subsidy rates, ceiling amounts, and grant levels in such programs have a complicated series of effects, some of which are counter to expectations. The effects differ depending upon the shapes of the indifference curves and upon the location of a government along the initial budget constraint. As an empirical matter, it is generally the case that one observes recipient governments to be spread out along such constraints, with different governments choosing different expenditure levels. Consequently the net effect of any particular policy change in the parameters of the program will in part depend upon the shape of the distribution of the governments over different parts of the constraint.

It is important therefore to be able to estimate what might be termed "structural" demand functions--that is, functions which allow for the non-linearities in response suggested by this analysis. Any reduced-form demand function that specifies "average expenditure" as a function of the parameters of the entire constraint will not be particularly useful, for the coefficients of such a function will be averages of the responses of different types of governments and hence will not be generalizable to any situation in which the distribution is different.

One such "structural" demand function is just the neoclassical demand function that results from utility maximization. Assume that a government maximizing the utility function $U(X,Y)$ subject to the linear constraint $M = PX + Y$ has the demand function $g(P,M)$. According to standard utility theory we know that $g_1 < 0$ and $g_2 > 0$ if X is a normal good, as will be assumed throughout. When the budget constraint is instead piecewise-linear, the constraint becomes:

$$M = P_1X + Y \qquad \text{if} \quad X \leqslant X^*$$

$$M = P_1X^* + P_2(X-X^*) + Y \qquad \text{if} \quad X > X^*$$

or

$$M = P_1X + Y \qquad \text{if} \quad X \leqslant X^*$$

$$\tilde{M} = P_2X + Y \qquad \text{if} \quad X > X^*$$

where $\tilde{M} = M + (P_2 - P_1)X^*$. $\tilde{M}$ is just the intercept of the linearized segment two, and is illustrated in the Figures. It will be convenient to assume that a government along segment 2 faces the linear segment with price P_2 and income $\tilde{M}$.

Given this constraint one natural approach is to attempt to estimate the demand function directly by assigning to each government the parameters of the segment on which it is observed to be located. As Hall (1973) noted in a well-known labor-supply study, an agent observed to be maximizing utility along a particular segment would presumably choose that same point if he faced a linear constraint with the same parameters --after all, the point is presumably the maximum maximorum. In the convex case there is a question regarding what to do with governments observed at the kink, but assume for the moment that these observations are not used. Then, adding an error term ε, we would estimate the function:

$$X = f(P_i, M_i) + \varepsilon \tag{2}$$

where P_i is the price along segment i and $M_1 = M$ and $M_2 = \tilde{M}$.

Perhaps the most important points to be made are that (a) estimation of this equation by ordinary least squares (OLS) will yield biased parameter estimates; (b) estimation by two-stage least-squares or instrumental variables is very difficult; and (c) that the coefficients of the equation by themselves are not of interest anyway. Consider first estimation by OLS. OLS estimates will be biased because the error term is correlated with the price and income variables in the equations, for two distinct reasons. These two reasons relate to different interpretations of what the error term represents, for there are two possibilities--it may represent unobserved variation in tastes (i.e., heterogeneity of preferences), or it may represent other factors--measurement error, "optimization" error, "disequilibrium" error, specification error, and so on. These latter types of error I will loosely term "random" error.[2]

If the error term represents unobserved variations in governmental indifference maps, then such variation will clearly be correlated with the price and income of the segment along which a government is located, for that segment with its price and income are chosen by the government subject to its indifference map. In fact, it is obvious from the figures that P_2 will be observed only if X is high, simply because of the nature of the budget constraint. But if all governments have the same preference maps and there is no heterogeneity error, then there will still be a problem because governments will not necessarily be observed on the segment they all (equally) "desire." The error term may be sufficiently large to move the observed value of X to a different segment, and the probability that this occurs will be related to the size of the error term. Hence the value of the error term will be correlated with the parameters of the observed segment.

These considerations suggest that the price and income variables in equation (2) be treated as endogenous, and that some sort of two-stage least squares or instrumental-variable technique be used to estimate the equation. This indeed has been the most common solution technique in the grants-in-aid literature (e.g., Feldstein, 1975; Orr, 1976). The conceptual difficulty with this approach is that it provides no concrete interpretation of why the variables are endogenous. If they are endogenous because of preference heterogeneity, are the instruments to be interpreted as reflecting some underlying utility-function transformation? If so, in what sense can the "choice of segment" be separated from the choice of X along a segment? Mirroring this conceptual difficulty is the empirical difficulty of determining variables that that are exogenous to the choice of X along a segment but not to the choice of segment itself. Likewise, if the error is interpreted as purely random error, it is difficult to obtain variables that affect such error which do not belong in the demand function in the first place. (Note that the two-stage lambda technique of Heckman is a separate matter--see below.)

Finally, there is the question of the usefulness of the coefficients of (2). The estimated parameters of the function g(•) cannot by themselves tell us anything about the important comparative statics discussed earlier--when a government will "jump" from one segment to another in the non-convex case, when it will "stick" at the kink in the convex case, or how it will change in response to changes in X^* (which is not even in the equation). The function g(•) is simply incomplete--it offers no theory of the choice of segment itself.

This discussion suggests that the choice of segment must be formally modeled and estimated along with the function g(•), which is only the demand function conditional upon choice of segment. In this paper the choice of segment will be assumed to arise from standard maximization of the utility function U(X,Y). But since grants-in-aid are the focus of the paper, it should be noted that these econometric difficulties would present themselves regardless of what objective function or what process of public choice is assumed to generate state and local expenditures. If the grant program offered to the locality is non-linear, the expenditure functions will be non-linear in any model of public choice.

The choice of segment therefore will be just the segment upon which utility is highest. Assuming now that ε can be decomposed into two separate error terms, ε_h (heterogeneity error) and ε_r (random error), the complete demand function for the convex case can be written:[3]

$$X = D_1[g(P_1,M) + \varepsilon_h] + D_2\ [g(P_2,\tilde{M}) + \varepsilon_h] + (1-D_1-D_2)X^* + \varepsilon_r \qquad (3)$$

where

$$D_1 = 1 \text{ if } \hat{D}_1 > 0;\ D_1 = 0 \text{ otherwise}$$

$$D_2 = 1 \text{ if } \hat{D}_2 > 0,\ D_2 = 0 \text{ otherwise}$$

$$\hat{D}_1 = X^* - g(P_1,M) - \varepsilon_h$$

$$\hat{D}_2 = g(P_2,\tilde{M}) + \varepsilon_h - X^*$$

and for the non-convex case:

$$X = D[g(P_1,M) + \varepsilon_h] + (1-D)[g(P_2,\tilde{M}) + \varepsilon_h] + \varepsilon_r \qquad (4)$$

where

$$D = 1 \text{ if } \hat{D} \geqslant 0;\ D = 0 \text{ if } \hat{D} < 0$$

$$\hat{D} = V(P_1,M;\varepsilon_h) - V(P_2,\tilde{M};\varepsilon_h)$$

Here $V(P_i,M_i;\varepsilon_h)$ is the indirect utility function along a segment with price P_i, imputed income M_i, and for a government with utility paramter ε_h.

The estimation of these functions cannot be performed with OLS. The functions are non-linear in their parameters and in the two error terms, ε_h and ε_r. However, the parameters can be estimated with maximum-likelihood techniques if the probabilities of observing X can be specified. In general form, the probabilities in the convex case can be written:

$$\begin{aligned} \text{Prob}(X) &= \text{Prob}[X = g(P_1,M) + \varepsilon_h + \varepsilon_r,\ D_1 = 1] \\ &+ \text{Prob}[X = g(P_2,\tilde{M}) + \varepsilon_h + \varepsilon_r,\ D_2 = 1] \\ &+ \text{Prob}[X = X^* + \varepsilon_r,\ D_1 = 0,\ D_2 = 0] \end{aligned} \tag{5}$$

and in the non-convex case:

$$\begin{aligned} \text{Prob}\ (X) &= \text{Prob}\ [X = g(P_1,M) + \varepsilon_h + \varepsilon_r,\ D = 1] \\ &+ \text{Prob}[X = g(P_2,\tilde{M}) + \varepsilon_h + \varepsilon_r,\ D = 0] \end{aligned} \tag{6}$$

Thus each probability is the sum of the probabilities of observing the particular value of X if utility maximization occurs on each segment or kink. The evaluation of these probabilities in terms of probabilities of ε_h and ε_r is straightforward in general and is presented in the Appendix to this paper for the AFDC application discussed in the next section.[4]

Since the econometric problem created by piecewise-linear constraints is one of selectivity bias, it may appear that the two-stage technique of Heckman (1979) can be used. Indeed, Heckman and MaCurdy (1981) have pointed out that such techniques can be applicable to the problem of kinked budget lines (see Welch, 1981, for an application to grants-in-aid). Unfortunately, however, the two-stage technique is easy to apply only when the budget constraint is globally convex and when there is only heterogeneity error. If the constraint is non-convex, the selection equation estimated in the first stage will be a complex function of direct and indirect utility functions and hence will be non-linear in parameters and error terms. In addition, if there is random error, then since the observed segment is not necessarily the "desired" segment the selection equation is much more complex--the probability of observing an observation on a particular segment includes the probability that other segments and kinks are "desired" (i.e., implied by the estimated parameters to be the utility-maximizing points). Given these difficulties, the fully efficient full-information maximization procedure is used here.

Finally, the estimation of a model with two error terms, seemingly both additive, should be noted. The variances of the two error terms in this model can be separately identified because one (ε_h) appears in the utility function and one (ε_r) does not. The first is essentially a random coefficient. As a result, different distributions of the data will be generated depending upon which error term has the larger variance. If most of the variance is a result of heterogeneity, governments will be observed to cluster around the kink point of a convex constraint and to be dispersed away from the kink point of a non-convex constraint. Indeed, in the extreme case of heterogeneity-error only, there will be a massing of observations exactly at a convex kink point and a region of zero observations around a non-convex kink point. (A discontinuity in the likelihood function will be consequently generated.) In the opposite extreme case of no heterogeneity error and only random error, there will be no clustering or dispersion about the kinks at all--the data will be scattered randomly around the single expenditure level "desired" by all since all objective functions are the same. Kinks in the rest of the constraint will have no effect on the distribution, and a change in the subsidy rate in other regions of the constraint could have exactly zero effect on mean expenditures in the population. Empirically, then, the variance of ε_h will be estimated to be large (small) relative to that of ε_r if the distribution of the data shows a high (low) degree of clustering and dispersion around the kinks. See Moffitt (1982a) for an extended discussion of this point.

II. FEDERAL GRANTS-IN-AID FOR STATE AFDC EXPENDITURES

The involvement of the federal government in subsidizing state welfare expenditures dates from the Social security Act of 1935, when the Aid to Families with Dependent Children (AFDC) program was enacted. AFDC, the major welfare program in the U.S., provides cash assistance to low-income families. Each state sets its own benefit level, but the federal government subsidizes each state in an amount determined by the level of the benefit per AFDC recipient in the state. The question of the effect of this subsidization on state AFDC benefit levels has been examined in several studies over the last 10-15 years. The best known paper is that of Orr (1976).[5]

Let the per capita AFDC benefit chosen by the state be B. If M is per capita income, Y is per capita expenditures on non-AFDC goods in the state, C is the size of the AFDC caseload (i.e., the number of recipients), and N is the size of the state population, then the budget constraint facing the median voter in the absence of federal matching is $M = (C/N)B + Y$. Here Y includes both private and public expenditures; the state tax rate is submerged. In the presence of federal matching, the price of the AFDC benefit becomes $(C/N)(1-s)$, where s is the marginal federal share.

Figure 3 shows the budget constraint facing states under current federal law. Over the first \$18 of B the federal government pays 83 percent of the benefit; from \$18 to \$32, s_f percent is paid, where s_f varies by state according to a federal formula; from \$32 to some level B*, no matching at all is provided; and beyond B*, a matching rate of s_m is paid, which also varies by state. This peculiar set of rates is a result of 1965 legislation that permitted states to opt for subsidization at the same rate as they receive for Medicaid payments, the rate s_m. Under the old AFDC formula, only the three subsidy rates of .83, s_f, and zero were offered. That formula was based upon the simple notion that low-benefit states should be given more inducement to raise benefits than high-benefit states; that is, the desire was to reduce the variance of benefits across states. The fact that the variance has always been fairly large has always been one of the major points of discussion in this literature. It can be shown as a general proposition of maximizing behavior that a convex budget constraint, whether it be piecewise-linear or "smoothly" non-linear (i.e., everywhere differentiable), will reduce the variance of chosen quantities relative to the variance under a linear constraint.[6] The same applies here. However, when in 1965 the states were offered the alternative Medicaid matching formula, which is also open-end but at a constant rate, this principle was violated and higher matching rates were provided at high benefit levels than at slightly lower benefit levels. (In fact, s_m is always greater than or equal to s_f as well.) Beyond point B* (the benefit level at which two formulas provide equal subsidies), the Medicaid formula is more generous.[7] As is clear from the diagram, the introduction of the Medicaid option should

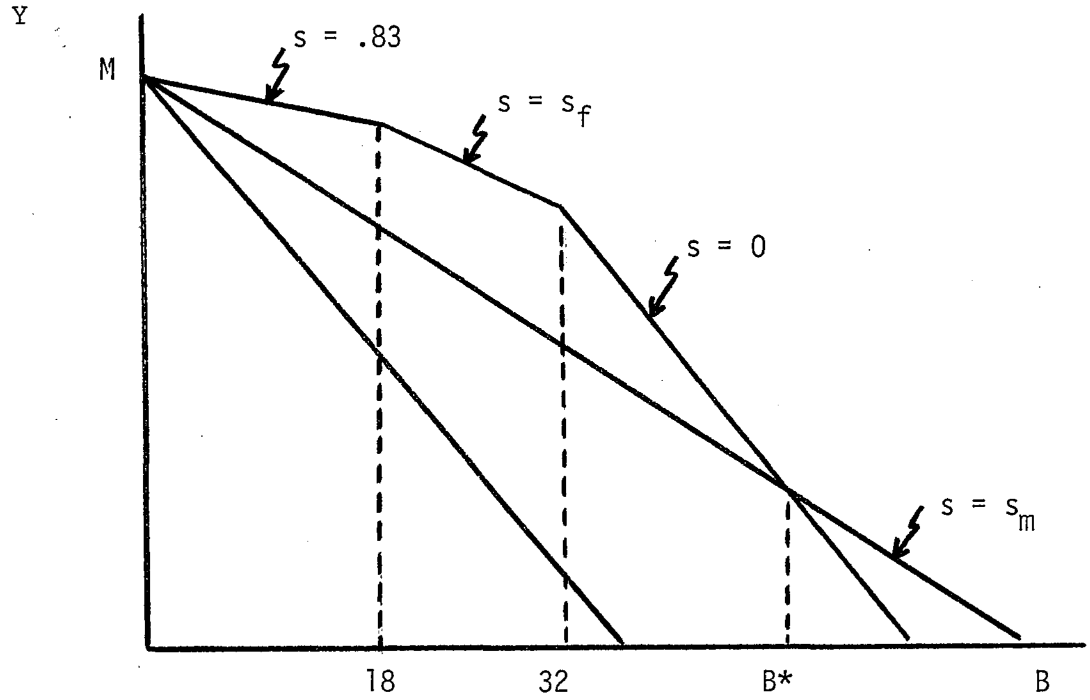

Figure 3. AFDC Budget Set.

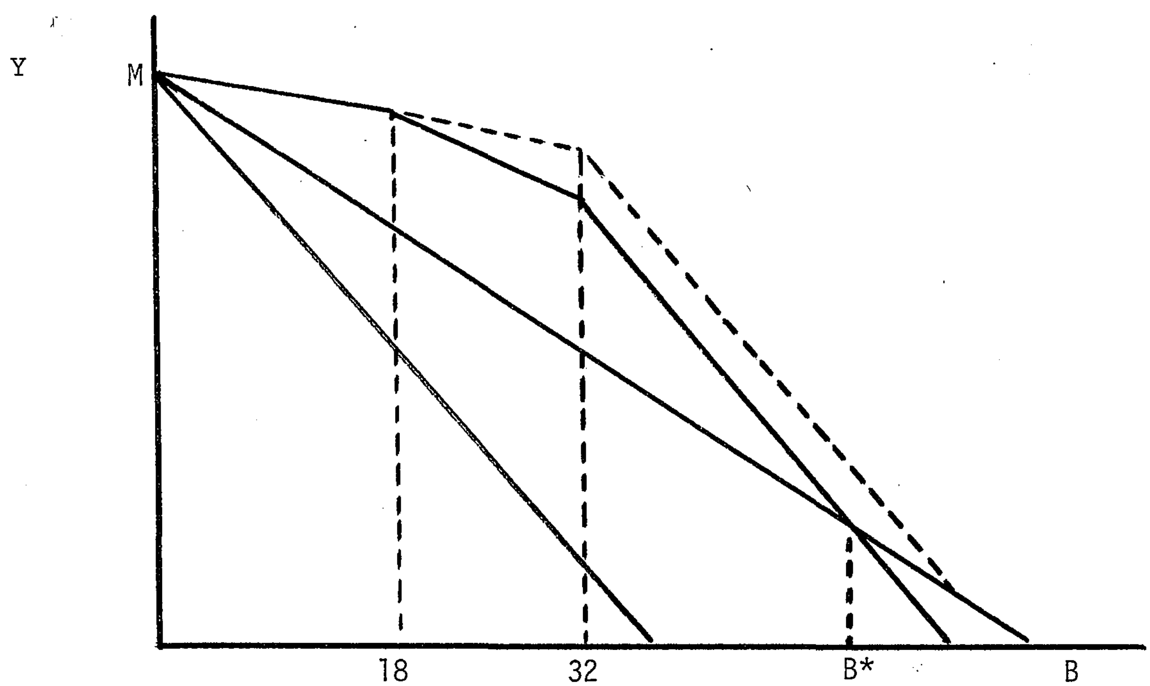

Figure 4. Increase in s_f.

induce some states to increase their benefit levels more than would be the case otherwise. But, since the general proposition noted above also works in reverse--non-convex constraints increase the variance of observed quantities--cross-state inequality should be higher than otherwise.

The presence of a non-convexity in the budget constraint also opens the door to the perverse comparative statics noted in the last section. For example, an increase in M and a consequent outward parallel shift in the entire constraint need not increase B. Also, an increase in s_f, the federal matching rate for AFDC alone, need not increase B. This latter case is illustrated in Figure 4, where it can be seen that reductions in B are possible if states are initially beyond B*.

When the 1965 legislation was introduced, most states were on the first or second segments of the constraint. Over time, states have moved to the right along the constraint as benefits have grown. The matching rates s_f and s_m have varied slightly, but, more importantly, the federal government has left the kink benefit amounts of $18 and $32 constant in nominal terms. Consequently, as a result of nominal benefit growth if nothing else, virtually all states are now on the fourth segment of the constraint. Consequently, whatever variance-reducing effect the original formula had is no longer present.

For present purposes, this means that data from early in the 1970's must be used to observe a suitable dispersion of states over the constraint. The data chosen for this study are from 1970, when 17 states were to the left of B*. It should be noted, however, that one state (Arizona) did not have a Medicaid program at that time and hence was not eligible for the optional Medicaid matching. Its global budget constraint therefore contained only three segments. In addition, five states had a sufficiently high s_m relative to s_f that the intersection point B* fell to the left of $32; hence these states also faced only a three-segment constraint.

Before implementing the econometric model with these data, it is useful to look at the distribution of benefits over the constraint. As noted above, heterogeneity of preferences should induce a systematic pattern of clustering and dispersion in the data. A direct examination of the data can thus provide a simple test for whether the heterogeneity hypothesis has any *prima facie* plausibility.

Figures 5 to 8 show the distribution of benefits across the 51 jurisdictions (50 states and the District of Columbia) in 1970. Figures 5 and 6 show the distribution of benefits and the log of benefits, respectively. The data show no sign of a clustering around $18, the first kink, but there is a noticeable bump around the kink at $32. The mode of the distribution is on the fourth segment, and there is also a slight drop in the distribution between $32 and the mode. It is not clear whether that drop is a result of the non-convex kink at B*, since B* is at different points for different states. Figures 7 and 8 provide evidence on this question, for they show the distributions of benefits and log benefits around a state's B*. There is indeed a slight dip in the distribution around the zero (B*) point, although the frequency is a bit lower just above B* than at B*.[8] So, on the basis of these unadjusted frequency distributions, it seems that there is some evidence of heterogeneity--the shape of the constraint *does* seem to be affecting governmental choices.

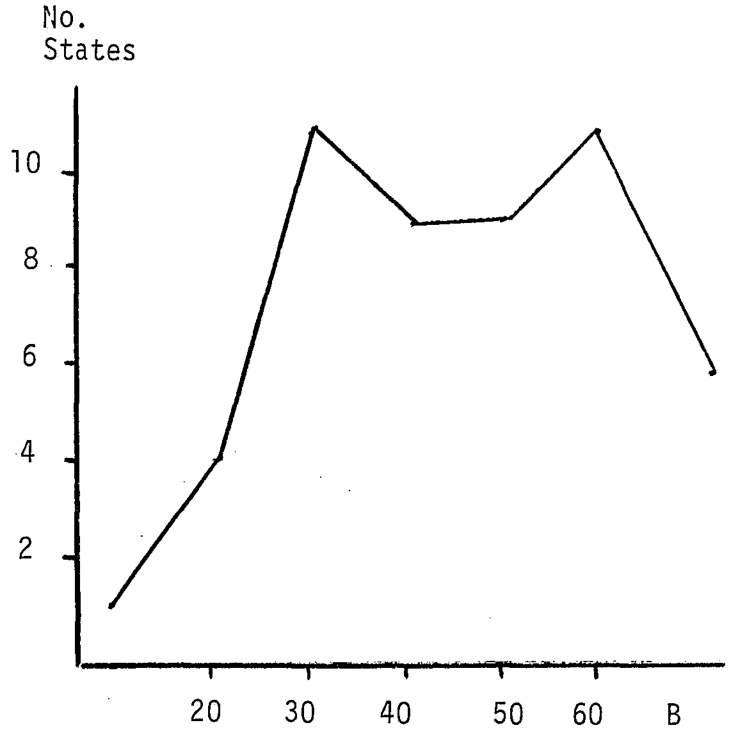

Figure 5. Distribution of B.

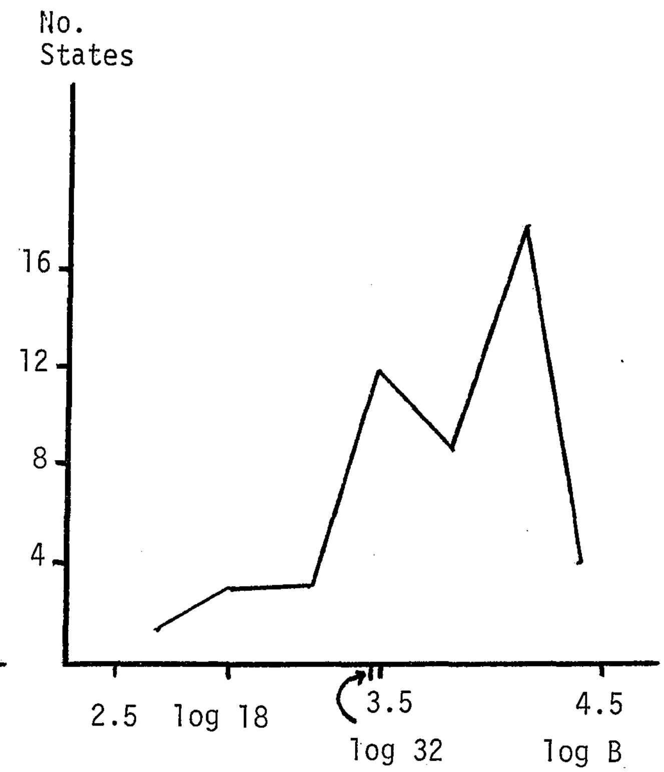

Figure 6. Distribution of log B.

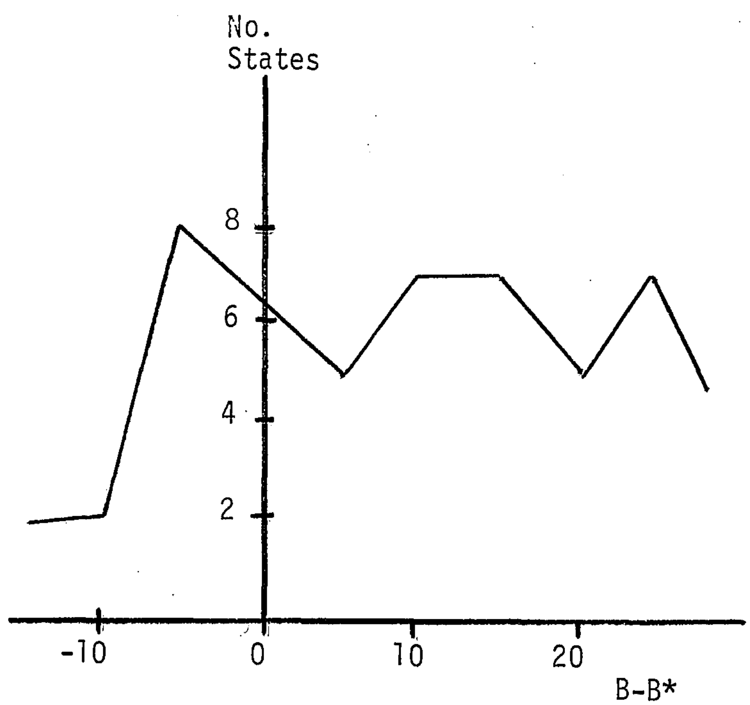

Figure 7. Distribution of (B-B*).

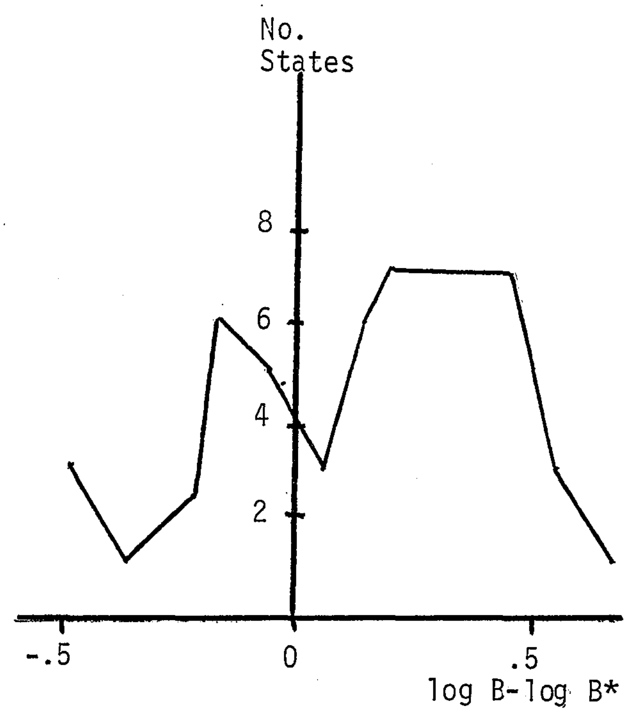

Figure 8. Distribution of (log B-log B*).

Implementation of the model discussed in the last section to the AFDC case is straightforward. With two convex kinks and one non-convex kink the probability statement for each observation is a combination of those shown in equations (5) and (6). It is given in detail in the Appendix.[9] As for functional forms of the utility function and demand function, I have chosen to test three different functional forms. Since different functional forms imply different restrictions on the form of price and income elasticities, the results of the estimation may be sensitive to the functional form assumed; hence this should be tested. It is assumed that the demand function is of one of the following three forms:

$$B = Z\alpha + \beta P + \delta M + \varepsilon_h \tag{7}$$

$$\ln B = Z\alpha + \beta P + \delta M + \varepsilon_h \tag{8}$$

$$\ln B = Z\alpha + \beta \ln P + \delta \ln M + \varepsilon_h \tag{9}$$

where Z is a set of exogenous variables in the model. In the linear equation (7), price and income elasticities vary inversely with the level of the benefit and positively with price and income. In equation (8), elasticities are independent of the level of B but again vary positively with price and income. In equation (9), elasticities are constant and independent of benefits, prices, and income. A disadvantage of these linear or quasi-linear forms is that the compensated substitution effect is not necessarily globally negative, an undesirable feature. But if the range of prices and incomes in the data fall in the permissible range, this is of little consequence. The equations are of interest in the first place because they are commonly used in studies of this type. For each of the functional forms of the three demand equations, the indirect utility functions can be derived by integrating up from the first order conditions for utility maximization. The utility functions are shown in the Appendix.

The independent variables included in the model are taken from those in the Orr study. They include the fraction of the caseload that is non-white, included on the hypothesis that the taxpayer's preferences for redistribution are affected by this fraction, and regional dummies for the Northeast, the West, the "old" South, and the "border" South.[10] The means and data sources of all variables used in the analysis are shown in Appendix Table B.1.

The results of estimating the linear model are shown in Table 1. The first column shows the results of estimating a simple, parsimonious specification in which no independent variables other than price and income are included in the benefit equation. The results show a price effect that is negative but of low significance, combined with a positive income effect of high significance. The mean price elasticity implied by the coefficient is approximately -.08, fairly low.[11] The elasticity with respect to the subsidy rate alone is .15 (Orr obtained .34). Consequently, an increase in the subsidy fraction of .10 would raise the benefit from its $45 mean by about $1. The income elasticity, on the other hand, is a large 1.57 at the mean.

The coefficient estimates in this model, as in any model (e.g., Tobit) in which coefficients on only a latent index are estimated, must be used with some caution. The coefficients are only those on a conditional demand function, as noted in the previous section, and cannot be used directly to make statements regarding the effects of changes in

Table 1

Maximum-Likelihood Estimates of Parameters of Linear Models

	(1)	(2)	(3)	(4)
Price effect (β)	-247.67* (1.36)	-271.01* (1.49)	-262.21* (1.34)	-378.76** (2.48)
Income effect (δ)	0.0193** (6.27)	0.0142** (4.58)	0.0140** (3.88)	0.0197** (5.65)
Fraction nonwhite	--	-14.45 (0.94)	-14.64 (0.88)	--
Northeast	--	5.57 (1.18)	5.26 (0.72)	
West	--	-6.56 (1.25)	-7.08 (1.19)	--
Old South	--	-14.85** (2.53)	-15.13** (2.50)	--
Border South	--	-10.50 (0.94)	-10.62 (0.93)	--
Flypaper effect (γ)	--	--	1.03* (1.38)	0.39 (.125)
Constant	-21.20** (1.98)	9.90 (0.98)	10.49 (0.84)	-15.38* (1.38)
σ_h	11.56** (5.80)	7.66** (5.25)	7.73** (4.56)	11.62** (6.81)
σ_r	2.86* (1.39)	3.08* (1.41)	3.08 (1.24)	1.84 (1.03)
Log Likelihood Function	-199.9	-181.6	-181.6	-198.2

Note: C = state AFDC caseload; N = state population; M = state per capita income.
Dependent variable = 1970 AFDC Benefit per recipient (mean = $45).
Mean (C/N) = .04
Mean M = $3712
Unsigned asymptotic t-statistics in parentheses
*Significant at the 20 percent level
**Significant at the 10 percent level

non-linear benefit formulas. Instead, the fitted utility function must also be used. These calculations are reported below.

The parameter estimate in column (1) for σ_h shows significant evidence of clustering in the data; that for σ_r shows some (though statistically weaker) evidence of random error. Again, however, the relative magnitudes of the two standard errors must be used with caution. The parameter σ_h represents the standard error on a latent index that is truncated at the convex kink points and dispersed from the non-convex kink point. As noted in the previous section, the variance of benefits will be lower than σ_h^2 in convex regions and higher in non-convex regions. The random error, on the other hand, is untruncated and hence translates directly into the benefit. Perhaps more relevent would be a demonstration of what the variance of benefits would be on the basis of the heterogeneity error and random error alone (see below).

The second column of the table shows the results of adding the independent variables noted above. The orders of magnitude of the price and income effects are unchanged. The extra variables themselves are often fairly large in magnitude and generally of the same sign as found in the Orr study, but are almost always low in significance. To a great extent this may be a result of the use of a single cross-section rather than a panel of cross-sections, which would increase efficiency and lower standard errors on the coefficients.

The parameters in the last two columns show the results of a test of the well-known "flypaper effect." The flypaper effect refers to studies that show that grant income has a larger effect on public expenditures than non-grant income (Gramlich, 1977; Inman, 1979). It should be stated at the outset that the concept of the flypaper effect is a bit cloudy in anything other than the block-grant case, for the amount of the grant is endogenous in all other grant forms and hence there is really no "effect" of the grant amount per se to compare with that of private income. It is clear, for example, that the effect of an increase in private income of an amount G will have smaller effects on expenditure than an open-end matching program that generates an equilibrium grant amount G for the same reasons that replacing an open-end matching grant with a block grant reduces expenditure (i.e., there are price effects). But more fundamentally in the piecewise-linear case, an increase in M has very different effects than an increase in subsidy rates, ceiling amounts, and other program parameters simply because they alter the budget constraint in very different ways. Consequently, finding a "flypaper effect" as it is usually estimated could be a result of the non-linearity of the budget constraint rather than of inherently different responses to different forms of income.

In the context of the present model, the flypaper hypothesis can be tested by allowing income from the federal AFDC subsidy to have different effects than M, the level of community income. Algebraically the budget constraint can be written:

$$M + \gamma S = (C/N)B + Y \tag{10}$$

where S is the grant amount and γ is the flypaper measure. It is equal to one if the two types of income are interchangeable and greater than one if grant income has a larger effect than private income. This specification of the flypaper effect is fundamentally ad hoc, for equation (10) obviously does not hold in dollar terms unless $\gamma = 1$. But it does lead to a demand equation of the type used in most tests of the flypaper effect, for the demand function implied by (10) has a net price equal to

$(C/N)(1 - \gamma s)$ and an imputed income variable (in the non-linear case) of $\tilde{M} = M + \gamma(C/N)B^{*}s$. Thus the specification simply implies that the income and price coefficients are different depending upon whether grants or private income is the source of the effect--that is, the parameter γ allows the coefficient on (C/N) and s, and on M and the imputed grant, to differ. Of course, a better approach would be to model some sort of flypaper effect formally, but that is difficult in this type of model.[12] The present specification should at least be sufficient to determine whether any such effect is present.

Surprisingly, as the results in column (3) indicate, the flypaper effect is estimated to be almost exactly zero! The parameter γ is estimated at 1.03, and its standard error easily encompasses 1.0. This is surprising enough that I have reestimated the simple model (i.e., without any other independent variables) to see if the basic price-income-alone model is providing this result. As column (4) indicates, the flypaper effect actually fell below zero in that exercise. Apparently, there is still no flypaper effect (except perhaps a negative one) in these results. These results provide some evidence for the hypothesis mentioned above--that past flypaper estimates could be caused by budget-constraint non-linearities.

Table 2 shows the results of estimating the model with different techniques and specifications. The first two columns show the results of estimating equation (2) above, the conditional demand function with no adjustment for the endogeneity of the price and income variables.[13] As the table indicates, the price coefficients are considerably overestimated relative to their counterparts in Table 1. Most of the states are on the fourth segment of the constraint, which has a lower price than

Table 2

Alternative Parameter Estimates

	OLS[a] (1)	OLS[a] (2)	Lagged Caseload	Log B	Iso-Elastic
β	-857.30** (3.53)	-448.2** (2.0)	-321.2 (1.2)	-5.19 (1.10)	-0.19** (1.79)
δ	0.026** (7.12)	0.016** (5.17)	0.014** (3.98)	0.00034** (4.38)	1.42** (3.77)
Fraction non-white	--	-3.13 (0.23)	-17.54 (1.05)	-0.59* (1.39)	-0.53* (1.40)
Northeast	--	4.67 (1.10)	4.26 (0.68)	0.06 (0.32)	0.79 (0.50)
West	--	-2.69 (0.58)	-7.73* (1.36)	-0.20* (1.33)	-0.16 (1.20)
Old South	--	-19.80 (3.67)	-14.98 (2.39)	-0.37** (2.53)	-0.36** (2.77)
Border South	--	-12.50** (2.21)	-10.98 (1.01)	-0.17 (0.83)	-0.15 (0.72)
γ	--	--	1.28 (0.95)	0.59 (1.22)	0.83* (1.57)
Constant	-35.4** (3.1)	3.27 (0.29)	10.58 (0.95)	3.04** (10.04)	-8.30** (2.77)
σ_h	--	--	7.58** (3.94)	0.20** (6.26)	0.17** (3.37)
σ_r	--	--	3.44 (1.44)	0.04 (0.98)	0.09 (1.09)

Note: Unsigned t-statistics in parentheses.

[a]Segment observations only. Deleted are observations within $2 of a kink.
*Significant at 20 percent level.
**Significant at 10 percent level.

segments two or three (where all but two or three of the other states are located). Consequently, a spurious negative correlation between benefits and price is introduced into the equation.

The third column shows the results of using the lagged AFDC caseload (1968) in the price variable instead of the current caseload. There has been some discussion in the literature of the possibility that the caseload is endogenous, inasmuch as participation in AFDC in a function of the benefit level.[14] The results in the table indeed show a somewhat stronger price effect when the lagged caseload is used. However, since caseloads were rising extremely rapidly in this period for reasons not entirely, or even mostly, related to the benefit level, this result is not as strong as it might appear. The lower caseload in 1968 explains most of the difference in the price coefficients just as a scaling difference.

The final two columns show the results of estimating the other two functional forms of the demand equation. The results are quite similar to those obtained in Table 1. Using the logarithm of the benefit as the dependent variable generates parameters with the same sign and general significance level as those using the simple benefit as the dependent variable. The price and income elasticities in the first equation in the table are -.09 and 1.27, respectively, quite close to those obtained previously. The iso-elastic equation in the last column generates a higher price elasticity (0.19) but still relatively low in significance. The income elasticity is 1.42, again similar to that obtained in the previous equations. The signs and significance levels of the other independent variables are also similar. These results are important, for they indicate that our findings are fairly robust with respect to the functional form of the utility function.

Simulations. As noted previously, the parameters of the conditional demand functions reported in Tables 1 and 2 cannot be directly translated into benefit effects because of the non-linearity of the model, which arises because changes in segment location are possible when any of the independent variables change. However, the mean and standard deviation of the benefit can be obtained by calculating the first and second moments of the distribution of benefits over any arbitrarily complex budget constraint (see the Appendix). The results of several such calculations are shown in Table 3. The linear-model results in column (2) of Table 1 are used for the simulation.[15]

The first line of the table shows that the mean AFDC benefit in the absence of any federal matching would be about $40, with a standard deviation of $8.26. Under the existing system the mean benefit is $45, implying that federal matching raises the benefit by about 11 percent.[16] However, the standard deviation of benefits is also increased by the federal matching formula. As noted previously, convex kinks in a formula act to reduce the variance of benefits and non-convex kinks act to increase it; since so many more states are on the fourth segment and are affected by the non-convex kink than are on the second or third segments and affected by the convex kink, the variance-increasing effect dominates. Thus the federal matching formula has the paradoxical effect of increasing interstate inequality in AFDC benefit levels.

The table next shows the standard deviation of benefits that would arise if there were no random error. Since this is close to that under the existing system, the implication of the untruncated variances that

Table 3

Simulation of Alternative Grant Formulas[a]

	Mean Benefit	Standard Deviation of Benefit
No Federal Grant	\$39.88	\$8.26
Existing System	45.52	9.46
Existing System with No Random Error (σ_r=0)	45.52	8.95
Marginal Changes in Existing System:		
s_f = .833	43.68	10.14
s_3 = .10[b]	43.81	9.94
B_2 = 34[c]	45.28	9.49
Reduction in s_m of .10	42.57	9.71
Non-Marginal Changes in Existing System:		
Elimination of Medicaid Formula	40.39	7.78
Elimination of Non-Medicaid Formula	46.52	8.26
New Systems:		
Open-end Matching at .50	45.30	8.26
Block Grant at Current Mean Benefit	39.89	8.26
Graduated Convex Schedule[d]	45.27	7.58
100-Percent Matching up to 65-Percent of Poverty Line[e]	48.82	6.57
100-Percent Matching up to 50-Percent of Poverty Line[e]	43.17	7.63

[a]All calculations performed using mean values of the exogenous variables.
[b]The matching rate is s_3 in the third segment, currently equal to zero (see Figure 3).
[c]The benefit is B_2 at the second kink, currently equal to 32 (see Figure 3).
[d]Schedule assumes that matching rate begins at .90 over first \$10 of B and falls by .10 in each succeeding \$10 benefit interval.
[e]In 1970 the annual poverty income for a four-person family was \$3968. On a per-recipient, monthly basis this is \$82. Sixty-five percent of this is \$53 and 50 percent is \$41.

heterogeneity is much more important than random error is confirmed. The next four rows show the effects of marginal changes in the existing system. Increasing the federal matching rate in the second segment to .833 (the rate in the first segment) actually lowers the mean benefit level. This is an example of the perverse price effects illustrated by the shift in Figure 4, arising because a significant number of states would move from segment four to segment three. Likewise, an increase in the matching rate in the third segment from zero to .10 would result in a reduction in the mean benefit, as would an increase in the benefit level at which matching ends (B_2). Thus a number of changes in the formula which ostensibly improve the generosity of matching would actually reduce benefits. This is summarized in one of the "non-marginal changes" shown in the table: complete elimination of the non-Medicaid portion of the grant formula would have the overall effect of raising the mean benefit level.

Lowering the Medicaid subsidy rate or elimination of the Medicaid formula would, as should be expected, lower the mean benefit. The latter would also reduce the inequality of benefits, inasmuch as the non-convex region of the constraint would be eliminated and only a convex piecewise-linear constraint would remain. The table also shows the effects of a number of new systems. Open-end matching at 50 percent would leave the mean benefit approximately unchanged, but would lower the standard deviation of benefits. A block grant is shown to reduce the mean benefit down to what it would be in the absence of matching altogether. Block grants always have this direction of effect, since price effects are eliminated, but the magnitude of the reduction here arises because the marginal share of state income devoted to AFDC is small. According to Table 1, about $.014 of every extra state dollar goes into AFDC benefits, implying that \$.986 of every block grant dollar would go into non-AFCC expenditures. This effect would be reduced if there were a flypaper effect, but none has been found here, as already discussed.

A graduated convex schedule could be constructed which would leave the mean benefit about the same, but would reduce the standard deviation of benefits; this is shown in the table as well. A special case of such a convex schedule is the two-segment schedule generated by 100-percent matching up to some specified benefit level, followed by zero matching thereafter, i.e., a closed-end grant. The effect of such minimum-benefit program would depend upon the level of the minimum benefit. If it were set relatively low, at 50 percent of the poverty line, the mean benefit would decline; if set higher, at 65 percent of the poverty line, the mean benefit would increase.[17]

III. CONCLUSIONS

The analysis in this paper has developed econometric techniques for properly estimating state and local expenditure equations when the grant formulas imposed by grant-in-aid programs create non-linearities in the budget constraint. The appropriate maximum-likelihood procedure was applied to the U.S. AFDC program. The paper also demonstrated the power of the new technique by illustrating the manner in which the response to any arbitrarily complex and non-linear grant-in-aid formula can be predicted. Among the more notable conclusions from the simulations here were (a) that the present federal AFDC grant formula actually increases, rather than decreases, cross-state inequality in state AFDC benefits; (b) that wholesale elimination of the subsidy rates in part of the federal

AFDC formula would actually increase the average national AFDC benefit; and (c) that a federal formula with steadily decreasing subsidy rates instead of the present one could successfully reduce cross-state inequality of benefits. These conclusions would have been difficult, if not impossible, to draw from the models estimated in previous studies.

Suggestions for new research are many. Substantively, for the case of AFDC, an examination of data more recent than 1970 would be of interest. More recent results would be of use in discussing recent federal block grant proposals (Chernick, 1982). In addition, the incorporation of food stamps into the model would be of interest, since there has been some discussion of the possibility that food stamps displace AFDC in state utility functions (Orr, 1979; Gramlich, 1982). Also, of course, it would be of interest to apply these techniques to other grant-in-aid programs. Methodologically, the techniques demonstrated in this paper should be useful in many other areas of public finance, wherever private agents face piecewise-linear budget constraints imposed by a higher level of government. This includes the impact of virtually all tax schedules, but also the effects of balanced-budget amendments and tax and expenditure limitation restrictions, both of which create convex kinks in the public sector's choice set.

APPENDIX

Derivation of the Likelihood Function

First consider the linear case. Let the demand function along a linear segment with price P_i and (possibly imputed) income M_i and its associated indirect utility function be:

$$B^d = g(P_i, M_i) + \varepsilon_h$$

$$V(P_i, M_i, \varepsilon_h) = U[g(P_i, M_i) + \varepsilon_h, M_i - (C/N)(g(P_i, M_i) + \varepsilon_h)]$$

where $U(B,Y)$ is the direct utility function and where B^d is demand without the random error term ε_r (i.e., "desired" demand). Now refer to Figure A.1, which shows diagrammatically the values of ε_h that delineate B^d into different segments and kinks. The diagram uses the function $k(i,B) = B - g(P_i, M_i)$, which is the value of ε_h that would make the desired benefit along segment i equal to the value B. The diagram also uses the function $m(i,j)$, which is the value of ε_h which equilibrates utility along two segments i and j, defined implicitly by:

$$V[P_i, M_i, m(i,j)] = V[P_j, M_j, m(i,j)]$$

Then the function describing the choice of B^d is:[18]

$$\begin{aligned}
B^d &= g(P_1, M_1) + \varepsilon_h && \text{if} \quad \varepsilon_h \leq k(1,18) \\
&= 18 && \text{if} \quad k(1,18) < \varepsilon_h \leq k(2,18) \\
&= g(P_2, M_2) + \varepsilon_h && \text{if} \quad k(2,18) < \varepsilon_h \leq k(2,32) \\
&= 32 && \text{if} \quad k(2,32) < \varepsilon_h \leq k(3,32)
\end{aligned}$$

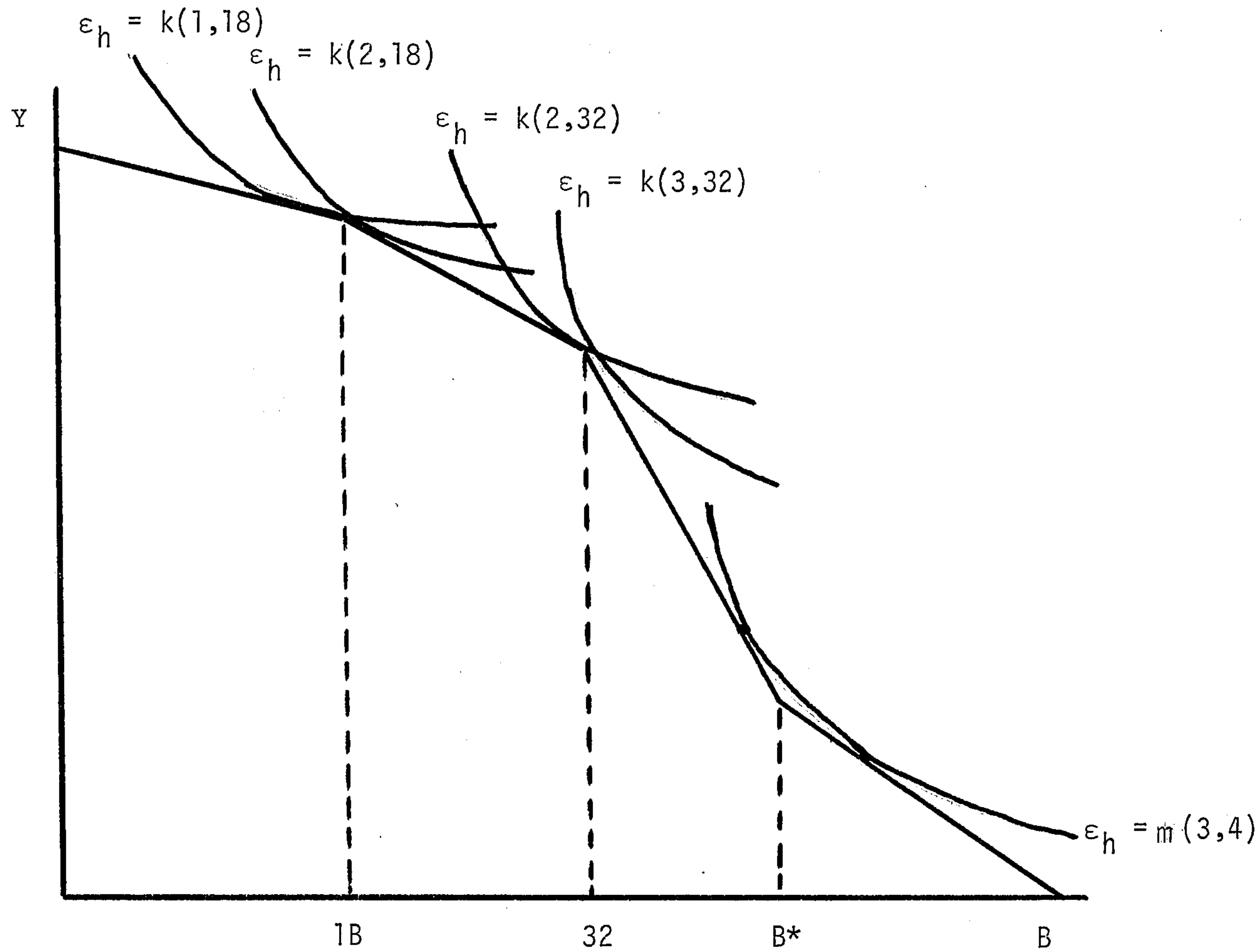

Figure A1. Boundary Indifference Curves.

$$= g(P_3,M_3) + \varepsilon_h \qquad \text{if} \qquad k(3,32) < \varepsilon_h \leq m(3,4)$$

$$= g(P_4,M_4) + \varepsilon_h \qquad \text{if} \qquad m(3,4) < \varepsilon_h$$

The observed benefit is $B = B^d + \varepsilon_r$. Letting ε_h and ε_r be distributed normally and independently with means zero and respective variances σ_h^2 and σ_r^2, the probability density of a given value of B is:

$$\begin{aligned}
\text{Prob }(B) = {} & \text{Prob}[\varepsilon_h + \varepsilon_r = k(1,B),\ \varepsilon_h \leq k(1,18)] \\
& + \text{Prob}[\varepsilon_r = B\text{-}18,\ k(1,18) < \varepsilon_h \leq k(2,18)] \\
& + \text{Prob}[\varepsilon_h + \varepsilon_r = k(2,B),\ k(2,18) < \varepsilon_h \leqslant k(2,32)] \\
& + \text{Prob}[\varepsilon_r = B\text{-}32,\ k(2,32) < \varepsilon_h \leqslant k(3,32)] \\
& + \text{Prob}[\varepsilon_h + \varepsilon_r = k(3,B),\ k(3,32) < \varepsilon_h \leqslant m(3,4)] \\
& + \text{Prob}[\varepsilon_h + \varepsilon_r = k(4,B), m(3,4) < \varepsilon_h] \\
& = F[y_1(1,18)]f(z_1)/\sigma_w + \{F[u(2,18)] - F(u(1,18)]\}f(v_{18})/\sigma_r \\
& + \{F[y_2(2,32)] - F[y_2(2,18)]\}f(z_2)/\sigma_w \\
& + \{F[u(3,32)] - F[u(2,32)]\}f(v_{32})/\sigma_r \\
& + \{F[y_3(3,4)] - F[y_3(3,32)]\}f(z_3)/\sigma_w \\
& + \{1 - F[y_4(3,4)]\}f(z_4)/\sigma_w
\end{aligned}$$

where f and F are the unit normal density and distribution functions, respectively, and where

$$\sigma_w^2 = \sigma_h^2 + \sigma_r^2$$

$$z_i = k(i,B)/\sigma_w$$

$$u(i,j) = k(i,j)/\sigma_h \quad \text{if} \quad j = 18,32$$

$$= m(i,j)/\sigma_h \quad \text{if} \quad j = 4$$

$$y_i(j,j') = [u(j,j') - \rho z_i]/(1-\rho^2)^{1/2}$$

$$\rho = \sigma_h/\sigma_w$$

$$v_i = (B-i)/\sigma_r$$

Thus the probability of observing a value of B is the sum of the joint probabilities that ε_h lies in the range needed to make each segment or kink "desired" and that ε_r moves the agent to the observed value at B. Probabilities denoting desired kink locations can be factored into separate ε_h and ε_r probabilities (since they are assumed independent)--see the second and fourth terms in the sum--and the rest of the bivariate probabilities can be factored into two univariate probabilities, one conditional (F(y)) and one unconditional (f(z)). This factorization uses common Gaussian formulas (see Maddala, 1977, p. 451).

A few additional technical notes are necessary. First, there is a possibility that segment 3 will be skipped altogether, which would occur if the value of ε_h equating maximum utility on segment 4 and utility at B = 32 were smaller than m(3,4). This occurs for a few states for whom segment 3 is very small. For this group the probability of locating on segment 3 does not appear, and the probabilities of desiring to locate at $B = 32$ and on segment 4 are, respectively, the probabilities that ε_h is less than and greater than n(32,4), where n(i,j) is the value of ε_h equating $U(B_i,Y_i)$ and $V(P_j,M_j)$.[19] Second, for Arizona, no segment four is present. The above probability is somewhat simpler in that case, but is not presented here for brevity. Also, for five states B^* lies to the left of 32, eliminating the second kink and third segment from possible choice. The probability also simplifies somewhat in that case, but is again not presented here. The log likelihood function for the sample is the sum of the logs of these probabilities.

For functional forms, in the linear model we have:

$$g(P_i,M_i) = Z\alpha + \beta P_i + \delta M_i$$

$$\log V(P_i,M_i,\varepsilon_h) = \delta P_i - \ln[-(\beta + \delta(g(P_i,M_i) + \varepsilon_h))]$$

In the second two models the error terms are assumed to be log-normally distributed and the benefit amounts in all of the above formulas are replaced by their logarithms. The indirect utility functions in those cases are:

$$\log V(P_i,M_i\varepsilon_h) = -\delta M + \ln[-(\beta + \delta\hat{B})]$$

$$V(P_i,M_i,\varepsilon_h) = [M^{1-\delta}/(1-\delta)] - [P^{1+\beta}/(1+\beta)]\exp[Z\alpha + \varepsilon_h)$$

where $\hat{B} = \exp(Z\alpha + \beta P_i + \delta M_i + \varepsilon_h)$.

<u>Simulations</u>. To simulate the benefit of a particular state to an arbitrary budget constraint, the expected value of B must be calculated.

If there are m kinks j and n segments i along a constraint, the expected value of B in general form is:

$$E(B) = \sum_{i=1}^{n} \text{Prob } (e_{i-1} < \varepsilon_h \leq e_i)[g(P_i,M_i) + E(\varepsilon_h \mid e_{i-1} < \varepsilon_h < e_i)]$$

$$+ \sum_{j=1}^{m} \text{Prob}(e_{j-1} < \varepsilon_h \leq e_j)B_j,$$

where B_j is the value of the benefit at kink j and where the e_i and e_j are the values of ε_h separating the different constraint location choices. Thus E(B) is just a weighted average of the kink values and the conditional means of the truncated distributions on each segment. The formulas for these conditional means are not written out, to save space. Likewise, the variance of the benefit is not written out--it is just a weighted average of the conditional variances of the truncated distributions over the constraint.

Table B.1

Means and Standard Deviations of the Variables Used in the Analysis

	Mean	Standard Deviation
Dependent Variable:		
B	45.7	16.5
Price Variables:[a]		
C/N	0.040	0.015
s_f	0.569	0.068
s_m	0.613	0.102
P_1[b]	0.007	0.002
P_2[c]	0.017	0.007
P_4[d]	0.016	0.008
Income Variables:[e]		
M	3711.8	615.2
M_2[f]	3712.0	615.3
M_3[g]	3712.7	615.2
Other Variables:		
Fraction nonwhite	0.470	0.210
Northeast	0.176	0.385
West	0.255	0.440
Old South	0.216	0.415
Border South	0.118	0.325

[a]Note that P_3 = C/N (caseload/population ratio)
[b]Note that $P_1 = (C/N)(1-.833)$
[c]Note that $P_2 = (C/N)(1-s_f)$
[d]Note that $P_4 = (C/N)(1-s_m)$
[e]Note that $M_1 = M_4 = M$
[f]Note that $M_2 = M + (C/N)18(.833-s_f)$
[g]Note that $M_3 = M + (C/N)[18(.833) + 14(s_f)]$

(Table continues)

Data Sources for Table B.1:

B: Average payment per recipient, December 1970. *Annual Statistical Supplement of the Social Security Bulletin*, 1970.

C: Number of AFDC recipients, December 1970. *Social Security Bulletin*, May 1971.

N: State population, 1970: *Statistical Abstract of the United States*, 1971, p. 12.

s: Matching rates. *Characteristics of State AFDC Plans*, 1971.

M: Per capita income, 1970. *Statistical Abstract of the United States*, 1972, p. 319

Fraction nonwhite: *Findings of the 1971 AFDC Study*, Part I, Table 2.

Footnotes

[1]Some governments initially above X* will locate on the first segment; other will move to the (new) kink; and some governments initially at the kink will stay at the kink and others will locate on segment 1. See Moffitt and Nicholson (1982) for a diagrammatic analysis of such shifts.

[2]Statistically speaking, of course, both errors are "random." But the first is random only to the analyst; it is presumed known to the government.

[3]It can be shown (Moffitt, 1982a) that the utility condition for picking the different segments and kinks shown in this equation is equivalent to a formal statement in terms of the utility function.

[4]The form of the probabilities shows that the system is very similar to that of a switching regression, whose specification and estimation are discussed by Heckman (1978) and Lee (1979). Here we have a two-equation system in which one of the endogenous variables is dichotomous and hence non-linear in its parameters. Maximum-likelihood is necessary because no linear reduced form can be obtained.

[5]See also Collins (1967), Gramlich (1982), Sloan (1977), and Tresch (1975).

[6]This has been recently demonstrated in the smoothly non-linear case (Moffitt, 1982b).

[7]One of the curious features of state behavior is that not all states immediately locate on the higher budget constraint. There is some tendency for states whose benefits have just risen past B* to stay under the

old formula temporarily, switching only eventually to the Medicaid formula. See the discussion by Spall (1978) and Orr (1978).

[8]This may be a result of the phenomenon discussed in note 7.

[9]As noted in note 7, not all states are located on the envelope of the two budget constraints. Most who are not, however, are not far from B*. In the work reported here, all states are assumed to be on the envelope. The fact that some are not is less serious in this model than in some others, for the actual segment location of a state is never used in the analysis. This follows from the presence of the random error term, which moves an observation away from its estimated utility-maximizing point. In the presence of non-envelope-locations, this error term will also capture non-utility-maximizing choices of such a type.

[10]Orr included the number of recipients in his equation on the assumption that taxpayers should obtain more utility, the greater the number of individuals assisted by AFDC. Such a variable does not enter the model here because taxpayer utility is a function only of B, the average benefit to all those "in need," rather than of the total amount of benefits transferred. The utility function U(BC,Y) would not generate the Orr hypothesis, since in such a utility function a higher level of C implies a lower marginal utility of B at any given benefit level, implying a smaller preferred benefit. Benefits and caseloads are substitutes. However, the Orr hypothesis would be generated by the utility function CU(B) + U(Y). In any case, it is not obvious that taxpayer utility is related to the number of recipients, since this would imply increases in utility if the number of individuals in poverty increases.

[11]Since most states are on segment four, I have used its subsidy rate in this calculation. The mean price $(C/N)(1-s_m)$ in the sample is .017.

[12]For example, there is no obvious way other than this to allow the utility of grant income to differ from the utility of non-grant income. The utility function includes expenditures, not income, and it is difficult to see how expenditures themselves have any inherently different utility if they are financed one way than another. The specification here implicitly assumes such, however, for the implicit utility function U(B,Y) contains, if Y were substituted in from equation (10), B and γ.

[13]Observations at a kink, defined as being within \$2 of any of the three kinks, were eliminated.

[14]See Orr (1978), Southwick (1978).

[15]The first several rows of Table 3 were also generated with the two alternative, logarithmic demand equations. The results were quite similar in all qualitative respects, although the magnitudes were somewhat smaller. In particular, all the "perverse" effects discussed below also are found in these alternative simulations.

[16]The \$5-\$6 increase is about 20 percent of the mean federal subsidy amount. Hence about four-fifths of the grant is substituted into non-AFDC expenditures.

[17]This conclusion differs from that of Orr (1976, pp. 366-367), who simulated the effect of a federal minimum benefit by just assuming that the marginal subsidy rate falls to zero. This results in an overestimate of the benefit reduction because may states would not reduce benefits below the kink.

[18]The slope of an indifference curve at any point (B*,Y*)--equal to $\frac{\partial U}{\partial B}(B^*,Y^*)/\frac{\partial U}{\partial Y}(B^*,Y^*)$--is positively related to ε_h. Hence low values of ε_h correspond to very flat indifference curves and high values correspond

to very steep curves. In Figure A.1 the curves can be obtained by starting at the upper left, with low values of ε_h (flat indifference curves), and pivoting the curves to make them steeper.

[19]There is no closed-form expression for these values, so they must be solved for numerically.

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