Edgar Olsen
Scott Bierman

SELECTION BIAS IN THE
ESTIMATION OF BENEFITS OF
IN-KIND TRANSFERS

DP #711-82
Selection Bias in the Estimation of Benefits of In-Kind Transfers

Edgar O. Olsen
Department of Economics
University of Wisconsin-Madison

Scott Bierman
Department of Economics
Carleton College

October 1982

This research was supported in part by funds granted to the Institute for Research on Poverty at the University of Wisconsin-Madison by the Department of Health and Human Services pursuant to the provisions of the Economic Opportunity Act of 1964.
ABSTRACT

In recent years information on individual households has been increasingly used to estimate sophisticated measures of the net benefit of government programs. Any good measure will depend in part on the preferences of affected households. Since many programs confront households with all-or-nothing choices or create kinks in their budget frontiers, the consumption patterns of households under these programs often provide little information about their preferences.

The typical approach to benefit estimation used in these recent studies is to (1) identify families who can be presumed to face the usual linear budget constraint, (2) divide these families into groups according to their characteristics such as family size and race, (3) posit a particular functional form for the indifference map of families of each type, (4) estimate its parameters by estimating the parameters of the implied system of demand equations, and (5) use the estimated indifference map for families of each type to estimate the net benefit of the program to similar households participating in it. The indifference-map parameters estimated are the means of the parameters for households of each type.

One problem with this procedure is that households that participate in a government program may have different tastes from households that do not participate. It is usually argued that the forementioned approach will typically lead to an underestimate of benefit to the recipients of an in-kind transfer because they have stronger than average tastes for the subsidized good. Our paper shows that this conclusion is not
warranted for a program which has an upper income limit for eligibility or which does not serve all eligible families willing to participate, and it estimates the direction and magnitude of selection bias for such a program. We find that the standard approach will underestimate the mean benefit of the public housing program by less than 5 percent.
Selection Bias in the Estimation of Benefits of In-Kind Transfers

In recent years information on individual households has been increasingly used to estimate sophisticated measures of the net benefit of government programs, for example, the unrestricted cash transfer that would be as satisfactory to a household as the program (DeSalvo 1975; Rosen 1978; Love 1978). Any good measure of the net benefit of a government program will depend in part on the preferences of affected households. Since many government programs confront households with all-or-nothing choices or create kinks in their budget frontiers, the consumption patterns of households under these programs often provide little information about their preferences.¹

The typical approach used in these recent studies is to (1) identify families who can be presumed to face the usual linear budget constraint, (2) divide these families into groups according to their characteristics, such as family size and race, (3) posit a particular functional form for the indifference map of families of each type, (4) estimate its parameters by estimating the parameters of the implied system of demand equations, and (5) use the estimated indifference map for families of each type to estimate the net benefit of the program to similar households participating in it. The parameters estimated are the means of the parameters for households of each type.

One problem with this procedure is that households that participate in a government program may have different tastes from households that do not participate.² The direction of the bias is clearest in the case of a per-unit subsidy for a particular good where participation is voluntary.
and funding is available to serve all eligible families. Participation rates will be highest for households with the strongest taste for the subsidized good.

In this paper we investigate the magnitude of selection bias in the estimation of the mean benefit of the public housing program, which has been the largest program of housing subsidies to low-income families in the United States for fifty years. The first section explains the estimation of mean benefit using the standard approach; the second presents the usual objection to this approach based on selection bias and the defects of this objection; the third explains the estimation of mean benefit accounting for selection bias. The results of the two approaches are compared in the concluding section.

I. STANDARD APPROACH TO BENEFIT ESTIMATION

Consider a family with income $Y$, which consumes housing and other goods at prices $P(h)$ and $P(x)$, respectively. Its initial position is shown in Figure 1. The budget line has intercepts $Y/P(x)$ and $Y/P(h)$, and the family consumes at point $m$. The option of participating in the public housing program adds one point to the family's budget space, for example, point $g$. The family has the option of renting a specific apartment for an amount below its market rent. A way of measuring the benefit from living in a public housing unit is to ask the question, What is the increase in income the family would need in order to gain the same level of utility which it enjoys as a participant in the public housing program? In Figure 1, the unrestricted cash grant necessary to attain the subsidized utility level is $B$. 
This benefit obviously depends upon the family's preferences, and so it is necessary to specify the functional form of its indifference map in order to estimate benefit. For this study we assume that each household has preferences that can be represented by a Stone-Geary, or displaced Cobb-Douglas, utility function, which for the \( i \)th family is

\[
U(i) = [Q(h,i) - \alpha(h,i)]\gamma(h,i)[Q(x,i) - \beta(x,i)]^{1-\gamma(h,i)},
\]

(1)

where \( Q(h,i) \) and \( Q(x,i) \) are the quantities of housing and other goods consumed by the \( i \)th household, and \( \gamma(h,i) \), \( \alpha(h,i) \), and \( \beta(x,i) \) are its indifference-map parameters. Notice that we allow the parameters of the indifference map to be different for different families. The benefit, \( B(i) \), from participating in public housing for this family is

\[
B(i) = \frac{[P(h)Q(h,g,i) - P(h)S(h,i)]}{\gamma(h,i)}\frac{[P(x)Q(x,g,i) - P(x)S(x,i)]}{[1-\gamma(h,i)]}^{1-\gamma(h,i)} + P(h)\beta(h,i)
\]

\[
+ P(x)\beta(x,i) - Y(i),
\]

(2)

where \( [Q(h,g,i), Q(x,g,i)] \) is the \( i \)th family's consumption bundle under the program.\(^3\) Since our data are for one market area at one point in time, we assume that all consumers face the same vector of market prices and so these prices do not have an \( i \) subscript.

In order to use this formula, we need the market rent of the public housing unit \( P(h)Q(h,g,i) \), expenditures on all other goods when living in public housing \( P(x)Q(x,g,i) \), estimates of the parameters of the utility function, and income.\(^4\) To estimate these magnitudes, we use data for individual families and housing units from the 1965 New York City Housing
and Vacancy Survey. This survey was conducted by the U.S. Bureau of the Census, and the data are of the sort collected in the Decennial Census of Population and Housing except that information was collected on whether the housing unit was in a public housing project or subject to rent control.

The market rents of public housing units are not observed. However, they can be predicted by estimating a statistical relationship between rent and characteristics of dwelling units using data for unsubsidized rental housing and then substituting the characteristics of each public housing unit into this estimated equation.

The survey contained information on about 10,000 unsubsidized rental dwellings. Since the Housing Authority did not provide furniture for its units, we excluded from our sample of private dwellings those for which the landlord provides furniture. Since no public housing units were in structures with one or two units, we excluded these units. We also excluded units with more than seven rooms, because the number of rooms was not reported in these cases and almost no public housing units were this large. The results in Table 1 were obtained using the remaining uncontrolled private rental dwellings for which all variables involved in the regression were reported. This equation was used to predict the market rent of each public housing unit in our sample.

Expenditures on all other goods under the public housing program may be directly calculated by subtracting the actual rent paid by the family for its public housing unit from its income.\textsuperscript{5}
Table 1

Estimated Relationships Between Annual Gross Rent per Room and Housing Characteristics

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Description of Regressors</th>
<th>Coefficients (Standard Errors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>Inverse of the number of rooms</td>
<td>1,023.88 (23.27)</td>
</tr>
<tr>
<td>$X_2$</td>
<td>1 if dwelling built in 1960-1965; 0 otherwise</td>
<td>110.92 (11.15)</td>
</tr>
<tr>
<td>$X_3$</td>
<td>1 if dwelling built in 1947-1959; 0 otherwise</td>
<td>1.89 (10.12)</td>
</tr>
<tr>
<td>($X_2 = X_3 = 0$ if dwelling built prior to 1947)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_4$</td>
<td>1 if condition of unit is sound; 0 otherwise</td>
<td>189.51 (34.62)</td>
</tr>
<tr>
<td>$X_5$</td>
<td>1 if condition of unit is deteriorating; 0 otherwise</td>
<td>110.69 (36.51)</td>
</tr>
<tr>
<td>($X_4 = X_5 = 0$ if condition of unit is dilapidated)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_6$</td>
<td>1 if dwelling located in Queens; 0 otherwise</td>
<td>-168.65 (8.29)</td>
</tr>
<tr>
<td>$X_7$</td>
<td>1 if dwelling located in Bronx; 0 otherwise</td>
<td>-195.31 (10.28)</td>
</tr>
<tr>
<td>$X_8$</td>
<td>1 if dwelling located in Brooklyn; 0 otherwise</td>
<td>-217.70 (8.89)</td>
</tr>
<tr>
<td>$X_9$</td>
<td>1 if dwelling located in Richmond; 0 otherwise</td>
<td>-180.55 (49.67)</td>
</tr>
<tr>
<td>($X_6 = X_7 = X_8 = X_9 = 0$ if dwelling located in Manhattan)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{10}$</td>
<td>Story of unit if it is less than 7;</td>
<td>-22.88 (4.56)</td>
</tr>
<tr>
<td>$X_{11}$</td>
<td>$X_{10} \times \text{ELEV}$ where ELEV is 1 if building has an elevator and 0 otherwise</td>
<td>32.02 (3.79)</td>
</tr>
</tbody>
</table>

(table continues)
Table 1 (cont.)
Estimated Relationships Between Annual Gross Rent per Room and Housing Characteristics

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Description of Regressors</th>
<th>Coefficients (Standard Errors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{12} )</td>
<td>1 if story of unit is 7 or greater; 0 otherwise</td>
<td>138.54 (11.18)</td>
</tr>
<tr>
<td>( x_{13} )</td>
<td>Proportion of rooms which are bedrooms</td>
<td>74.60 (24.82)</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>102.35</td>
</tr>
<tr>
<td>Coefficient of determination</td>
<td></td>
<td>.61</td>
</tr>
<tr>
<td>Standard error</td>
<td></td>
<td>187.40</td>
</tr>
<tr>
<td>Number of observations</td>
<td></td>
<td>4,260</td>
</tr>
</tbody>
</table>
The standard approach to estimating the indifference maps of public housing tenants involves the assumption that each family has an indifference map that is typical of similar unsubsidized families. A family with a Stone-Geary indifference map and paying market prices for all goods will spend a fraction

\[\frac{P(h)Q(h,i)/Y(i)}{Y(i)} = y(h,i) + [1-y(h,i)]\beta(h,i)\left[\frac{P(h)}{Y(i)}\right]
- y(h,i)\beta(x,i)\left[\frac{P(x)}{Y(i)}\right]\] (3)

of its income on housing. Since the utility function is not defined for \(Q(h,i) < \beta(h,i)\) or \(Q(x,i) < \beta(x,i)\), a positive displacement parameter is usually interpreted as the subsistence quantity of the good. We assume that all families of the same size have the same displacement parameters \(\beta(h)\) and \(\beta(x)\) but different marginal propensities to spend on housing \(y(h,i)\). Let \(y(h)\) be the mean of the \(y(h,i)\) for all families of a particular size. Then, for any family of this size, \(y(h,i)\) can be written as the sum of \(y(h)\) and some new variable \(w(i)\) which has mean zero, and equation (3) can be rewritten as

\[\frac{P(h)Q(h,i)/Y(i)}{Y(i)} = y(h) + [1-y(h)]\beta(h)\left[\frac{P(h)}{Y(i)}\right]
- y(h)\beta(x)\left[\frac{P(x)}{Y(i)}\right]
+ \left[1-[P(h)\beta(h) + P(x)\beta(x)\left[\frac{1}{Y(i)}\right]]\right]w(i).\] (4)

Since we assume that all consumers face the same vector of market prices, we cannot estimate the parameters \(y(h), \beta(h),\) and \(\beta(x)\) in a straightforward way. So we proceed in two steps.
First, we rewrite equation (4) as

$$P(h)Q(h,i)/Y(i) = \gamma(h) + \alpha(h)[1/Y(i)] + u(i), \quad (5)$$

where

$$\alpha(h) = [1-\gamma(h)]\beta(h)P(h) - \gamma(h)\beta(x)P(x)$$

and

$$u(i) = \{1-[P(h)\beta(h) + P(x)\beta(x)][1/Y(i)]\}w(i).$$

We assume that the random variables $Y(i)$ and $w(i)$ are independent. This implies that $1/Y(i)$ and $u(i)$ are uncorrelated and hence that the OLS estimators are consistent.

Equation (6) reports the OLS estimates based on the 1,422 two-person households in the sample living in unfurnished uncontrolled private rental housing for which the values of the required variables are reported.

$$P(h)Q(h,i)/Y(i) = .0787 + 1213.32[1/Y(i)] \quad R^2 = .69 \quad (6)$$

The numbers in parentheses are standard errors.

The second step in estimating the parameters of the indifference map involves using the estimate of $\alpha(h)$ and an independent estimate of subsistence expenditure on housing $P(h)\beta(h)$ to estimate $P(x)\beta(x)$. (Since all families in private uncontrolled housing are assumed to face the same set of prices, the analysis is simplified by measuring quantities such that both prices are unity.) If subsistence housing expenditure is the same for all two-person households, if the population of such households in New York City in 1965 contained a household at subsistence, and if the rents of all uncontrolled, privately owned rental apartments are accurately reported and reflected neither public nor private charity, then the sample minimum rent of such units occupied by two-person households is a consistent estimator of $P(h)\beta(h)$, and its upward bias declines to zero as
the sample size approaches the population size. The smallest annual rent among the 1,422 households whose behavior underlies equation (6) is $240.7 Therefore, our best estimates of $\gamma(h)$, $\beta(h)$, and $\beta(x)$ for unsubsidized two-person households are .0787, 240, and -12,607, respectively.8

The standard approach involves substituting these estimated indifference-map parameters together with predicted market rent, nonhousing expenditure under the program, and income into equation (2) for each family in public housing in order to estimate its benefit. The estimated mean benefit for the 235 two-person households in the sample who live in public housing and have all the required information reported is $1,124 per year.

II. OBJECTION TO STANDARD APPROACH

It has been suggested that this estimate is subject to selection bias because households who participate in this program have tastes that differ systematically from the tastes of other households. Specifically, it has been argued that $\gamma(h)$ has been underestimated for public housing tenants and hence so have their benefits from the program. The reasoning is as follows. Each eligible family would receive some net benefit (possibly negative) from participating in the public housing program. The families with the largest benefits participate. Since public housing is a program which distorts consumption towards housing, the largest benefits would be received by families with the largest marginal propensities to spend on housing $\gamma(h,i)$. Therefore, the mean of this parameter for participants will be greater than its mean for others.
Since public housing tenants are not selected at random from the set of all families, the existence of selection bias is undeniable. However, the defects of the preceding model are many and serious such that there is no strong a priori reason to believe that the bias is in a particular direction.

First, the existence of upper income limits for eligibility combined with the preceding argument makes the direction of the bias indeterminate. Consider the following example. Assume that each family's unsubsidized housing expenditure $R$ is given by the equation

$$R = 500 + \gamma(h)(Y-1000)$$

where $Y$ is the family's income. The population consists of three eligible families each with an income of $5,000 and three ineligible families each with an income of $10,000. The three values of the marginal propensity to spend on housing $\gamma(h)$ in each group are .04, .10, and .16. Suppose that the two eligible families with the largest $\gamma(h)$ participate in the public housing program. The mean $\gamma(h)$ for these families is .13. Suppose that we attempt to estimate this mean based on random samples from the population of eligible nonparticipants and ineligible families. Since $E(R \mid Y)$ for this group is a linear function of $Y$ with a slope equal to .148, we will overestimate the mean $\gamma(h)$ for participants on average.

A second important way in which the simple selection bias argument deviates from reality is the assumption that public housing tenants are the eligible families who have the most to gain. This assumption ignores the fact that there have always been many fewer public housing units than...
eligible families and hence that participation depends upon who is selected as well as who is willing to participate. The waiting list in New York City contained 89,200 families in November 1967 (National Commission on Urban Problems [1969, p. 131]) and others were undoubtedly deterred from applying due to its length. Even if families with stronger than average tastes for housing are overrepresented among eligibles willing to participate, they may not be overrepresented among participants. Federal legislation has long directed local authorities to give special consideration to families in the worst housing, and the New York City Housing Authority did give a preference to such families. This could result in the mean of the \( \gamma(h,i) \) for public housing tenants being less than the mean for all eligible families.

These considerations make clear that not only the magnitude of the selection bias but also its direction are in doubt.

III. BENEFIT ESTIMATION ACCOUNTING FOR SELECTION BIAS

To investigate the importance of the bias, we applied the procedures recommended by Heckman (1979) to data for two-person households eligible for public housing in New York City in 1965. The selection bias argument of the preceding section implies that the expected value of \( w(i) \) in equation (4) and hence \( u(i) \) in equation (5) are not zero for eligible nonparticipants and that this mean is smallest for types of families with the highest participation rates in public housing. Heckman's results imply that under a certain set of assumptions, if an additional variable \( \lambda(i) \), which is a function of the estimated participation rate of
similar families, is added to the regression (5), OLS estimators of $\alpha(h)$ and $\gamma(h)$ based on a random sample of eligible nonparticipants are consistent.

The first step in the analysis is to estimate a probit relationship explaining the participation rate in public housing. This is done using data on the 690 eligible two-person families in the survey for whom the required information is reported.

Participants in the program are the people who are both willing to live in public housing and selected by the housing authority. For families living in public housing, willingness to continue to participate depends upon the consumption bundle made available by the program, the family's budget constraint in its absence, and the family's preferences. For other eligible families, it depends upon the expected consumption bundle in addition to the family's budget constraint and preferences.

We have already described how the market rents of the units occupied by public housing tenants are predicted. To predict the market rent of the public housing unit that an eligible nonparticipant could expect to occupy under the program, we estimated a relationship between the predicted market rents of public housing units and the characteristics of their occupants (see Table 2) and substituted the characteristics of each eligible nonparticipant into this equation. Since the rent paid for public housing units in New York City depends on income and family size, and since nonhousing expenditure is calculated as the excess of income over this subsidized rent, expenditures on nonhousing goods under the public housing program for two-person households depends on income. To capture some of the differences in taste that account for different
Table 2

Estimated Relationship between Market Rent of Public Housing Units and the Characteristics of Their Occupants

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Description of Regressors</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONS</td>
<td>Constant</td>
<td>1872.24</td>
<td>432.05</td>
</tr>
<tr>
<td>INC</td>
<td>Annual income (in thousands)</td>
<td>72.80</td>
<td>91.38</td>
</tr>
<tr>
<td>INCSQ</td>
<td>INC*INC</td>
<td>1.03</td>
<td>10.94</td>
</tr>
<tr>
<td>AGE</td>
<td>Age of head of household (in tens)</td>
<td>1.66</td>
<td>158.07</td>
</tr>
<tr>
<td>AGESQ</td>
<td>AGE*AGE</td>
<td>1.48</td>
<td>14.58</td>
</tr>
<tr>
<td>WHITE</td>
<td>l if head is white; 0 otherwise</td>
<td>-195.71</td>
<td>75.75</td>
</tr>
<tr>
<td>MALE</td>
<td>l if head is male; 0 otherwise</td>
<td>-50.32</td>
<td>79.41</td>
</tr>
</tbody>
</table>

Note: $R^2 = .09$.
Standard error = 508.17.
Number of observations = 235.
participation rates, we also include variables for the age, race, and sex of the head of the household.

Our data contain few of the attributes of households used by the New York City Housing Authority in selecting tenants. For example, it does not tell us whether a family has been displaced as a result of action by a public agency or whether the head of the household is a veteran. Our analysis accounts for such variables only to the extent that they are correlated with the household characteristics previously mentioned.

Table 3 reports the estimates of the probit relationship explaining participation in the public housing program. The signs of the estimated coefficients show how the participation rate differs for different types of families. For example, we conclude that it is lower for white households headed by males. Since it is not important for our purposes, we did not determine the ranges of income and age of the head over which the participation rate rises with increases in these variables. Since the predicted market rent of the public housing unit is a linear function of the other variables in the probit relationship for eligible nonparticipants, its insignificance is not particularly surprising.

The second step in the analysis is to use the estimated probit relationship to create a new variable $\lambda(i)$ for each family that is a nonlinear function of the estimated participation rate of similar families. Specifically,

$$\lambda(i) = \frac{f[Z(i)]}{1-F[Z(i)]}, \quad (7)$$

where $f$ and $F$ are the density and distribution function for a standard normal variable, $Z(i) = bX(i)$, $b$ is the vector of coefficients in Table
Table 3
Estimated Probit Relationship Explaining Participation in Public Housing

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Estimated Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONS</td>
<td>-1.472</td>
<td>.671</td>
</tr>
<tr>
<td>MKRT</td>
<td>.002</td>
<td>.161</td>
</tr>
<tr>
<td>INC</td>
<td>-.144</td>
<td>.172</td>
</tr>
<tr>
<td>INCSQ</td>
<td>.006</td>
<td>.021</td>
</tr>
<tr>
<td>AGE</td>
<td>.711</td>
<td>.208</td>
</tr>
<tr>
<td>AGESQ</td>
<td>-.046</td>
<td>.020</td>
</tr>
<tr>
<td>MALE</td>
<td>-.205</td>
<td>.124</td>
</tr>
<tr>
<td>WHITE</td>
<td>-.825</td>
<td>.141</td>
</tr>
</tbody>
</table>

Note: MKRT is the predicted annual market rent (in thousands) of the public housing unit that was or would have been occupied by the family. The other variables are defined in Table 2. In this sample, -2 times the logarithm of the likelihood function is 128.88, and the number of observations is 690.
3, and $X(i)$ is a vector of the values of the corresponding variables for the $i$th family.9

Heckman shows that under a certain set of assumptions if $\lambda(i)$ is included as a regressor in equation (5), the OLS estimators of $\gamma(h)$ and $\alpha(h)$ will be consistent. Equation (8) reports such estimates for the 455 two-person families in the sample who are eligible for public housing but living in unfurnished uncontrolled private rental housing and for whom the values of the required variables are reported.

$$P(h)Q(h,i) = .1061 + 1213.36[1/Y(i)] - .043\lambda(i) \quad R^2 = .60 \quad (8)$$

For purposes of comparison, equation (9) reports the results for this sample when $\lambda(i)$ is excluded.

$$P(h)Q(h,i)/Y(i) = .0981 + 1167.66[1/Y(i)] \quad R^2 = .60 \quad (9)$$

The numbers in parentheses below these equations are standard errors. Greene (1981) has shown that the OLS estimators of the standard deviations of the estimators of the parameters in equation (8) can be asymptotically biased in either direction. Since we believe that the existence of selection bias is undeniable, we did not attempt to obtain better estimates of the standard errors which would enable us to test the hypothesis that the coefficient of $\lambda(i)$ is zero. Instead, we used the results of this regression to obtain an estimate of the mean benefit of public housing that can be compared with the estimates resulting from the standard approach.
The constant term in equation (8) is an estimate of the mean of the \( \gamma(h,i) \) for the entire population that accounts for selection bias. The slope coefficient is an estimate of an expression that involves this mean as well as the displacement parameters \( \beta(h) \) and \( \beta(x) \), which are assumed to be the same for all families of this type. Using these estimates and our previous estimate of \( \beta(h) \) discussed in the first section, we can make an estimate of \( \beta(x) \) that accounts for selection bias. This estimate is -9,415 as compared with our previous estimate of -12,607.

The mean of the \( \gamma(h,i) \) for the population is not used directly to estimate the benefits of the public housing program because we have reason to believe that it differs from the mean for participants. Indeed, under our assumptions, the mean of the \( \gamma(h,i) \) for participants varies with the characteristics listed in Table 3. Specifically,

\[
E[\gamma(h,i) \mid X, \text{participants}] = \frac{E(R \mid X, \text{participants}) - \beta(h)}{Y - \beta(h) - \beta(x)}, \quad (10)
\]

where \( E(R \mid X, \text{participants}) \) is the mean housing expenditure in the absence of the program by public housing tenants with characteristics \( X \). In turn,

\[
E(R \mid X, \text{participants}) = \alpha(h) + \gamma(h)Y - \delta[f(Z)/F(Z)]Y, \quad (11)
\]

where \( \delta \) is the coefficient of \( \lambda(i) \) in the stochastic model underlying equation (8).

To predict the mean of the \( \gamma(h,i) \) for public housing tenants with characteristics \( X \), we substitute our estimates of the parameters into equations (10) and (11). This is our estimate of \( \gamma(h,i) \) for each family.
in public housing with these characteristics. The estimates range from .1070 to .1414. Recall that the standard approach led us to use a value of .0787 for all families.

Finally, we substitute the estimates of $\beta(x)$ and $\gamma(h,i)$ discussed in this section, the estimates of $\beta(h)$ and the market rent of the public housing unit discussed in the first section, and data on income and nonhousing expenditure under the program into equation (2) for each family in public housing in order to make an estimate of the benefit from the program that accounts for selection bias. The mean of these estimated benefits is $1,175 compared with $1,124 using the standard approach.

IV. CONCLUSION

Selection bias is a common problem in estimating the benefits of government programs. Any good measure of net benefit will depend in part on the preferences of affected households. In cases where data on participants cannot be used to estimate their preferences, data on nonparticipants with the same observed characteristics are often used. It is usually argued that this approach will typically lead to an underestimate of the mean benefit to recipients of an in-kind transfer because the recipients have stronger than average tastes for the subsidized good.

Our paper shows that this conclusion is not warranted for a program which has an upper income limit for eligibility or which does not serve all eligible families willing to participate, and it estimates the direction and magnitude of selection bias for such a program. We find that
the aforementioned approach will underestimate the mean benefit of the public housing program by less than 5 percent. This suggests that selection bias is not an important defect in the standard approach to estimating the benefits of this program.
NOTES

1Public housing, for example, offers each family that reaches the top of the waiting list a particular dwelling at a below-market rent. The Food Stamp program creates a kink in the budget frontier at the consumption bundle that can be obtained by spending the food stamp on food and all cash on other goods.

2Another problem is that it ignores variation in tastes among affected households with similar objective characteristics. Since, for any good, some households have stronger than average tastes and others weaker than average tastes, it might appear that there would be no bias in the estimate of mean benefit. Overestimates of net benefit for some households would be offset by underestimates for others. Unfortunately, the formulas for calculating net benefit are rarely, if ever, linear, and so the absence of bias is the exception rather than the rule. Olsen and Caniglia (1982) have shown that the bias can be in either direction and have estimated that the mean benefit of a program of unrestricted cash grants would be underestimated by $245 per year in 1965 New York City prices using the standard procedures. Olsen and Agrawal (1982) have estimated that the mean benefit of the public housing program would be overestimated by 10 to 30 percent due to this aggregation bias.

3The formula for calculating $Y(i) + B(i)$ can be obtained by deriving the expenditure function corresponding to the utility function (see, for example, Henderson and Quandt, 1980, pp. 44-45) and substituting in the quantities of housing and other goods consumed when the family lives in public housing.
We assume that the public housing program has no effect on market prices. This is surely close to the truth, because the program has had little effect on the total quantity of housing services. Even in New York City, which has an unusually large program relative to the size of its housing market, the quantity supplied has been increased by less than 2 percent (Olsen and Barton, forthcoming). We also assume that the program has had no effect on the incomes of public housing tenants.

This assumes that households spend their entire income in each period. Since this is an unsatisfactory model of intertemporal choice, we did estimate intertemporal indifference maps under a number of sets of assumptions. In some cases estimates of the structural parameters could not be recovered from estimates of the reduced form parameters; in other cases the estimates were implausible. Since we have limited confidence in the intertemporal indifference maps estimated, and since using them to estimate benefits would be difficult, we did not pursue this matter further. See Olsen and Barton (forthcoming) for a discussion of some attempts to estimate intertemporal indifference maps using the data underlying this study and Hammond (1982) for a policy analysis in which such indifference maps are estimated and used to calculate benefits of several government housing programs.

Normally, no problem would be created by allowing all three parameters to be different for different households. The regression results would yield estimates of their means. Due to a shortcoming in our data, we are forced to obtain an estimate of one parameter outside of the regression and our estimator only makes sense if there is relatively little variation in $\beta(h,i)$ among families of the same size. We consider this to be a reasonable assumption.
Using the second smallest housing expenditure, $540, as the estimate of \( P(h)\beta(h) \) has little effect on our results.

Negative estimates of \( \beta(x) \) are typical of attempts to estimate the Stone-Geary indifference map with data on individual households (Cronin 1979; Hammond 1982, pp. 102-113; Olsen and Barton, forthcoming). In light of the theoretical and statistical reasons for expecting difficulties in estimating subsistence expenditures (Olsen and Barton, forthcoming), it is best to think of our estimates as yielding a reasonable approximation of the true indifference map over that part of consumption space containing the consumption bundles in our sample rather than yielding reliable estimates of subsistence.

Our expression for \( Z(i) \) does not contain a minus sign because our probit relationship explains participation whereas Heckman's explains nonparticipation. The probit relationship explaining nonparticipation is obtained from Table 3 by changing the signs of the coefficients.
REFERENCES


