University of Wisconsin-Madison

Institute for Research on Poverty

Discussion Papers

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LIFE-CYCLE CONSUMPTION AND WILLINGNESS TO PAY FOR INCREASED SURVIVAL

DP #676-81

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March 1982

An earlier version of this paper was presented at the Conference on Valuation of Life and Safety, Geneva, Switzerland, March 30 - April 1, 1981. Shepard's research was supported by National Science Foundation Grant 77-SOC16602. Zeckhauser's research was supported by funds granted to the Institute for Research on Poverty by the Department of Health and Human Services pursuant to the provisions of the Economic Opportunity Act of 1964 and by a grant from the Richard King Mellon Foundation to the Division of Health Policy Research and Education, Harvard University. The authors are grateful to Michael Jones-Lee, John Pratt, Howard Raiffa, Eugene Smolensky, and Michael Stoto for helpful comments on previous versions of the paper.

ABSTRACT

Many health and environmental interventions affect the probability of death for persons of different ages. Willingness to pay (WTP) is a theoretically attractive method for estimating a monetary value for a death avoided, but is hard to assess directly. We develop an indirect method to estimate willingness to pay for a reduced probability of death through a life cycle model on consumption. In this model, a consumer seeks to maximize expected lifetime utility by choosing his level of consumption subject to alternative constraints on income and wealth that represent two polar cases of societies. Under our Robinson Crusoe case, an individual must be entirely self-sufficient, and annuities are not available. Under the perfect markets case, actuarially fair annuities are not available, and an individual can borrow against expected future earnings.

Under one set of plausible assumptions (constant proportional risk aversion on consumption, average age-specific earnings, and a five percent real discount rate), WTP to avoid death was derived for a financially independent American male from the age of 20 onward. The model was calibrated to 1978 data, in which average annual income (excluding persons with no income) for men in their peak decade of earnings, age 45-54, was \$18,874. Under the Robinson Crusoe case, WTP increases from \$500,000 at age 20 to a peak of \$1,250,000 at age 40, declining to \$630,000 at age 60. In the perfect markets case, WTP is highest (\$1,260,000) at age 20 and declines thereafter, reaching \$830,000 at age 60. These results

indicate that individuals would be willing to pay several times the pro-rata share of their future earnings to avoid a probability of death. The age pattern they portray is consistent with earlier direct assessments of utility for survival.

Life-Cycle Consumption and Willingness to Pay for Increased Survival

1. INTRODUCTION

Many actions--walking a mile, going to the doctor, tightening regulations on air quality, or eating a chocolate eclair--may be viewed as purchases or sales of survival. The quantity purchased may be negative (survival is sold), particularly when unhealthy benefits are being pursued (the eclair is a case in point). Most of the important purchases are made by individuals on their own behalf; but parents frequently buy for their children, and society undertakes a variety of programs for all of us. It is not clear, however, what the basis for these decisions is or should be. This analysis addresses the question: How should we think about purchases of survival?

Purchasing survival boils down to reducing risk. Our central question, then, is how to value reductions in risk. There are at least three justifications for examining the various approaches to this problem:

1. We may be better able to understand the decisions that individuals make when choosing occupations and lifestyles.
2. We can help individuals organize their probability assessments and value structures so that they make superior choices.
3. We can improve public policies that affect probabilities of loss of life.

Given space constraints, this paper focuses on the third objective, not because the individual's choice problem is insignificant, but because we believe that educating policy makers and/or the analysts who support them will provide greater immediate returns than educating individuals. We emphasize the prescriptive side of the problem. Thus, following the basic approach of all decision theory, we abstract information about preferences and opportunities from simple situations and then apply these generalizations to determine the actions appropriate in complex choice situations.

We do not claim that our models explain or reproduce individuals' survival choices in the real world. Survival choices are extraordinarily complex, involving consequences that stretch out over many years; these decisions must invariably be based on poor information, frequently involve low-probability events, and often are tied to emotion and to choices made predominantly for non-survival reasons. With such handicaps, why bother to develop analytic models? We think it is critical that the effort be made. If we cannot decide what we would want to do if all relevant information were available to guide decisions, and if all decisions could be analyzed dispassionately and at leisure, it is unlikely that we are making sensible decisions in the imperfect and confusing real world. Social decisions about survival may be more distorted still, because solid information is not available to hold in check the perverse effects of politics.

Merely by addressing questions as we have, we have prejudged a considerable range of policy issues. We have suggested that risk analyses can and should be undertaken, that discussions of risk to life--a commodity that in itself is sacred--are not humane or profane, and that tradeoffs between resources and survival probability should be examined explicitly. We recognize, of course, that it will sometimes be more comforting to ignore or even suppress some of these considerations. Given the extraordinary expenditures that are made to affect survival, however, we believe that society must think seriously about these issues, especially as the methodologies for making public policy decisions are formulated.

Our analysis is in the tradition of the burgeoning literature on the valuation of life. That literature has numerous limitations due to the delicate nature of the subject matter and the imprecision of existing techniques for estimation. Nevertheless we believe that work done in this area has raised the general level of discussion of lifesaving policy.

Approaches to Valuing Lives

Acton (1976) has identified two methods as most promising for valuing the benefits of health programs: the human capital, or livelihood, approach, and the willingness-to-pay approach, based on explicit statements of value by individuals. As Acton noted, however, both methods have inherent difficulties. The livelihood approach assigns valuations in direct proportion to income. Even when adjustments are made for home production (such as housewives' services), the method favors males over females, working persons over retirees, and higher-paid over lower-paid persons in a way that may not reflect individual or social preferences.

The willingness-to-pay criterion, discussed by Schelling (1968), rests on the principle that living is a generally enjoyable activity for which consumers should be willing to sacrifice other activities, such as consumption. While conceptually elegant--see overview by Jones-Lee

(1976)--this approach has been fraught with practical difficulties. The better-known pilot surveys (Acton, 1973; Fischer and Vaupel, 1976; Jones-lee, 1976; Keeler, 1970) show variability and inconsistencies in the responses. Quite simply, individuals have difficulty responding to disturbing questions.

In an effort to circumvent the problems of valuing health benefits in dollar terms, several analyses have computed person-years of life as a measure of benefits (Murray and Axtell, 1974; Preston, Keyfitz, and Schoen, 1972; Cole and Berlin, 1977; Neuhauser, 1977; Acton, 1975). In some cases these person-years have been discounted because they will come in the future (Berwick, Keeler, and Cretin, 1976). These types of assessments are valuable for some cost-effectiveness analysis, but to prescribe actual survival choices further refinement is needed. The value assigned to a year of life should take account of the health status or level of consumption in that year.

Some studies have adjusted life years to take account of health status (Torrance, 1976; Weinstein, Pliskin and Stason, 1977). Others have gone further and attended to time preference (or discounting) as well as health status adjustments (Weinstein and Stason, 1976; Zeckhauser and Shepard, 1976; Pliskin, Shepard and Weinstein, 1980).

Our Model of Analysis

In this analysis, we take the logical next step and focus on the way other age-related attributes, specifically income and consumption, can be incorporated into a utility function for life. But if we are to proceed further and predict, inform, or prescribe actual decisions affecting increased survival, we can no longer avoid monetary valuations. For individuals and for society, survival purchases are not always made out of a fixed budget. They compete directly against other sources of welfare, such as food or entertainment. Here we employ our assessments of utility for life and consumption to compute actual dollar amounts for willingness to pay for increased survival at various ages.

We shall address a series of issues in turn, using both simple models designed to develop intuition and more complex models that could ultimately be employed to test theories of human behavior, estimate parameters of interest, and yield values for policy-relevant variables that are not observable directly.

We shall frequently resort to the common analytic technique of dealing with polar cases. In part this is to gain tractability and eliminate clutter that might obscure results. We do not believe that our readers would be led seriously astray if they interpolated qualitative results between the poles.

The individual decision-maker in our models devotes his resources to two purposes: purchasing probabilities of survival, and consuming. Many goods, such as food, serve both functions. A central concept considered below is willingness to pay for increased survival probability. That is, how much would an individual threatened with a risk of losing his life at some age be willing to pay to reduce the probability of loss?

Anyone engaged in lifetime consumption allocations must first be concerned with what capital and insurance markets are available for trade. Can he borrow at fair rates against future earnings? Can he use his wealth to purchase annuities that will guarantee a given consumption level over an unexpectedly long life? We consider the two extremes: the "Robinson Crusoe" (or autarky) case in which there are no markets, and the case of perfect markets.

We also consider briefly a third case, which we term the pensioner situation, which is relevant to social insurance policies. It assumes that the individual has a per-period consumption level that is purchased at a price guaranteed by the government, and that the price does not vary with his survival portfolio over productive and nonproductive years.

In Part 2 we shall lay out a heuristic model which incorporates considerations of survival probability, consumption, and utility. The model introduces both our Robinson Crusoe and perfect markets cases.

Parts 3-5 show that, in conjunction with data on earnings, wealth, and survival probability, our methods can yield realistic estimates of implicit valuations of small changes in survival probability. Our analysis there is primarily illustrative; in particular, we assume a form for the period utility function rather than develop one based on information gleaned from experiment or observation.

Two other issues should be raised before proceeding to our analysis. The first is the importance of examining life valuation issues in a probabilistic context. Most risks to life involve low probabilities, and our ability to ameliorate those dangers also involves small reductions in risk. Moreover, thinking in terms of lost whole lives, rather than risks to life, engulfs the discussion in emotional considerations. The methods we shall outline for valuing risks to lives rely on the fact that we are concerned with marginal changes in probability. In an earlier analysis, to highlight the concern with marginal changes, Zeckhauser (1979) defined a risk unit or RU as a .001 change in the probability of survival, and suggested that discussions of the valuation of changes in survival be conducted in terms of RUs.

The second issue--society's provision for annuities and capital markets--provides a preface for our discussion of the Robinson Crusoe and perfect markets cases.

Consideration of Probabilistic Losses

Ever since the value-of-life discussion came into its own, it has been subjected to a continuing charge of insensitivity and immorality: Lives are not for sale, should not be for sale, and should not have a price tag attached to them. The counter-debate has proceeded on three fronts. First, as argued by Schelling (1968), we are far less affronted by the idea of dollar valuation if an individual is purchasing his own life, or placing his own life at risk in return for compensation. Second, it has been observed that we feel quite differently about identified versus statistical lives (i.e., hypothetical unknown potential victims). Third, we make decisions all the time that implicitly put a price tag on life.

We could formulate our analysis in greater ethical comfort if we restricted ourselves to situations where individuals purchased only their own survival. We do not, however, wish to limit the scope of the problem in this way. Many of the most consequential decisions on lifesaving are made through collective processes. For reasons good or bad, most developed nations subsidize the health care services their citizens receive. A principal component of many public goods provided by government, including national defense and environmental protection, is their effect on individuals' probability of survival. In theory, many government decisions could be made in a manner to compensate those who put their lives at greater risk, thus extricating the government from the life-valuation business--at least if one thought fair payment was fair play. But rarely is such compensation provided when a toxic waste facility is sited or an air pollution standard is set.

Indeed, governments have often gone further and involved themselves in private contractual decisions. The United States, for example, has established several government agencies, such as the Occupational Safety and Health Administration and the Consumer Product Safety Commission, whose job it is to police the marketplace for risk. From an economic perspective, such agencies might be justified if it could be demonstrated that information about risks flowed poorly and that the government could improve such flow. The implication would be that if the magnitudes of risks were better understood, individuals might not accept them voluntarily.[1]

Many ethical issues become entwined in the life-valuation issue once we involve the government in the choice process. The government may be spending A's dollars to improve B's survival. If on some ex ante basis A and B are similarly situated, then difficulties may be avoided. The citizens of a town may be happy to use tax dollars to support the local hospital or provide fire services free of charge, in that way providing some further risk spreading. But what if some individuals lived in wooden houses and others in houses made of brick? The brick dwellers might feel that equal contribution toward the fire department was unfair. Moreover, it would be inefficient, since individuals would have insufficient incentive to install fire control devices and might even be induced to build their homes out of the wrong material.

In the life-valuation area, this cluster of issues is likely to take on particular salience vis-à-vis lifestyle choices. Given that we have health insurance, how much should we charge people for smoking? Does it matter whether smoking is voluntary? Indeed, could we even demonstrate what voluntary might mean?

We shall not involve ourselves in these issues except to point out that few real-world policy problems arise in the pristine form that would make the methods developed below most applicable. Quite contrary to popular wisdom, we would claim that the more complicating factors there may be, the more important it is to have models such as the one below to get our thinking straight on the issues that can be handled.

We shall be looking at variations in the valuation of life along two dimensions, wealth and age. The former may be quite controversial. Just because an individual is poor, should his life receive lower valuation, say when pollution control decisions are made? We do not wish to tackle that class of issue in this paper. We think of our analysis as addressing the question of how a community that is relatively homogeneous as to wealth--and the community could be a country--should think about undertaking government programs that promote health and survival. We believe that willingness to pay provides a useful guide, and that the methods outlined below provide an appropriate complement to survey and market methods for assessing willingness to pay.[2]

Society's Provision for Annuities and Capital Markets

Our analysis examines two polar models with respect to the availability of actuarially fair annuities. We demonstrate that there are significant differences between the two models in the value of life, and in the time pattern that that valuation follows. We also conclude that the provision of such annuities can provide major benefits to society.

The literature on the economics of information is filled with discussions of why such annuities rarely exist in practice. The central elements of that analysis are the nonenforceability of contracts, and the asymmetry of information between the insured and the insurer. Given these market imperfections, society has evolved a series of elaborate mechanisms for achieving some form of substitute mechanisms, most notably through long-term employment relationships.

There are fairly reliable substitues for fair annuity and capital markets built around labor markets, i.e., during the years of one's employment. What can and should be done about the pre- and post-employment years? In traditional societies, provision for those years was a responsibility of a family or clan. With the breakdown of the intergenerational family in developed Western societies, such responsibilities have increasingly become those of the government. (Presumably causality runs in both directions.)

Most Western societies provide free education for children at least up to the years of productive employment. In this way they compensate for the absence of markets on which young people might borrow. They also provide a risk-spreading element; those with low earning opportunities are not denied. (A question worthy of examination is how equally education should be provided in a society of unequal resource endowments.)

When a person reaches old age, he should have already had the opportunity to build up his capital stock. The difficulty, however, is in risk spreading. Some individuals will live substantially longer than will others, and medical needs will be dramatically unequal. There may be an additional difficulty in a humane society if individuals are destitute, and must be provided for by society.

The policy response to this class of situations, at least in the United States, has been to develop social security and medical coverage programs for the elderly. Most other wealthy Western nations have programs for the elderly that are at least as broad in their coverage.

In sum, though problems of contracting and information asymmetries plague the operation of fair and efficient capital and annuity markets, and though developments within contemporary affluent societies have tended to destroy private insurance mechanisms that revolve around families, most societies have developed mechanisms for providing their members resources in those situations where individuals would have arranged to procure them if capital and insurance markets were functioning effectively.

2. HEURISTIC MODELS OF CONSUMPTION AND UTILITY

Introduction

In this section, we present a series of heuristic models to suggest how a rational individual would allocate his wealth between buying consumption and buying survival. Our present purpose is to illustrate salient features of these choices and to show, qualitatively, that the model gives plausible results. To avoid clutter, we deliberately oversimplify and define polar cases in this section. The next section gives a more fully developed and more realistic model. Two polar cases will be carried through the analysis in subsequent parts; one offers no markets, the other perfect markets. The third case captures some of the elements of social insurance schemes. These are the three cases:

1. Robinson Crusoe: Each individual is entirely self-sufficient. He must support himself entirely from his own wealth and earnings. He has no heirs or dependents. There are no markets on which he can trade.

2. Perfect Markets: Each individual must provide for himself out of his wealth and earnings. Perfect markets are available for trading claims across time periods (i.e., capital markets), and for insuring against variability in length of life (i.e., annuities).

3. Pensioner: Each individual is guaranteed a fixed consumption amount per period. This level is not adjusted as survival changes. One possible interpretation of the pensioner case would be that all individuals are identical, and that the total product of society is divided evenly among its members. When buying survival, each individual ignores the effect his survival will have on the overall level of resources.

Lifetime Utility

We assume that an individual starts with initial wealth of 100 units. Wealth earns no interest, and the individual has no earnings over his life. For all cases, the model assumes that the individual is concerned with choices in three periods: development (say, ages 0 through 29), middle age (ages 30 through 59), and senior age (ages 60 through 89). Everyone lives to at least the end of the first period, development. The probability of death at the end of the development period is 0.100; the probability of death at the end of middle age, given survival to that point, is 0.333. No one lives past the end of period three.

Let $\ell(i)$ denote the probability of being alive during period i, and q_i be the probability of death at the end of period i. We set $\ell(1)$ equal to 1 and compute successive values of $\ell(i)$ by the principles of life table construction as

$$\ell(i) = \ell(i-1)(1-q_i), \text{ for } i = 2,3. \tag{1}$$

The values of the baseline parameters in this example are given at the top of Table 1.

The individual's consumption in period i is denoted by c_i. Being alive and consuming in period i confers a period utility, $u(c_i)$, which in our examples in this part we take to be

$$u(c_i) = c_i^{0.2}.$$

The individual's objective at the beginning of any period is to maximize his expected utility over his remaining life. This utility, denoted by v(i), is defined recursively at his utility during the current period, plus his utility at the beginning of the next period times his probability of surviving to the beginning of the next period. Thus,

$$v_i = u(c_i) + (1-q_i)v_{i+1} \quad \text{for } i = 1,2.3. \tag{2}$$

Since nobody is alive after the third period, we define

$$v_4 = 0.$$

At the beginning of his life, period 1, an individual plans his future life-time consumption so as to maximize expected lifetime utility, v_1.

In this example, the period utility function does not change with age and exhibits diminishing positive marginal returns with increases in

Table 1

Solutions to the Three-Period Utility Model

Variable	Period 1: Development	Period 2: Middle Age	Period 3: Senior Age
	Given Values and Life Table Solution		
Period, i	1	2	3
Mortality probability, q_i	0.100	0.333	1.000
Survival probability, ℓ_i	1.000	0.900	0.600
Life expectancy at beginning of period	2.500	1.667	1.000
	Robinson Crusoe Solution		
Period consumption, c_i	41.59	36.45	21.96
Expected future utility, v_i	5.059	3.290	1.855
WTP_i [a]	290	110	n.a. [b]
	Perfect Markets Solution		
Period consumption, c_i	40.00	40.00	40.00
Expected future utility, v_i	5.228	3.485	2.091
Gross WTP_i [c]; WTP for pensioner	320	200	n.a.
Marginal annuity cost imposed by increased survival [d]	67	40	n.a.
Net WTP_i [e]	253	160	n.a.

[a] WTP to avoid infant mortality is 502.

[b] Not applicable.

[c] Calculated as $v_i/u'(\bar{c})$. WTP to avoid prenatal mortality would be 500.

[d] Calculated as (life expectancy at beginning of period i+1) x $\bar{c}$. WTP to avoid prenatal mortality is 200.

[e] Calculated as gross WTP less marginal annuity cost. Net WTP to avoid prenatal mortality is 300.

period consumption. Note also that the individual does not discount happiness (utility) to be received later in life.

Willingness-To-Pay

One of the central concepts that emerges from these models is how much an individual should be willing to pay, at the margin, to purchase additional survival. Schelling's (1968) famous essay laid the conceptual groundwork for willingness-to-pay analyses, noting that the amount that would be paid to reduce the probability of death would depend on how high the probability was, who would be paying for the reduction, who would receive the benefits, and whether the beneficiary could be identified beforehand. This section discusses the simplest of these paradigms--where the individual spends his own resources to achieve a finite reduction in his own probability of death at a given age.

Willingness to pay (WTP) is defined as the breakeven payment, per unit reduction in the probability of death, that leaves an individual's overall expected utility unchanged (see Jones-Lee, 1976). That is, his increased utility in longer survival is exactly counterbalanced by the reduction in consumption required to purchase the improved survival. Let dq_i and dc_i denote small changes (negative values indicate reduction) in the probability of death, q_i, and consumption, c_i, respectively in period i. The individual's overall utility at the beginning of period i is unaffected provided that the total derivative of v_i is zero:

$$dv_i = \frac{\partial v_i}{\partial q_i} dq_i + \frac{\partial v_i}{\partial c_i} dc_i = 0. \tag{3}$$

His willingness to pay is the derivative dc_i/dq_i, which indicates the breakeven change in consumption per unit change in the probability of death at the end of the period. Substituting (2) into (3) and solving gives

$$\text{WTP} = \frac{dc_i}{dq_i} = \frac{v_{i+1}}{u'(c_i)} . \tag{4}$$

Equation (4) shows that willingness to pay depends on an individual's lifetime pattern of consumption (of which c_i is part) as well as his future survival prospects (contained in v(i)). Our analysis relates willingness to pay to an individual's period utility function and his purchases of survival. We seek to complement other studies, such as Jones-Lee (1974), Weinstein, Shepard, and Pliskin (1980), and Thompson (1980), which show how willingness to pay relates to an individual's risk posture and to whether survival is being bought or sold, the amount of survival being bought or sold, and the individual's foreknowledge in the transaction.

If an individual faces a single opportunity for purchasing survival, he should make the purchase if its marginal cost per unit probability is less than his willingness to pay per unit probability. He is indifferent when the marginal cost equals his willingness to pay. This indifference rule can test whether any existing set of survival purchases is optimal.

Case 1. Robinson Crusoe. The individual must determine his consumption in each period, c_1, c_2, and c_3. His constraint on wealth requires that his total consumption not exceed his initial endowment,

$$c_1 + c_2 + c_3 \leq 100. \tag{5}$$

The optimal expenditure pattern is obtained by optimizing v(1) in (1) subject to (2) and (5). The optimal expenditure pattern and resulting WTP are given in the second section of Table 1. The solution is a special case of the more general results derived in Part III. Note that consumption declines over time, and that total consumption is 100 over the three periods. Where there is no interest or time preference, more consumption is allocated to periods where the probability of being alive is higher. Willingness to pay for survival declines with age for two reasons: (1) survival is less valuable, and (2) consumption falls with age and each marginal reduction in consumption entails greater sacrifice.

To indicate the monetary value of an entire lifetime from birth, we compute WTP to avoid "perinatal" mortality. Imagine that a person with the mortality risks indicated in the top of Table 1 were also faced with a potential risk of perinatal death. By using part of his wealth, he could make an expenditure to avoid the risk. The amount he would be willing to pay per unit probability is 502 units.

Case 2: Perfect Markets. With perfect markets, an individual can make fair actuarial trades for his own destiny. If, as we assume throughout, the interest rate and discount rate for utility coincide and the per-period utility function is stable, the individual will choose a level consumption stream, denoted by $\bar{c}$. Here we shall only be allocating an initial stock of wealth. Any expenditure on health to improve survival probabilities entails two types of charges against the stock: (1) the cost of purchasing survival, and (2) a reduction in the level consumption stream because it is more likely to be received, and hence costs more on an actuarial basis.[3]

The formal maximization problem is to choose $\bar{c}$ to maximize v(1) in (2) subject to

$$\bar{c} \times \text{PRICE} \leq 100. \tag{6}$$

PRICE, the price of an actuarially fair annuity of 1 unit per period, is merely life expectancy; it is given by

$$\text{PRICE} = \ell_1 + \ell_2 + \ell_3. \tag{7}$$

The solution is given in the third section of Table 1, where PRICE is 2.500.

In the perfect markets case, each survival purchase causes a marginal increase in the price of an annuity, and hence a reduction in subsequent consumption. These effects are traced through in the analysis of WTP. Gross WTP is future expected utility divided by the marginal utility of consumption. Increasing survival raises the annuity price, hence leads to a reduction in per-period consumption. Net WTP is gross WTP less the marginal annuity cost that is imposed per unit reduction in mortality.

$$\text{Net WTP} = \frac{\text{future expected utility}}{\text{marginal utility of consumption}} - \text{annuity cost.}$$

The perfect markets case can be given a quite different interpretation. It produces the same outcome that an ideally coordinated centralized society would reach. Each individual, presumably identical to his fellow citizens, would be compelled to make the optimal survival expenditures and no more. And that leads us to our next case, the pensioner.

Case 3: The Pensioner. The pensioner is sold an annual income stream by the society, but the price he is charged does not change with improvements in his survival. The health enthusiast does not find that his annual social security benefits re reduced; smokers are not awarded a bonus payment for shortening their lives. They typify the pensioner.

A centralized society that allowed its citizens to spend their own dollars on survival--assuming that they were not constrained in other ways such as lack of access to markets--might find itself in the pensioner's dilemma, which is in fact a prisoners' dilemma. No individual takes account of the effect his own survival expenditures have on the feasible size of consumption streams. This implies that if the pensioner case used the same annuity price as perfect markets, and if individuals could purchase nontrivial amounts of survival, the system would lose money.

Willingness to pay under the pensioner formulation is equal to Gross WTP from perfect markets, rather than net WTP. That is, he would pay 500 as opposed to 300 to avoid perinatal death, 320 as opposed to 253 for improved survival in period 1 and 200 as opposed to 160 in period 2.[4] The individual is inclined to spend more to improve survival because the fixed price of annuities provides a subsidy for extensions of life. This subsidy is not justified by any market imperfection; indeed, it represents just such an imperfection. Note also that the pensioner will have a higher willingness to pay than an individual confronting perfect markets; it is because he need not reduce his consumption in response to increased survival.

The loss of efficiency associated with the pensioner case--in contrast to perfect markets--is precisely parallel to losses of efficiency in a number of familiar economic contexts. The basic problem is that individuals are being rewarded with average results--the level consumption stream that is available per dollar of initial capital on average when individuals optimize their survival expenditures--when their marginal return would be appropriate for efficiency. The metaphor of the commons comes to mind, say excessive fishing in a common pool. Each identical fisherman receives the average product of a fishing boat. If he were paid the marginal product--and would have to subtract out the reduction in catch to other fisherman--he would fish far less. So too, if the individuals in our world were charged the amount their increased survival expenditures cost society, they would purchase an efficient amount of survival, which would be less survival, and a higher consumption stream.[5]

3. THE GENERAL LIFE-CYCLE MODEL OF CONSUMPTION

The Consumption-Allocation Model

The consumption-allocation model combines some of the strengths of the willingness-to-pay and person-years approaches. One heroic assumption underlies our analysis: an individual's utility over life-spans of different lengths can be represented as a weighted sum of period utilities. By invoking this assumption, we join with most previous literature on lifetime consumption patterns. We shall not be formally concerned with quality arguments other than consumption in any period, but we could readily incorporate health status if it were desired.

The weights we assign to the utility of the same consumption in different periods decline geometrically. Some analysts believe the discount rate should be zero. We shall not join the debate over discount rates, and will follow the prevailing tradition of employing the real interest rate (with effects of inflation removed). The model derives a value function for lives saved as a function of age, giving the value of a life at one age compared with the value at another, and the tradeoff between improved survival and enhanced consumption.

There is an interesting theoretical limitation to the approach: the value function, defined as a weighted sum of period utility functions, is itself a utility function only for small perturbations in the survival function. The effects of large changes must be obtained by solving a complex problem in the calculus of variations, and cannot be represented by a simple function. Similar limitations have been discovered in deriving the utility of other attributes from a consumption function, such as return on a risky investment (Meyer, 1970) or future wealth (Spence and Zeckhauser, 1972). The reason for the limitation is the same in all these examples: decisions must be made before uncertainty is resolved, so the utility of an attribute in one period depends on the probability distribution of the likely amount of that attribute in a future period.

In 1930, Irving Fisher set in motion a continuing research effort concerned with the allocation of consumption over one's lifetime. The interplay between the "impatience" to consume and the productivity of resources has given rise to models offering insights about consumption, saving, investment, portfolio selection, purchase of life insurance and annuities, aggregate savings and growth, and choices between leisure and work (Elton and Gruber, 1974; Hakansson, 1969; Meyer, 1970; Thurow, 1969; Tobin, 1967; Yaari, 1965). Conley (1976) has applied this approach to consider willingness to pay for safety in relation to future earnings and consumption. Numerous court decisions related to torts such as medical malpractice or injury of bystanders have also developed valuation procedures from models of the type developed here. The attraction of models such as these is that they permit inferences about important, observable behaviors from fundamental assumptions or deductions about consumers' preferences and investment opportunities. In essence, we shall attempt to build predictions about valuations for a larger magnitude--risks to life--from smaller observations about the valuation of consumption.

We assume that a consumer maximizes his expected utility of consumption over an uncertain lifetime subject to wealth and solvency constraints. A person's lifetime utility is an additive function of his period utility functions, a formulation that Yaari (1965) refers to as the "Fisher problem." The consumer's period utility depends on whether he is alive or dead, and on his rate of consumption in that period if he is alive. The period utility function is scaled so that the period utility of being dead is zero and the utility of consumption (conditional on being alive) is non-negative, monotonically increasing, and risk averse over relevant values of consumption. Furthermore, we assume that the intertemporal utility function displays impatience (positive time preference) or discounting. While there is no <u>necessary</u> reason for discounting, empirically most people tend to be risk averse on lotteries involving longevity; this behavior is consistent with discounting (see McNeil, Weichselbaum, and Pauker, 1978). Like Yaari (1965) and most other economists analyzing intertemporal consumption, we assume that the period utility function remains the same over all periods.[6]

After our analysis was largely completed, Arthur (1981) published a model which also constructed an individual's lifetime utility as the discounted sum of his period utilities, and calibrated it to U.S. mortality data. Although the specifications and assumptions differ, many similar issues are addressed. Our model is distinguished by our explicit treatment of the dependence of earnings rates on age, and our treatment of an individual's life-cycle savings and consumption decisions as explicit endogenous variables.

Notation and Assumptions

We denote the rate of consumption at time t by $c(t)$ and the period utility function at x by $u(t)$. The full notation developed in this part is presented in Table 2.

The probability of being alive at time t is given by an actuarial survival function, $\ell(t)$, where $\ell(0) = 1$ and $0 \leq \ell(t) \leq 1$. Other than learning that he is still alive, and his age, we shall assume that the individual does not update $\ell(t)$ as time moves forward.[7] (Time t need not be normal age. We can set the time origin arbitrarily as long as we normalize the survival function to $\ell(0) = 1$ and are consistent.) Let T be the maximum possible survival time, so $\ell(t) = 0$ for $t > t$.

An individual's utility at time t of his remaining life after age t, denoted by $y(t)$, is given by his expected discounted utility of consumption for each year in which he is alive from time t on. It is defined by

$$y(t) = \int_t^T e^{-r(\tau-t)}\ell(\tau)u(c(\tau))d\tau. \tag{8}$$

An important special case, $y(0)$ corresponding to $t = 0$, gives the utility from the initial age onward. Thus (8) is a joint utility function for a consumption trajectory and a survival function.

Implicit in (8) are some strong assumptions about the multiperiod utility function.

Assumption A: Death has a utility of zero relative to the utility $u(c)$ for being alive with consumption c. Since we have not yet placed

Table 2

Notation for Model of Full-Scale Estimation

Notation	Interpretation
τ, t	Time in individual's life, from 0 to T
t_0	Time at which individual first begins to save (accumulate wealth)
$c(t)$	Rate of consumption at time t
$w(t)$	Level of wealth at time t
$\dot{w}(t) \geq 0$	Solvency constraint
$m(t)$	Rate of earnings at time t
$y(t)$	Utility of survival at age t of life at age t, <u>not</u> conditional on survival at age t
$v(t)$	Value of remaining life at age t, conditional on survival at age t
$R(t)$	Marginal change in optimal v(t) per unit reduction in the force of mortality at time t
$E(t)$	Discounted expected remaining years of life following age t (not conditional on survival at age t)
$\bar{c}$	Constant rate of consumption (in perfect markets and pensioners cases)
H	Scaling factor for utility function
$f(t)$	Net amount received by an annuitant alive at age t
$N(t)$	Discounted expected earnings following age t, discounted to age t and conditional on survival at age t
$G(t)$	Discounted expected consumption following age t, discounted to age t and conditional on survival at age t

any restrictions on the form or scaling of the period utility function, Assumption A is relatively benign. Its effect is to require that death cannot have a period utility of negative infinity. That is, our model would not apply if a person would trade off an infinite amount of money for a probabilistic improvement for just an instant more of life. As long as the utility of death is not infinitely negative, the period utility function can be rescaled (by adding a constant) so that the utility of death is set to zero.

Assumption B: The utility of consumption at one time is independent of past consumption. This assumption, termed the marginality assumption, implies the additive (integral) form for total expected utility.[8]

Assumption C: Legacies or bequests are not valued--the consumer is assumed indifferent to remaining wealth at the time of death.[9] Assumption C, necessary to keep the mathematics manageable, is plausible under two conditions: (a) the consumer has no economic dependents, or (b) he has purchased paid-up life insurance to satisfy the basic needs of any dependents and is not interested in a further bequest to them (e.g., he feels it might weaken their will to work).

The objective is to maximize $y(0)$ in (8), subject to a feasible trajectory on consumption.[10] A feasible trajectory on consumption is constrained by non-negativity constraints on consumption

$$c(t) \geq 0 \quad \text{for all } t. \tag{9}$$

Analyses of Three Cases

1. Robinson Crusoe Case

In this case, the individual is self-reliant. He cannot borrow against future earnings, and no actuarial markets for the purchase of annuities are available. Therefore the individual faces a solvency (no debt) constraint on wealth, $w(t)$,

$$w(t) \geq 0 \quad \text{for all } t,^{11} \tag{10}$$

and an initial condition

$$w(0) = w_0. \tag{11}$$

In general, we assume that when a person is first able to allocate consumption over his lifetime, he has no accumulated wealth and set $w_0 = 0$. Wealth is related to consumption by the differential equation

$$\dot{w}(t) = rw(t) + m(t) - c(t), \tag{12}$$

where $m(t)$ is the rate of earnings at time t. In this model, the level of earnings depends only on whether the individual is alive at time t. Equation (12) states that the rate of change in wealth is equal to the interest at rate r earned on a risk-free investment (here the rate assumed to equal the discount rate for future utility[12]) less consumption, plus earnings. We begin from a perspective of time zero, so our problem is to maximize $y(0)$ defined by (8), subject to (9) and (12).

Solution. The problem will be solved by the calculus of variations. The control variable is $c(t)$, the consumption rate, and the state variable is $w(t)$, wealth. If the utility function on consumption is strongly risk-averse near zero consumption, as will be the case in our analyses, then the consumer's preferences will keep c positive and (9) becomes redundant. We are left considering two cases: those intervals in which the debt constraint (10) is binding (i.e., an exact equality holds) and those in which it is not (a strict inequality applies). The dividing point between these cases is the time t_0 at which solvency ceases to bind, i.e.,

$$t_0 = \min_{(0,T)} \{t \text{ such that } w(t) > 0\}. \tag{13}$$

In this analysis we assume, as will be the case with a single-peaked earnings function used in our empirical example, that initially (10) is binding but it ceases to be binding at t_0 for the rest of the lifetime. This behavior will prevail whenever period earnings early in life are lower than ideal period consumption based on lifetime income from that point forward. The model could be extended, through the use of inequality constraints, to have multiple transitions between the two cases.

The first case, where (10) is binding, applies for τ between 0 and t_0. To maximize (8), we raise consumption to the limit allowed by the solvency constraint, so that $w(\tau)$ is identically zero over the interval 0 to t_0. In this interval, $w(\tau)$ is identically zero over the interval 0 to t_0. In this interval, $w(\tau) = 0$ and $w(\tau) = 0$. Substituting into (12) gives

$$c(\tau) = m(\tau) \quad \text{when (10) is binding.} \tag{14}$$

The second case, where the solvency constraint (10) is not binding, holds for $\tau > t_0$. Solving it is equivalent to maximizing $y(t_0)$ in (8) subject to (12). To proceed, we form the Hamiltonian

$$H(c,w,\lambda,\tau) = e^{-r(\tau-t_0)} \ell(\tau)u(c(\tau)) + \lambda(\tau)[rw(\tau) + m(\tau) - c(\tau)]. \quad (15)$$

Following the first Euler-Lagrange equation, we set the partial derivative of H with respect to the control variable equal to zero:

$$\frac{\partial H}{\partial c} = e^{-r(\tau-t_0)} \ell(\tau)u'(c(\tau)) - \lambda(\tau) = 0.$$

Solving for the optimal consumption trajectory $c^*(\tau)$, we find it must satisfy

$$\left.\frac{du}{dc}\right|_{c^*(\tau)} = e^{r(\tau-t_0)} \lambda(\tau)/\ell(\tau). \quad (16)$$

To find $\lambda(\tau)$, we use the second Euler-Lagrange equation,

$$\frac{\partial}{\partial w} H(c,w,\lambda,T) = -\frac{d}{d\tau}\lambda(\tau),$$

which gives

$$\lambda(\tau)r = -\frac{d}{d\tau}\lambda(\tau).$$

Solving this ordinary differential equation gives

$$\lambda(\tau) = \lambda(t_0)e^{-r(\tau-t_0)}. \quad (17)$$

We may interpret $\lambda(\tau)$ as the marginal value (utility) to the individual at time t_0 of wealth at time τ. Equation (17) indicates that this value is highest at time t_0 and declines with time at the discount rate. Thus the marginal value of a unit of wealth at time t is proportional to its present value at time t.

Combining (16) and (17), we find that for the nonbinding cases $c(\tau)$ must satisfy[13]

$$u'[c^*(\tau)] = \lambda(t_0)/\ell(\tau). \tag{18}$$

Equation (18) shows that the marginal utility of consumption, under the optimal pattern, is inversely proportional to the probability of being alive. As a result, once the individual has positive wealth, his consumption must decline monotonically over time. In the Robinson Crusoe case an individual does not know when he will die, and there is no insurance. Allocating consumption to a later as opposed to an earlier period involves a greater risk of not enjoying it. Therefore, in this case an individual consumes more money earlier.

Multiplying both sides of (18) by $\ell(\tau)$ gives another important result:

$$u'[c^*(\tau)]\ell(t) = \lambda(t_0). \tag{18a}$$

The <u>expected</u> marginal utility of consumption (the marginal utility times the probability of being alive) is constant for all ages at which the subject is not on the brink of insolvency.

To solve (18) for the actual level of consumption, we must determine the constant $\lambda(t_0)$. It obviously depends on earnings possibilities and is found by the end point and solvency constraints on w, (10) and (11). To use these constraints we solve the ordinary linear nonhomogeneous differential equation (12). We obtain

$$w(t) = e^{rt} \int_{t_0}^{t} e^{-r\tau}[m(\tau) - c^*(\tau) - c^*(\tau)]d\tau, \tag{19}$$

which is the future value (at time t) of the difference between earnings and consumption (under the optimal path) from time t_0 (defined by (13)) to time t. Since it cannot be optimal to have wealth remaining at the maximum possible age, T (which may be infinite), we must have

$$w(T) = 0,$$

so

$$\int_{t_0}^{T} e^{-r(T-\tau)}[m(\tau) - c^*(\tau)]d\tau = 0. \tag{19a}$$

We will present solutions for particular functions u(c), $\ell(\tau)$, and $m(\tau)$ in subsequent sections.

Value of remaining life, v(t). The value of remaining life at age t is needed to assess the utility of reductions in the probability of death at age t. We will show that this value is the same as the utility at age t of remaining life beyond age t conditional on survival at age t for an individual following the optimal consumption pattern. Furthermore, we shall show that the value function v(t) based on the optimal consumption pattern behaves like a utility function for small changes in survival probabilities from $\ell(t)$.

In order to state these results more precisely, we introduce some additional terminology. To formalize the notion of the utility of remaining life, we recall that y(t) for $t \neq 0$ in (8) is the utility at age t of all expected years of life beyond age t. The expected utility y(t) in (8), for $t \neq 0$, is not conditional on survival at age t; that is, it includes the possibility that the subject may already have died at age t. For most purposes the more relevant measure is the utility of life following age t, conditional on survival at age t. We define this value, v(t), by dividing y(t) by the probability of survival to age t:

$$v(t) = y(t)/\ell(t). \tag{20}$$

To formalize the definition of the utility of a reduction in the probability of death, we first must define the probability of death and the meaning of a reduction in it. The force of mortality at any age τ is given by

$$\mu(\tau) = \frac{d}{d\tau} \log_e \ell(\tau).$$

Suppose there is a small reduction in the force of mortality at some age t; i.e., $d\mu(t) < 0$. It lowers $\mu(\tau)$ for $\tau = t$, but leaves $\mu(\tau)$ unchanged for other values of τ. The perturbation changes the survival function from $\ell(\tau)$ to $\ell^{\Delta}(\tau)$, where

$$\ell^{\Delta}(\tau) = \begin{cases} \ell(\tau) & \text{for } \tau < t \qquad (21) \\ e^{d\mu(t)}\ell(\tau) & \text{for } \tau \geq t. \qquad (22) \end{cases}$$

For small $d\mu(t)$, (22) is approximately

$$[1 + d\mu(t)]\ell(\tau). \tag{23}$$

We define R(t) as the marginal utility at time t, conditional on survival at that time, per unit reduction in the force of mortality at time t. In other words, for a perturbation $d\mu(t)$, the change in utility is $R(t)d\mu(t)$. Our result is stated formally and proved as Proposition 1:

Proposition 1: The marginal utility of survival probability in the Robinson Crusoe case is equal to the value of remaining life. If a consumer acts to maximize (8) subject to (9) through (12), then at every age t,

$$R(t) = v(t). \tag{24}$$

Proof: To evaluate $R(t)$, we recall that by definition $y(0)$ is based on an optimal consumption trajectory so that reallocations of consumption satisfying (9) to (12) have no first-order effect on $y(0)$. Thus we can consider $c^*(t)$, and hence $u(c^*(t))$, as functions which do not change with perturbations in $\ell(\tau)$. Letting $dy(0)$ be the increase in $y(0)$ resulting from the reduction in mortality $d\mu(t)$, we find on substituting (21) and (23) for the perturbed survival function and simplifying that

$$dy(0) = e^{-rt}\, y(t)d\mu(t).$$

Since our model discounts utilities at rate r per year, the future value at time t of this change in utility is

$$dy(t) = y(t)d\mu(t). \tag{25}$$

Finally, we need to express this change in utility in terms of a unit change in $\mu(t)$ and to make it conditional on survival at time t. We divide both sides of (25) by the probability of the conditioning event, $\ell(t)$, and by $d\mu(\tau)$, obtaining:

$$\frac{1}{\ell(t)}\,\frac{dy(t)}{d\mu(t)} = \frac{y(t)}{\ell(t)}. \tag{26}$$

The left side of (26) is by definition $R(t)$. The right side is $v(t)$ by (20). This proves the proposition.

Notice that in the proof above the critical element is the age at which mortality is changed. For an infinitesimal change, the age at which one learns of this change is not important because marginal readjustments in the consumption rate have only second-order effects, i.e., they go to zero with the square of $d\mu(t)$, a very small number. For small changes in mortality, $v(t)$ defined by (20) gives their value to the consumer at time t.

Implicit willingness to pay. Willingness to pay measures a person's willingness to sacrifice one desired attribute, wealth for future consumption, in order to obtain another desired attribute, improved survival. It is a theoretically pure, although practically difficult, measure for establishing the consumer demand for improved survival. A major advantage of our model is that it yields estimates of WTP for marginal changes in the probability of death. Furthermore, these estimates can be derived from period and intertemporal preferences on consumption. If these preferences are consistent with the utility functions in our model, then the unreliable, and somewhat anxious, process of trying to assess WTP directly can be avoided.

To calculate WTP, we let $d\mu$ denote a marginal change in the force of mortality, and dw denote the marginal change in wealth (or dc the marginal change in the rate of consumption) that the person will just accept as compensation to leave his overall conditional utility at age t constant. That is, dw or dc is the WTP for a reduction $d\mu$ in the probability of death.[14] We first consider the case where solvency is non-binding and wealth is positive. Then WTP is expressed in terms of dw rather than dc.[15]

WTP is determined by the indifference relation that the total derivative of v(t) is zero. Thus

$$dv = \frac{\partial v}{\partial \mu}\, d\mu + \frac{\partial v}{\partial w}\, dw = 0. \tag{26a}$$

Using Proposition 1 and the substitutions described below, this equation becomes

$$R(t)d\tau + [\lambda(t)e^{rt}/\ell(t)]dw = 0. \tag{27}$$

The term $R(t)d\mu$ measures the marginal utility at age t of the change in survival, $d\mu$, conditional on survival to that age. The second term is the marginal conditional utility of the change in wealth. The term in brackets, the shadow price of the change in wealth, is derived by the following sequence of multiplications. The marginal contribution of wealth at time t to utility at time zero is the shadow price $\lambda(t)$ of wealth in the optimization of y(0). To express this price in terms of utility at time t we convert to future value by multiplying by e^{rt}. Finally, to make the utility of the change in wealth conditional on survival at age t, we divide by the probability of the conditioning event, $\ell(t)$. The product of these factors is the second term in (27). The marginal WTP is

$$\text{WTP} = \frac{dw}{d\mu} = \frac{R(t)}{\lambda(t)e^{rt}/\ell(t)} \tag{28}$$

i. <u>Solvency constraint nonbinding</u>. In the case where the solvency constraint is not binding, we substitute (17) for $\lambda(t)$ in (28), obtaining

$$\text{WTP} = \frac{R(t)}{\lambda(t_0)/\ell(t)}\ . \tag{29}$$

Solving (18) for $\lambda(t_0)$ and substituting the result into (29) gives

$$\text{WTP} = \frac{R(t)}{u'(c^*(\tau))} \ . \tag{30}$$

This important result says that willingness to pay is proportional to $R(t)$, the expected utility of remaining life at age t conditional on survival at that age, and inversely proportional to the marginal utility of consumption at age t.

ii. Solvency constraint binding. In the case where the solvency constraint is binding, payments to reduce the probability of death must come out of immediate consumption. Then the indifference relation for WTP is

$$-R(t)d\mu + u'(c)dc = 0. \tag{31}$$

Thus, marginal WTP is

$$\text{WTP} = \frac{dc}{d\mu} = \frac{R(t)}{u'(c(t))} \ . \tag{32}$$

Equation (32) is formally equivalent to (29). In both cases, WTP is inversely proportional to the marginal utility of consumption on the optimal trajectory. The difference is that in (28) the consumption level is an internal optimum, whereas in (32) it is determined by the solvency constraint and is equal to earnings, $m(t)$, by (14).

2. Perfect Markets Case

Insurance annuities offer protection against outliving one's wealth. In return for payments according to some specified schedule during certain years, the insurer promises to pay some stated income beginning at a specified age and continuing indefinitely. Payments and receipts are both conditional on the insured's being alive at the specified age. In the present analysis we assume that annuities are actuarially fair; i.e., for any contract the insurer's expected receipts equal his expected disbursements. Annuities increase the range of consumption allocations available to a consumer and thereby increase his expected utility of living to any age, and of his remaining life beyond any age. As shown in part 2 of this paper, however, annuities have an indeterminate effect on willingness to pay for reductions in the chance of death. With perfect markets, individuals can borrow money. Thus, there will be no need to constrain consumption in early low-earning years as we did in the Robinson Crusoe model.

In terms of our formal model the availability of fair annuities replaces the wealth equation (12) by

$$\dot{w}(t) = rw(t) + m(t) - c(t) + f(t), \tag{33}$$

where $f(t)$ is the net amount received by the annuitant at age t. (Thus $f(t)$ is negative for net payments by the annuitant (premiums), positive for net receipts, and zero if the annuity is inactive at that age.) The constraint of actuarial fairness requires

$$\int_0^T e^{-r\tau}\ell(\tau)f(\tau)d\tau = 0. \tag{34}$$

Since the type of annuity postulated does not require prepayment, it includes borrowing with a life insured loan; i.e., borrowing against human capital during years with low earnings and no wealth.

Solution. To maximize y(0) in (8) subject to (9), (10), and (33), we form the Hamiltonian as in (15), but with the right side of (33) substituted for the term in square brackets. Using the Euler-Lagrange equations, we find that the optimal consumption stream is a constant rate of consumption regardless of age, which we will call $\bar{c}$. It cannot be optimal to hold positive wealth at any age, because the chance of death means that wealth would become worthless. It is always better to invest wealth in an annuity, which provides a higher consumption rate in the case of survival.

Under perfect markets (actuarially fair annuities and enforceable contracts are available and create no disincentive on work), an individual should exchange his lifetime wealth for a level lifetime annuity. Hence the solvency constraint (10) is binding at every age t, so $w(t) = \dot{w}(t) = 0$ for all t. Solving (33) for f(t) gives

$$f(t) = \bar{c} - m(t). \tag{35}$$

Thus the level of the annuity is the deficit in earnings below the constant level of consumption. Before finding the consumption level that the earnings will support on an expected value basis, we develop some notation. We define E(t) as discounted life expectancy (expected remaining years of life) at age t (conditional on survival at age t). Thus

$$E(t) = \frac{1}{\ell(t)} \int_t^T e^{-r(\tau-t)} \ell(\tau)d\tau. \tag{36}$$

Analogously, we define N(t) as discounted expected earnings following age t, discounted to age t and conditional on survival to age t.

$$N(t) = \frac{1}{\ell(t)} \int_t^T e^{-r(\tau-t)} \ell(\tau)m(\tau)d\tau. \tag{37}$$

Substituting (35) and (37) into (34) and rearranging terms gives

$$\int_0^T e^{-r\tau} \ell(\tau)\bar{c}d\tau = N(0). \tag{38}$$

Equation (38) says that discounted expected lifetime consumption equals discounted expected earnings--the annuity is actuarially fair. Substituting (36) and solving for $\bar{c}$ gives

$$\bar{c} = N(0)/E(0), \tag{39}$$

which says that the maximum attainable level of consumption is the ratio of discounted lifetime earnings to discounted life expectancy.

Marginal utility of survival probability. We substitute (39) and (8) into (20), where

$$v(t) = u(\bar{c})E(t) \tag{40}$$

and E(t) is discounted remaining life expectancy at age t. Thus, with perfect markets, the utility of remaining life is equal to the utility of consumption per year times discounted life expectancy. It will be useful to define E(t) as the discounted expected remaining years of life at age t.

Proposition 2: The marginal utility of survival probability in the perfect markets case is the sum of a financial surplus term and a direct utility-gain effect.

Proof: To find the marginal utility of survival probability at age t, R(t), we must evaluate the effect of a perturbation $d\mu(t)$, or $d\mu$ for short, in the mortality function under annuities. To do so, we differentiate the lifetime utility function with respect to $\mu(t)$:

$$\frac{d}{d\mu}(y(0)) = [\frac{d}{d\mu}(u(\bar{c}))]E(0) + u(\bar{c})\frac{d}{d\mu}(E(0)). \tag{41}$$

In contrast to the Robinson Crusoe case, the derivative of $u(\bar{c})$ does not vanish, since a change in mortality affects not only the number of years over which consumption can be enjoyed, but also the total amount of consumption available. Since annuities were assumed actuarially fair, the annuity schedule must be readjusted to consumer any additional earnings. Since by (23)

$$\frac{d}{d\mu}\ell^{\Delta}(\tau) = \ell(\tau) \quad \text{for } \tau > t, \tag{42}$$

differentiating (39) yields

$$\frac{d}{d\mu}(\bar{c}) = \frac{1}{E(0)} e^{-rt}[N(t) - \bar{c}E(t)].$$

Thus the first term in (41) is

$$e^{-rt} u'(\bar{c}) [N(t) - \bar{c}E(t)]. \tag{43}$$

To evaluate the second term in (41), note that by (42) the derivative of $E(0)$ is $e^{-rt}E(t)$. Substituting this result and (43) into (41) gives

$$\frac{d}{d\mu}\, y(0) = e^{-rt}\{u'(\bar{c})[N(0) - \bar{c}E(t)] + u(\bar{c})E(0)\}. \tag{44}$$

But the derivative in (44) is not quite the same as the value of saving a life, $R(t)$. The variable $R(t)$ measures the utility at age t of saving a life, conditional on survival at age t. To obtain this we must convert $\frac{dy(0)}{d\mu}$ to future value at time t and divide by the probability of survival at that age. Thus

$$R(t) = e^{-rt}\, \frac{1}{\ell(t)}\, \frac{d}{d\mu}\, [y(0)]. \tag{45}$$

Substituting (44) and (26) into (45) gives

$$R(t) = u'(\bar{c})[N(t) - \bar{c}E(t)] + u(\bar{c})E(t). \tag{46}$$

The first term in (46) measures the financial effect of a reduction in the probability of death: the marginal utility of consumption times expected earnings net of consumption following age t, given survival at that age. The second term measures the pleasure of additional life with quality held fixed. This completes the demonstration.

An equivalent expression to (46) is

$$R(t)_{(\text{annuities})} = u'(\bar{c})N(t) + [u(c) - \bar{c}u'(c)]E(t),$$

which states that the marginal value of saving a life at age t is equal to the sum of the product of the marginal utility of consumption times discounted expected earnings after age t conditional on survival at that age, and the product of the consumer surplus in each year times discounted remaining life expectancy. The second term in brackets, $-\bar{c}u'(c)$, enters because of the requirement that annuities be actuarially fair. An extension of life must be accompanied by a reduction in the rate of consumption, so the utility of an extension of life with a wealth constraint is the consumer surplus in extra years.

<u>Implicit willingness to pay.</u> To evaluate WTP with perfect markets, we note that the solvency constraint on wealth is always binding in the sense that one always expends his entire wealth on actuarially fair annuities. Thus the compensating variations that define WTP are given by (31) and WTP itself is given by (32). We substitute (46) for R(t), its value in the perfect markets case, obtaining

$$\mathrm{WTP} = [N(t) - \bar{c}E(t)] + \frac{u(\bar{c})}{u'(\bar{c})}E(t). \tag{47}$$

The term in brackets is the impact on net lifetime income (income net of consumption) of improved survival. The second term is WTP for remaining life at a constant level of consumption.

Notice that at time 0 the term in brackets vanishes, and WTP equals

$$\frac{u(\bar{c})}{u'(\bar{c})}\, E(t).$$

This is the total lifetime utility of consumption divided by the marginal utility of consumption. A similar quotient applies for the Robinson Crusoe case; see (30). The difference in WTP between Robinson Crusoe and the perfect markets case is indeterminate, since both numerator and denominator are greater for perfect markets.

For a more meaningful expression, we define the amount of consumption equivalent to the consumer's surplus from being alive in any year with this consumption level as

$$\bar{c}_s = \frac{u(\bar{c})}{u'(\bar{c})} - \bar{c}. \tag{48}$$

Substituting (48) into (47) gives

$$WTP = N(t) + \bar{c}_s E(t). \tag{49}$$

The first term in (49) is discounted expected additional earnings conditional on survival at age t, the human capital or livelihood at age t. The second term is surplus consumption times discounted life expectancy. Thus livelihood provides a lower bound on willingness to pay. There is also a consumer surplus from being alive which is valued.

3. Pensioner Case

An important special case is one where readjustments in the rate of consumption are ignored (i.e., annuities could not be readjusted). This case would arise if an individual were choosing for himself and his consumption level were guaranteed by some external agency. (If this is true for everyone, it is a prisoner's dilemma situation. This implies, for example, that to the extent there is social security, individuals purchase too much survival when old.) To value the marginal reduction in the chance of death at age t in this case, we ignore the first term in (46). Then

$$\underset{\substack{\text{(ignoring} \\ \text{readjustments)}}}{R(t)} = u(\bar{c})E(t) = v(t). \tag{50}$$

The similarity between this result and (24) should not be surprising. Under the Robinson Crusoe case we did not adjust our consumption level with small perturbations in the survival function. The assumption that $\bar{c}$ remains fixed is the analogue in this model.

In the pensioner case, WTP is obtained by dividing the utility of remaining life for a person alive at age t by his marginal utility of income. Thus,

$$\text{WTP} = \frac{u(\bar{c}) \cdot E(t)}{u'(\bar{c})}. \tag{50a}$$

Substituting (48) into (50a) gives

$$\text{WTP} = \bar{c}_s E(t) + \bar{c}\ E(t). \tag{50b}$$

In the pensioner case, WTP is the sum of annual consumer surplus (c_s) times discounted life expectancy plus average consumption ($\bar{c}$) times discounted life expectancy. The former product is lifetime consumer surplus from being alive; the latter product is discounted consumption.

Figure 1 presents a graphical method for calculating and displaying these two products.[16] The curved line is the utility function u(c), the dot shows $u(\bar{c})$, with a tangent to the curve drawn at that point. The horizontal

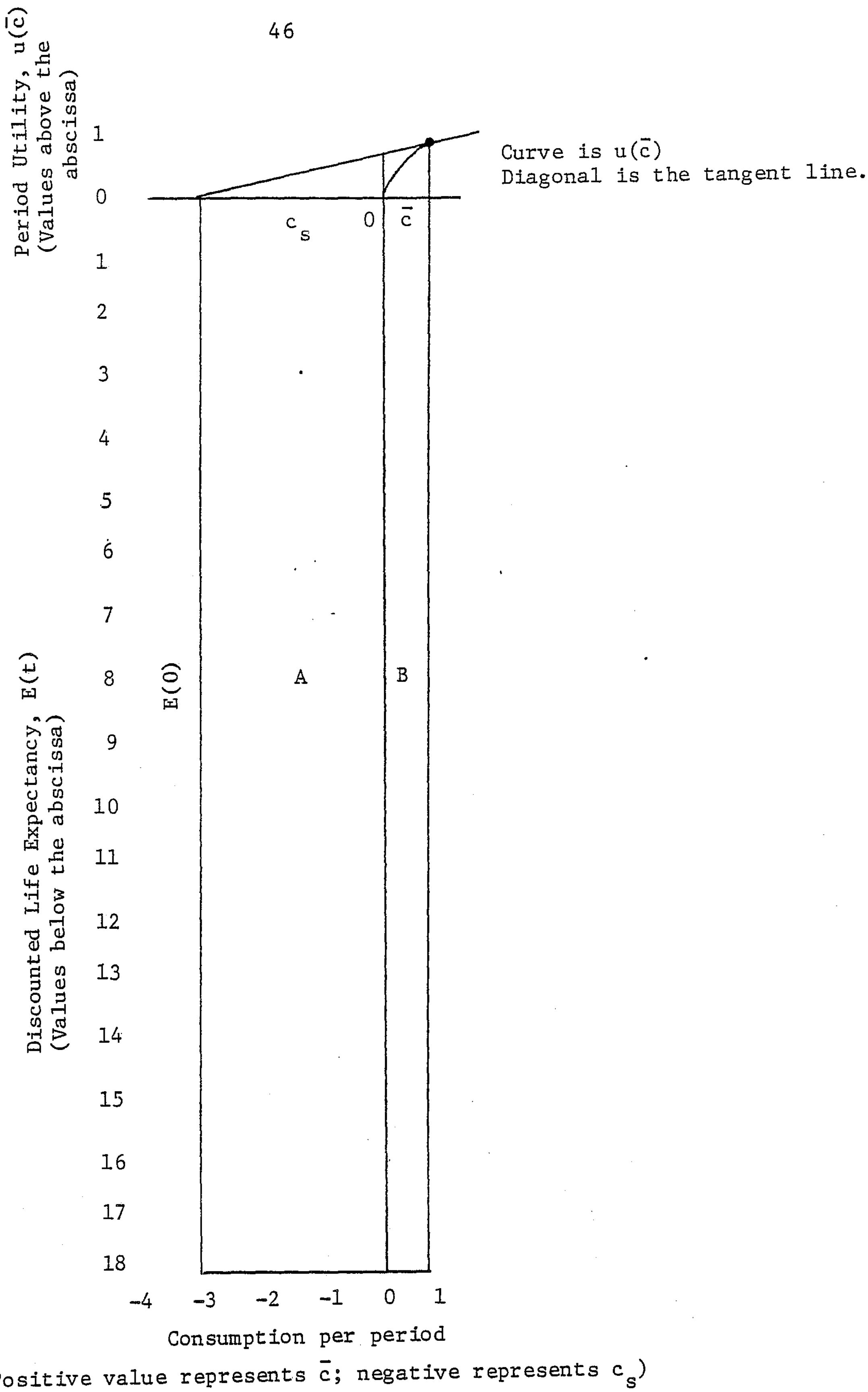

Figure 1. Graphical calculation of WTP in pensioner case.

distance from the intersection of the tangent to the origin is c_s. The distance from the origin to the right is $\bar{c}$. On the lower vertical axis E(t) is measured. Area A corresponds to the consumer surplus, and area B to discounted consumption. The total, A + B, is the total willingness to pay.

4. RESULTS FOR CONSTANT PROPORTIONAL RISK AVERSION

To obtain some numerical results, let us now solve explicitly for an interesting special class of period utility functions on consumption, namely constant proportional risk aversion (CPRA) with constant m.[17] We further assume that the utility functions are time-invariant and scaled such that u(0) = 0, which is identical to the utility of death. Under these conditions, an individual's u(c) must satisfy

$$u(c) = Hc^{1-m}, \tag{51}$$

where H is an arbitrary scaling factor of the same sign as the exponent of c.[18] The greater the parameter m, the greater the risk aversion on consumption. With this function, the elasticity of period utility with respect to consumption is the constant 1-m, regardless of the level of consumption.

Sensible results require the composite range restriction $0 < m < 1$, which applies to the balance of this paper. First, the assumption that the individual is risk-averse on consumption levels requires that $m > 0$.[19]

Second, the condition that any level of consumption is preferred to death requires that $m < 1$. The composite restriction means that survival without consumption is no better than death, and utility is finite for all finite levels of consumption. Since the scaling of utility units is arbitrary, without loss of generality, we set $H = 1$ for simplicity. To find optimal consumption paths, we differentiate (51) with respect to c, obtaining:

$$u'(c) = (1-m)c^{-m}. \tag{52}$$

1. Robinson Crusoe Case

Optimal consumption path. To find $c^*(t)$ where the solvency constraint (10) is not binding but actuarial markets are not available, substitute (52) into (18), obtaining[20]

$$c^*(t) = K[\ell(t)]^{1/m}. \tag{53}$$

Here K is a constant, which spreads earnings over one's lifetime. If we have already found $\lambda(t_0)$ to satisfy the endpoint conditions on $w(t)$, then we can find K by

$$K^m = \frac{(1-m)}{\lambda(t_0)}, \tag{54}$$

where t_0 is defined by (13). If $\lambda(t_0)$ is not known, we evaluate K directly from the endpoint conditions on $w(t)$, setting $w(t_0) = w(T) = 0$ and $w(t) \geq 0$ for all t. See the Appendix. Thus as long as the solvency constraint is not binding, optimal consumption is proportional to a power $(1/m > 1)$ of survival.

Relations between discounted consumption, utility, and willingness to pay. We shall establish some operational relationships between utility, willingness to pay, and discounted consumption. First we need more notation. We define $G(t)$ as the present value at time t of consumption from time t onward under the optimal path, discounted to age t, and conditional on survival at age t:

$$G(t) = \frac{1}{\ell(t)} \int_t^T e^{-r(\tau-t)} c^*(\tau) d\tau. \tag{55}$$

Three propositions follow for the time $t > t_0$ at which the individual is following his optimal consumption path, and has passed age t_0, at which the solvency constraint relaxes. Proposition 3 says that the utility of saving a life at age t (conditional on being alive at age t) is proportional to discounted consumption divided by the probability of survival to age t. Proposition 4 says that marginal WTP is equal to discounted consumption times the factor $1/(1-m)$, the reciprocal of the exponent in the period utility function. Finally, Proposition 5 indicates that the multiple $1/(1-m)$ (greater than one) of expected discounted future earnings (livelihood) provides a lower bound on willingness to pay for reduced chances of death. In other words, the WTP to save a life is always greater than discounted expected earnings.

Proposition 3: Let t_0 be the minimum age at which the solvency constraint (10) is not binding, as defined in (13). If $u(c)$ is given by (51), exhibiting CPRA, and $t \geq t_0$, then utility at age t conditional on being alive at age t, $v(t)$, satisfies

$$v(t) = K^{-m}\ G(t). \tag{56}$$

Utility is proportional to discounted future consumption of a person now alive.

Proof: We transpose sides in (53), replace t by τ as the variable, and raise both sides to the power m, obtaining

$$K^{m}\ell(\tau) = c^{*m}(\tau). \tag{57}$$

Multiplying both sides of (57) by the positive factor

$$\left(\frac{1}{1-m}\right) K^{-m} c^{*1-m}(\tau) e^{-r(\tau-t)},$$

cancelling powers of K on the left, and reordering factors gives

$$e^{-r(\tau-t)}[(\frac{1}{1-m})c^{*(1-m)}(\tau)\ell(\tau) = (\frac{1}{1-m})e^{-r(\tau-t)}K^{-m}c^{*}(\tau). \tag{58}$$

The factor in brackets on the left is u(c), by (51). Substituting u(c) into (58) and dividing both sides by $\ell(t)$ gives

$$\frac{1}{\ell(t)}\ e^{-r(\tau-t)}\ell(\tau)u(c^{*}(\tau)) = \frac{1}{\ell(t)}\ e^{-r(\tau-t)}[K^{-m}c^{*}(\tau)]. \tag{59}$$

Integrating both sides of (59) with respect to τ over the limits t to T, and using the definitions in (8), (20), (36), and (55), gives (56). Q.E.D.

Proposition 4: If u(c) is given by (51), then marginal willingness to pay (per unit reduction in the probability of death) for $t \geq t_0$ is

$$\text{WTP} = \frac{1}{1-m}\ \ell(t)G(t). \tag{60}$$

This proposition says that WTP is proportional to discounted expected consumption, averaged over everybody in the initial cohort. The more inelastic is the utility function of consumption, the greater is WTP.

Proof: We divide both sides of (56) by $u'(c^*(\tau))$, use (24) to replace $v(t)$ by $R(t)$, substitute the result in the right side of (30) and substitute (52) for $u'(c^*(\tau))$, giving

$$\mathrm{WTP} = [K^{-m}G(t)] \cdot [(1-m)(c^*(\tau)^{-m}]. \tag{61}$$

Multiplying the numerator and denominator in (61) by (57) gives (60). Q.E.D.

Proposition 5: Let $N(t)$ denote the present value of earnings from some age t throughout the balance of one's working life conditional on survival at age t, as in (37). If $u(\cdot)$ satisfies (51), and $t > t_0$, then marginal

$$\mathrm{WTP} \geq \frac{1}{1-m}\, \ell(t)N(t). \tag{62}$$

This indicates that livelihood provides a lower bound on WTP.

Proof. First we will establish the following result for every age t, which does not depend on CPRA:

$$G(t) = N(t) + w(t). \tag{63}$$

To show this, note that by the definitions of t_0, $w(t_0) = 0$. Substituting this into (19) and using the definitions of $N(t)$ and $G(t)$ from (37) and (55) gives

$$N(t) = G(t) \tag{64}$$

for $t = t_0$. But by equation (14), (64) also holds for $t < t_0$. This verifies that (63) holds for $0 \leq t \leq t_0$. Writing (19) as the difference between an integral with m(t) and one with c*(t), and expressing each of these integrals as the difference between the integral over $[t_0,T]$ and the integral over [t,T] gives

$$w(t) = \ell(t)[e^{r(t-t_0)} N(t_0) - N(t)] - \ell(t)[e^{r(t-t_0)} G(t_0) - G(t)].$$

Substituting (64) for $t = t_0$, simplifying, transposing, and dividing both sides by $\ell(t)$, gives (63). Notice that this preliminary result has not depended on CPRA.

Since $w(t) \geq 0$,

$$G(t) \geq N(t) \quad \text{if } t \geq t_0. \tag{65}$$

Substituting (65) into (60) yields (62).

2. Perfect Markets Case

In the perfect markets case, R(t) is given by (32) and WTP is given by (49). Substituting the CPRA utility function c^{1-m} into (48) gives

$$\bar{c}_s = \frac{m}{1-m}\bar{c} .$$

Using this result in (32) and (49) and simplifying gives

$$R(t) = (1-m)\bar{c}^{(-m)}[N(t) - \bar{c}E(t)] + \bar{c}^{(1-m)}E(t), \tag{66}$$

and

$$WTP = N(t) + \left(\frac{m}{1-m}\right)[\bar{c} \cdot E(t)]. \quad (67)$$

The factor in brackets is discounted future consumption for a person alive at age t. Thus, in the perfect markets case, WTP is the sum of livelihood plus a multiple of discounted future consumption. The multiple depends on the degree of risk aversion, and degenerates to 0 when $m = 0$ (the period utility function is linear in consumption).

5. APPLICATION: CALCULATION FOR MALES AGE 20 ONWARDS

To illustrate these formulas, we provide numerical calculations for a representative financially self-sufficient individual--defined here as a twenty-year-old male. We assume that consumption in retirement is supplied only by savings accumulated during years of earnings. The type of factors omitted in this assumption may, as an approximation, be treated as cancelling. On the one hand, our earnings measure counts only money earnings, excluding employer-provided fringe benefits (insurance and pension contributions) and the value of home production. On the other hand, work-related expenses (commuting and meals away from home) are also excluded.

Assumptions and Data

Calculations require specification of $\ell(\tau)$, $m(t)$, r, and m. For $\ell(t)$, we rescaled a U.S. male life table (National Center for Health Statistics, 1975, Table 5-1) so that for age 20, $\ell(0) = 1$. To simplify calculations, we grouped ages into five-year intervals. Figure 2 displays this survival curve.

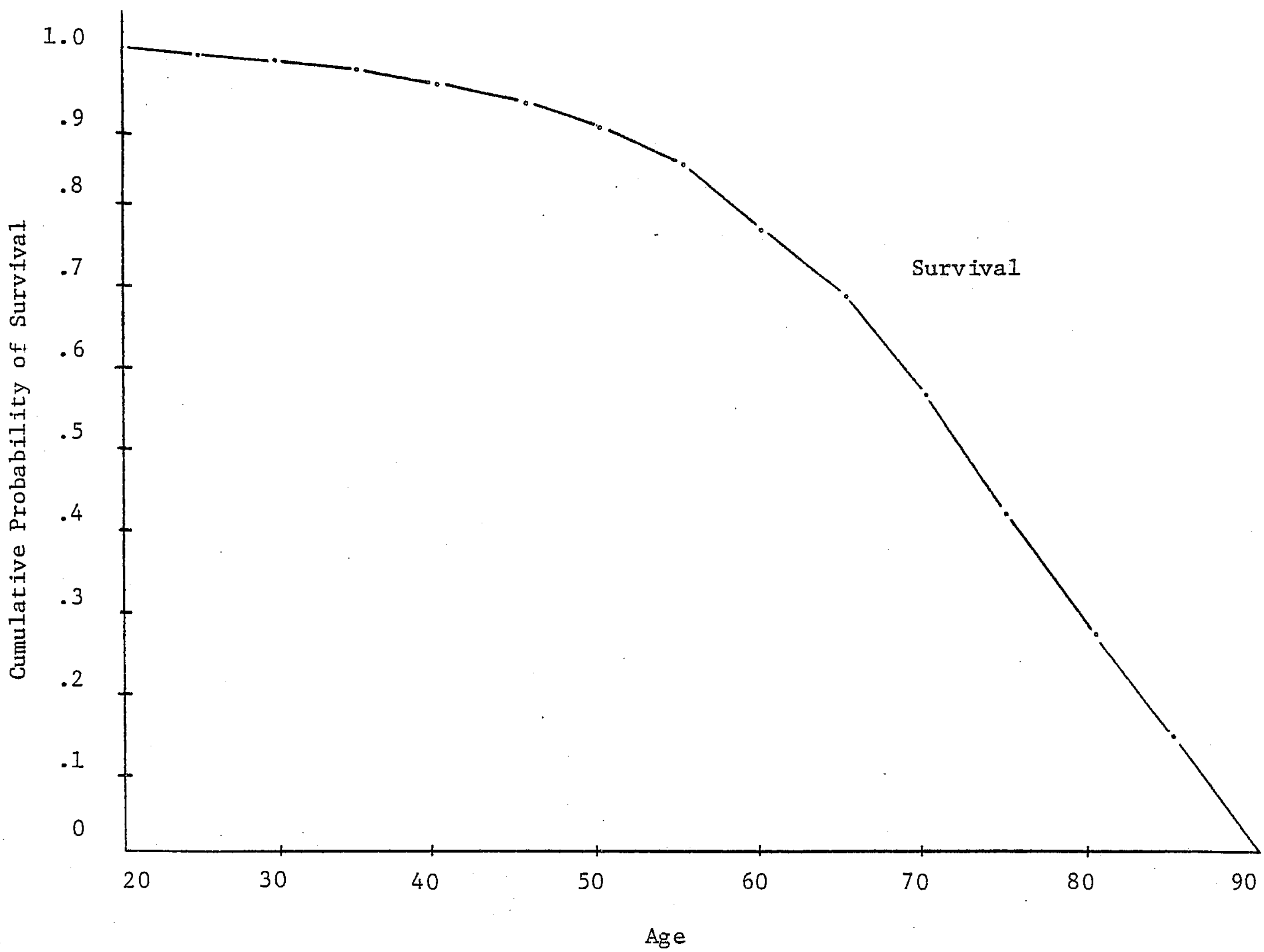

Figure 2. Survival function, $\ell(t)$.

For earnings we used an average profile from a sample of Social Security enrollees and assumed that earnings ceased at the system's normal retirement age of 65 (U.S. Senate, 1976, p. 54). In this profile, past earnings were adjusted to constant dollars by an index of wage rates to avoid confounding age effects with general increases in wage levels. Finally, we expressed all earnings relative to earnings during the peak year, which occurred at age 50. The earnings profile is shown in Figure 3.

Because earnings are expressed in constant dollars with the effects of wage increases due to inflation and productivity gains removed, the interest rate r should be a comparable real rate. For this analysis we set $r = 0.05$. As assumed previously, the period utility function is c^{1-m}, which implies constant proportional risk aversion. The value of m must lie between 0 and 1. We assume implicitly that any positive consumption level is superior to death. This utility function makes assumptions about the <u>scaling</u> of utility as well as the shape. With this function, the utility of zero consumption is set equal to the utility of not being alive. The results for CPRA presented earlier show that this assumption leads to powerful results. Equally important, it is a reasonable assumption under either of two alternative interpretations.

In the first interpretation, we consider c to be the absolute level of consumption per period. As c approaches zero, consumption approaches zero. Since part of consumption is for necessities--food, clothing, and shelter--a level of zero consumption is not possible. Values of c that are possible must all exceed some positive threshold. It seems reasonable

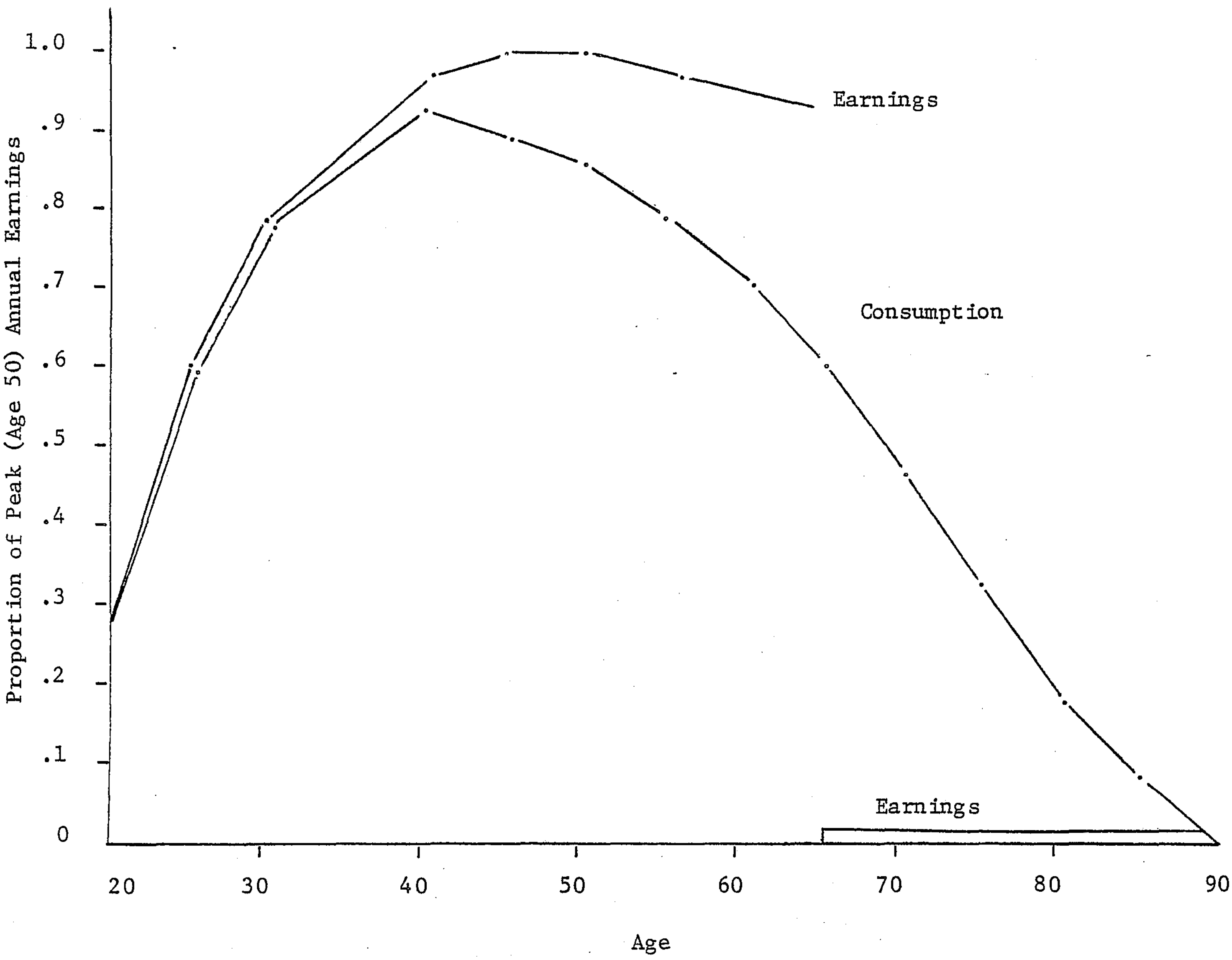

Figure 3. Earnings function, m(t), and optimal consumption function, c*(t).

that the period utility function for consumption levels above this threshold could be approximated by a function in the family $u(c) = c^{1-m}$ for some value of m. Although the approximation may not apply to consumption levels below this threshold, such values would not arise in practice. The approximation works where it is needed.

An alternative interpretation is that c is not the absolute level of consumption per period, but the excess above a level that we term a "minimum amenities level" of consumption. A year at this minimum amenities level is as bad as a year in which one is not alive at all. Similarly, what we term "earnings" in the model represents a flow of purchasing power beyond that required to meet the minimum amenities level in that year. Governmental and private welfare services, not counted elsewhere in this model, tend to provide such a floor on consumption. If this second interpretation is chosen, then "wealth" represents money wealth above that required for the minimal level. These minimum amenity amounts would have to be subtracted from actual earnings to compute the net earnings function m(t) required in our model. In a numerical calculation, a minimum amenities level needs to be chosen. For the numerical calculations in this paper, we have chosen the first interpretation, equivalent to setting minimum amenities level in the second interpretation equal to zero.

We set the risk aversion parameter $m = 0.8$, so that utility is proportional to consumption to the 0.2 power. With this value, a consumer faced with a lottery giving equal chances of consumption rates of \$10,000 and \$20,000 per year would have a certainty equivalent of about \$14,300, reflecting a risk premium of \$700, or 4.7 percent of mean consumption.

Together, the scaling of utility and the choice of m imply that the level of consumption is relatively unimportant compared to survival itself. To avoid a 1 percent chance of death now, the representative consumer would be willing to cut his consumption over the entire remainder of his life by 5 percent. Note that the units for consumption are the same as those for earnings--peak year earnings.

Analysis of Polar Cases

1. Robinson Crusoe Case

The optimal consumption function is shown by the consumption curve in Figure 3. Notice that it is identical to the earnings curve from age 20 to 35, because increasing earnings make savings not worthwhile. Beyond age 35 savings begin to accumulate as consumption drops below earnings. These relationships are shown more clearly in the wealth curve (Figure 4). Wealth is identically zero up to age $t_0 = 35$. Then it gradually increases, reaching a maximum at age 65 when earnings cease. Thereafter wealth declines as it is depleted by consumption. The most important function is discounted remaining consumption, G(t). In Figure 5 we have plotted $K^m v(t)\ell(t)$ on the same graph as $G(t)\ell(t)$. As required by Proposition 3, these two expressions are identical to one another at age 40 and beyond, the age at which the solvency constraint relaxes. (The interval between ages 35 and 40 is a mixture, arising because consumption and survival were considered step functions over each five-year interval.)

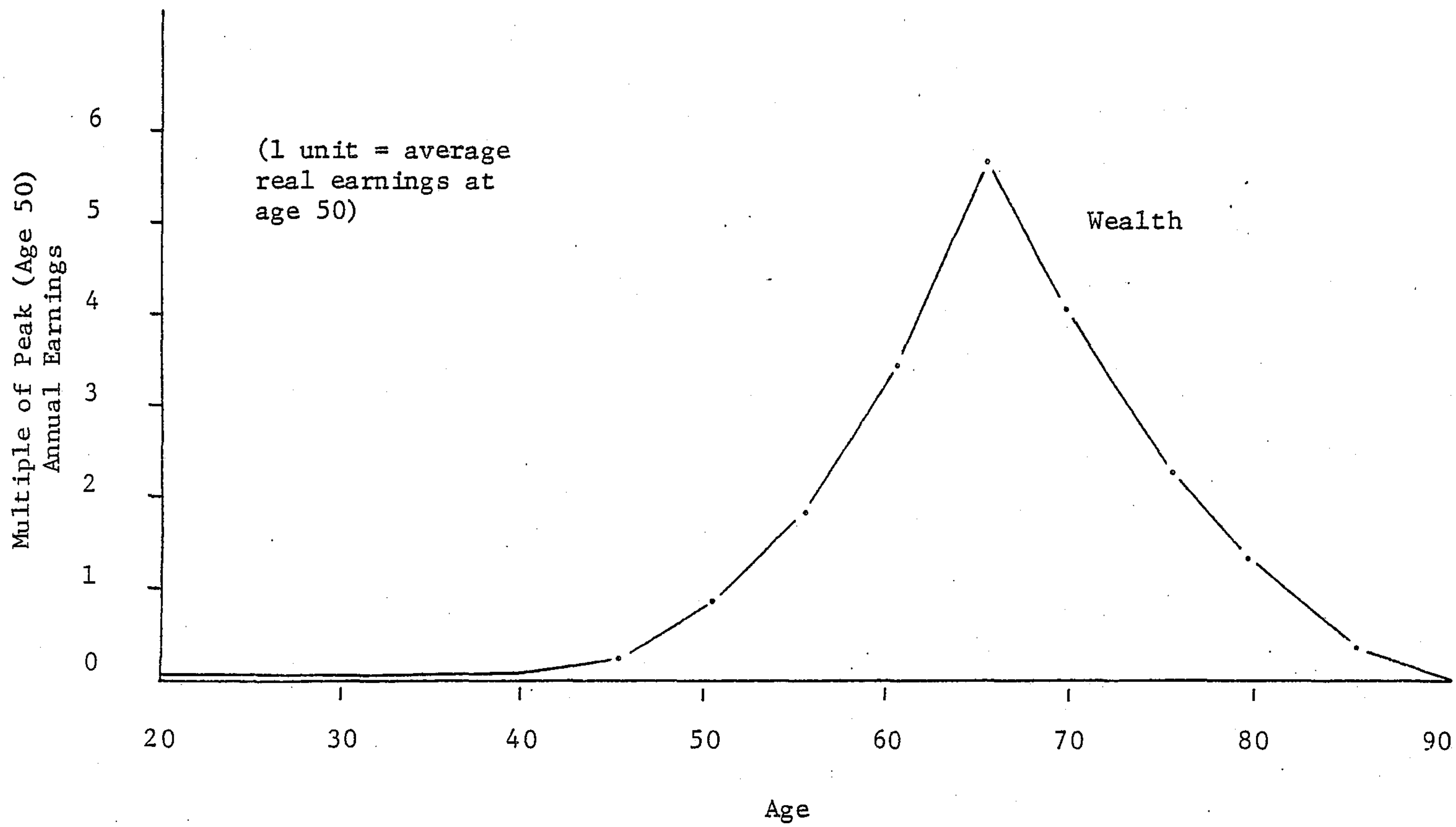

Figure 4. Wealth function, w(t).

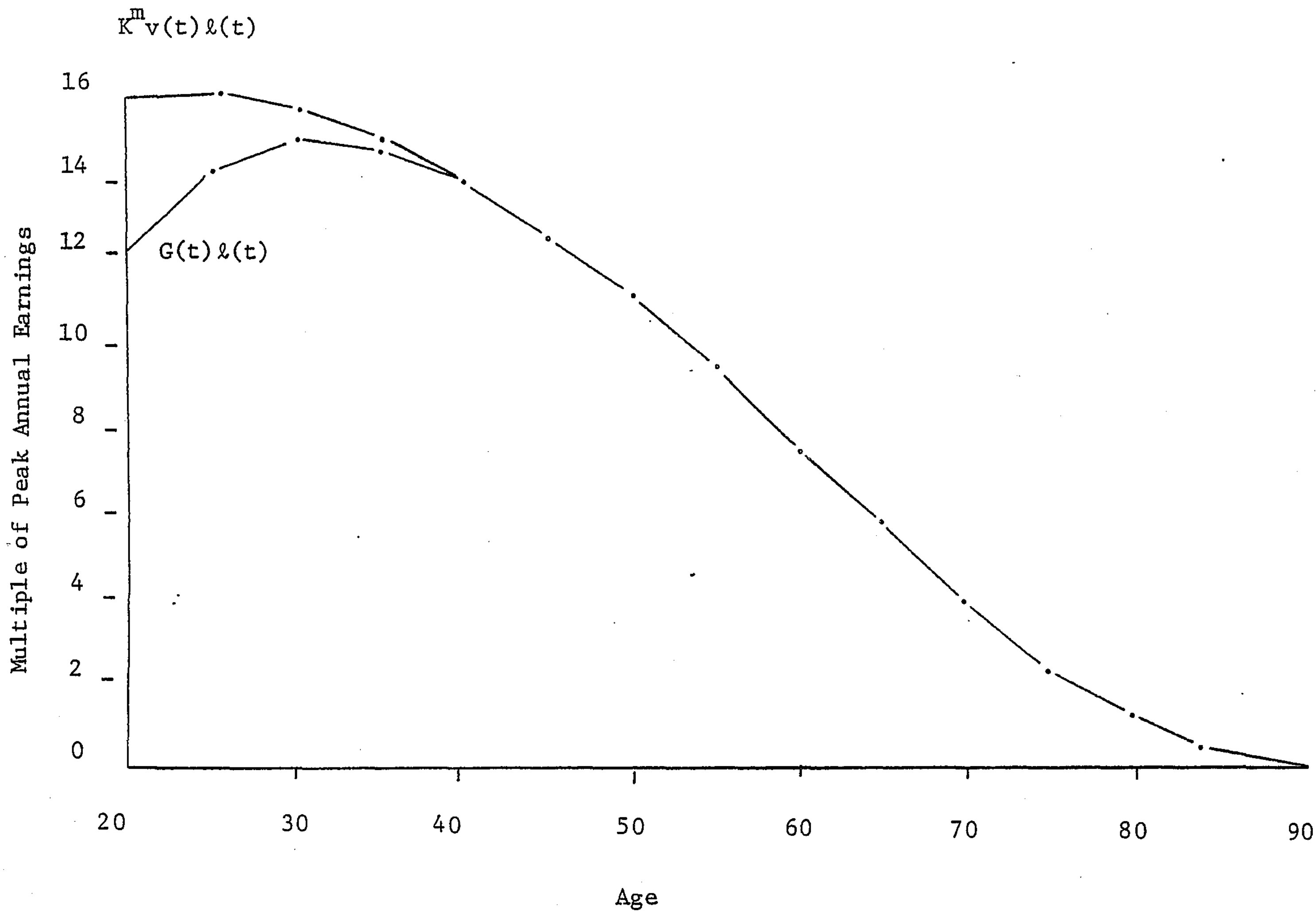

Figure 5. Total discounted consumption, $G(t)$, and expected utility conditional on survival, $v(t)$ in Robinson Crusoe case.

Note: The curves coincide when the solvency constraint ceases to bind.

In Figure 6 we present remaining lifetime utility, $v(t)$. By Proposition 1, this function equals $R(t)$, the value of saving the life of the representative male at age t. Notice that the peak occurs at age 25. Interestingly, from 20 through 35 the function is almost constant; from ages 35 through 85 it can be approximated by a straight line that would intersect the abscissa at age 95. For purposes of approximate calculation, we can treat age 95 as a horizon, and assume that $v(t)$ is composed of two straight lines that meet at age 35. Using this approximation, the value of saving a life decreases linearly with age after age 35, reaching zero around age 95. Thus, the utility of remaining life to a 35-year-old man is twice that of a 65-year-old.

The curvature in $v(t)$ means that a straight line cannot fit exactly. The departures from linearity are in the direction of upward convexity. In the ages 20 through 50, utility falls less rapidly than the straight line approximation would indicate; thereafter it falls more rapidly. The reason is that the loss in additional years of survival as one moves from age 20 to age 50 are of relatively little import--they are far in the future, and hence heavily discounted, and they are years of relatively low quality anyway because of the low level of anticipated consumption. Lifetime utility begins to fall much more steeply at age 50 because the years of life lost come much sooner, and the quality of the more immediate years is declining noticeably with age because of lower consumption rates.[21]

It is interesting to contrast the utility $v(t)$ with discounted remaining life expectancy of a male based on the same life table--a procedure which

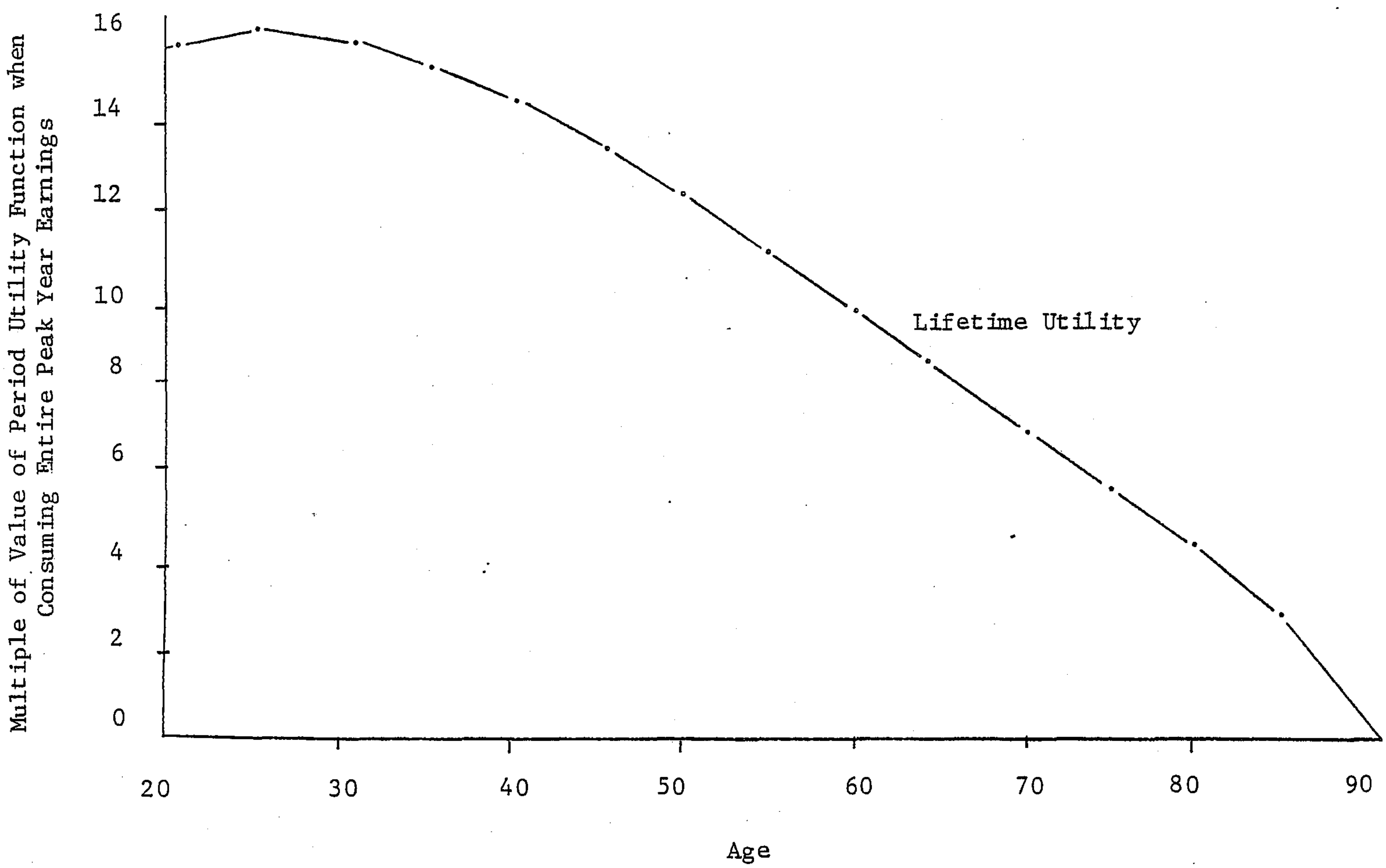

Figure 6. Utility conditional on survival, v(t).

treats all years of survival equally, regardless of level of consumption. Although a straight line from ages 35 to 95 would provide a very rough approximation, this curve departs from linearity in the direction of concavity upwards. A perfect linear relationship would indicate a survival pattern consistent with a fixed, deterministic age at which life ceases. Life expectancy would be the number of years remaining until that age. The upward concavity arises because survival is not deterministic. Each year of survival indicates that certain chances of death have been successfully overcome, and one's expected age of death (the projected intercept of a linear relationship) is constantly pushed outward.

Finally, in Figure 7 we present willingness to pay for reductions in the probability of death. For $t < t_0 = 40$, WTP is given by (32), and for $t \geq t_0$, WTP is given by (30). In the latter case, we found $\lambda(t_0) = 0.203$ by equating (18) and (52). In this case, WTP is a constant multiple (4.924) of $v(t)$ given in Figure 5. At low ages WTP is particularly depressed because the shortage of current earnings compared with current and future consumption preferences is particularly acute.

To translate these multiples into dollars, we need actual "earnings" for a recent year. In 1978, the mean income of men aged 45 to 54 with money income was $18,874. We assume that the age profile is similar to that for 1975. Assuming that the distribution of their income by source was similar to the aggregate for all families with income, only 4.4 percent or $830 is attributable to interest, dividends, rents, and royalties (U.S. Bureau of the Census, 1980, pp. 462, 457). The remainder, about $18,000,

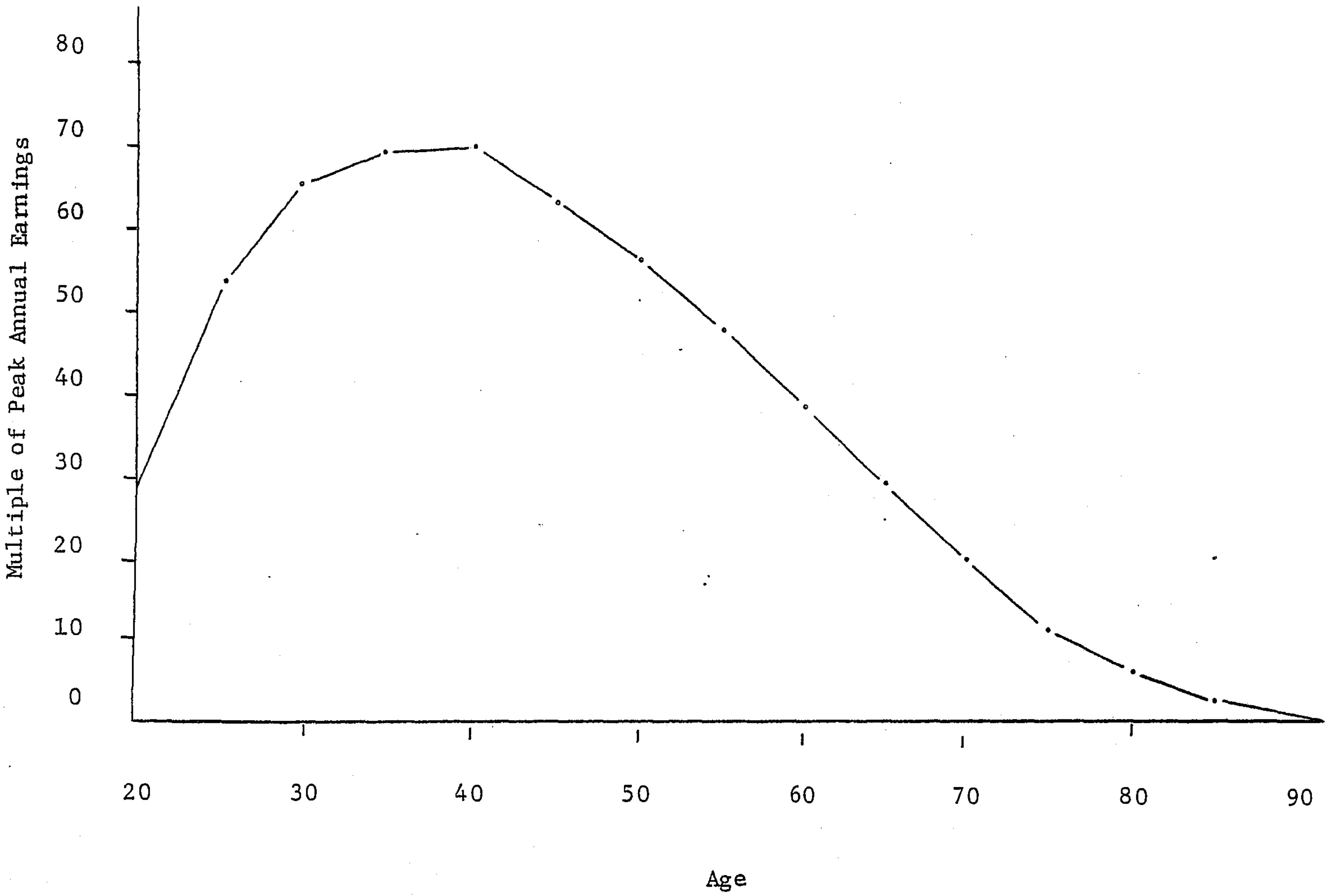

Figure 7. Willingness to pay as a function of age: Robinson Crusoe Case (no annuities available).

is from earnings (including self-employment) and transfers. Thus, to translate the multiples in Figure 7 into 1978 dollars, we multiply by \$18,000. This gives a peak value of WTP for the Robinson Crusoe case of about \$1.25 million. Other values are shown in the Robinson Crusoe column of Table 3.

2. Perfect Markets Case

If perfect markets exist (annuities are available), the optimal consumption function is a constant. For the survival and earnings functions in Figures 2 and 3, we find $E(0) = 17.94$, $G(0) = 11.70$, so by equation (38) $\bar{c} = 0.652$, and by equation (49) $\bar{c}_s = 3.260$. All the results are summarized in Figure 8.

Curve A shows willingness to pay per unit reduction in the probability of death at each five-year age interval, as given by (49). Since the rate of consumption is constant at all ages, the marginal utility per unit of reduction is the constant multiple, $u'(\bar{c})$ times WTP (where $u'(\bar{c}) = 0.142$). Curves B and C show the components of WTP. Curve C shows the livelihood component, the first term in (49), discounted expected future earnings. Curve B shows the consumer surplus component, the second term in (49). Notice that by (50), three variables are all proportional to curve B: marginal utility of a change in probability in survival (ignoring adjustments in annuities), $R(t)$; discounted remaining life expectancy, $E(t)$; and utility of remaining life, $v(t)$. Comparing Curve A with Figure 7, we see that annuities make WTP flatter as a function of age. Annuities raise WTP considerably before age 33 and after age 55 but depress WTP

Table 3

Valuations of Life at Various Ages Derived from Willingness to Pay for Males with 1978 Average Income Profile[a] (in millions of dollars)

Age	Robinson Crusoe Case	Perfect Markets Case
20	$0.50	$1.26
40	1.25	1.15
60	0.63	0.83
80	0.10	0.37

[a]Average earnings of 45-54 year-old male (excluding persons with no income) was $18,000 per year.

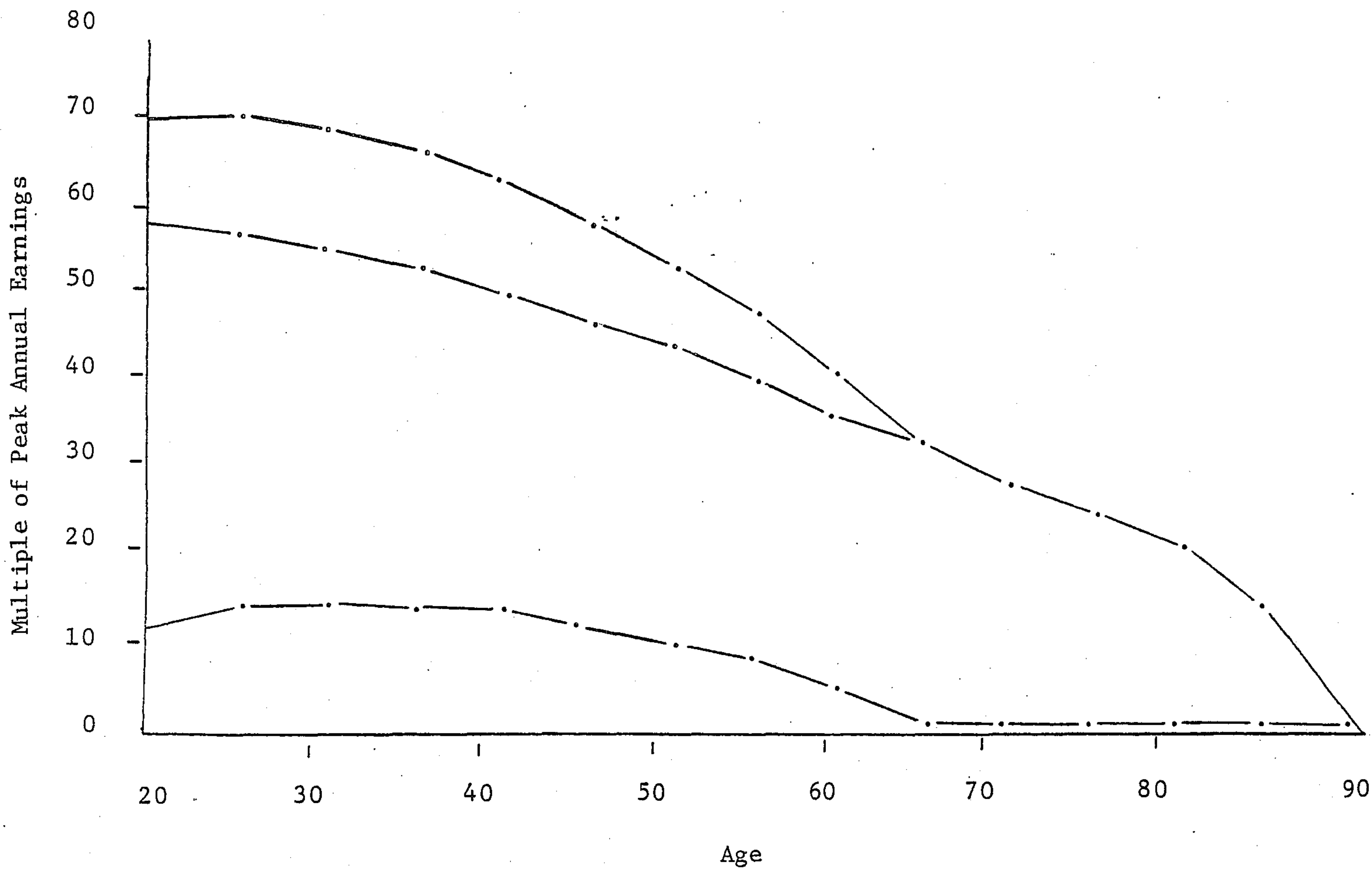

Figure 8. Components of willingness-to-pay to reduce the probability of death, as a function of age in perfect markets case:
A = Total
B = Consumer surplus with existing wealth, $\bar{c}_s E(t)$.
C = Discounted expected future earnings, $N(t)$.

somewhat within this high-earnings interval. Using the peak earnings rate of $18,000 per year cited previously, WTP is $1.26 million for males aged 20 to 30, and declines with age thereafter. Values for selected ages are given in Table 3. These results show that the availability of perfect markets has different effects on WTP, depending upon age. At very young ages, where consumers would otherwise be limited to their current income, and at advanced ages, where self-reliant consumers would have very little assets left, perfect markets increase WTP. For middle ages, WTP is less because peak consumption is lower.

On the average, earnings of women are only 43 percent of those of men (U.S. Bureau of the Census, 1980, p. 462). Although the earnings pattern of women has not been examined in detail in these calculations, it is likely that WTP for females would be slightly less than half of that for males. Thus, peak WTP for females in 1978 prices would be about $600,000.

Validation of Models

We have focused on two polar models. One assumes perfect markets, the other one no markets. The real world lies somewhere in between, which suggests that in a rough and ready way, if our models explain behavior for their idealized worlds, behavior in the real world should follow some in-between pattern. Thus, for example, we should observe life-cycle savings patterns, but they should be somewhat less pronounced than the Robinson Crusoe model would suggest.

Life-cycle savings. With departures that do not diminish the validity of the model for our purposes, observed patterns of savings are generally consistent with the life-cycle model. Individuals obviously vary in their survival probabilities and utility structures, and the relationships of our model are certainly nonlinear. Therefore, extrapolations from our calculations for a representative individual should not necessarily match population data. Nevertheless, the comparison at least indicates that our model is plausible.

To obtain testable relationships from our model, we weighted the age-specific earnings (Figure 3) and wealth (Figure 4) by the 1970 age distribution for males in the United States aged 20 and over to give an average earnings rate of 0.68 peak earnings and an average wealth holding of 1.00 peak earnings (the equivalence is coincidental). Thus the ratio of wealth to earnings is 1.47. We can compare this ratio with that of the actual counterparts in the economy--that is, the ratio of personal wealth to noninvestment personal income (all personal income except dividends, interest, and rental income) for a typical individual.

This overall ratio is not really the appropriate test of our model, however, because of the substantial inequality in wealth. Top wealth holders, persons with gross assets of $60,000 or more, constituted 9.7 percent of the 1972 adult population (age 20 and above) of 132 million, but owned 66 percent of the country's personal wealth (U.S. Bureau of the Census, 1975, pp. 6, 408, 409).[22] As an attempt to approximate the ratio of average lifetime wealth to average annual income for a person of more typical circumstances, we exclude the wealth owned by the top wealth holders and, for rough comparability, the 27 percent of earnings we attributed to this group.[23] For the remaining 90.3 percent of the adult population the ratio of net wealth (34 percent of $3,447 billion) to noninvestment income (87 percent of $813 billion) was 1.97. This is close to the ratio from our model of 1.47.

A second procedure for validating our model is to begin with the ratio of the median net personal wealth to the median total income in 1972, a ratio of 1.28.[24] We multiply this ratio of medians by the ratio of total income to total noninvestment income in 1972, 1.16 (U.S. Bureau of the Census, 1973). The resulting estimate of the ratio of median wealth to median earnings, 1.49, agrees remarkably well with the ratio in our model of 1.47.

The 1972 survey of wealth presents a distribution of net worth by age category that further corroborates our model. Median household net worth is $760 when the head is less than age 35, $6,600 for ages 35-44, $10,500 for ages 45-54, $13,200 for ages 55-64, and $9,700 for ages 65 and over.[25] Although wealth is not identically zero up to age 35, as predicted by our model, it is low and is held mainly in the forms of cash and equity in consumer durables such as automobiles. The minor discrepancy arises because our model omitted the transactions and precautionary needs for wealth, as well as institutional constraints and higher interest rates on borrowing against the full value of all assets.

Ability to infer willingness to pay. It is instructive to compare the results of our model with pilot surveys of willingness to pay cited earlier. Only the study of Fischer and Vaupel (1976) permits the effects of age to be assessed. In that study, subjects were asked to assign utilities to lifespans with specified rates of consumption and specified ages of death. These utilities assumed that an individual was certain to live to exactly age t and consumption would be at a specified constant level c. Assuming the period utility function u(c) is a time-invariant function, lifetime utility y(0) becomes

$$y(0) = \int e^{-rt} u(c)dt = 1/r[1 - e^{-rt}]u(c).$$

This function depends on both the specified rate of consumption, c, and the age of death, t. To compare this function with those assessed by Fischer and Vaupel, we rescale it by a positive linear transformation so as to make the utility for the lowest combination of c and t ($4,000 and 30 years, respectively) zero, and the utility for the highest combination ($24,000 and 80 years, respectively) unity.[26] Letting u(c) be the function defined by (51) with m = .8, we determined the utilities in Figure 9. For comparison we reproduce Figure 10 from Fischer and Vaupel. It shows utility as a function of age for their 65 "Type I" subjects, those who assigned the lowest utility to c = $4,000 and t = 30 years. They represented 78 percent of all respondents.

The patterns of utilities assessed by subjects and derived by our model are similar in general shape. Both utilities increase with age and consumption level; both figures show about the same utilities with the highest consumption level and shortest lifespan (0.2) and with the

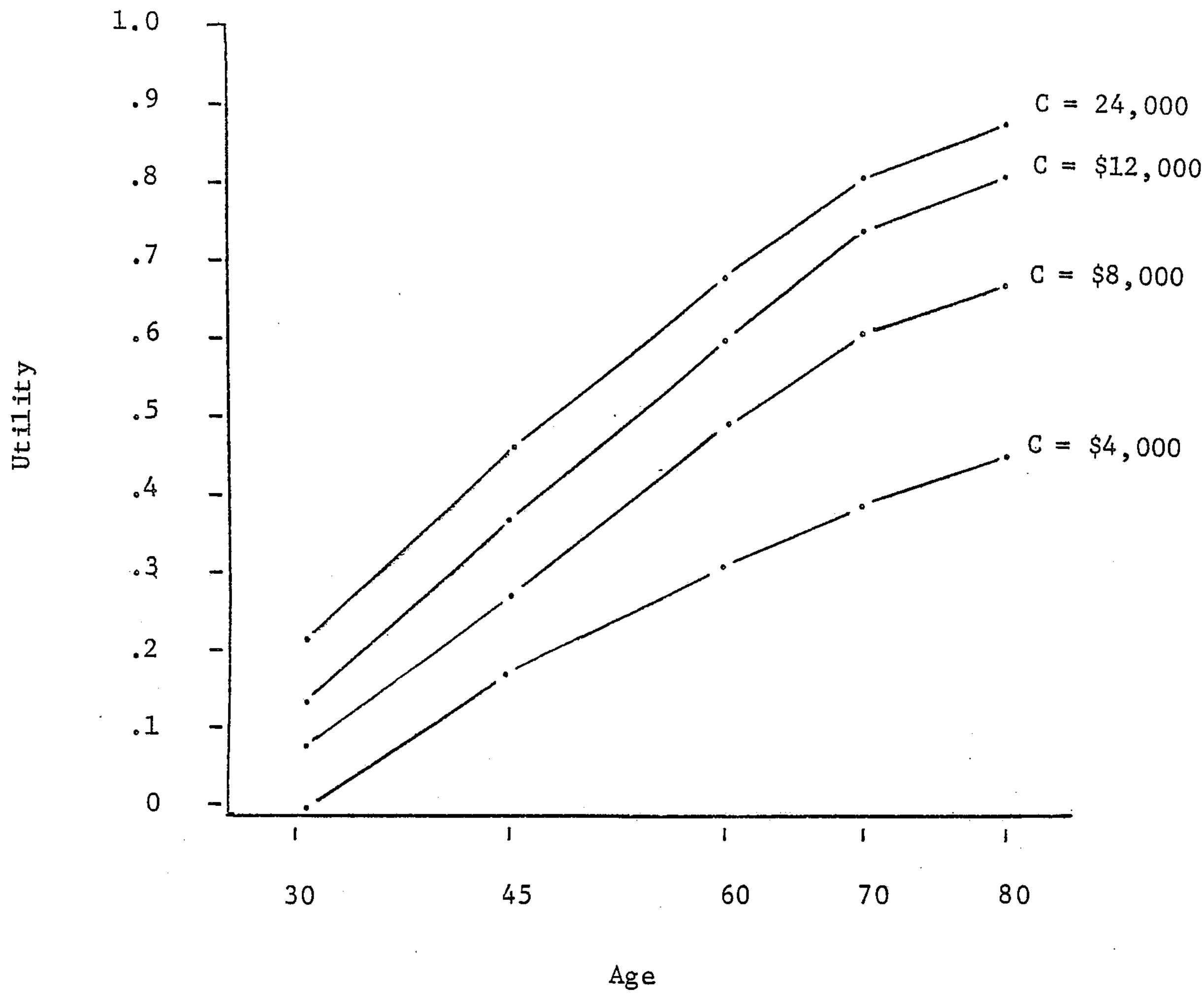

Figure 9. Mean utilities assessed directly by 65 subjects. Source: Fischer and Vaupel, 1976, Figure 1.

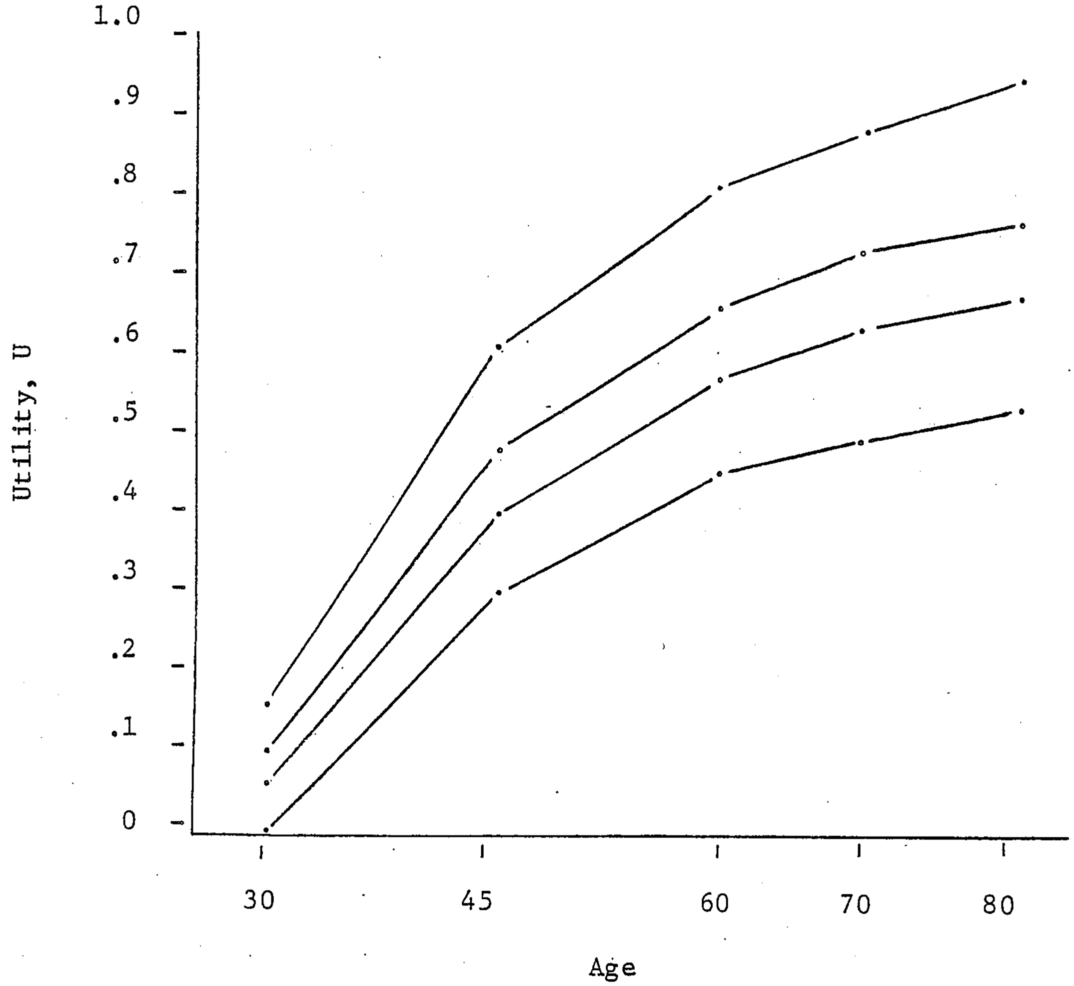

Figure 10. Utilities derived from our model for fixed lifetimes with constant consumption levels, using parameters from our application.

lowest consumption level and longest lifespan (0.5). At a more detailed level, certain differences are apparent.

First, utility is closer to a linear function of age holding consumption level fixed, in Figure 9, than in Figure 10. (The lines in Figure 9 are straighter than those in Figure 10.) This difference implies that subjects discounted future utility at a rate less than the r = .05 that we used.

Second, the spacing between lines at different consumption levels at any given age in Figure 9 indicates that the 65 subjects were more risk-averse in consumption than implied by the proportional risk-aversion parameter m = 0.8. Values of m obtained from another source were also greater than unity. Richard (1972) assessed the certainty-equivalent level of consumption for hypothetical lotteries on consumption for two subjects. Although his own analysis used the sumex utility function on consumption (the sum of two exponential functions), we have used his data to fit constant proportional risk-aversion utility functions for utility on consumption at ages 51-56. The values of m ranged from 1.1 to 2.4

We should not require our model nor select our parameters to agree with the subjects in Figure 9 or those considered by Richard. In both cases the subjects were university students and faculty, and not necessarily representative of the general population. Nevertheless, it is reassuring that the values of our preference parameters produce willingness-to-pay values that roughly correspond to the ones that would be generated by their assessments; were we to calibrate our model to their empirical findings, the agreement could be almost complete.

Additional Considerations

Many important considerations could be included in our model, imposing varying levels of mathematical complexity. Among the more important are the following:

i. Other sources of income. Our model could be made more realistic by replacing age-specific earnings with age-specific noninvestment income. The change would include nondiscretionary sources of income which an individual could use to accumulate wealth or rely on for support in retirement--mainly employer fringe benefits, proprietor income, and transfer payments (e.g., social security). Since these other sources, particularly transfer payments, tend to reduce the variability in income with age, they diminish savings due to the life-cycle pattern of earnings, and they increase consumption and willingness to pay at all ages, but particularly at post-retirement ages when these other sources of income are relatively most important.

ii. Families versus individuals. The preceding analysis has considered the utility of an individual isolated from family ties, both emotional and financial. An attempt to relax this assumption introduces a very complex constellation of interdependencies. The joint survival of all members of the family is important because of the effect both on earnings and on the utility of consumption at different periods. Finally, the current consumption versus bequest motive becomes important for each family member. The complications of attempting to develop such a model would probably be prohibitive.

Instead, we can include the bequest motive by assuming that an individual concerned with his legacy provides for it by the purchase of term life insurance. The decision about how much term life insurance to purchase is assumed exogenous to our model, and the annual premium for this protection is subtracted from earnings. (Whole life insurance would be considered a combination of term life insurance plus savings.) In view of the institutional arrangements by which life insurance is obtained, this treatment of legacies seems plausible. Half of the life insurance in force in 1974 was group, industrial, or credit, where subsidies or implicit provision give the insured little real discretion about obtaining insurance. The amount of ordinary (individually purchased) life insurance in force is only \$12,000 per adult (U.S. Bureau of the Census, 1975, pp. 6, 482).

The fact that our model has excluded the bequest motive for holding wealth is somewhat justified by the relatively low amounts of wealth held: In 1972, when per-adult personal income was \$7,000, net worth per adult was only \$12,000, and *median* net wealth per household was around \$14,000.[27]

6. CONCLUDING REMARKS

At the outset we identified three possible objectives for a study of this type: understanding the choices affecting survival that individuals actually make, helping them make superior choices in this complex area, and improving public policies that purchase survival. It is in this last area that our methods are likely to be most controversial. The final product of this study is a mechanism for valuing small changes in risk levels to individuals of various ages who have particular preferences and earnings opportunities. Thus, we have a means to compute the value to an individual of the benefits a public program provides to him.

But faced with the question of how we should use such numbers, the world divides into a number of camps. Some appear to find such numbers useful only to the extent they can be held up to ridicule, in effect to help discredit the whole cost-benefit approach. This hostile group in turn consists of two factions, at opposite ends of the political spectrum. Those who enshrine individual rights argue that money should not be taken from A to benefit B, no matter how great the benefit to B. More scientific-sounding calculations in support of such programs just lead to extra mischief. At the other extreme, those who see government as an instrument of social change and societal betterment are likely to argue that government programs should be judged by their ends, and that well-meaning programs should not be held up to the yardstick of efficiency.

The cost-benefit approach also has its passionate partisans, who ask us merely to plug in the numbers, and to select those policies that offer the highest total net value. There is a rich group of qualifying sects in the middle. They would agree that it is legitimate to discriminate among various changes in survival probabilities according to some criteria but not others. Most would accept the overall wealth level of society as an appropriate variable to acknowledge. A smaller group would accept differential valuations based on age, but would reject those based on income, in part on the theory that we all get a chance to pass through various ages but not through various income groups. Finally, many accept cost-benefit calculations based on age and wealth for such matters as travel time or recreation, but reject them for health-related issues, on the grounds that health is a merit good or even a right.

Many individuals support the goal of valuing survival outputs on cost-effectiveness grounds: We should get the greatest value of output for the dollars we devote to survival. Implicit in their argument is the belief that calculating values will have more effect on redirecting resources among programs than in increasing or decreasing total dollars spent. As with cost-benefit analysis, support for the cost-effectivenss route may be qualified as to what variables are acceptable in computing output measures.

Recognizing, then, that our results could be employed to support or oppose a great variety of positions, what do our numbers say about the two most controversial issues, willingness to pay for increased survival as a function of income (or wealth) and age?

Income or Wealth and Willingness to Pay

The application of our model discussed above considered only an individual at one earnings level and with one survival curve. In effect, an average 20-year-old male was considered a representative individual. To make appropriate predictions about individual behaviors at different earnings levels, we would have to account for differences between the rich and the poor in preferences and present probability of survival, as well as in wealth or earnings opportunities. To simplify, we shall deal with the case in which rich and poor on average are alike except in terms of earnings.

Before turning to calculations on rich versus poor valuation, it is worth inquiring why rich people value their lives, or more accurately risk to their lives, more highly than poor people. It is often asserted that they simply have more money to spend, and that money hence means less. This is an insufficient explanation. Although rich people have more money, for example, they do not value carrots or potatoes (in money terms) more highly than poor people do.

What are the critical differences between the carrots-and-potatoes case and the lives-valuation case? There seem to be two. First, lives are not bought and sold on markets. Some might say that this is because no one would sell his life. Leaving aside Faustian bargains, that might be true if only whole lives could be sold. But the market could be more refined. It might be perfectly rational for a poor person to sell a chance on his life, say a 1 percent probability of death, to a rich man if the transfer could be carried out. Occasionally an economist or a horror writer (is this tautological?) will posit a hypothetical lottery in which poor men sell a chance on their hearts or other vital organs.

Leaving moral scruples aside, the lottery process does not seem very realistic. The transaction costs associated with heart transplants, or most forms of health transfer, are great. This is not to suggest that we do not sell health in any form. Medical care is still available for purchase. Individuals with broader opportunity sets will generally take less risky jobs. Richer people choose to live in areas with less crime and less pollution, at the expense of higher mortgage payments. It is no surpise that rich people on average live longer than the poor.[29]

The second reason why rich and poor do not value their lives equally at the margin is that marginal probabilities are involved, not marginal purchases. Assume for the moment that survival could be sold on markets, and consider two situations. In one, life years would be for sale. In the other, the commodity offered would be probabilities of survival.

In the life-years situation, let us assume that both rich and poor start off with a 70-year entitlement. To simplif , assume they have an endowment of wealth that is independent of how long they live. Presumably both rich and poor have a utility function of the form U(W,L) for wealth, W, and longevity, L. The competitive market might, for instance, trade 11 years from poor to rich in return for $300,000. When trade is opened, as is usual when preferences are similar, the gap is narrowed in the good for which endowments were unequal (wealth) and a differential is established in the good of initial equality (longevity). From the standpoint of market function, however, the important point is that, at the equilibrium, both rich and poor would pay the same amount for another life year.

The situation is quite different when one has a chance to purchase probabilities of gaining or losing an entire lifespan. In this case, the rich and poor are buying quite different commodities. The rich man is getting a life of affluence, the poor man one of relative hardship. Would it not be reasonable to pay more for the rich man's life? The point can be made for a single individual pursuing a career whose economic success is chancy. If he were able to distribute his survival probability among various future career scenarios, he would probably assign more survival probability to the richer scenario. Indeed, if the tradeoff were linear, all probability of death would be attached to low-wealth outcomes.[30]

To make our previous example more concrete, consider the case in which the rich man earns twice as much as the poor man in each year, and both have level consumption in each period. (Their earnings may be level, or they may have well-functioning actuarial and capital markets on which to trade.) For simplicity, let us assume that consumption levels are not adjusted according to survival probability. (This could be our pensioner case, starting with a given initial wealth, or a situation in which future consumption and earnings just balance.) Consumption in each period is the constant $\bar{c}$.

As we saw earlier, a small change in survival probability will yield a benefit computed by multiplying that probability times the individual's discounted expected lifetime utility. Represent his period utility as

$u(c)$. In equation (47), we saw that under perfect markets (or other conditions of constant consumption),

$$WTP = [N(t) - \bar{c}E(t)] + \frac{u(\bar{c})}{u'(\bar{c})} E(t). \tag{68}$$

Here $E(t)$ is discounted life expectancy, a value that depends on age, survival probabilities, and the discount rate. If earnings $N(t)$ match future consumption $\bar{c}E(t)$ at every age, the first term above vanishes, leaving

$$WTP = \frac{u(\bar{c})}{u'(\bar{c})} E(t) \ . \tag{69}$$

An intuitive derivation of (69) is provided by the observation that the disutility to an individual of foregoing an amount of money M to purchase survival, i.e., M times his marginal utility of money, must just equal his utility gain from the reduction in mortality. This yields

$$Mu'(c) = \delta\mu \cdot E(t)u(\bar{c}). \tag{70}$$

Thus willingness to pay is

$$WTP = \frac{M}{\delta\mu} = E(t) \ \frac{u(\bar{c})}{u'(\bar{c})} \ ,$$

as we observed in (69). Since discounted future earnings were assumed always to equal discounted future consumption, $E(t)\bar{c}$, we can compute the ratio of WTP to future earnings. It is

$$\frac{WTP}{E(t)\bar{c}} = \frac{u(\bar{c})}{\bar{c}u'(\bar{c})} \ .$$

This ratio varies with $\bar{c}$ depending on the shape of $u(\bar{c})$, the degree of risk aversion. For the special case of CPRA, this ratio is constant for all consumption levels. Consider the willingness to pay of a rich and a poor person, where the rich person earns J times as much as the poor one. Under CPRA, the rich person will pay J times as much for a given reduction in mortality as the poor person.

<u>Age Patterns and Willingness to Pay</u>

In both the Robinson Crusoe and perfect markets cases, willingness to pay for improvements in survival as a function of age follows an inverted-U pattern. At younger ages, it is low because of low earnings and the discounting of years of higher earnings. At older ages, willingness to pay is reduced because of shorter remaining life, as well as lower levels of consumption in the Robinson Crusoe case. In both cases, willingness to pay reaches a maximum of about 70 times peak annual earnings during young adulthood--age 25 in the perfect markets case and age 40 in the Robinson Crusoe case. In 1978 prices, this is about $1.26 million for a male with earnings. The age that maximizes willingness to pay is younger in the perfect markets case because the perfect capital and contingent claims markets allow a young adult to borrow against future earnings in purchasing survival.

Beyond age 55, willingness to pay is also higher in the perfect markets case because consumption is at a higher level, conferring a higher utility on advanced ages. Also, the need to hoard wealth against the chance of a very long life is reduced. Finally, the perfect markets case assures that each individual consumes his entire wealth and earnings before his death, on an expected value basis; thus his level of consumption is higher, on average. Therefore, on average the utility in each year of life is higher, and the marginal utility of wealth is lower. As a result, the perfect markets case motivates an individual to spend more on buying survival, and allows him to do so.

A Summing Up

Our task in this paper was to determine how much individuals with specified preferences, survival opportunities, and earnings schedules would pay to increase their survival. We considered two polar models, one with perfect annuity and capital markets, the other with no such markets. Such models can be applied to explain observed patterns of nonmarginal purchases of survival, as we showed in part 2. We also discovered that if individuals are guaranteed a consumption level that is independent of their survival purchases, they will spend a greater-than-optimal amount--from the standpoint of society--on survival.

Part 3 of the paper developed our models in substantially greater detail. Part 4 utilized a constant proportional risk-aversion period utility function and empirically observed survival and earnings functions. For the case in which markets for annuities were absent, we observed life cycles in savings and in willingness to pay for survival. With perfect markets there are no savings as such--though one's market value would follow a cyclical pattern--but willingness to pay first rises, then falls. The maximum is reached much earlier than in the Robinson Crusoe case but not--significantly--at the beginning of earning years. It is slightly better (i.e., an individual would pay more) in the perfect markets case to postpone one's survival purchase until the gap between earnings and consumption widens. Getting closer to the peak gap more than compensates for the fact that the number of years giving utility has been reduced.

The principal objective of our analysis was not to present specific numbers for WTP, but to demonstrate the feasibility of a methodology. If this approach becomes accepted, substantial effort will have to be expended in estimating utility functions. Assessments of data about individual choices as well as survey work will be helpful in assessing period utility functions. If ultimate relevance is our goal, we should also be willing to undertake further empirical investigations of utilities over lifetimes of different length. (Different qualities of life years--as determined say by consumption levels--should also be considered, but given our lack of knowledge on the first issue, only as a second-order priority.) Thus we might ask, what chance of death at age 35 would an individual with a constant consumption level of $15,000 just exchange for a 1 percent chance of death at 60? At present, we have little intuitive feel for the way individuals would answer such questions, whether in a laboratory situation or when making real-world choices.

Part 5 addressed two of the most controversial areas related to the purchase of survival--differing valuations based on age and on earnings opportunities. To what extent should societal decisions reflect such differentials? Other factors being equal, should we pay more to clean up pollution in wealthy as opposed to poor sections of the city? Should we favor medical expenditures for individuals at the start of their prime earning years, as opposed to expenditures for those who are near to retirement, which may be somewhat more effective in terms of reduction of instantaneous mortality?

Merely considering these issues would mire us in such debates as whether health is a right or a good to be purchased, or whether cash or in-kind transfers are to be preferred. Much passion is devoted to such issues, particularly in the valuation of the lives of the rich versus those of the poor. In many instances, we believe, the disagreement stems from differences in values, rather than differences in predictions. Our purpose was to provide a methodology that would help make better predictions of what individuals in various circumstances would pay to increase their own survival. At the least, we hope that that information will help focus the debate over values.

We also hope that our approach will contribute to the continuing debate over what mechanisms are appropriate for valuing lives. We have taken the consumerist viewpoint that what you would pay is what you are worth, at least to you. This might seem strongly inconsistent with a variety of human capital approaches. It is not. For instance, in the perfect markets case--the usual starting point for economics discussion--if an individual has level earnings, his willingness to pay will be proportional to his discounted expected length of life. So too would it be under a human capital formulation that relied on discounted expected earnings.

When earnings vary over the life cycle, the individual himself will focus on human capital considerations. Two terms will enter his willingness to pay computation at a particular age: his lifetime utility, and his net contribution to his own stock of available resources. The latter is computed at each age as discounted future earnings less discounted future consumption. If we treat consumption as the cost of sustaining the individual, the second term is in some sense his net human capital. In sum, the human capital approach provides one key component of the willingness-to-pay formulation. The other is the net consumer surplus of being alive.

APPENDIX

Evaluation of the Constant, K

We could evaluate the constant K in (53) in terms of $\lambda(t_0)$ using (54), but it is easier to evaluate K directly from terminal conditions on w. Multiplying both sides of (19a) by the constant $e^{r(T-t_0)}$ and inserting (53) yields

$$A - KB = 0$$

where

$$A = \int_{t_0}^{T} e^{-r(\tau-t_0)}\, m(\tau)d\tau$$

is the present value at age t_0 of subsequent earnings, and

$$B = \int_{t_0}^{T} e^{-r(\tau-t_0)}\, [\ell(t)]^{1/m}\, d\tau.$$

Solving for K gives

$$K = A/B. \tag{A}$$

Equation (A) says that the constant of proportionality is the ratio of present value of earnings to the weighted sum of a power of the survival function. The most important feature of (A), however, is simply that K is a constant relating survival to earnings.

NOTES

[1]Many supporters of these intervention efforts express a quite different philosophy. They believe that the government should intervene to keep the maximum level of risk below the level that well-informed individuals would accept. Their views might be interpreted as reflecting (1) a belief that health is a merit good, (2) that risk acceptance is not really voluntary for individuals having poor employment opportunities, or (3) that by imposing health floors they could shift resources towards disadvantaged workers. Accepting the first argument, we should inflate individuals' valuations of the risks they confront. The second argument is a difficult one to confront, since we do not believe individuals should be denied opportunities or the right to make tradeoffs just because they are already disadvantaged. The third would suggest that the whole process is a strategic struggle. Willingness-to-pay calculations would be a weapon in the struggle.

A final argument might be that since we all share in the health and social support costs for the ill, a citizen's valuation of his own good health will understate its value to society as a whole.

[2]The case of uneven wealth among beneficiaries boils down to the problem of finding the appropriate means by which to transfer income.

[3]In Part 3 we shall allow for earnings. Then, increased survival in a period in which earnings exceed consumption will raise the stream, and vice versa. Rarely will it be possible to purchase more survival for early years without simultaneously providing later-year survival.

[4]These calculations apply assuming only incremental changes in survival can be purchased. The reduction that the pensioner formulation imposes on the consumption constraint therefore need not be considered.

[5]We remarked earlier that if a centralized society is to reach a first-best outcome it must restrict purchases of survival. This is not a pie-in-the-sky consideration. One of the justifications that is frequently advanced for mandatory social insurance schemes is that information asymmetries foster adverse selection; e.g., only those who expect long lives will buy annuities. This stymies the effective operation of private markets. If society does intervene, say with the provision of social security, in order to achieve efficiency it must intervene in another market as well and limit survival purchases.

Note also the redistributional aspects of any social security plan. It helps those who expect long lives at the expense of those who expect short ones. What the baseline for comparison should be is not clear, and is made less so because the private market may have multiple equilibria. Consider a society where a handful of individuals expect to live a long time. One equilibrium has only the long livers buying annuities. No normal person would choose to participate in the pool. But if all the normals were in, the excess cost of the long livers would be swamped by the risk-spreading gains due to having the normals participate in the insurance pool.

[6]We could add a generalizing wrinkle to our model by allowing the period utility function u to be a function of time and age, as well as

health (alive or dead) and consumption. To do this, we would subscript u by age τ recognizing that $u_\tau(c) = u(c)$ is a special case. This age dependence incorporates the tendency of consumption preferences to vary with such age-dependent factors as lifestyle or family responsibilities.

[7]An individual's knowledge of risks associated with his occupation or health characteristics (e.g., whether he is free of life-threatening diseases) provides opportunities for updating survival probabilities. We could allow $\ell(t)$ to be revised, though that would complicate our model considerably.

[8]This additive separable form, and a multiplicative form, are the two cases for which mutual utility independence between consumption in different periods applies.

[9]An alternative assumption that can be shown to be consistent with our model is that bequests are worth some constant fraction of that same wealth if alive.

[10]Given the stability of the utility function, it will always produce a trajectory that is optimal in a forward-looking framework. Once having survived to a particular age, however, you are likely to regret consumption expenditures made in the past, for we attribute no utility to memory or retrospection.

[11]Our analysis is restricted to a financially independent adult. For persons financially supporting or supported by another, the joint survival and utility patterns must be specified.

[12]The assumed equality of these two rates yields the very reasonable result that the rate of consumption would be constant over time in the absence of mortality and debt constraints.

[13]Interestingly, this result is independent of the discount rate, r, because the declining present cost of future consumption is offset by its declining contribution to present utility.

[14]For large purchases of survival, if the solvency constraint is not binding, the individual will be willing to pay less out of his present consumption than out of his wealth, for the latter offers opportunities for reallocating the reduction across future time periods.

[15]If $d\mu$ is positive, then dw is negative since both survival and wealth are valued attributes. In that case the negative of dw is a buying price of an improved lottery on survival. For a further discussion, see Weinstein, Shepard, and Pliskin (1980).

[16]This graph is drawn to scale for the example in a later section. There $u(c) = c^{0.2}$, $\bar{c} = 0.652$, $c_s = 3.260$, and $E(0) = 17.94$ years. Thus area A = 58.5; area B = 11.7; their total is 70.2, representing total WTP in the pensioner case.

[17]CPRA means that the level of consumption (c) times the local risk aversion at that level is the constant m (Keeney and Raiffa, 1976).

[18]This function in (51) could be generalized without losing CPRA by adding a constant FH. The constant FH is the pure utility in being alive, regardless of consumption. Thus, if an individual were alive with rate of consumption 0, he would risk odds of 1 to F of being dead in order to raise his period consumption rate to unity if he should live. CPRA is still preserved if the scaling factor H depends on age.

[19]If the individual is not risk-averse, the optimal solution degenerates to an infinite consumption rate in the first instant of life.

[20]As long as T is finite, the solution exists for arbitrary $\ell(t)$. If T is infinite, a solution exists provided life expectancy is finite.

[21]In this analysis, declining utility with age results solely from consumption patterns, making no assumptions about infirmity affecting utility.

[22]For the economy as a whole, in 1972 personal net wealth was $3,447 billion and noninvestment income was $813 billion, so their ratio was 4.24 (U.S. Bureau of the Census, 1975, pp. 410, 386).

[23]To obtain this estimate, we estimated the mean annual income of adults in the top 9.7 percent of the income distribution at $20,000, based on the 1972 distribution of income (U.S. Bureau of the Census, 1973). Thus they received 27 percent of total personal income of $945 billion. We assumed this same proportion applied to noninvestment income. Although the individuals with the highest gross assets are not necessarily those with the highest income, a close correspondence is likely. In any case, it seemed appropriate to exclude the same proportion of individuals from the tops of the income and wealth distributions.

[24]For that year the ratio of mean net worth to mean income by households was 3.17, comparable to the aggregate 1972 ratio.

[25]If there is technological progress over time, then younger people will have spent a greater portion of their working lives at higher productivity levels. This will tend to flatten the upward-sloping portion of the savings versus age curve. If workers' technological capabilities

are frozen at the time they enter the labor force, as opposed to the assumption that workers of any vintage will keep pace with progress, the flattening will be more extreme. Patterns of technological progress will already be reflected in the age versus earnings profile.

[26]The transformation is to subtract u ($4,000, 30 years) and divide the result by [u($24,000, 80 years) - u($4,000, 30 years)].

[27]Estimate obtained by multiplying median net worth in 1962, $6,700 (from Projector and Weiss, 1966), by the ratio of median family income in 1972 to median family income in 1962, 2.09 (from U.S. Bureau of the Census, 1975).

[28]Undoubtedly some causality runs in the opposite direction, since those with superior health presumably have superior earnings opportunities.

[29]Say a fixed price were charged per increment in survival probability at a given age, with all other aspects of survival the same for rich and poor. It is quite likely we would get a corner solution, with the rich man buying as much survival as possible, and the poor man nothing.

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