University of Wisconsin-Madison

Institute for Research on Poverty

Discussion Papers

Yves Balcer
Efraim Sadka

BUDGET SHARES AND OPTIMAL COMMODITY TAXES WITHOUT COMPUTATION

DP #634-80

Budget Shares and Optimal Commodity Taxes without Computation: A Note on Deaton's "Equity, Efficiency and the Structure of Indirect Taxation"

Yves Balcer
University of Wisconsin-Madison

Efraim Sadka
University of Wisconsin-Madison
Tel Aviv University

November 1980

The research reported here was supported by National Science Foundation Grant DAR-7917376 and by funds granted to the Institute for Research on Poverty at the University of Wisconsin-Madison by the Department of Health and Human Services (formerly HEW) pursuant to the provisions of the Economic Opportunity Act of 1964. The opinions expressed are those of the authors. We are grateful to Bernard Martin for his computational expertise in deriving the numerical results.

ABSTRACT

This paper shows for the linear expenditure system that the optimal tax rates increase across commodities in the ratio of the respective budget shares of the rich to those of the poor--i.e., the more luxurious good has a higher tax. These budget shares can be calculated either at pre-tax or post-tax price levels.

Budget Shares and Optimal Commodity Taxes Without Computation:
A Note on Deaton's "Equity, Efficiency and the
Structure of Indirect Taxation"

Introduction

In "Equity, Efficiency and the Structure of Indirect Taxation" (1977), Deaton has established a criterion to determine those goods that should be taxed more heavily than the average marginal tax rate and those that should be taxed less. The beauty of the criterion is that it depends solely on the relative budget shares of two individuals: one with the average income of society, and the other with a "socially representative" income. The socially representative income depends on the income distribution and the coefficient of inequality aversion (see Atkinson, 1970), and is smaller than the average income of society. More intuitively stated, the goods to be more heavily taxed than average are those for which the budget shares of the rich are larger than are those of the poor--in other words, luxuries. Conversely, the less heavily taxed goods are those for which the budget shares of the poor are greater than are those of the rich--necessities. The value of the criterion as a policy guide is limited by the fact that the budget shares of these two representative individuals depend on the tax rates; they therefore cannot in principle be ascertained before computing the optimal taxes explicitly, as noted by Deaton.

This paper will establish for the linear expenditure system that (1) at pre-tax prices, the tax rates increase in the ratio of the budget shares of the rich to those of the poor--roughly speaking, the more luxurious the good, the higher the tax levied on it; (2) the ranking of goods by the ratio of the budget shares of the rich to those of the poor at pre-tax prices (which are assumed to be 1 from here on, without any loss) is the same as their ranking by the ratio of budget shares at optimal post-tax prices; (3) the marginal tax rate, averaged over all commodities, is greater than the uniform tax, which is equal to the government revenue rate (revenue needs as a fraction of total income).

Model

We start by specializing Deaton's model to the linear expenditure system, as he did for his numerical example of Britain's direct taxes (Deaton, 1977, eqs. 18, 19). This restriction implies that the individual's utility of consumption (x_1, ..., x_n), up to a monotone transformation, is

$$U(x_1, \ldots, x_n) = \sum_{j=1}^{n} \beta_j \ell n(x_j - \gamma_j'), \tag{1}$$

with the following demands:

$$x_i(q,I) = \beta_i(I - \Sigma q_j \gamma_j') q_i^{-1} + \gamma_i', \tag{2}$$

where q are prices, I is money income, and γ_i' is the minimum amount of good i that must be consumed before deriving any utility. The term β_i can be interpreted as the budget share of the rich (individual with infinite income) for good i, and the term $\gamma_i = \gamma_i' / \sum_{j=1}^{n} \gamma_j'$ is the budget share of the poor (individual with income $I^P = \sum_{j=1}^{n} \gamma_j'$, the subsistence level) for good i, at pre-tax prices. Let V(q,I) be the indirect utility associated with (1) and (2).

The government seeks to maximize the sum of the utilities over all income groups while meeting its revenue requirement. Thus the government problem reduces to

$$\underset{q}{\text{Max}} \ \frac{1}{1-\varepsilon} \int [V(q,I)]^{1-\varepsilon} f(I) dI \tag{3}$$

such that

$$\sum_{j=1}^{n} (q_j - 1) x_j (q, \bar{I}) \geq R, \tag{4}$$

where ε is Atkinson's coefficient of inequality aversion, R is government revenue per capita, and f is the distribution of income. Note that $q_j - 1$ is simply the tax rate on commodity j. Since the demands are linear in income, the revenue constraint can be expressed in terms of the average income, $\bar{I}$.

From (2), it follows that the budget shares of an individual with income I can be expressed as the weighted sum of the budget shares of the rich and the poor:

$$\frac{q_i x_i (q,I)}{I} = \beta_i \frac{(I - \Sigma q_j \gamma_j I^P)}{I} + \frac{q_i \gamma_i}{\Sigma q_j \gamma_j} \frac{\Sigma q_j \gamma_j I^P}{I}, \tag{5}$$

where the weights depend on income. The larger the income, the more the budget shares resemble those of the rich, i.e., as income increases, the weight of the rich's budget shares increases up to 1. Note that the budget shares of the rich are independent of taxes, while those of the poor $(q_i \gamma_i / \Sigma q_j \gamma_j)$ depend crucially on them; also that the income necessary for a poor man to consume his subsistence bundle $(\gamma_1 I^P, \ldots, \gamma_n I^P)$ varies with taxes.

Results

Since the budget shares for any income level depend solely on those of the poor and the rich, Deaton's result (1977, eq. 15), to the effect that the relative budget shares of the average individual (with income $\bar{I}$) and the socially representative individual (with income $I_o < \bar{I}$) determine which goods are taxed at a rate greater or less than the average marginal tax rate, reduces to

$$(q_i-1)/q_i \lesseqgtr \sum_{j=1}^{n} \beta_j(q_j - 1)/q_j \qquad \text{as} \qquad \beta_i \lesseqgtr q_i\gamma_i/\sum_{j=1}^{n} q_j\gamma_j. \tag{6}$$

Since the price of commodity i before tax is 1, $(q_i-1)/q_i$ is the tax rate on consumer price. The term $\Sigma\beta_j(q_j-1)/q_j$ is the weighted average tax rate, the weights being $\beta_j = q_j\partial x_j/\partial I$, which are the marginal propensities to consume. As pointed out earlier, the criterion depends on the optimal tax rates, and can only serve to confirm that the optimal taxes obtained are indeed optimal. It does, however, affirm that the goods more in favor with the rich are taxed more heavily than those in favor with the poor, but it does not tell us a priori which ones belong to each of the two groups. We shall now prove that

$$q_i > q_j \qquad\qquad \text{if } \beta_i/\gamma_i > \beta_j/\gamma_j . \tag{7}$$

This result is an a priori characterization of the optimal tax structure which can be obtained before calculating the optimal tax rates.

Our point of departure is the first-order condition of the welfare maximization represented by (3) and (4). It is essentially the first-order condition derived by Deaton (1977, eq. 12) and specialized to the linear expenditure system. After a series of simple arithmetical operations, it reduces to

$$A + B\beta_i^{-1}\gamma_i q_i = C[D - (q_i-1)q_i^{-1}] \tag{8}$$

where A, B, C, and D are independent of i. Specifically, they are the following:

$$A = \left[1 - \sum_{j=1}^{n} \beta_j (q_j-1)q_j^{-1}\right]\left[(I_o - \sum_{j=1}^{n} q_j\gamma_j I^P)I_o^{-1} - (\bar{I} - \sum_{j=1}^{n} q_j\gamma_j I^P)\bar{I}^{-1}\right],$$

$$B = \left[1 - \sum_{j=1}^{n} \beta_j (q_j-1)q_j^{-1}\right]\left[I^P I_o^{-1} - I^P\bar{I}^{-1}\right] > 0,$$

$$C = \bar{I}^{-1}\left[\bar{I} - \sum_{j=1}^{n} q_j\gamma_j I^P\right] > 0,$$

$$D = \sum_{j=1}^{n} \beta_j (q_j-1)q_j^{-1}.$$

Since the tax rates are measured with respect to consumer prices, they can never exceed 1, and since the β's sum to 1, it follows that the first expression in the brackets in B is positive. Since $I_o < \bar{I}$, the second term in brackets in B is positive. Also, C is positive as, by assumption, average income is larger than the income needed to buy the subsistence level at consumer prices.

When we note that $(q_i-1)q_i^{-1}$ is monotone, increasing in q_i, an examination of (8) proves (7). Thus, we conclude that for a set of optimal tax rates, the tax rate increases across commodities as the ratio of pre-tax rich/poor budget shares, β/γ, increases.

Also, considering $\beta_i^{-1}\gamma_i q_i$, an examination of (8) proves that

$$q_i > q_j \qquad \text{if } \beta_i/q_i\gamma_i > \beta_j/q_j\gamma_j. \tag{9}$$

We conclude that for a set of optimal tax rates, the tax rates increase across commodities as the ratio of post-tax budget shares, $\beta_i/q_i\gamma_i$, increases. Ranking the commodities in such a way that $\beta_i/\gamma_i < \beta_{i+1}/\gamma_{i+1}$, we obtain from (7) and (9) that

$$1 < q_{i+1}/q_i < (\beta_{i+1}/\gamma_{i+1})/(\beta_i/\gamma_i). \tag{10}$$

In particular, the relative budget shares of the rich to those of the poor are ranked identically before and after tax; thus, the variations in tax rates from one commodity to the next are bounded by the ratios of their relative pre-tax budget shares.

To prove the third point, we must define the revenue needs of the government, R, in terms of the average income in the economy. Let r be the uniform tax rate on all commodities--or equivalently on income--which meets the government revenue constraint, i.e., $r/(1+r)\bar{I} = R$. Using (5), we can derive total tax revenue and equate to it revenue needs, as follows:

$$\sum_{i=1}^{n}(q_i-1)\beta_i q_i^{-1}(\bar{I} - \sum_{j=1}^{n} q_j\gamma_j I^P) + \sum_{i=1}^{n}(q_i-1)\gamma_i I^P = r/(1+r)\bar{I}. \tag{11}$$

Dividing both sides by $\bar{I}$ and defining α as $\sum_{j=1}^{n} q_j\gamma_j I^P/\bar{I}$, (11) becomes

$$\sum_{i=1}^{n}\beta_i(q_i-1)q_i^{-1} - \alpha\left[\sum_{i=1}^{n}(q_i-1)q_i^{-1}(\beta_i - q_i\gamma_i/\sum_{j=1}^{n} q_j\gamma_j)\right] = r/(1+r). \tag{12}$$

Since both β_i and $q_i\gamma_i / \sum_{j=1}^{n} q_j\gamma_j$ sum to 1 and are positive, it follows from (6) that the term inside the square brackets on the left hand side of (12) is positive. This implies that the average marginal tax rate, $\sum_{i=1}^{n} \beta_i(q_i-1)q_i^{-1}$, is larger than the average revenue rate, $r/(1+r)$.

The tax rates obtained by Deaton (1977) in his Tables 2, 3, and 4 violate the main result of this paper, the monotonicity of the tax rates in the relative budget shares, as stated in eq. (7) above. We suspect that there is a calculation error in Deaton's paper. We recalculated some of the optimal tax rates reported by Deaton: government's revenue of 10% of total consumer expenditures, $\varepsilon = 0.75$, and a log-normal distribution of income with $\sigma^2 = 0.8$ approximated by 10 income deciles. The results are reported in Table 1. The first column presents the budget shares of the poor; the second column presents the budget shares of the rich; the third column presents the ratio of the budget shares at pre-tax prices; the fourth column presents the ratio of budget shares at post-tax prices; and the fifth and sixth columns present the tax rates. Table 1 clearly illustrates property (7): The ranking of commodities by their optimal tax rates is the same as their ranking by the pre-tax ratios of budget shares (β_i/γ_i). Property (9) is verified too.

Table 1 can serve also to illustrate the impracticality of Deaton's main theoretical result that goods should be taxed more heavily than the average tax rate if their budget shares are higher for the rich than for the poor at post-tax prices (luxuries). The average tax rate in Table 1, $\Sigma\ \beta_i(q_i - 1)/q_i$, is 0.107. Only two goods, "Housing" and "Travel and communication," are taxed more heavily than average. These two goods

Table 1

Extremal Budget Shares, Budget Ratios, and Optimal Commodity Taxes
(Assuming Government Revenue Needs are 10% of Income and the Coefficient of Inequality Aversion is .75)

Commodity	Budget Share		Ratio of Budget Shares		Optimal Tax Rates	
	Poor γ_i	Rich β_i	Pre-tax β_i/γ_i	Post-tax $\beta_i \Sigma q_j \gamma_j / q_i \gamma_i$	Producer Prices $q_i - 1$	Consumer Prices $(q_i - 1)/q_i$
6. Travel and communications	.0255	.2271	8.906	7.090	.233	.189
3. Housing	.0476	.2428	5.101	4.109	.219	.180
7. Other goods	.0975	.1101	1.129	.991	.119	.106
4. Fuel	.0507	.0503	.992	.882	.104	.094
2. Clothing	.0931	.0885	.951	.850	.099	.090
5. Drink and tobacco	.1534	.1156	.754	.692	.069	.065
8. Other services	.1793	.1114	.621	.585	.042	.040
1. Food	.3529	.0542	.154	.194	-.223	-.287

are indeed the only ones which have post-tax ratios of budget shares in excess of one, confirming Deaton's theoretical result. At <u>pre-tax</u> prices there is, however, another commodity, "Other goods," which is a luxury ($\beta_i/\gamma_i = 1.129 > 1$). Thus, one cannot distinguish a priori (before calculating the optimal tax rates) those goods which should be subjected to higher than average tax rates.

REFERENCES

Atkinson, A.B. 1970. On the measurement of inequality. Journal of Economic Theory, 2, 244-263.

Deaton, Angus. 1977. Equity, efficiency and the structure of indirect taxation. Journal of Public Economy, 8, 299-312.