University of Wisconsin-Madison

Institute for Research on Poverty

Discussion Papers

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MEASURE OF HOUSEHOLD
INCOME EQUIVALENCE

DP #611-80

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April 1980

This research was supported by funds granted to the Institute for Research on Poverty at the University of Wisconsin-Madison, pursuant to the provisions of the Economic Opportunity Act of 1964. The conclusions expressed are my own. I thank Sheldon Danziger, Arthur Goldberger, Efraim Sadka and Eugene Smolensky for useful comments on an earlier draft.

On Lazear and Michael's Measure of

Household Income Equivalence: A Note

In a paper in the American Economic Review, Lazear and Michael (1980) present a procedure to estimate "per capita income equivalence among families of different sizes." They acknowledge the similarity between their approach and Barten's (1964), but argue that their "research strategy" differs.

In this note we will show to what extent the Lazear and Michael (henceforth LM) procedure is similar and to what extent it differs from applying Barten's approach to the Linear Expenditure System. Because of the deviations (for example, the LM approach drops the budget constraint), the LM model turns out to be unnecessarily complicated. More seriously, in an attempt to solve the problems they have created, the authors make a conceptual error. Consequently the LM measure cannot be used to adjust income for households of different sizes. We conclude by suggesting an alternative approach.

INCORPORATING BARTEN'S APPROACH IN THE LINEAR EXPENDITURE MODEL

To ease the exposition we first briefly discuss Barten's approach to derive true (constant utility) household equivalence scales. Assume that households maximize a utility function under a budget constraint.[1] Households then

$$\text{maximize } U = U\left\{\frac{x_1}{m_1}, \ \ldots \ \frac{x_I}{m_I}\right\} \tag{1}$$

$$\text{subject to } \sum_i x_i = C \tag{2}$$

where x_i is the quantity consumed of good x_i,

m_i is a good specific weighting factor, and

C is total consumer expenditures (exogeneously determined).

Following Barten, the m_i's are assumed to be a function of household characteristics. For example, comparing only one-person households with two-person households, we specify

$$m_i = 1 + \delta_i h \qquad i = 1, I \qquad (3)$$

with h = 1 for a two person household or

= 0 for a one person household.

It can easily be shown that for U specified as

$$U = \sum_i \beta_i \log \left(\frac{x_i}{m_i} - \gamma_i\right), \text{ with } 0 < \beta_i < 1,\ x_i > \gamma_i \text{ and } \Sigma\beta_i = I \qquad i = 1, I \qquad (4)$$

the maximization yields

$$x_i = \alpha_i + \beta_i C + \mu_i h \qquad i = 1, I \qquad (5)$$

$$\text{with } \alpha_i = \gamma_i - \beta_i \Sigma\gamma_j \qquad i = 1, I \qquad (6)$$

$$\text{and } \mu_i = \gamma_i \delta_i - \beta_i \Sigma\gamma_j \delta_j \qquad i = 1, I \qquad (7)$$

which is the familiar Linear Expenditures System, with market prices constant across observations. All information on the effect of household characteristics is contained in an intercept shift, $\mu_i h$.

The estimation of true (constant utility) household equivalence scales then proceeds as follows (Muellbauer, 1974). We

- obtain estimates of α_i, β_i and μ_i in (5)
- use these estimates in (6) and (7) to solve for the I unknown γ_i's and I unknown δ_i's respectively

- calculate $E = \frac{e(U/h = 1)}{e(U/h = 0)}$ at different values of utility U,

where e is the expenditure function dual to the utility function, i.e., it gives the total dollar amount a household of given composition needs to obtain a given utility level. The expenditure function has the form:

$$e(U/h) = \sum_i \gamma_i (1 + \delta_i h) + \exp \left[U - \Sigma\beta_i \log\beta_i + \Sigma\beta_i \log (1 + \delta_i h) \right]. \quad (8)$$

Consequently the constant utility ratio E will generally differ for different utility levels.

As is well known, the procedure just outlined suffers from an identification problem. The budget constraint implies (summing equation 5 over all commodities):

$$\Sigma\alpha_i = 0 \text{ and } \Sigma\mu_i = 0. \quad (9)$$

Equations (6) and (7) contain only 2(I - 1) independent equations from which the 2I unknown γ_i's and δ_i's cannot be solved.

THE LAZEAR-MICHAEL APPROACH

LM do not want to impose the "considerable restrictions" of a formal demand system on their data. Instead, they use "a reduced form approach which requires much less of the data." The gain (a less restricted model) actually comes at an excessive cost. All that is needed to obtain an equivalence measure in the above sketched procedure is to determine <u>one</u> γ_i and <u>one</u> δ_i[2] from outside the system. The other γ_i's and δ_i's can then be obtained. The LM procedure, however, is both more complicated and less conceptually defensible. LM specify the demand for good i for a single male person as (LM equation 6):

$$x_{im} = \alpha_o + \alpha_1 P_1 + \ldots. + \alpha_I P_I + b_i Y_m \tag{10}$$

where the P_i's are prices, and Y_m is nominal income.

The demand equations in the LES have the form

$$\begin{aligned} x_{im} &= \gamma_i - \beta_i P_i^{-1} \Sigma\, P_j \gamma_j + \beta_i P_i^{-1}\, Y_m \\ &= \gamma_i - \beta_i \gamma_1 P_i^{-1}\, P_1 - \beta_i\, \gamma_2 P_i^{-1} P_2 - \ldots - \beta_i \gamma_I P_i^{-1} P_I + \beta_i P_i^{-1} Y_m \\ &= \alpha_o + \alpha_1 P_i^{-1} P_1 + \ldots + \alpha_1 P^{-1} P_I + \beta_i P_i^{-1}\, Y_m \end{aligned} \tag{11}$$

where the α's are implicitly defined. One of the differences between (10) and (11) is that the marginal propensity to consume item i is a constant in equation (10), but depends on P_i in the LES equation (11). This deviation from LES makes things more complicated, since a couples' demand for good i is specified by LM as:

$$\begin{aligned} x_{imf} = \Bigg[\alpha_0 &+ \alpha_1\left\{\frac{P_1}{1 + J_1}\right\} + \ldots + \alpha_I \left\{\frac{P_I}{1 + J_I}\right\} \\ &+ b_1\, Y_{mf}\Bigg] / (1 + J_i) \end{aligned} \tag{12}$$

where the factor $(1 + J_i)$ is similar to the m_i in our notation, and the subscript mf refers to a couple. So in the LM procedure both the intercept and the marginal propensity to consume change from one type of household to another.[3]

LM arrive at estimates for the J_i's by first assuming that all price and income elasticities are known.[4] They then compare, using (10) and (12), the actual expenditure on each item i by a couple with a nominal income of Y_{mf}, with the estimated amount which each two persons would have spent if they lived separately, earning incomes Y_m and Y_f (Y_m being the male's income and Y_f the female's).

It can easily be seen that this comparison cannot lead to an estimate of the J_i's, which can be interpreted as the effect of household composition on the transformation of quantities of goods into service flows, if we assume that all the true J_i's equal zero (in our notation all δ_i's equal zero). Consequently expenditure patterns between singles and couples with the same income will not differ--see our equations (5) and (7), or LM, equation (5) and (5'). The LM procedures, however, will generally yield differences in expenditure between one and two person households (and consequently will give J_i's that are not equal to zero) because two singles with income Y_m and Y_f are compared with a couple with a <u>different</u> income Y_{mf}.

The LM measure of equivalence is thus a mixture of the Barten's measure and information on the household income distribution that would have existed if all couples had lived as singles. As such, it differs considerably from the measures usually employed. In fact, in the LM-approach equivalence is not defined. If we interpreted the sum of the J_i's (weighted with the budget shares of a couple) in the same way as Barten's measure, as if different types of households faced different prices, the result would be misleading.

In the next section we will return to the Barten approach, and suggest a way to circumvent the identification problem.

BARTEN AND THE EXTENDED LINEAR EXPENDITURE SYSTEM

Using the Barten approach in Lluch's Extended Linear Expenditure System (ELES; Lluch, 1973) poses the following consumer choice problem:

$$\text{maximize } V = \int_0^{\infty} e^{-pt}\, U\left(\frac{x_1(t)}{m_1}, \ldots, \frac{x_I(t)}{m_I}\right) dt \tag{13}$$

under a lifetime wealth constraint. The p is a time-preference parameter and t is time. If U has the form as given in (4), the demand equations for the first period (dropping the subscript t) read:

$$x_i = \alpha_i + \beta_i^* z + \mu_i h \qquad i = 1, I \tag{14}$$

with z as total income in the first period[6] and

$$\mu_i = \gamma_i - \beta_i^* \Sigma\gamma_i \qquad i = 1, I \tag{15}$$

$$\mu_i = \gamma_i\delta_i - \beta_i^* \Sigma\gamma_i\delta_i \qquad i = 1, I. \tag{16}$$

In this formulation, total consumer expenditure, C, is determined endogenously. Since C is not necessarily equal to z, equations (15) and (16) contain 2I independent equations from which the γ_i's and δ_i's can be derived, once estimates of α_i, β_i^* and μ_i are obtained from (14).

A constant utility equivalence scale can readily be obtained using (8). Note that this scale is based on present consumption expenditures only. Utility derived from future consumption, or savings, is ignored.

As an example, we estimated equation (14) by OLS specifying h as a vector of household characteristics, using data from the Consumer Expenditure Survey 1960/61. See Table 1 for the household characteristics distinguished, and for the estimation results. The regression coefficients in Table 1 are used to derive estimates of the γ_i's and δ_i's, using (15) and (16). Finally (8) is used to derive constant utility equivalence scales. These results are given in Table 2.

CONCLUSION

We have compared the LM procedure with the Barten approach as applied to the Linear Expenditure System. Lazear and Michael's defend their procedure as one that avoids restrictions stemming from a formal demand model so that "much less" is required from the data. However, their procedure both is more cumbersome and needs more information than the original Barten approach. More seriously, the LM procedure leads to some undefined measure of equivalence that cannot achieve their objective, which is to obtain "comparable per capita income measures across families of different sizes." Their procedure comes close to the one obtained by applying the Barten approach to Lluch's Extended Linear Expenditures System (ELES). As we have shown, the derivation of constant utility equivalence scales <u>based on present consumption behavior only</u> is then straightforward.

The plausibility of the assumptions upon which ELES is based, and the usefulness of equivalence scales derived from current consumption only (ignoring savings, leisure, household dynamics, future income, past consumption, etc.), are worth discussing, but are beyond the scope of this comment.

Table 1

Regression Coefficients of the Demand Equations, and R^2

	Food	Housing	Clothing	Transportation	Other
After tax income, z	.095	.173	.070	.087	.132
Household characteristics, h[a]					
Two children, aged:					
<6	58	348	-32	-81	.1*
6-11; <6	228	204	37	193	15*
6-11	163	131	89	-70*	119
12-17	213	22*	199	9*	140
12-17; <6	299	42*	144	-194	71
12-17; 6-11	325	37*	223	-114	141
18+; <6	399	-21*	336	94*	202
18+; 6-17	242	-85	324	223	353
18+	146	-141	129	249	272
Age of head					
<35	-55	5*	33	112	4*
55-64	-43	-91	-66	-125	-60
65+	-109	-57	-115	-272	-84
Female head	- 76	96	48	-191	-152
Log family size	177	22	35	21*	-16*
Constant	402	412	21	345	306
R^2	.564	.494	.528	.214	.476

*Is not significant at a 10% level.

[a] h is a vector of dummy variables taking the value 1 if a household has the characteristic and zero otherwise. The last element of h is log (family size). The specification of h is partly hampered by data limitations: it is known whether one or more children in a given age class are present. But we do not know exactly how many children per class.

Table 2

True Constant Utility Household Equivalence Scales

	Age of Head			
	35	35–54	55–64	65+
One person				
Male	65	61	45	35
Female	54	50	34	23
Two persons				
Husband and wife,	72	68	52	42
Female head, child aged 6–11	78	74	59	48
Three persons				
Husband, wife, child aged:				
<6	88	84	68	58
6–11	94	90	74	63
12–17	101	97	81	71
18+	103	99	83	73
Four persons				
Husband, wife, 2 children aged:				
<6	91	87	–	–
6–11; <6	91	87	–	–
6–11	97	92	77	–
12–17; <6	94	90	74	–
12–17; 6–11	104	100	84	–
12–17	103	99	83	–
18; <6	120	116	101	90
18+; 6–17	122	118	102	92
Five persons				
Husband, wife, 3 children aged:				
6–11	99	95	79	–
12–17; 6–11	106	102	87	–
12–17	105	101	85	–
18+; 6–17	124	120	105	–
Six persons				
Husband, wife, 4 children aged:				
6–11	101	96	81	–
12–17; 6–11	108	104	89	–
12–17	107	103	87	–
18+; 6–17	126	122	107	–

Note: Based on a 4 person family, head 35–54, one child aged 6–11 and one aged 12–17.

NOTES

[1]We will assume that all households face the same prices; quantity measures are transformed to set prices equal to 1.

[2]In the case H household characteristics are distinguished, one vector δ_i of length H should be prespecified.

[3]The factors $2\alpha_o$, $2\alpha_1$, etc. in LM's equation (9) are probably a mistake. See LM equation (5').

[4]Stemming from an augmented Linear Expenditure System estimated by Abbott and Ashenfelter (1976). LM treat these elasticities as constants.

[5]These expenditures and income data are estimates based on "reduced-form" epxenditure functions (income is not a variable in these functions) and income functions based on individual personal characteristics.

[6]See Lluch et al., (1977) for the additional assumptions needed to arrive at this result.

[7]Since the scales turned out to be about the same for a large range of values of U, only one scale is given. More detailed information in a paper discussing the equivalence scale derived is available from the author.

REFERENCES

Abbott, M. and Ashenfelter, O. 1976. Labor supply, commodity demand and the allocation of time. Review of Economic Studies.

Barten, A.P. 1964. Family composition, prices and expenditure patterns. In P.E. Hart, G. Mills and J.R. Whitaker, eds., Economic analysis for national economic planning. Butterworth.

Lazear, E.P. and Michael, R.T. 1980. Family size and the distribution of real per capita income. American Economic Review, 70, 91-107.

Lluch, C. 1973. The extended linear expenditure system. European Economic Review, 4.

Lluch, C., Powell, A.A., and Williams, R.A. 1977. Patterns in household demand and saving. New York: Oxford University Press.

Muellbauer, J. 1974. Household composition, Engel curves and welfare comparisons between households. European Economic Review, 5.