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INCOME TESTING AND SOCIAL WELFARE

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ABSTRACT

This paper compares income-tested vs. non-income-tested transfer programs. Although a general conclusion about the welfare superiority of one program over the other could not be established, simulation results with plausible parameter values suggest that non-income-testing is more often than not superior to income testing.

Income Testing and Social Welfare

In the United States in 1979 most Americans would agree with the view that it is the responsibility of government to ensure a certain minumum level of living below which no one should be allowed to fall. (This is not, of course, to say that there is agreement concerning what that level should be.) Government can meet this responsibility in two ways: (1) by providing minimum standards of income, goods, and/or services for only the poor--an income-tested approach, or (2) by providing them for everyone regardless of income--a non-income-tested approach.

The income support system of the U.S. today follows both strategies. AFDC, Supplemental Security Income (SSI), Food Stamps, and Medicaid are restricted to those with low incomes. Public education, Social Security, and Unemployment Compensation are open to people regardless of income.

Until recently the consensus of economic experts was that income-tested programs are more efficient than non-income-tested programs. This consensus apparently stemmed from the widespread use of the target efficiency measure--a conceptually flawed measure of technical rather than economic efficiency.[1] For example, the authors of Setting National Priorities: The 1973 Budget argue:

> "...universal payment systems are a very inefficient means for helping those with low incomes, since the benefits are not concentrated where the need is greatest. Large numbers of families would receive allowances and at the same time have their taxes increased to pay for the allowances. Tax rates would have to be raised simply to channel money from the family to the government and back to the family again." [Schultze, et al., 1972, p. 200].

The authors fail to note, however, that universal payment systems imply lower tax rates for the poor. To analyze the efficiency of income testing both taxes and transfers must be considered.

In a recent paper, Kesselman and Garfinkel (1978), establish the possibility that non-income-tested transfer tax regimes are more efficient than income-tested ones. By an income-tested transfer-tax regime Kesselman and Garfinkel mean one in which marginal tax rates on the poor exceed those on the nonpoor. Higher tax rates on the poor are a consequence of limiting transfer payments to poor people, that is, of income-testing benefit payments. In a continuous world where there are finer divisions than the poor and nonpoor, income testing occurs when marginal tax rates—both implicit and explicit—decrease as income increases. A transfer-tax system is non-income-tested if marginal tax rates are either constant or increase as income increases. Put in these more general terms the issue of income testing becomes simply what is the pattern of optimal income tax rates by income class.

Kesselman and Garfinkel found that the efficiency of income testing depends upon how the compensated wage derivative of labor supply varies by income class. If this derivative either increases, or remains constant as income increases, then income testing is economically efficient. If it decreases as income increases, depending upon the rate of decrease, income testing may be inefficient. They also found that for reasonable values of the differences across income classes of the compensated wage derivative of the labor supply, the economic efficiency gains or losses from income testing are likely to be so small as to be inconsequential for policy purposes.

The purpose of this paper, therefore, is to reexamine the welfare aspect of income testing within the framework of the general equilibrium optimal income tax literature. The Kesselman-Garfinkel analysis relies upon a model with only two skill classes and makes pure efficiency comparisons without recourse to a social welfare function. But this advantage is achieved at a cost. Specifically, Kesselman and Garfinkel fix the utility level of the poor and examine the effect of income testing on the utility level of the rich. Their results are not invariant to the level at which the poor man's utility is fixed. In contrast, in this paper we ask the question whether income testing improves or reduces total social welfare. We also examine the robustness of our results to alternative specifications of the social welfare function. In those cases in which income-testing is an optimal policy, we offer a measure of the welfare loss incurred in the absence of income-testing. Similarly when non-income-tested regimes are optimal, we calculate the welfare loss incurred by adopting a particular income-tested regime.

In the following section we discuss the optimal income-tax-transfer model. The third section explains how the calculations are made and the fourth presents our results. The paper concludes with a brief summary and policy implications section.

THE MODEL

The Individuals

We consider an aggregate model of our economy in which there are only two commodities, consumption (x) and labor services (y). There are I individuals in this economy, all having the same preference over bundles of x and y. These preferences are represented by a twice continuously differentiable utility function $u(x, y)$, where $\frac{\partial u}{\partial x} > 0$ and $\frac{\partial u}{\partial y} < 0$. Consumption is assumed to be a normal good. Individuals differ in their wage rates. We denote w_i the wage rate of person i. The individuals were arranged in the following order: $w_1 < w_2 < \ldots < w_I$.

Our model differs from others in that it allows households to have some unearned incomes (such as rent, interest, etc.), in addition to earned income. The unearned income of person i is denoted by A_i. It should be noted here that since individuals were ranked according to their wage rates, it is not necessarily true that $A_1 < A_2 < \ldots < A_I$. Indeed, we do not assume the latter. As we shall see in the next section, the presence of unearned incomes invalidates the standard result from the optimal tax literature that gross income is an increasing function of the wage rate.[2] This could potentially complicate the calculations of optimal tax rates.

<u>The Tax-Transfer System</u>

The gross income of individual i is denoted by z_i. This includes both earned and unearned income so that $z_i = w_i y_i + A_i$, where y_i is the amount of labor services supplied by household i. The income tax in this model is piece-wise linear and it does not distinguish between earned and unearned income. Because there are only two linear pieces (income-brackets) to this income tax, it can be described by four parameters, two parameters for each of the two linear pieces. These are denoted by G_P, τ_P, G_R and τ_R as follows: $(1 - \tau_P)$ and $(1 - \tau_R)$ are the marginal tax rates at the low and at the high income brackets, respectively: $- G_P$ and $- G_R$ are the (usually negative) lump-sum taxes at the low and at the high income brackets, respectively. (G_P is a guaranteed income for the poor; G_R may be thought of as a "shadow" guarantee in the sense that if the rich man's income ever fell to zero he would collect not G_R but G_P. Geometrically G_R is simply the intercept obtained by projecting the linear tax at the high income bracket back to the vertical axis.) The gross income level at which the two linear parts of the income tax intersect is denoted by $\bar{z}$. Thus, $\bar{z}$ is defined implicitly by

$$-G_P + (1 - \tau_P)\,\bar{z} = - G_R + (1 - \tau_R)\,\bar{z},$$

which can be solved explicitly to obtain

$$\bar{z} = \frac{G_R - G_P}{\tau_P - \tau_R}\,. \tag{1}$$

Employing (1), the income tax may be formally written as

$$T(\bar{z}) = \begin{cases} - G_P + (1 - \tau_P)\,z & \text{if} \quad z \leq \dfrac{G_R - G_P}{\tau_P - \tau_R} \\[2ex] - G_R + (1 - \tau_R)\,z & \text{if} \quad z \geq \dfrac{G_R - G_P}{\tau_P - \tau_R} \end{cases} \tag{2}$$

After-tax income, $z - T(z)$, is therefore

$$z - T(z) = \begin{cases} G_P + \tau_P\, z & \text{if} \quad z \leq \dfrac{G_R - G_P}{\tau_P - \tau_R} \\[2ex] G_R + \tau_R\, z & \text{if} \quad z \geq \dfrac{G_R - G_P}{\tau_P - \tau_R} \end{cases} \tag{3}$$

When the kink in the income tax occurs exactly at the break-even income, i.e., $T(\bar{z}) = 0$, we say that the tax system is fully integrated. Otherwise, it is not fully integrated. In terms of the four tax parameters, the restriction for a fully integrated income tax is

$$- G_P + (1 - \tau_P) \frac{G_R - G_P}{\tau_P - \tau_R} = - G_R + (1 - \tau_R) \frac{G_R - G_P}{\tau_P - \tau_R} = 0. \qquad (4)$$

Since (4) can be solved for, say, G_R in terms of the other three parameters, we can thus define a fully integrated tax as a tax satisfying the constraint

$$G_R = \frac{1 - \tau_R}{1 - \tau_P} G_P. \qquad (5)$$

If the income tax imposes a higher marginal tax rate on the poor than on the rich, i.e., $\tau_P < \tau_R$, we call it a negative income tax (NIT). A fully integrated NIT is a NIT which satisfies (5). Fully integrated and nonfully integrated NITs are illustrated in Figures 1 and 2. By an income-tested tax-transfer program we mean a NIT program (a fully or nonfully integrated one). Finally, we call an income tax with a constant or increasing marginal tax rate and a (usually) negative lump-sum tax (namely, $\tau_P \geqq \tau_R$ and $G_P \leqq G_R$) a credit income tax (CIT). In the text we analyze only the fully integrated case. In Appendix A, we explain that in our model the nonfully integrated case does not shed much light on the income-testing issue.

The Household Income-Leisure Choice

Each individual i is assumed to choose his (x_i, y_i) bundle by maximizing his utility $u(x_i, y_i)$, subject to his budget constraint.

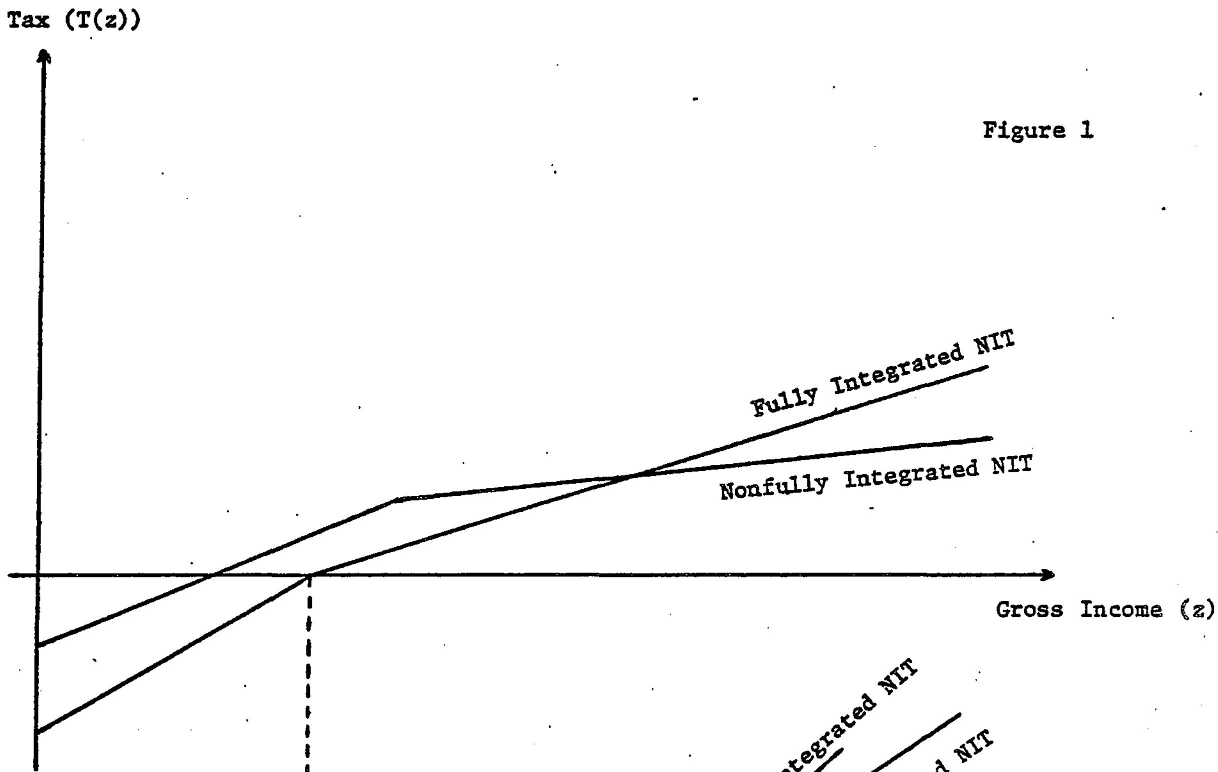


Figure 1

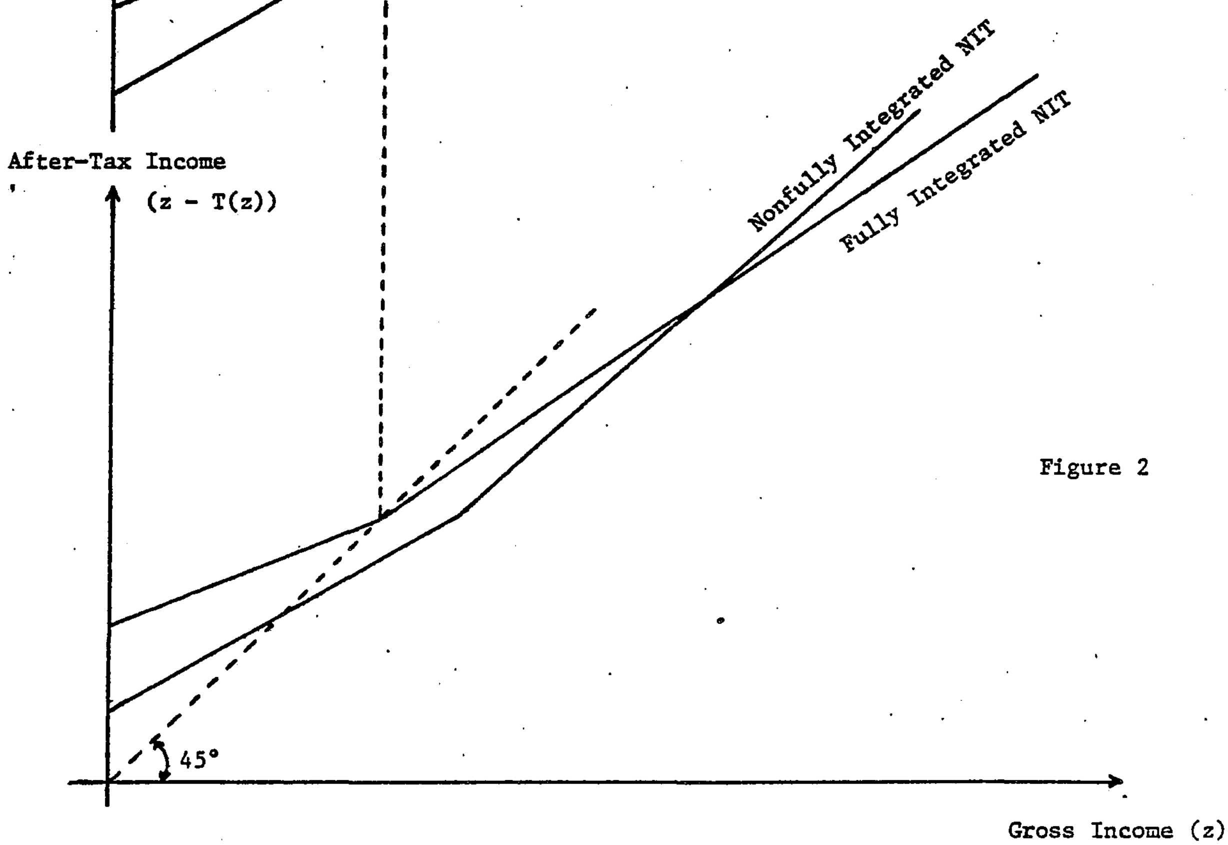


Figure 2

Without any taxes the budget constraint is

$$x_i = w_i y_i + A_i. \tag{6}$$

His choice of x_i and y_i are then functions, $X(w_i, A_i)$ and $Y(w_i, A_i)$, respectively, of his wage rate, w_i, and lump-sum income, A_i. These functions are the same for all individuals. With a piece-wise linear income tax, his budget constraint is either

$$x_i = \tau_P w_i y_i + \tau_P A_i + G_P \tag{7}$$

if individual i is "poor" (i.e., at the low income bracket), or

$$x_i = \tau_R w_i y_i + \tau_R A_i + G_R, \tag{8}$$

if individual i is "rich" (i.e., at the high income bracket). Thus, if we can be sure that individual i is poor then his x_i and y_i are $X(\tau_P w_i, \tau_P A_i + G_P)$ and $Y(\tau_P w_i, \tau_P A_i + G_P)$, respectively, where $\tau_P w_i$ is his net wage rate and $\tau_P A_i + G_P$ is his net lump-sum income. Similarly, if we are sure that individual i is rich, then his x_i and y_i are $X(\tau_R w_i, \tau_R A_i + G_R)$ and $Y(\tau_R w_i, \tau_R A_i + G_R)$, respectively. However, whether an individual will be poor or rich depends not only on his wage and unearned income, but also on all four tax parameters (τ_P, G_P, τ_R and G_R).

Therefore, in our calculations, the demand x_i and supply y_i are determined in two steps. First, for any combination of τ_P, G_P, τ_R and G_R we determine the sets $P(\tau_P, G_P, \tau_R, G_R)$ and $R(\tau_P, G_P, \tau_R, G_R)$ of poor and rich individuals, respectively. An individual i belongs to $P(\tau_P, G_P, \tau_R, G_R)$ if his gross income $z_i = w_i y_i + A_i$ is strictly less than $\bar{z}$. He belongs to

$R(\tau_P, G_P, \tau_R, G_R)$ if $z_i > \bar{z}$. When $\tau_P < \tau_R$, it is possible for an individual to be indifferent between being poor and being rich. In this case we arbitrarily pick $z_i < \bar{z}$ and classify this individual as poor. When $\tau_P > \tau_R$, an individual may choose to be exactly at the kink in the income tax schedule (i.e., $z_i = \bar{z}$). The set of such individuals is denoted by $K(\tau_P, G_P, \tau_R, G_R)$. In Appendix B we shall describe how to find the sets $P(\cdot)$, $R(\cdot)$ and $K(\cdot)$ and show that is is possible to do so before first finding x_i and y_i. Once these sets are determined, the choice of individual i of x_i and y_i is then given by

$$x_i = X(\tau_P w_i, \tau_P A_i + G_P) \text{ and } y_i = Y(\tau_P w_i, \tau_P A_i + G_P) \text{ if } \quad i \in P(\tau_P, G_P, \tau_R, G_R); \tag{9}$$

$$x_i = X(\tau_R w_i, \tau_R A_i + G_R) \text{ and } y_i = Y(\tau_R W_i, \tau_R A_i + G_R) \text{ if } \quad i \in R(\tau_P, G_P, \tau_R, G_R);$$

$$x_i = \bar{z} \quad \text{and } y_i = \frac{\bar{z} - A_i}{w_i} \quad \text{if } i \in K(\tau_P, G_P, \tau_R, G_R).$$

Optimality

The social welfare function is assumed to be

$$W = \sum_{i=1}^{I} \frac{u(x_i, y_i)^{(1-\varepsilon)}}{1 - \varepsilon} \tag{10}$$

Roughly speaking, ε on the RHS is an index of inequality aversion. The higher the ε, the higher is the aversion to inequality. The Arrow-Pratt measure of absolute risk aversion for $u^{1-\varepsilon}/(1 - \varepsilon)$ is increasing in ε

and one may expect higher marginal tax rates $(1 - \tau)$ and lower negative lump-sum taxes $(-G)$ as ε increases [see Helpman and Sadka (1978)]. As ε goes to infinity, the criterion (10) approaches the Rawlsian max-min criterion. In our calculations we consider several alternative values for ε and the max-min criterion.

The government is assumed to have fixed revenue needs, so that its budget constraint is

$$\frac{1}{I} \sum_{i\varepsilon P(\tau_P,G_P,\tau_R,G_R)} \{-G_P + (1 - \tau_P)(w_i y_i + A_i)\} + \frac{1}{I} \sum_{i\varepsilon K(\tau_P,G_P,\tau_R,G_R)} [-G_P + (1 - \tau_P)\bar{z}]$$
$$+ \frac{1}{I} \sum_{i\varepsilon R(\tau_P,G_P,\tau_R,G_R)} \{-G_R + (1 - \tau_R)(w_i y_i + A_i)\} \geqq B \tag{11}$$

where B is the government's revenue requirement per individual.[3]

To find the optimal piece-wise linear tax, one has to find τ_P, G_P, τ_R and G_R which maximizes (10) subject to constraint (11), taking into account that x_i and y_i are as defined in (9). The optimal fully-integrated tax is found by further adding the constraint (5).

Finally, we estimate the welfare loss incurred by using a CIT when and NIT is optimal and vica versa. To do this we first find the optimal fully integrated piece-wise linear tax. Denote by $\overline{W}$ the level of W obtained with this optimal tax. Suppose that this optimal tax is an NIT (i.e., $\tau_P > \tau_R$). Next we must compare this NIT to some CIT. The choice is arbitrary. But one CIT that has great appeal for comparison purposes is where $\tau_P = \tau_R$. This is the case of maximum tax simplification and also represents a tipping point. While we focus on other cases below, we shall explain our technique here by continuing with this example.

If we try to design a CIT in which $\tau_P = \tau_R$ and which sustains a level $\bar{W}$ for our social welfare function, we shall not be able to generate the required revenue B. Thus, if we want to achieve $\bar{W}$ via this CIT, we shall have to surrender some amount of revenue. This loss in revenue can serve as a reasonable measure of the welfare cost of replacing the NIT with a CIT in which $\tau_P = \tau_R$. Formally, after finding $\bar{W}$, we solve

$$\max_{\tau, G} \frac{1}{I} \sum_{i \varepsilon I} \{- G + (1 - \tau) Y(\tau w_i, \tau A_i + G)\} \tag{12}$$

subject to

$$\sum_{i=1}^{I} \frac{1}{1-\varepsilon} u [X(\tau w_i, \tau A_i + G), Y(\tau w_i, \tau A_i + G)]^{1-\varepsilon} \geq \bar{W}. \tag{13}$$

Denote the maximum level of (12) by $\bar{B}$. Then $B - \bar{B}$ is the welfare cost per individual of adopting the CIT rather than the NIT.[4]

CALCULATIONS

Data

The wage and unearned income distribution were taken from the 1976 Current Population Survey (CPS) for heads of households (male or female) who are not retired, full-time students, or handicapped. These individuals were arranged first in an increasing order according to their wage rates and then grouped into five quintiles. For each quintile the average wage rate and average unearned income were calculated. Our economy was then composed of five individuals who were given the average wage rate and the average unearned income of each of the five quintiles. It should be noted that the quintiles were constructed according to the wage rate distribution and not the unearned income distribution, so that while we have $w_1 < w_2 < \ldots < w_5$, we do not necessarily obtain $A_1 < A_2 < \ldots < A_5$. Unearned income included interest, dividends, rent, veteran payments, unemployment compensation, pensions, alimony, and child support. The average wage rates and unearned incomes are presented in Table 1.

We also present in Table 1 the average annual labor supplies of each of the quintiles. The average labor supply increases with the wage rate up to the fourth quintile and then drops for the highest quintile. This pattern is very similar to our calculations of the labor supplies under the optimal tax coefficients for the constant elasticity of substitution (CES) utility function (see Table 3 below). A comparison between Tables 1 and 3 also indicates that the magnitude of the labor supplies which we have calculated are very similar to the average labor supplies in the 1976 CPS.

To choose an appropriate value for B (the government's revenue need per each head of household who is not retired, a full-time student or handicapped), it was assumed to be the average actual tax collected in calendar 1975 from each such head of household. To calculate this figure we had to decide how to treat the Social Security tax (employee's share). We followed two alternative approaches. One approach was to assume that individuals treat their payments to Social Security as an income tax.

Table 1

The Wage and Unearned Income Distribution

	Hourly Wage Range (Dollars)	Average Hourly Wage Rate (Dollars)	Average Annual Unearned Income (Dollars)	Average Annual Labor Supply (Hours)
Bottom quintile	0 - 2.86	1.52	782	1782
Second lowest quintile	2.87 - 4.27	3.56	681	1958
Third lowest quintile	4.28 - 5.77	4.98	693	2044
Fourth lowest quintile	5.78 - 7.69	6.63	733	2065
Top quintile	7.70 - 333.33	11.74	1348	1978

In this case the effective income tax which determines their labor supply is the sum of the personal income and the Social Security taxes. In calendar year 1975, the personal income tax (less the earned income tax credit) was $121.34 billion and the Social Security tax added $41.40 billion for a total of $162.74 billion. In order to obtain B, we first divided the latter figure by 56.963 million, which is the number of heads of households who are not retired, full-time students or handicapped. The result was then multiplied by the ratio of total taxes paid in 1975 by these heads of households to total taxes paid by all individuals. Our estimate for this ratio was 0.726, which was the ratio of the total income of these households to the national personal income. In this way we obtained a value of $2074 for B. It should be understood that the optimal tax coefficients which we calculated for this B are the effective coefficients. For instance, the optimal marginal tax rate which we found should be regarded as the sum of the personal income and the Social Security marginal tax rates.

An alternative approach is to assume that individuals believe that their payment to Social Security is essentially an old-age insurance and hence that they do not view these payments as a tax at all. Therefore, the $41.40 billion Social Security tax should not be added to the personal income tax in calculating B. If this is done, we find a value of $1546 for B. Optimal tax coefficients were also calculated for this value of B.

Finally, we have calculated our optimal tax coefficients for five alternative values of ε: 0.2, 0.4, 0.6, 0.8 and 1.4 Often, we also carry out the calculations for the max-min criterion ($\varepsilon = \infty$). Occasionally, we also consider the other extreme case where $\varepsilon = 0$. In the latter case the social marginal utility of full income (inclusive of the value of leisure) is constant and the society becomes indifferent to full-income inequality.[5] However, the social marginal utility of actual income (exclusive of the value of leisure) is usually diminishing and the society aspires for a more equal distribution of actual income.[6] Thus, the case when $\varepsilon = 0$ does not usually imply that no minimum income should be guaranteed to the poor, as we shall see in the next section (see Table 4).

Specification of the Utility Function and Its Parameters

In calculating the optimal tax coefficients, we employ a CES utility function

$$u(x,y) = \left[\beta x^{\rho} + (1-\beta)(\bar{y}-y)^{\rho}\right]^{\frac{1}{\rho}} \tag{14}$$

where the elasticity of substitution is $\sigma = 1/(1-\rho)$. $\bar{y}$ is the individual's endowment of leisure, or the maximum amount of hours that he can work; it is the same for all individuals--60 hours per week, 52 weeks per year for a total of 3120 hours per year. Our CES utility function yields the following consumption demand and labor supply function:

$$X(\tau w,\ \tau A + G) = \frac{\tau w\bar{y} + \tau A + G}{\tau w\left[\frac{1-\beta}{\beta\tau w}\right]^{\frac{1}{1-\rho}} + 1} \tag{15}$$

and

$$Y(\tau w,\ \tau A + G) = \bar{y} - \frac{\tau w\bar{y} + \tau A + G}{\left[\frac{\beta\tau w}{1-\beta}\right]^{\frac{1}{1-\rho}} + \tau w}. \tag{16}$$

With the CES utility function the values of two parameters, β and σ, have to be determined. Here we considered two cases: (a) β and σ vary across income classes; (b) β and σ are the same for all individuals.

In case (a) we employed the findings of Masters and Garfinkel (1978). They studied, among other things, the dependency of the compensated wage elasticity of the labor supply on various demographic variables. In our terminology this elasticity can be written as

$$\theta = \frac{W}{Y}\left(\frac{\partial Y}{\partial W} - Y\frac{\partial Y}{\partial I}\right) \tag{17}$$

where $W = \tau w$ is the net wage rate and $I = \tau w\bar{\bar{y}} + \tau A + G$ is full income. Given the demographic composition of each of our quantiles, we employed Masters and Garfinkel's findings in order to estimate the θ of each quantile. These estimates are presented in Table 2. We then calculated β and σ for each quantile by requiring them to yield our estimates of θ and the labor supplies in the last column of Table 1.

Specifically, to find the pair (β, σ) for a certain quantile, we solved these two simultaneous equations:

$$\theta(\tau w,\ \tau A + G;\ \beta,\ \sigma) = \text{Estimated } \theta \text{ (from Table 2)}; \tag{18}$$

$$Y(\tau w,\ \tau A + G;\ \beta,\ \sigma) = \text{Average Labor Supply (from Table 1)}. \tag{19}$$

In order to solve (18)-(19) for (β, σ) we first had to determine the actual τ and G faced by each quantile in 1975. For all but the bottom quantile, τ and G were calculated exclusively from the federal income tax tables (for married filing jointly, claiming a total number of three exemptions and taking the standard deduction). Since the federal income tax schedule is not linear, $1 - \tau$ and $- G$ were taken as, respectively, the slope and the intercept of the line which is tangent to the tax schedule at the point where the quantile in question actually was in 1975. This procedure is justified in view of the quasi-concavity of the individual's utility function which gives rise to well-behaved indifference curves. In determining the actual τ and G for the bottom quantile, we also took into account various income-tested cash and in-kind transfer programs (such as Food Stamps, AFDC, and the earned income credit).

Table 2 presents the actual values of τ and G and our estimates of β and σ. In fact, we calculated two alternative values of (τ, G) and hence of (β, σ), depending upon whether the employee contribution for Social Security (FICA) is viewed as a tax or not. Recall that a similar distinction was made with respect to B, the government's revenue requirement. In case I, FICA was considered as a tax and we found B to be \$2074. Correspondingly, when we calculate optimal tax rates for this value of B we employed our estimates of β and σ under the assumption that FICA is indeed viewed as a tax. Similarly, in case II we had B = \$1546 and we used our estimates of β and σ under the assumption that FICA is not regarded as a tax.

In case (b) where β and σ are the same for all individuals, we followed the suggestions of Stern (1976) that β is between 0.95 and 0.995 and σ between 0.35 and 0.50 (see also his references). In the text, we present results for $\beta = 0.98$ and $\sigma = 0.50$.[7]

RESULTS

In Table 3 we present the optimal tax rates (τ), guarantees (G), break-even levels of income ($\bar{z}$), labor supplies (y), and before- and after-tax

Table 2

The Compensated Wage Elasticity (θ) and the Elasticity of Substitution (σ)

		Case I (B = \$2074)[a]				Case II (B = \$1546)[b]			
	θ	τ	$\bar{G}$ (Dollars)	β	σ	τ	$\bar{G}$ (Dollars)	β	σ
Bottom quantile	0.26	0.79	1916	0.9245	0.4653	0.85	1916	0.9245	0.4689
Second lowest quantile	0.21	0.67	1697	0.9363	0.4976	0.73	1656	0.9367	0.5014
Third lowest quantile	0.15	0.75	1060	0.9807	0.4092	0.81	1019	0.9818	0.4108
Fourth lowest quantile	0.11	0.755	1012	0.9979	0.3108	0.815	967	0.9982	0.3118
Top quantile	0.10	0.72	1822	0.9999	0.2589	0.72	2668	0.9999	0.2553

[a] FICA is viewed as a tax.

[b] FICA is not viewed as a tax.

Table 3

Optimal Tax-Transfer Programs for a CES Utility Function with Variable β and σ

	ε = 0		ε = 0.2		ε = 0.4		ε = 0.6		ε = 0.8		ε = 1.4		Max-Min (ε = ∞)	
	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546
τ_P	.92	.95	.51	.50	.41	.65	.48	.55	.43	.49	.42	.40	.14	.14
τ_R	.92	.95	.62	.66	.57	.39	.43	.35	.38	.33	.22	.26	.10	.11
G_P	-1070	-919	3142	3566	4012	3554	4241	4455	4727	4951	5226	5732	6673	7100
G_R	-1070	-919	2437	2425	2924	6195	4649	6435	5142	6504	7028	7069	6983	7348
$\bar{z}$	-13,380	-18,380	6413	7131	6799	10,155	8156	9900	8293	9707	9011	9552	7759	8256
y_1	2341	2302	1476	1365	1157	1422	1151	1154	958	972	792	616	0	0
y_2	2157	2141	1879	1887	1825	1751	1639	1635	1550	1554	1465	1389	513	404
y_3	2122	2118	1984	1995	1959	1900	1822	1848	1769	1810	1670	1734	1332	1295
y_4	2086	2099	2047	2068	2043	1930	1989	1923	1972	1923	1829	1884	1746	1777
y_5	1948	1953	2020	2022	2037	2053	2066	2071	2082	2083	2100	2104	2171	2178
z_1	4340	4280	3025	2857	2541	2942	2531	2536	2238	2258	1986	1718	782	782
z_2	8359	8301	7371	7398	7178	6915	6514	6501	6198	6212	5895	5624	2505	2119
z_3	11,259	11,242	10,573	10,628	10,449	10,155	9765	9898	9502	9707	9011	9331	7326	7141
z_4	14,560	14,648	14,305	14,441	14,275	13,531	13,921	13,485	13,808	13,484	12,860	13,221	12,307	12,517
z_5	24,216	24,273	25,060	25,084	25,256	25,452	25,606	25,663	25,789	25,801	25,997	26,050	26,836	26,913
Mean Income	12,547	12,549	12,067	12,082	11,940	11,799	11,667	11,617	11,507	11,492	11,150	11,189	9951	9894
x_1	2922	3147	4685	4994	5053	5467	5456	5850	5690	6057	6060	6419	6782	7210
x_2	6620	6967	7007	7307	7015	8049	7368	8031	7392	7994	7702	7981	7024	7397
x_3	9288	9761	8992	9439	8879	10,155	8848	9899	8753	9707	9011	9464	7698	8100
x_4	12,324	12,997	11,306	11,956	11,061	11,472	10,635	11,155	10,389	10,953	9858	10,506	8214	8725
x_5	21,209	22,140	17,794	18,980	17,320	16,121	15,659	15,417	14,941	15,018	12,748	13,842	9667	10,308
Percentage of Net Beneficiaries	0	0	20%	20%	20%	40%	40%	60%	40%	40%	40%	60%	60%	60%

income (x and z, respectively) for the variable β and σ case. The results are presented for seven values of ε, the inequality aversion parameters, and two values of B, the government's revenue need.

In all but three cases, the tax rate on the nonpoor exceeds or is equal to that on the poor. The CIT is optimal. The difference in tax rates ranges from zero to a high of 26 percentage points. Except in the $\varepsilon = 0$ case, both tax rates are quite high. In the max-min case the tax rates are virtually confiscatory. Even in the other cases, the tax rates range from a minimum of 34% to a maximum of 78%. As expected, the tax rates more or less increase as aversion to inequality (ε) increases.

The guarantee to the poor is relatively high. Except for the case where $\varepsilon = 0$, the guarantee varies from a low of about $3000 a year to a high of about $7000 a year. [When $\varepsilon = 0$, the tax rates on the poor and on the rich are equal to each other and are very small. The guarantees to the poor and to the rich are, of course, equal to each other and negative. Essentially, we have a head tax in this case.] As expected, the guarantees to the poor and to the rich both increase with ε.[8] Similarly, the lower the value of taxation required to finance other government provided goods and services (namely B), the higher are usually the guarantees.

The optimal income tax, or more approximately, the optimal tax-transfer system has a break-even level of income below the mean income. The difference between the break-even and the mean income shrinks as ε increases to a low level of less than $2000. Consequently, the number of the net beneficiaries in the optimal tax-transfer system (those whose after tax-transfer incomes exceed their before tax-transfer incomes) increases with ε. Most commonly, 20% to 40% of the population are net beneficiaries.

With one exception (namely, when $\varepsilon = 0$), the optimal labor supplies increase as the wage rate increases. (In three other cases, the fifth quantile's optimal labor supply is slightly lower than the fourth.) The poorest wage class works substantially less than the rest of the population. (Again, the case of $\varepsilon = 0$ is an exception.) Furthermore, the greater the aversion to inequality as measured by ε, the less the poor work and hence the greater the divergence between their life styles and that of the rest of the population. Indeed, in the max-min case, the poor do not work at all; and the near-poor work very little. The explanation for this result is simple. As the guarantee and tax rate increase, the ability of the poor to afford to forego work increases while the rewards they derive from work decreases. Whether such a large divergence in life styles is consistent with a broader notion of equality is an important question which unfortunately cannot be addressed within the confines of our formal model.

As mentioned earlier, we also simulated optimal income taxes for the constant β and σ case. Alternative values of β between 0.95 and 0.995 were combined with various values of σ between 0.35 and 0.50. Table 4 presents the optimal tax rules and guarantees for $\beta = 0.98$ and $\sigma = 0.50$. In all cases, the optimal tax rate on the poor exceeds that on the nonpoor. This result holds for all our simulations with constant β and σ, including the case of a Cobb-Douglas utility function which is a special case of the CES function (i.e., $\sigma = 1$). Thus, the NIT is optimal when both β and σ are constant. Another interesting feature of the results in Table 4 is that the guarantee to the poor and to the rich are no longer negative in the case where $\varepsilon = 0$. In fact, G_P can reach as high as \$5000 a year. This is not surprising in view of our earlier discussion of this situation.

Table 4

Optimal Tax Coefficients for $\beta = 0.98$ and $\sigma = 0.5$

	$\varepsilon = 0$		$\varepsilon = .2$		$\varepsilon = .4$		$\varepsilon = .6$		$\varepsilon = .8$		$\varepsilon = 1.4$		Max-Min ($\varepsilon = \infty$)	
	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1456
τ_P	0.44	0.42	.36	.32	.30	.32	.30	.42	.29	.39	.35	.34	0.12	0.12
τ_R	0.61	0.63	.55	.61	.52	.61	.52	.49	.52	.46	.39	.41	0.22	0.23
G_P	4287	4916	5211	5714	5781	5714	5781	5535	5845	5858	5913	6366	7404	7776
G_R	2989	3136	3664	3277	3964	3277	3964	4867	3951	5186	5549	5691	6562	6804

A comparison between the tax rates and the guarantees in Table 3 and Table 4 shows that the poor face a significantly higher tax rate in the constant (σ, β) case than in the variable (σ, β) case. The difference is around 15% to 20%, except in the $\varepsilon = 0$ case, where it jumps to 50% to 56%. The guarantee to the poor is significantly lower in the variable (σ, β) case. Except in the case $\varepsilon = 0$ where it reaches \$6000 a year, the difference between the guarantees to the poor in the two cases is between \$1000 to \$3000 a year. The fact that both the marginal tax rate on the poor and the guarantee received are higher in the constant (σ, β) case than in the variable (σ, β) case, explains why the NIT is optimal in the former case, while the CIT is optimal in the latter case. Comparing τ_R and G_R in the two cases does not suggest any clearcut pattern.

Next, we consider the magnitude of the welfare losses incurred by adopting an income-tested tax-transfer schedule when a non-income-tested system is optimal and vice versa. The choice of the nonoptimal income-tested (non-income-tested) system to stimulate is, of course, arbitrary. Recall that we generally found non-income-tested systems to be optimal when the substitution elasticities more or less declined with the wage rate and income-tested systems to be optimal when the elasticity was constant. A natural comparison then, which highlights the importance of whether the substitution elasticity declines with the wage rate, is to use the difference in the optimal tax rates parameters derived from the constant elasticity case as constraints in the maximization problem in the declining elasticity case and vice versa. We also compare the optimal tax in each case with the linear tax (i.e., $\tau_P = \tau_R$). The welfare losses are then calculated as explained earlier. In Tables 5 and 6 we present welfare losses in absolute terms and as percentages of government's revenue and national gross earnings.

When β and σ are variable (where the optimal tax is usually a CIT), the welfare losses of adopting a linear tax (Table 5A) are generally low (0.3 to one billion dollars); in three cases they reach about \$3.5 billion and in one case they even exceed \$6 billion. The welfare losses of adopting a NIT (Table 5B) are substantially higher. They are especially high for high levels of ε and in one of the two maximin cases they even reach \$35 billion!

When β and σ are constant (where the optimal tax is a NIT), the welfare losses of adopting a linear tax are usually between one to three billion dollars and most often they are less than \$2 billion. The maximin case is an exception (\$16 to \$21 billion). Adopting a CIT with the value of $\tau_P - \tau_R$ as in the variable (β, σ) case result in quite high welfare losses.

Finally, a brief comparison of our results with those of Garfinkel and Kesselman (1978) is in order. They argue that the compensated wage derivative of the labor supply function[9] is lower for the rich than for the poor. They also found that if this derivative falls sufficiently fast with the wage rate, then a fully integrated NIT is inefficient. Recall that our findings show that the variable (β, σ) is more favorable to the CIT while the constant (β, σ) is more favorable to the NIT. Thus, for our results to be consistent with those of Garfinkel-Kesselman, it must be the case that the compensated wage derivative of the labor supply falls, roughly speaking,

Table 5

Welfare Losses Caused by Nonoptimal Tax Transfer Programs when β and σ are Variable.

	$\varepsilon = 0$		$\varepsilon = 0.2$		$\varepsilon = 0.4$		$\varepsilon = 0.6$		$\varepsilon = 0.8$		$\varepsilon = 1.4$		Max-Min ($\varepsilon = \infty$)	
	B = 2074	B = 1546	B = 2075	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546
A. Welfare Losses of a Linear Tax ($\tau_P = \tau_R$ and $G_P = G_R$)														
Total (in billions of dollars)	0	0	0.48	1.04	0.55	0.10	0.32	2.47	0.41	3.94	3.32	6.44	3.55	2.91
As percentage of government's revenue	0	0	0.3	0.9	0.3	0.1	0.2	2.0	0.2	3.2	2.0	5.3	2.2	2.4
As percentage of gross national earning	0	0	0.05	0.12	0.06	0.01	0.04	0.29	0.05	0.47	0.41	0.79	0.50	0.41
B. Welfare Losses of a Tax with a Value of $\tau_P - \tau_R$ as in the Optimal Tax for the Case of Constant β and σ														
Total (in billions of dollars)	0	0	0.25	0.99	0.21	1.53	3.22	3.62	9.93	5.48	4.96	9.16	8.46	34.97
As percentage of government's revenue	0	0	0.2	0.8	0.1	1.3	2.0	3.0	6.1	4.5	3.0	7.6	5.2	28.8
As percentage of gross national earnings	0	0	0.03	0.1	0.02	0.2	0.4	0.4	1.1	0.7	0.6	1.1	1.1	4.9

Table 6

Welfare Losses Caused by Nonoptimal Tax-Transfer Programs when β and σ are Constant.

	$\varepsilon = 0$		$\varepsilon = 0.2$		$\varepsilon = 0.4$		$\varepsilon = 0.6$		$\varepsilon = 0.8$		$\varepsilon = 1.4$		Max-Min ($\varepsilon = \infty$)	
	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546	B = 2074	B = 1546
A. Welfare Losses of a Linear Tax ($\tau_P = \tau_R$ and G_R)														
Total (in billions of dollars)	1.01	1.85	2.07	2.34	2.78	1.64	2.93	0.62	2.26	0.77	0.44	1.14	16.31	21.05
As percentage of government's revenue	.6	1.5	1.3	1.9	1.7	1.4	1.8	0.5	1.4	0.6	0.3	0.9	10.0	17.3
As percentage of gross national earnings	.1	.2	.2	.2	.3	.2	.3	.07	.2	.08	.05	.1	2.0	2.6
B. Welfare Losses of a Tax with a Value of $\tau_P - \tau_R$ as in the Optimal Tax for the Case of Variable β and σ														
Total (in billions of dollars)	1.01	1.85	0.40	1.49	0.20	11.62	3.73	9.56	3.26	8.26	12.52	9.91	31.36	27.08
As percentage of government's revenue	0.6	1.5	0.2	1.2	0.1	9.6	2.3	7.9	2.0	6.8	7.7	8.2	19.3	22.3
As percentage of gross national earnings	0.1	0.2	0.04	0.2	0.02	1.3	0.4	1.0	0.3	0.9	1.4	1.1	3.9	3.4

more rapidly with the wage rate when β and σ are variable than when they are constant. Strictly speaking, whether the compensated wage derivative falls more rapidly in the variable (β, σ) case than in the constant (β, σ) case is an ambiguous question because the compensated wage derivative depends on τ and G. It is not obvious what τ and G should be employed in investigating this question. We calculated the compensated wage derivatives for our various wage classes both at the actual τ and G in 1975 (see Table 2) and at the various optimal τ and G. Although a clearcut pattern did not exist, our calculations certainly suggest that this derivative is falling more rapidly in the variable β and σ case. For instance, at the actual τ and G in 1975, the compensated wage derivative is about 20 times as high for the poorest individual as it is for the richest one in the variable (β, σ) case, while the same ratio is only about 5 in the constant (β, σ) case.

SUMMARY, QUALIFICATIONS AND POLICY IMPLICATIONS

Our results are broadly consistent with the Garfinkel-Kesselman findings in favor of the CIT. For most values of our inequality-aversion parameter (ε), the CIT is optimal, when the elasticity of substitution between leisure and consumption (σ) falls across wage classes (and β rises). Making use of the best available labor supply estimates for a variety of demographic groups, we found that σ indeed falls and β indeed rises across wage classes, starting from the lowest wage class and moving upward.

Higher elasticities than those found by Masters-Garfinkel are somewhat less favorable to the CIT although even here the non-income-tested tax-transfer schedules were optimal far more often than income-tested ones. (For space limitation, these results are not presented in this paper.) Only when the elasticity was constrained to be the same for all wage classes were income-tested tax-transfer schedules consistently optimal.

In general, the welfare losses of adopting a non-income-tested regime (such as a linear tax) when the income-tested regime is optimal, are not very large. If this result continues to hold up in future research, the choice between income-tested and non-income-tested tax-transfer schedules will depend much more heavily on other criteria. Nearly all of these other noneconomic considerations favor non-income-tested programs.[10] Here we discuss only two: equality of opportunity and the dignity and self-respect of beneficiaries.

Taxation reduces the opportunity of individuals to improve their own lot through hard work and sacrifice. The higher the tax rate, the greater the reduction in opportunity. Placing the highest tax rates on the poor via income-testing transfers, therefore, exacerbates already existing inequalities of opportunity.

A cost to beneficiaries of participating in welfare programs is loss of pride. So much stress in this country is placed on economic success and "making it," that to declare oneself poor is as good as proclaiming oneself a failure. As a consequence, many who are eligible for welfare benefits do not claim them and among many who do, a negative self-image is fostered.

Because the noneconomic considerations favor non-income-tested programs, the results presented in this paper which also favor non-income-tested programs should be subjected to careful scrutiny.

For example, the model in this paper is unrealistic in several respects which could affect the results. Perhaps the two most important are that the model consists of individuals rather than families and no account is taken of the effect of taxation on savings. The labor supply literature indicates that wives of all income groups have higher substitution elasticities than husbands. The substitution elasticity of all family members, therefore, might decrease less rapidly with family income than the substitution elasticity of family heads decreases as wage rates increase. Similarly, if savings is more responsive to taxation than labor supply, this would tend to make the optimal tax rates on the well-to-do lower. We intend to incorporate these and other similar considerations in future work.

Still, the results presented in this paper are sufficient to call into question the consensus among economic experts that transfer programs which provide benefits only to those with low incomes are more efficient than those which provide benefits to all regardless of income. At the very least, this paper serves the function of shifting the grounds of debate away from preoccupation with the concept and measure of target efficiency to a concern with "real" economic efficiency.

APPENDIX A: Nonfully Integrated Tax-Transfer Systems

Our calculations for nonfully-integrated tax-transfer systems are presented in Table A1 for a CES utility function with $\beta = 0.98$ and $\sigma = 0.5$.

In this case, the marginal tax rate faced by the richest individual is zero ($\tau_R = 1$). In essence, the income tax on the richest quintile becomes a head tax (equals $- G_R$) which ranges in value from about $8000 to about $17,000. As might be expected, the head tax increases with ε and it is usually higher for the higher value of β. All the other quintiles are "poor" in the sense that their gross income is below the kink in the tax schedule. (This, however, does not mean that all of them are net transfer recipients because the kink in the tax schedule occurs above the break-even point in the nonfully integrated case). They have a guaranteed income (G_P) which ranges in value from about $5000 to about $8000 and they face a marginal tax rate ($1 - \tau_P$) which lies between 50% to about 85%. As expected, the income guarantee and marginal tax rate are increasing with the inequality-aversion parameter ε.

Strictly speaking, the results presented in Table A1 indicate the superiority of the NIT over the CIT. We further present in the bottom panel of Table A1 the welfare losses incurred when a linear CIT (namely, a CIT with $\tau_P = \tau_R$) replaces the NIT, as the percentages of government's revenue losses from total revenues and from GNP. These losses are quite significant: 7.3% to 16.7% of total government's revenue or 1.1%

Table A1

Tax Parameters for Optimal Nonfully-Integrated Systems and CIT Welfare Costs (A CES Utility Function with β = 0.58 and σ = 0.50)

	ε = 0.2		ε = 0.4		ε = 0.6		ε = 0.8		ε = 1.4		Maxi-min ε = ∞	
	B=2074	B=1546	B=2074	B=1546	B=2074	B=1546	B=2074	B=1546	B=2074	B=1546	B=2074	B=1546
τ_P	0.42	0.50	0.42	0.50	0.42	0.44	0.40	0.40	0.31	0.32	0.14	0.15
τ_R	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
G_P	5500	5000	5500	5000	5500	5700	5700	6100	6600	6900	7760	8080
G_R	-10512	-8511	-10512	-8511	-10512	-9566	-10767	-10089	-12390	-11587	-17244	-16362
Percentage of loss from government's revenue	7.3	9.0	9.2	9.7	10.3	12.9	10.9	13.6	13.1	16.7	39.6	52.9
Percentage of loss from gross earning	1.21	1.11	1.53	1.20	1.71	1.61	1.82	1.72	2.24	2.16	7.46	7.50

to 2.2% of gross earned income (except in the maxi-min case where they are even much higher). These losses are much higher than those incurred when the optimal fully integrated NIT in the constant (β, σ) case was replaced by a linear CIT (see Table 6).

However, we feel that a nonfully integrated system is not an appropriate framework for evaluating the relative merit of a linear CIT compared to a NIT. Therefore, we also believe that the figures in Table A1 are not good indicators of the welfare costs of the linear CIT. Our calculations of nonfully integrated systems rather emphasize the importance of the result about the optimality of a zero marginal tax rate at the top end of the income ladder. Sadka (1976) has shown that any tax system which taxes the richest individual with a positive rate at the margin will be improved (according to any individualistic social welfare function) by reducing this rate to zero. Thus, any linear CIT can be improved by adjusting the marginal tax rate at the top end of the income range to zero. Performing such an adjustment results in a rather strange NIT system where the four lowest quintiles face the same (positive) marginal tax rate and the highest quintile faces a zero marginal tax rate. Such a strange NIT system is not exactly a "conventional" one; it is not what people usually have in mind when they talk about a NIT system. A "conventional" NIT looks more like our fully integrated NIT where the first or, perhaps, also the second lowest quintiles faced one marginal tax rate and the rest faced another (lower) marginal tax rate. Limiting (as we did) the number of brackets in the tax-transfer system to only two, one can choose between either a conventional NIT or a nonfully integrated NIT which places the same marginal tax rate on all, except on the richest person. Thus, it seems to us more accurate to interpret the costs presented in Table A1 as indicating that adjusting a linear CIT in order to comply with the principle of zero marginal tax rate at the top end of the income ladder is far more important than changing a linear CIT to a conventional (fully integrated) NIT: Given the choice between a zero marginal tax rate on the richest individual and a conventional NIT, the former altnerative is an overwhelming winner.[11]

APPENDIX B: The Determination of the Sets P, R and K.

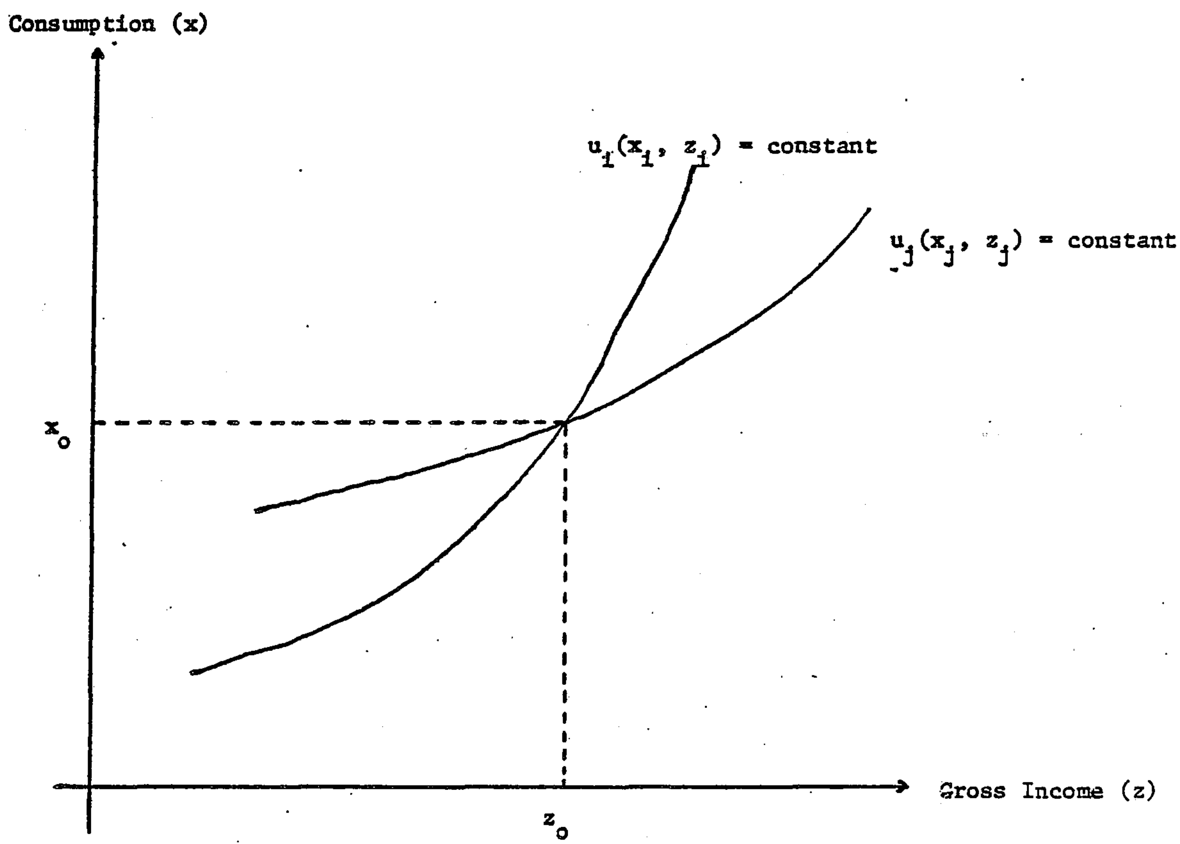
We show here how we determine the sets P(•), R(•) and K(•). To do this we employ a transformation of the utility function suggested by Sadka (1976). Suppose that we try to use the standard indifference curve and budget line diagram in order to depict the individual's choice of an optimal consumption bundle. Individuals have the same map of indifference curves in the (x, y)-space. But because they do not all have the same wage and unearned income, they will not have the same budget line. It is therefore convenient to work in an (x, z)-space, where all have the same budget line $x_i = z_i - T(z)$. But now because $y_i = (z_i - A_i)/w_i$ and A_i and w_i are not constant over i, their maps of indifference curves over (x_i, z_i) are not the same. Specifically, for $z_i \geqq A_i$ we describe the i^{th} individual's preferences over (x_i, z_i) by the utility function u_i:

$$u_i(x_i, z_i) = u(x_i, \frac{z_i - A_i}{w_i}). \tag{20}$$

Sadka (1976), as does most of the literature on optimal taxation, assumed that $A_i = 0$ for all i and was able to show that the indifference curves, $u_i(x_i, z_i)$ = constant, become flatter as w_i increases. Formally, for any point $(x°, z°)$ in the (x , z)-space (see figure 3):

$$-\frac{\frac{\partial u_i}{\partial z_i}(x°, z°)}{\frac{\partial u_i}{\partial x_i}(x°, z°)} > -\frac{\frac{\partial u_j}{\partial z_j}(x°, z°)}{\frac{\partial u_j}{\partial x_j}(x°, z°)} \quad \text{if } i < j \text{ (and hence } w_i < w_j). \tag{21}$$

Now, if (21) were true even when $A_i \neq 0$, then the derivation of the sets P(•), R(•) and K(•) would have been very simple. In particular,



Consumption (x)
$u_i(x_i, z_i)$ = constant
$u_j(x_j, z_j)$ = constant
x_o
z_o
Gross Income (z)

Figure 3

$(i < j)$

we would have been able to determine to which of these sets a certain i belongs before actually finding y_i and $z_i = w_i y_i$. To see this, consider first the case $\tau_P < \tau_R$ (Figure 4). We look for an individual i_o who is just indifferent between being poor and being rich. Then $i \in P(\tau_P, G_P, \tau_R, G_R)$ if $i \leqq i_o$, while $i \in R(\tau_P, G_P, \tau_R, G_R)$ if $i > i_o$. In the case where $\tau_P > \tau_R$ (Figure 5), we look for two individuals, i_1 and i_2. Individual i_1 has his indifference curve tangent to the steep portion of the income tax schedule exactly at the kink point M, while individual i_2 has his indifference curve tangent to the flat portion of the income tax at point M. Then $i \in P(\tau_P, G_P, \tau_R, G_R)$ if $i < i_1$, $i \in K(\tau_P, G_P, \tau_R, G_R)$ if $i_1 \leqq i \leqq i_2$, and $i \in R(\tau_P, G_P, \tau_R, G_R)$ if $i > i_2$.

In general, (21) is no longer true when $A_i \neq 0$. This could potentially complicate the derivations of the sets $P(\cdot)$, $R(\cdot)$ and $K(\cdot)$. However, we can show that for the specific joint distribution of w_i and A_i that we employ in our calculations (21) is still correct. Hence, the procedure described above for determining the sets $P(\cdot)$, $R(\cdot)$ and $K(\cdot)$ remains valid.

To see this, use definition (20), to obtain

$$\frac{\partial u_i}{\partial x_i}(x^\circ, z^\circ) = \frac{\partial u}{\partial x}\left(x^\circ, \frac{z^\circ - A_i}{w_i}\right)$$

and

$$\frac{\partial u_i}{\partial z_i}(x^\circ, z^\circ) = \frac{1}{w_i}\frac{\partial u}{\partial y}\left(x^\circ, \frac{z^\circ - A_i}{w_i}\right)$$

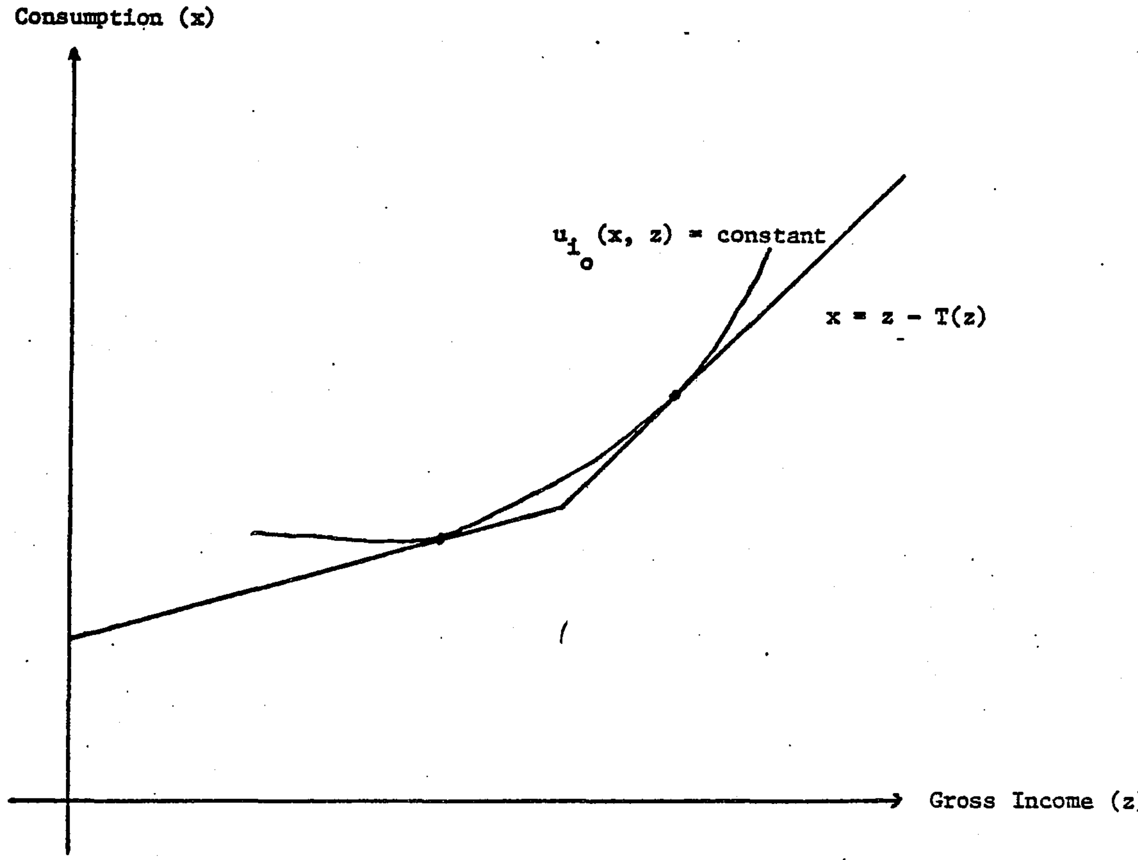
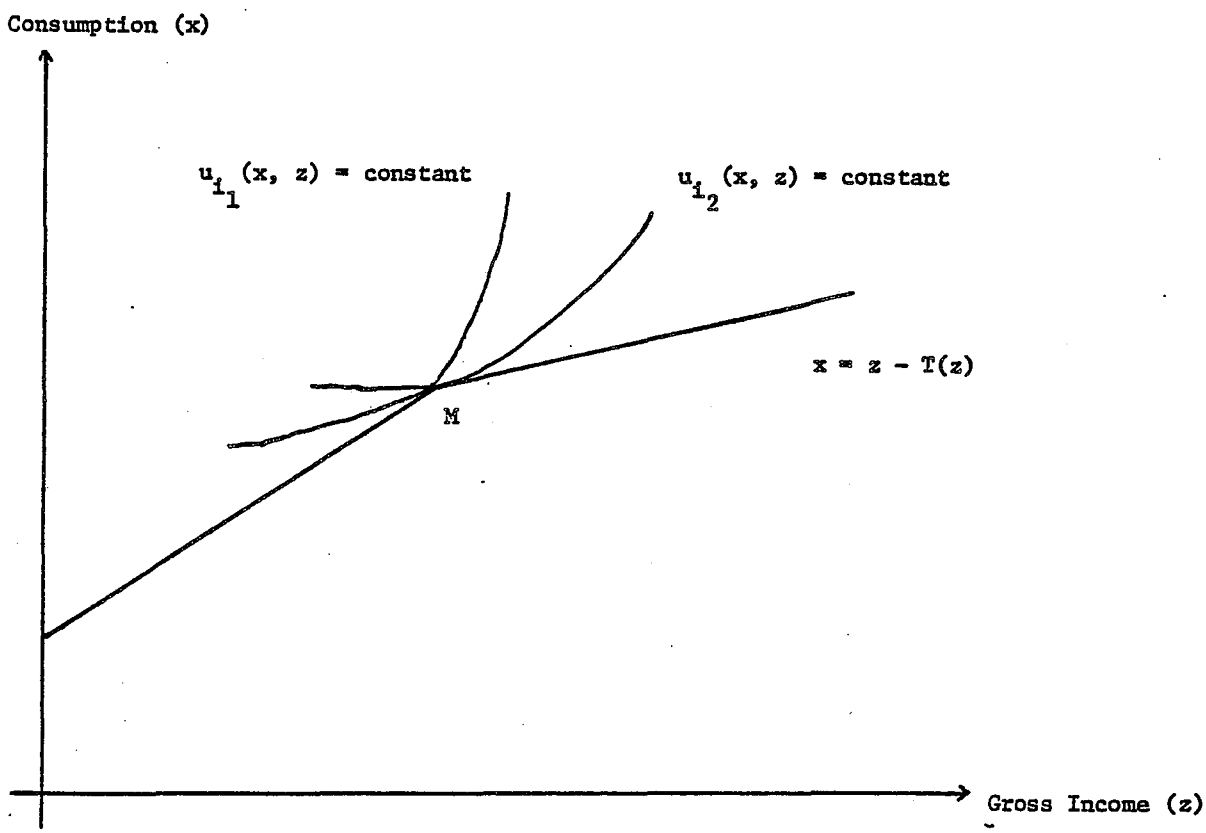


Figure 4



Consumption (x)
$u_{i_1}(x, z)$ = constant
$u_{i_2}(x, z)$ = constant
x = z - T(z)
M
Gross Income (z)

Figure 5

Hence,

$$- \frac{\frac{\partial u_i}{\partial z_i}(x^\circ, z^\circ)}{\frac{\partial u_i}{\partial x_i}(x^\circ, z^\circ)} = \frac{1}{w_i}\left(- \frac{\frac{\partial u}{\partial y}\left(x^\circ, \frac{z^\circ - A_i}{w_i}\right)}{\frac{\partial u}{\partial x}\left(x^\circ, \frac{z^\circ - A_i}{w_i}\right)} \right) \tag{22}$$

The normality of consumption implies that if

$$\frac{z^\circ - A_i}{w_i} > \frac{z^\circ - A_j}{w_j} \tag{23}$$

then

$$- \frac{\frac{\partial u}{\partial x}\left(x^\circ, \frac{z^\circ - A_i}{w_i}\right)}{\frac{\partial u}{\partial y}\left(x^\circ, \frac{z^\circ - A_i}{w_i}\right)} > - \frac{\frac{\partial u}{\partial x}\left(x^\circ, \frac{z^\circ - A_j}{w_j}\right)}{\frac{\partial u}{\partial y}\left(x^\circ, \frac{z^\circ - A_j}{w_j}\right)} \tag{24}$$

Therefore, if whenever $w_i < w_j$ we have also (23), then it follows from (22) and (24) that (21) holds. Thus, a sufficient condition for (21) to be valid in general is that:

$$(z^\circ - A_i)/w_i > (z^\circ - A_j)/w_j \ , \quad \text{whenever } w_i < w_j \ . \tag{25}$$

The latter condition clearly holds if $A_i \leq A_j$, whenever $w_i \nless w_j$. On the other hand, if $A_i > A_j$ while $w_i < w_j$, then (25) does not have to hold and (21) may be violated.

In the data we have used (see Table 1), there are indeed cases where $A_i > A_j$ although $w_i < w_j$. Nevertheless, it can be verified that (25) still holds for these data and hence we can employ (21).

NOTES

[1]Target efficiency has been used by some of the most prominent economists in the field of income maintenance to evaluate alternative transfer programs. (See Barth, 1972; Haveman, 1973; Musgrave, Heller and Peterson, 1970; and Rea, 1974.) Only Rea presents measures of both target efficiency and economic efficiency.

Target efficiency is defined as the proportion of total transfer benefits which accrue to some target group--usually the pretransfer poor. Target efficiency thus refers not to economic efficiency but to some notion of technical efficiency.

Even as a measure of technical efficiency, though, the target efficiency ratio is flawed. Its denominator, total transfer benefits, is not necessarily a useful measure of inputs or costs. In an income-tested program, total transfer benefits paid are a measure of the cost to government and might approach the net cost of the program to nonbeneficiaries. In a non-income-tested program, while total transfer benefits are a measure of the cost of the program to government, they do not gauge the net cost of the program to the net losers. Thus, as long as ultimate interest lies in the well-being of people rather than the accounts of government, target efficiency ratios will not be a good measure of technical efficiency.

[2]See Mirrlees (1971) or Sadka (1976).

[3]Notice that in the fully integrated case the second summation in the LHS of (11), of course, vanishes.

[4]This measure of the welfare loss is inspired by the works of Diamond and McFadden (1974) and, especially, Pazner and Sadka (1978).

[5]The constancy of the social marginal utility of full income follows from the linear homogeneity of the utility functions (14) and (17) in $(x, \bar{y} - y)$.

[6]See Sadka (1976b).

[7]We also calculated results for all possible combinations of $\beta = 0.95$, 0.98, 0.99, 0.995 and $\sigma = 0.35$, 0.40, 0.45, 0.50, but found that the pattern of the results was more or less invariant with respect to this range of values of β and σ. We also computed optimal tax rates for a Cobb-Douglas utility function (i.e. $\sigma = 1$).

[8]For a general result of this sort, see Helpman and Sadka (1978).

[9]In our terminology, this derivative is

$$\left.\frac{\partial \bar{y}}{\partial(\tau w)}\right|_{u\,=\,\text{const.}}$$

and it can be calculated from (16) by applying the Hicks-Slutsky equation.

[10]See the proceedings of the Conference on Universal vs. Income-Tested Transfer Programs, Institute for Research on Poverty, University of Wisconsin, March 15-16, 1979 (Forthcoming).

[11]This result is in sharp contrast to Mirrlees (1976) who understates the importance of having a zero marginal tax rate at the top end of the income distribution.

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