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INCOME MAINTENANCE SCHEMES UNDER WAGE-RATE UNCERTAINTY

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ABSTRACT

The theory of taxation and labor supply is examined in a model with exogenous wage rate uncertainty in order to reappraise conventional views regarding incentive effects of income maintenance schemes. A negative income tax and various versions of a wage subsidy scheme are compared in a simplified model on the basis of incentive criteria. The results one obtains depend crucially on wage-rate uncertainty or the possibility of being unemployed.

Income Maintenance Schemes Under Wage-Rate Uncertainty

1. INTRODUCTION

While there is a very large theoretical and applied literature on taxation and labor supply under conditions of perfect certainty, comparatively little attention has been paid to the case of uncertainty.[1] In so far as labor is supplied under a regular fixed-wage contract with no risk of involuntary unemployment, the neglect of uncertainty is possibly quite reasonable. When one applies taxation theory to income maintenance schemes for the poor, however, the certainty assumption may be misleading. This paper examines the way in which uncertainty on the seller's side in the labor contract modifies standard results in taxes, subsidies, and labor supply.

We shall concentrate specifically on wage-rate uncertainty. This can affect incomes of poor people in a number of ways. (1) Those who are self-employed receive a random return to the weekly hours they devote to their job. (2) "Piece work" or other productivity-related employment contracts clearly have an uncertainty component. (3) Promotion and bonus schemes can have a similar effect in the life cycle of poverty. Many currently low paid workers know that the prospect of a higher wage through career-ladder advancement may be obtained by currently working more hours or at a greater intensity. At the time that the labor supply decision is made, of course, the returns to this marginal effort are uncertain. (4) The possibility of unemployment can also be interpreted in the context of a random wage--though more care is needed here. If the hours-of-work decision is made collectively (for example via union representation), although the going wage-rate may be known with certainty, for each individual it may be uncertain whether he will be able to find employment or not, or indeed keep his job at the going rate.

Before we proceed to formal analysis let us see why a problem might arise. Suppose the wage rate is certain, and let the worker be a conventional utility maximizer confronted with a linear income tax schedule. Suppose now that the tax is made more progressive, by increasing both the marginal tax rate (m.t.r.) and the exemption level of the linear tax schedule. Under fairly weak conditions we can show that there is an unambiguous decrease in labor supply. This is because the adjustment in the exemption level offsets the income effect of the change in the m.t.r., leaving us with almost a pure substitution effect.[2] However, now suppose the marginal wage is stochastic with a given probability distribution. Then the decision problem can be interpreted in the following way. The person has 24 hours a day to allocate, each of which could be devoted to leisure (with a determinate return in terms of utility) or to work, with a stochastic return. This looks rather like a portfolio composition problem, and in that case we would expect an increase in the m.t.r. to increase the amount of resources "invested in the risky asset"--i.e., to increase the time devoted to risky work--by reducing the effective dispersion of the reward. In fact both conventional labor supply effects and portfolio effects are present in a model with wage rate uncertainty, and sections 2 to 4 of this paper are devoted to analyzing their opposing effects.

Since poor people often have uncertain budget constraints and since income maintenance programs can obviously be discussed in the context of taxation theory, the informal results of the last paragraph raise some intriguing questions, which we shall examine later. (1) Does increased progression still reduce work incentives? (2) What is the effect of social insurance payments on work effort? (3) Can we still assert that wage subsidy schemes will be more efficient than negative income taxes in terms of the implicit labor supply disincentives?[3]

2. THE MODEL

Since our informal discussion suggested that there were two offsetting effects of tax-rate progression when the wage rate is stochastic, we might guess that the outcome depends crucially on the stochastic specification. This guess is confirmed by the general results proved formally in the Appendix. It is, however, rather difficult to get much insight from a very general exposition, so we shall employ an extremely simple version of the model; it will readily be seen that the qualitative results will carry over to more complex and realistic specifications. Hence, let the combined tax and income maintenance systems be represented by a tax function:

$$T = my - B, \tag{1}$$

where m is the uniform m.t.r., B is a transfer which may or may not be exogenously determined, T may be positive or negative, and y is original income which is given by:[4]

$$y = Wh. \tag{2}$$

Here $h \in [0, 1]$ is the proportion of total time available that is actually spent at work, and W is the stochastic wage rate. The simplification we shall introduce for the distribution of W is to take this as a simple 2-state variable:

$$W = \begin{Bmatrix} w \text{ with probability } 1\text{-}p \\ w' \text{ with probability } p \end{Bmatrix} \tag{3}$$

with $w' < w$, so that we may think of w and w' as, respectively, success and failure in the risky labor contract. All the results of this paper can be obtained using (3), but we shall simplify further by putting $w' = 0$. The advantages of doing this are that it saves a good deal of messy algebra, and has a natural interpretation in terms of the fourth kind of uncertainty we shall consider.[5]

Disposable income c is obviously y - T, so that we have the stochastic budget constraint:

$$\left.\begin{array}{ll} c^1 = B + vhw & (\text{"success"}) \\ c^0 = B & (\text{"failure"}) \end{array}\right\} \qquad (4)$$

where $v = 1 - m$. We turn now to consumer preferences. Once again we expect the particular shape of the utility function to be crucial in determining the outcome of the two approving forces cited in the introduction, and once again we shall simplify in order to be able to interpret the parameters easily. Let us take a form which has been shown to give a variety of plausible shapes for the labor supply function:[6]

$$U = -\frac{1}{\beta\varepsilon}\left[(c-k)(\ell-h)^{\beta}\right]^{-\varepsilon} \qquad (5)$$

where the parameters ℓ, k, β, ε satisfy $0<\ell\leq 1$, $\beta>0$, $\varepsilon>1$. When the parameter k is positive, it may be interpreted as "subsistence minimum" consumption. This utility function yields strictly declining absolute risk aversion, and increasing/constant/decreasing relative risk aversion depending on the sign of k; thus although simple in form, it gives us a large variety of interesting cases.

As a final piece of preliminary work we need to discuss the distinction between uncertainty of types (1) to (3) and uncertainty of type (4) mentioned in the introduction. Under types (1)-(3) the person obviously makes a decision to work h hours, and has to accept whatever W is realized: Leisure is measured by ℓ-h whatever the realized value of W. What happens with type (4) uncertainty? Here we imagine workers collectively offer a contract of h units labor for a given wage w. However, for each man it is uncertain whether he will be able to secure a job at that wage (if unemployed) or keep his job (if employed) and thus secure the wage w for his effort. Then p becomes the probability of being unemployed.[7] There remains a difficulty concerning the amount of leisure realized. Two extreme assumptions are possible. (a) The disability of being involuntarily unemployed exactly matches the disability of work if employed--in which case leisure is effectively given by ℓ-h as before. This is the position taken by Sjoquist (1976), and can be justified by noting that if unemployed search costs in the form of time and goods will usually be incurred by those wishing to work, and that higher search costs are presumably incurred by those willing to work more. (b) The disability of involuntary unemployment is nil. In this case:

$$\text{leisure} = \begin{Bmatrix} \ell\text{-h with probability 1-p} \\ \ell \text{ with probability p} \end{Bmatrix} \tag{6}$$

This is the position taken by Hartley and Revankar (1974) and by Raniv (1979). It has the rather unattractive implication that time when one is laid off is pure leisure, thus destroying the distinction between voluntary and involuntary unemployment. The truth probably lies between the two extremes, and though we shall work principally with case (a) which corresponds to uncertainty of types (1)-(3), we shall also state results for case (b).

In case (a), maximizing EU subject to (4) we get an interior solution h* given by:

$$z(h^*) = -\beta \frac{[1+x]^{-\varepsilon} + qx^{-\varepsilon}}{[1+\alpha][1+x]^{-\varepsilon} + q\alpha x^{-\varepsilon-1}} \tag{7}$$

where for convenience we have defined the following new variables $z(h^*) \equiv [h^*-\ell]/h^*$, $x \equiv [B-k]/vwh^*$, $q \equiv \frac{p}{1-p}$, $\alpha \equiv \frac{1}{vw}\,\partial B/\partial h$. We note z is an increasing function of h*, that x has the interpretation of "excess transfer income" as a proportion of net earnings, and that x will depend on the type of tax or maintenance program being considered.

In case (b) the corresponding relation is:

$$z(h^*) = -\beta \frac{[1+x]^{-\varepsilon}}{[1+\alpha][1+x]^{-\varepsilon-1} + \alpha qx^{-\varepsilon-1}\left[1-\frac{1}{z}\right]^{\beta\varepsilon}}. \tag{8}$$

These two rather opaque formulae, (7) and (8), yield fairly simple results for the various particular schemes we shall now examine.[8]

3. INCOME TAXATION

The simplest case of a tax scheme which incorporates negative income taxation is found by requiring β to be a constant in all states of nature, so that the transfer depends neither on the decisions of the worker, nor whether "success" or "failure" occurs. This implies that α is zero, and under these circumstances if case (b) were relevant it is clear that if unemployment arises the person would enjoy maximum leisure combined with a lump sum guarantee, whatever work contract he makes. Hence the probability of unemployment would have no effect on the labor contract in this extreme case. This can be checked by putting α=0 in (8) and noting that q disappears. If the reward for failure is non-zero, however, or if the world operates like case (a) (or some intermediate case between (a) and (b)) this will not hold. Hence, we need to see what happens with case (a). Putting $\alpha=0$ in (7) yields:

$$z = -\beta \left[1+x\right] \left[1+q\right] \left[1+ \frac{1}{x}\right]^{\varepsilon}. \tag{9}$$

Observe also that:

$$\frac{\alpha z}{\alpha x} = z_x = -\beta-\beta q \left[1+ \frac{1}{x}\right]^{\varepsilon} \left[1- \frac{\varepsilon}{x}\right]. \tag{10}$$

Note first an implication of (9) using the definition of the variables x and z. Simple rearrangement of (9) reveals that q is a decreasing function of h* and vice versa.[9] So increased risk of the employment contract (interpreted as a higher probability of failure) reduces labor supply. Now consider (10). Notice that this is in the form of [constant] + [expression involving q]. As $q \to 0$ (perfect certainty), obviously only the constant remains on the RHS of (10). We may thus think of the first term of the RHS as a "certainty component" and of the second term as an "uncertainty component" in the analysis of a small change in x. While the former is of unambiguous sign, the latter is ambiguous. The uncertainty component only has the same sign as the certainty component if x, the ratio of excess transfer income to net earnings exceeds ε. Now consider what may happen to the sign of z_x. Suppose that at $q=0$, $\varepsilon > x$. Then allow q to increase: for a certain interval $(0, \underline{q})$ the "certainty component" may dominate, so that $z_x < 0$. Thereafter, for a certain interval, $(\underline{q}, \bar{q})$, the "uncertainty component" dominates, so that $z_x > 0$. However, as q is increasing, h* is falling, and x is rising. Assuming there is a finite $\bar{\bar{q}}$ such that $h^* \to 0$ as $q \to \bar{\bar{q}}$, then x is bound to exceed ε eventually, and so for

$q>\bar{q}$ we have $z_x<0$ once again. Thus for a high probability of failure the qualitative effect of an increase in the x-ratio on h* is the same for low values of q, while for intermediate values the effect may be reversed (see Figures 1 and 2).

Let us see what this implies for changes in tax parameters. First, differentiate (z) to obtain:

$$\frac{\ell}{h^{*2}} \, dh^* = z_x dx. \tag{11}$$

If B is kept constant and m is increased, then $dx = x\left[\frac{1}{v}dm - \frac{1}{h^*}\,dh^*\right]$. On substituting this in (11) and rearranging we have:

$$\frac{\partial h^*}{\partial m} = \frac{h^*}{v}\left[1 + \frac{\ell}{h^* x z_x}\right]^{-1}. \tag{12}$$

This is negative if $-\frac{\ell}{xh^*} < z_x < 0$, and is positive elsewhere.[10] Hence (12) may have either sign for different values of q, which is in sharp contrast to the certainty case for this model. For if we substitute for x and let $q\to 0$ we find the limiting value of (12) is $\left[h^*-\ell/(1+\beta)\right]/v$ which is strictly negative.[11] Second, if m is kept constant, and B increased, re-evaluation of dx in (11) yields:

$$\frac{\partial h^*}{\partial B} = \frac{h^*}{B-k}\left[1 + \frac{\ell}{h^* x z_x}\right]^{-1}. \tag{13}$$

Evidently the same conditions on the sign prevail as in the case of (12), provided that B>k; and in the case of certainty expression (13) becomes $-\beta/\left[vw\,(1+\beta)\right]$ which is again always negative. Thus, even though the labor supply curve under certainty is of a conventional, upward-sloping type, for certain values of the probability of unemployment and of the elasticity of utility w.r.t. consumption (such that $z_x \notin (-\ell/xh^*,0)$) we have the unconventional result that labor supply increases with an increase in the m.t.r. or the lump sum transfer.

However, single parameter changes are of only limited interest. We should also like to know what happens if both tax parameters are manipulated so that the system is made more progressive. Following Cowell (1975) there appear to be two interesting and practical ways to define an increase in progression: a positive dm accompanied by a dB such that dc = 0 (so expected disposable income remains unaltered), or a positive dm accompanied by a dB such that EdT = 0 (so expected tax liability or expected net benefit entitlement remains unaltered). Evaluating dx in each case,[12] using manipulations similar to those above, we get in each of these two cases:

$$\left.\frac{dh^*}{dm}\right|_{Edc = 0} = \frac{h^*}{v}\left[1 + \frac{\ell}{z_x h^* \; x+1-p}\right]^{-1} \tag{14}$$

and

$$\left.\frac{dh^*}{dm}\right|_{EdT = 0} = \frac{h^*}{v}\left[1 + \frac{1}{x+1-p}\left[\frac{\ell}{z_x h^*} - \frac{1-p}{v}\right]\right]^{-1}. \tag{15}$$

Note that under certainty expressions (15) and (16) become, respectively, $\left[h^*-\ell\right]/v$ and $\left[h^*-\ell\right]/\left[v+\beta\right]$ each of which is always negative, as we would expect. However, if the uncertainty component "outweighs" the certainty component in (10), so that $z_x > 0$, we find the remarkable result that (14) is negative. This is also true in (15) if $\frac{v\ell}{h^*\left[1-p\right]} > z_x > 0$. So even though leisure and consumption are superior goods there are values of the probability of failure, and of the risk aversion parameter such that we get the perverse effect of increased progression _increasing_ labor supply!

More formally we can state: _For certain values of q and ε the effect of an increase in the m.t.r. combined with a change in the transfer such that expected tax payments/receipts are kept constant is to increase work incentives. Whenever this occurs for the expected tax-compensated case, it also occurs for the expected consumption-compensated case and the uncompensated cases._[13]

So far we have only demonstrated the possibility of wage uncertainty yielding perverse results on progression and work incentives. Let us then consider more closely under what conditions this unusual result will occur. Clearly the three quantities which together determine whether the "certainty" or "uncertainty" effect dominates are ε (risk aversion minus), p and x the ratio of excess transfer income to earned income. For the purposes of illustrating how these three interact it is more convenient to use $\xi = xh^* = \frac{B-k}{vw}$. Clearly we will be interested in critical values of (ε, p, ξ) such that z_x is exactly zero. These values form the boundary points of a convex set, within which cases of perverse labor supply response are to be found, and outside which one finds cases of conventional labor supply. The contours of this set are illustrated in Figures 1 and 2 for the special case $\ell=\beta=1$. Notice that the upper part of each contour is labelled $\bar{p}$, and the lower part $\underline{p}$. The reason for this is that at any given value of ε and ξ, as we increase p from zero the first time the boundary is crossed at $\underline{p}$ indicates the point at which we switch from conventional to unconventional supply response; the second crossing at $\bar{p}$

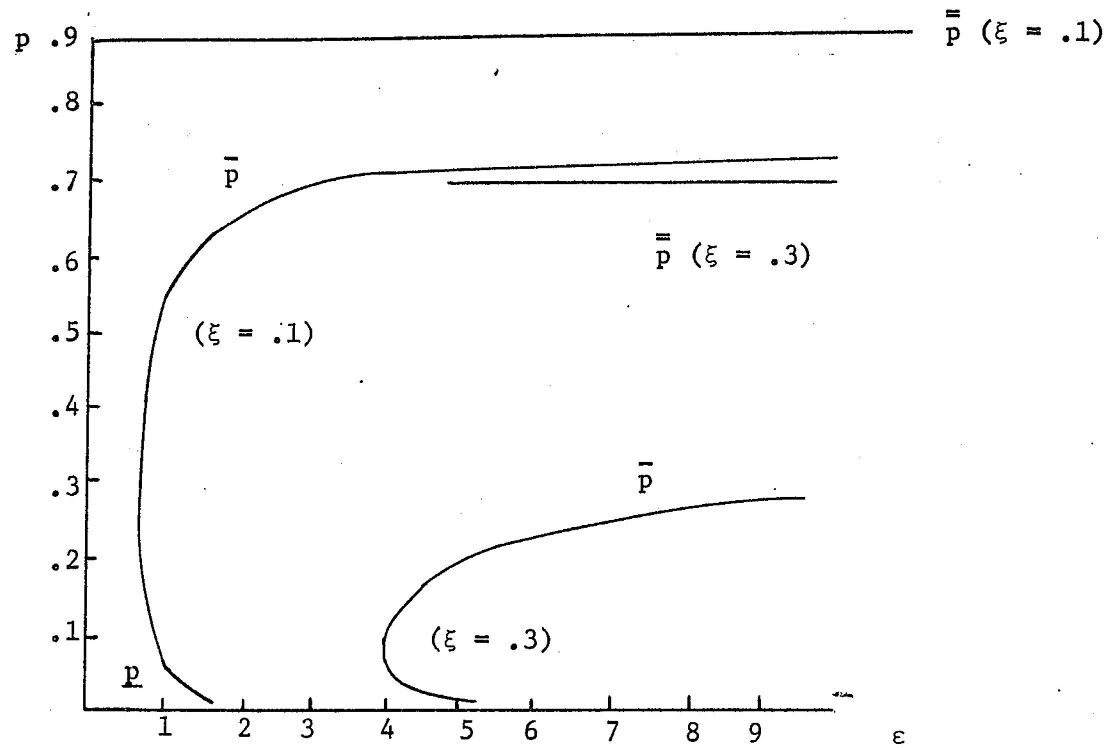

Figure 1. Critical Values of $\bar{p}$ and ε

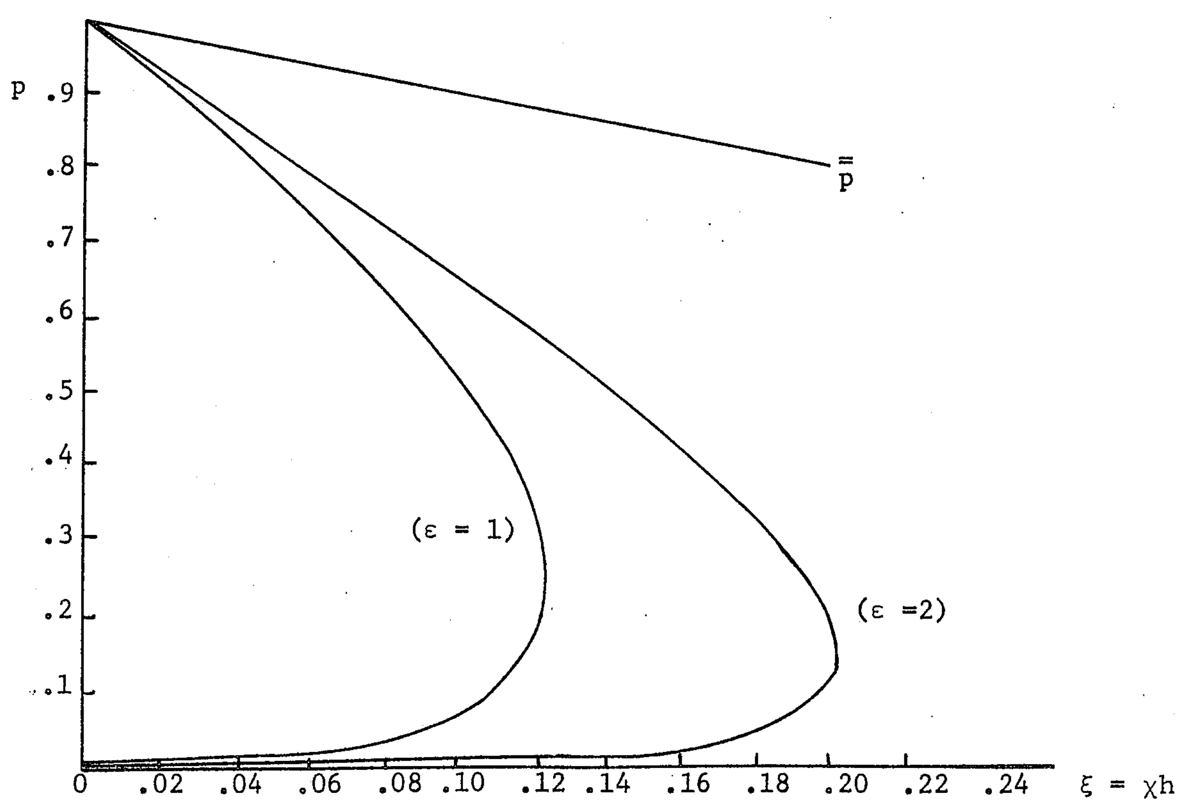

Figure 2. Critical values of ε and χh

indicates the point at which we revert to conventional behavior. Thus, for example, given $\xi=.1$ (excess transfer income is 10% of the net wage rate), and $\varepsilon=1$, if the probability of "failure" (unemployment) lies between about 7% and 52%, we find the perverse case of labor supply increasing with an increase in tax progression. From either picture it is readily seen that the higher the elasticity of utility with respect to consumption, and/or the lower the ratio of transfer income to net earnings, the larger is the interval $(\underline{p}, \bar{p})$ in which the perverse result arises.

Also in Figures 1 and 2 is plotted the set of values (ξ, ε, p) such that $h^*=0$; this is marked as $\bar{\bar{p}}$. Notice that as the ratio ξ is increased, two possibly undesirable effects occur. (i) As we have just seen, the range within which it is possible to get both increased progression and greater work incentives at the margin shrinks. (ii) The probability level at which no work whatsoever is offered also falls. This is given by $\bar{\bar{p}} = 1 - \frac{\beta\xi}{\ell}$ so that in our case, if $B-k = vw$, then $h^*=0$, whatever the labor market conditions. One obvious implication of this is that as we look at different sections of the labor market, given that the tax/transfer scheme is common to all, the higher is the wage rate, the lower is the "cutoff" probability $\bar{\bar{p}}$ at which the individual will not seek work at all.

Hence, for poor people who face wage uncertainty of types (1) and (3) (or type (4) when assumption (a) holds) we have a surprising result for the negative income tax--increased progression can accompany increased work incentive even when the person is exactly compensated. The segment of the population that is potentially in such a position is quite large.

Even if we ignore the problem of unemployment (and hence that of whether assumption (a) or (b) is relevant) we find that about 22% of poor American families are headed by partially or wholly self-employed persons, while about 18% of all poor families are headed by wholly self-employed persons.[14] What this means is that for risk-averse families with low nonemployment income we can manipulate the tax/income maintenance scheme to secure a more equitable distribution of disposable income without incurring a reduction in social product through the labor supply response. This is independent of the labor supply response of the non-poor and relies solely on the variance-reducing effect of the policy instruments.

4. WAGE TAXATION AND SOCIAL INSURANCE

By contrast to section 2 we now wish to use the <u>wage rate</u> as the base for a taxation and income maintenance program to see if similar conclusions apply. The obvious way to do this is to write $B = Dh$ in (1), with D a constant.[15] Using (2) this then yields a wage tax/subsidy scheme with marginal tax rate m and a breakeven point at $w = D$.

However, if the analysis is to continue to cover the possibility of unemployment (as the fourth source of uncertainty) the wage subsidy must be complemented by some social insurance scheme when the person has no job. Contrast this with the NIT which could be discussed in isolation since a minimum level of consumption was guaranteed in <u>all</u> states of the world by the NIT alone. The way the unemployment insurance is administered will crucially affect the labor supply decision.

Consider first of all a system which ties the unemployment compensation to the normal contracted hours. Disposable income is then:

$$\left.\begin{aligned} c^1 &= Dh^* + vh^*w, \text{ if employed,} \\ c^o &= Dh^*, \qquad\qquad \text{if unemployed.} \end{aligned}\right\} \tag{16}$$

As before we may distinguish case (a) where leisure is always given by ℓ-h (applicable to uncertainty types (1) to (3) and capturing one polar case of type (4) uncertainty--unemployment) and case (b) where leisure is determined according to equation (6) (the other extreme assumption about unemployment).

In addition to these two cases, however, we must consider an alternative way of running the insurance system.[16] Suppose benefits are <u>not</u> related to regular earnings, or work hours, but are a given sum $\overline{B}$, then we have:

$$\left.\begin{aligned} c^1 &= Dh^* + vh^*w, \text{ if employed,} \\ c^o &= \overline{B}, \qquad\qquad\quad \text{if unemployed,} \end{aligned}\right\} \tag{17}$$

where I assume $\overline{B} < c^1$ in order to avoid the problem of sybaritic excesses on social security. Once again we could distinguish the two rival assumptions about leisure under type (4) uncertainty. However, budget constraint (17) together with the assumption of equation (6) turns out to yield the same result as our case (a) and therefore, does not need separate analysis. The three cases for consideration under wage subsidy are therefore: (a) budget constraint (16) and leisure = ℓ-h; (b) budget constraint (16) and leisure given by (6); (c) budget constraint (17) and leisure = ℓ-h.

The expressions that one derives for the optimal solution of substituting from (16) or (17) into (7) or (8), respectively, turn out to be quite complicated except in case (a). Hence we shall make the simplifying assumptions that $k = 0$ (so that $x = D/vw$) and that $B = \overline{B} = Dh^*$. The more general version is unlikely to provide substantially more insight, and unconventional results are far more easily discussed using this simple and conventional model. Optimal labor supply in the three cases is then given, respectively, by:

$$z = -\beta, \tag{18a}$$

$$z = -\beta\left[1+qx^{-\varepsilon}\left[1-1/z\right]^{\beta\varepsilon}\right]^{-1}, \tag{18b}$$

$$z = -\beta\left[1+q\left[1+1/x\right]^{\varepsilon}\right], \tag{18c}$$

with the corresponding formulae for z_x being:

$$z_x = 0, \tag{19a}$$

$$z_x = \frac{z^2}{x\beta}\left[s - \frac{x^{\varepsilon}}{q\varepsilon}s^{\beta\varepsilon}\right]^{-1} \tag{19b}$$

$$z_x = \beta q\varepsilon\left[1+1/x\right]^{\varepsilon-1}/x^2, \tag{19c}$$

where $s \equiv z/z-1$.

We want to see what effect uncertainty has on labor incentives. Note that in this simple model with Cobb-Douglas utility, the certainty case is given by (18a, 19a) -- setting $q = 0$ elsewhere confirms this well known result. In general, under perfect certainty, whether a parameter change increases or decreases work incentives depends simply on (i) whether the net wage is increased, (ii) whether the regular labor supply function is forward- or backward-blending. With uncertainty we find the picture rather more complex since the parameter change will alter the effective dispersion of rewards. In case (c), we may use the definitions of x and z to deduce immediately that $h^*_m, h^*_D \gtrless 0$ as $\varepsilon \gtrless 0$. Hence, in this case, if the probability of unemployment is positive, an increment in either the m.t.r. or the guaranteed wage increases/decreases labor supply according as the elasticity of utility w.r.t. consumption is positive or negative. In other words, quite risk averse people ($\varepsilon > 0$) will increase their labor supply when the wage tax/subsidy scheme is made more progressive (D and m go up). Less risk averse people (for whom $-1 < \varepsilon < 0$) decrease their labor supply. Manipulation of (19b) is obviously a little harder, although if $\varepsilon < 0$, we evidently have $z_x > 0$ and hence h^*_m, $h^*_D > 0$. If ε is sufficiently large and positive, then we may find z_x, h^*_m, $h^*_D < 0$.[17] Hence in this case increased progression of the wage tax/subsidy makes high-risk-averse people work less and low-risk-averse people work more.

Cases (b) and (c) because of their different treatment of leisure combined with their different treatment of the unemployment-guarantee evidently produce "mirror-image" results. In each case, increased risk (a higher q) increases the magnitude of the incentive effect.

The appropriate answer will depend on the institutional set-up, the nature of the uncertainty, and on preferences. The institutional set-up determines whether (16) or (17) is the relevant constraint. The nature of the uncertainty and workers' preferences will determine which treatment of leisure is appropriate. Personal preferences, in turn, fix the degree of risk aversion. In any case, it is clear that as with the income tax the policy instruments may be maneuvered to yield higher income support for the poor without <u>necessarily</u> producing a concomitant reduction in labor supply and social product.

5. COMPARISON OF THE SCHEMES

So far we have shown that wage rate uncertainty drastically alters the relationship between progressivity and labor supply in a certain section of the population for each of two income maintenance programs.

To compare the performance of the two programs correctly requires a thorough consideration of the welfare economics of this simple two-good economy. However, let us examine one straightforward issue which has been prominent in the literature--which scheme provides greater work incentives? Clearly, the answer depends on the type of model we use and the basis for comparison of the schemes. We deal first with uncertainty of types (1)-(3), and type (4) with the assumption that leisure = ℓ-h; and we shall consider three bases for comparison, as follows.

I. Universal Equivalence. Here we assume in each state of nature either that the schemes each produce the same level of disposable income, or that they each produce the same tax revenue/subsidy requirement, or that they each produce the same utility level. Notice that since the probabilistic structure is very simple, it is possible to guarantee such universal equivalence with only the small number of tax parameters available. Call optimal labor supply under NIT, under wage subsidy scheme (a), and under wage subsidy scheme (c) h^*, h^{**}, h^{***}, respectively.

Since it is clear from (9), (18a), (18c) that $h^{**}>h^*$ and $h^{**}>h^{***}$ whatever the x-ratios of the three schemes, we do not need to include wage-subsidy scheme (a) in our comparisons, since this will dominate the other two schemes. We may restrict ourselves to a comparison of h^* and h^{***}.

Consider first universal equivalence on a disposable income (consumption) basis. Requiring that c^0 be uniform under all three schemes evidently means $B=\overline{B}$. If we also require that c^1 be uniform under each scheme:

$$vwh^* + B = \hat{v}wh^{***} + Dh^{***}, \tag{20}$$

where $\hat{v}$ is the value of v under wage-subsidy scheme (c). Assuming for convenience that under scheme (c) $\overline{B}=Dh^{***}$, as in the previous section, then dividing (20) by c^0, immediately reveals that we have identical x-ratios (given $k=0$), and using equations (9) and (18c) we have the result that for this simplified utility function $z(h^{***}) > z(h^*)$ so that $h^{***} > h^*$. A similar argument establishes an identical result for equivalence on a utility basis.

Next consider the case of identical tax receipts/subsidy payments under each scheme. Obviously we must again have $B=\overline{B}$, and we shall assume again that for the (c)-program $\overline{B}=Dh^{***}$. Then if tax revenues are also equal,

$$mwh^* - B = \left[\hat{m}w - D\right]h^{***}, \tag{21}$$

where $\hat{m}$ is the value of m under wage-subsidy scheme (c). It is evident from (21) that the x-ratio for the NIT is $x=B/\left[\left[1-m\right]wh^*\right]$ and for the wage-subsidy scheme is $\hat{x}=D/\ 1-\hat{m}\ w=B/\ wh^{***}-wmh^*$, so that $\hat{x}\gtrless x$ as $h^{***}\lessgtr h^*$. However, from (9) and (18c) it can be shown that $h^*>h^{***}$ would imply $\hat{x}<\chi$ which is a contradiction. Hence $h^{***}>h^*$ once again.

Thus, under universal equivalence the level of labor incentives ranks the schemes in the following order: type-(a) wage-subsidy, type-(c) wage-subsidy, NIT.

II. *Expected Equivalence*. Suppose that instead of requiring equivalence under each state of nature we require only that *expected* disposable income, taxes or utility be the same in each case. Once again we may argue that wage-subsidy scheme (a) provides higher work incentives than does scheme (c) or the NIT. When we come to a comparison of scheme (c) and the NIT, however, the comparison is not so straightforward. For example, let us suppose that it is required that expected disposable income at the optimum be the same for each scheme. Clearly this requires:

$$vwh^* + \left[1+q\right]\overline{B} = \left[\hat{v}w + \left[1+q\ D\right]h\right]^{***} \qquad (22)$$

where $Dh^{***}=\overline{B}$. Notice that there is only one constraint on the value of the tax parameters rather than the two constraints under comparison II. Hence, given $\hat{v}$ and D, it will in general be possible to choose a NIT scheme with parameters v, B such that (22) is satisfied, $x<\hat{\chi}$ and $h^*>h^{***}$. Hence if only expected equivalence rather than universal equivalence is required we can *construct* a NIT with more favorable labor incentives but, in general, a different m.t.r.

III. *Marginal Comparisons*. So far we have considered simply the effect of a given set of tax parameter values on the *total* quantity of labor supplied. An important additional question concerns the *change* in labor supply in response to a small change in taxation. The motivation for this is easy to see. Suppose we design an anti-poverty program with some given level of transfer and marginal tax rate. Furthermore, suppose that it is subsequently decided that the m.t.r. should be increased along with the guaranteed transfer in order to "direct public assistance more selectively"--i.e., to increase the scheme's progressivity. We should be interested to know whether the particular administrative structure of the scheme will imply that such increases in the m.t.r. will be accompanied by significant disincentives to work at the margin. For this comparison we are interested in the relative magnitude of a quantity such as $\partial h/\partial m$ at the optimum under the different schemes.

For ease of comparison we examine the three schemes under a situation where each has an identical x-ratio, and consider the effect on optimum labor supply of a small increase in progression--i.e., of an increase in this ratio of transfer income to disposable earnings. This can be done by examining z_x at the optimum, which provides a convenient way of representing the effect of an increase in the progessivity of the maintenance scheme upon labor supply.

Now it is evident from (19a, 19c) that $z_x^{**} = 0$ and $z_x^{***}>0$. So if $\varepsilon>0$ an increase in x has a more favorable effect on incentives under wage-subsidy scheme (c) than under scheme (a); the reverse conclusion is true if $\varepsilon<0$. By using (10) a similar argument holds for a comparison of the NIT with the type (a) wage-subsidy scheme. There remains the comparison of wage-subsidy scheme (c) with NIT. From (19c) and (10) it is apparent that $z_x^{***} \gtrless z_x^{*}$ as:

$$q\varepsilon\left[1+1/x\right]^{\varepsilon-1} \gtrless \left[x^2+q\left[1+1/x\right]^{\varepsilon-1}\left[1+x\right]\left[\varepsilon-x\right]\right]$$

$$\text{or as } q\left[1+1/x\right]^{\varepsilon-1}\left[1+x-\varepsilon\right]+x^2 \gtrless 0. \tag{23}$$

Hence, we expect that for low values of ε (risk aversion), wage subsidy type (c) provides superior incentives to those of NIT; for large values of ε, the situation will be reversed. However, if q vanishes, then NIT always provides lower incentives at the margin.[18]

Finally, let us consider the case where the uncertainty relates to unemployment (type 4) and leisure is determined by relation (6). Once again we may compare the NIT, a wage subsidy with earnings-related unemployment benefit (case (b) of the last section), and a wage subsidy with flat rate unemployment benefit. As we noted before, this last program turns out to have the same labor supply function as case (a) (see equations (18a), (19a)). From equations (9), (10) it is clear once again that, except for marginal comparisons, this program will produce greater work incentives than either of the other two programs. Let us then consider the NIT versus a type (b) wage subsidy.

Clearly z_x is negative for the NIT, and from (19b) we know that it will be positive for the wage subsidy if $\varepsilon<0$. Hence we may infer that for low inequality aversion the wage subsidy provides greater work incentives than the NIT. This conclusion does not necessarily hold for high risk aversion.

Notice how the choice between the two wage subsidy schemes depends on the way leisure is modelled under the different states of nature. If leisure is $\ell - h^*$ then an earnings-related unemployment compensation program elicits maximum work hours. If leisure is determined as equation (6) then a flat rate unemployment compensation is to be preferred. In each case the wage subsidy is preferred to the NIT, except for marginal comparisons when there is a positive probability of failure.

5. CONCLUDING COMMENTS

In a simplified model we have seen how dramatically different results on taxation and labor incentives are found if wage uncertainty is present, because of the uncertainty-reducing role of the tax and income maintenance scheme. The phenomenon of greater progressivity producing increased work arises in each of the models of risky labor contracts we have considered for one or more of the income maintenance programs. Aside from the issues of comparison of the schemes (section 4) what are the policy implications?

Clearly, redistribution and incentives may go hand-in-hand for some segment of the population regardless of the instrument used. This segment would be characterized by relatively high risk aversion, significant but not enormous wage-risk, and not too high a ratio of nonemployment income to earnings. It would be interesting to know how these characteristics are distributed within the population--in particular the possible correlation of risk aversion and incomes. However, it seems that persons for whom the perverse results are particularly likely to hold will be the moderately poor with little income other than earnings (the very poor will have very low potential earnings and therefore high x-ratios), with fairly high income variability, and fairly strong aversion to risk (which may in turn depend on family size). If these groups can be identified in practice then the perverse incentive results may provide the basis for a valuable development in income maintenance programs.

APPENDIX

Let $U(c,h)$ be a concave utility function satisfying $U_c>0$, $U_h<0$, $U_c(0,h) = U_h(0,1) = \infty$, and let $c = y - T$ where T and y are given by (1) and (2). Maximization of EU gives:

$$\frac{dEU}{dh} = vE(U_c W) + EU_h = 0 \tag{A1}$$

for an interior solution. Write $\bar{w} \equiv EW$ and differentiate equation (A1). We get:

$$vE(U_{cc}Wdc^*) + E(U_{ch}dc^*) + \left[vE(U_{ch}W) + EU_{hh}\right] dh = E(U_c W)dm, \tag{A2}$$

where $dc^* = vWdh - h^*Wdm + dB$. Now if we put $Edc=0$, we have $dB = h^*\bar{w}dm - v\bar{w}dh^*$. Substituting in (A2) gives:

$$\left[v\left[E(U_{cc}W^2) - \bar{w}E(U_{cc}W)\right] + \left[E(U_{ch}W) - \bar{w}EU_{ch}\right]\right]\left[vdh^* - h^*dm\right]$$
$$+ \left[vE(U_{ch}W) + EU_{hh}\right]dh^* = E(U_c W)dm \tag{A3}$$

Noting that $E(U_{cc}W^2) - \bar{w}E(U_{cc}W) = \text{cov}\ (U_{cc}W,W)$ and writing $Q = \text{cov}\ (W, U_{ch} + vW\ U_{cc})$, we find (A3) implies:

$$\left.\frac{dh^*}{dm}\right|_{Edc\ =\ 0} = \frac{E(U_c W) + h^*Q}{vE(U_{ch}W) + EU_{hh} + vQ} \tag{A4}$$

Secondly, if we put $EdT=0$, we have $dB=h^*\bar{w}dm - v\bar{w}dh^*$, which upon substitution in (A2) by a similar method yields:

$$\left.\frac{dh^*}{dm}\right|_{EdT\ =\ 0} = \frac{E(U_c W) + h^*Q}{v\bar{\bar{w}}E(U_{cc}W) + \bar{\bar{w}}EU_{ch} + vE(U_{ch}W) + EU_{hh} + vQ} \tag{A5}$$

Clearly if Q=0 the signs of (A4) and (A5) can be determined simply by the signs of their respective denominators. The sufficiency conditions for these to be negative turn out to be identical to the conditions for the case of certainty referred to in footnote 2 of the text. Otherwise, the sign and magnitude of the covariance term Q will be crucial in determining the effect of tax progression. The sign of this clearly depends on the dispersion of W, the magnitude of $v\bar{w}$, and the "curvature" of the utility function--risk aversion. In particular, Block and Heineke (1973) show that the reasonable assumptions of declining absolute risk aversion, separable utility (implying that risk aversion is independent of total hours worked), and the superiority of leisure guarantee that Q is strictly positive. Hence it is clearly <u>possible</u> to get the perverse results of a compensated increase in progression leading to <u>greater</u> labor supply in a quite general model with a conventional utility function. The text identifies more carefully the conditions under which this arises in a more convenient special case.

NOTES

[1]Examples of the theory of labor supply under certainty are Allingham (1972), Cassidy (1970). One paper that has dealt with the general problem of wage rate uncertainty is Block and Heineke (1973).

[2]To make this precise we need to know the size of the changes in the change in the exemption level relative to the change in the m.t.r. If this is constructed so that the person is left on the same indifference curve, clearly we isolate a pure substitution effect. If the change is such that the person enjoys the same consumption level as before then the increased progression reduces work effort as long as consumption is a superior good. If the change is such that the person has the same tax liability as before, then work effort is reduced as long as leisure is a superior good.

[3]On this subject see Garfinkel (1973), Kesselman (1969, 1976), Schlenker (1973), Zeckhauser (1971), Barth and Greenberg (1971).

[4]Unearned income is an unnecessary complication at this stage.

[5]If w' is non-zero the essential difference in what follows is that the ratio w'/w further modifies the conditions under which 'perverse' results are obtained.

[6]See Barzel and MacDonald (1973). Consider the no-tax case under perfect certainty and let there be unearned income I. Then it is easily seen that:

$$\frac{w}{h}\frac{\partial h}{\partial w} = \frac{\beta\left[I-k\right]}{w\ell-\beta\left[I-k\right]}$$

$$\frac{I}{h}\frac{\partial h}{\partial I} = \frac{-\beta I}{w\ell-\beta\left[I-k\right]}$$

each of which may be either positive or negative if $k \neq 0$. We could also consider $0 > \beta > -1$, but this makes (5) slightly more complicated.

[7]Taking p as parametrically given is a strong assumption, since it implies that it is unaffected by the optimal amount of labor supplied or of effort in job search, and is thus unaffected by the tax system. It is, however, a device used elsewhere in the literature--see, for example, Hartley and Revankar (1974)--and to relax it satisfactorily would probably involve a thorough-going model of temporal risk, a complication which I have been anxious to avoid. A second difficulty concerning p may be dealt with more easily: It is evidently reasonable to suppose that this probability be different for an employed man from that for an unemployed man. This presents no problem for the formal analysis as long as the person has a clear idea of what the probability of employment ("success") is at the time he makes the labor supply decision. It is true that this probability may differ from that facing his neighbor, but that will not alter the qualitative results of the paper since it deals only with individual behavior and subjective probabilities (of any magnitude). However, this difficulty would become significant if an attempt is made to aggregate over individuals, and to interpret p as an observable rather than a subjective probability.

[8]One further note of caution: for some maintenance schemes the value of B, and hence α, will depend on the particular state (that is realized (e.g. whether the person is unemployed)). This additional complication which we will consider later has not been written into (7) and (8).

[9]This result is similar to that obtained by Sjoquist (1976) and is true for <u>any</u> concave utility function U.

[10]Recalling that β and $1+\varepsilon$ are necessarily positive, examination of (9) and (10) reveals that the lower constraint cannot be violated for any q.

[11]The first order condition guarantees that $1-\ell/h^*<-\beta$.

[12]Edc=0 implies:

$$-whdm+vwdh+\left[1+q\right]\,dB = 0$$

Now we have as an identity:

$$\frac{dx}{x} = \frac{dB}{B-k} + \frac{dm}{v} - \frac{dh}{h}$$

Substituting for dB from the first relation in the second and noting that $1/\left[1+q\right] = 1-p$ yields the result. A similar manipulation produces the condition for EdT=0.

[13]The justification of this last remark is simple. The "perverse result" is obtained in the uncompensated cases, etc., if z_x is positive; the result is obtained in the expected-tax-compensated case if z_x lies within a given subset of the positive halfline. This result generalizes to <u>any</u> concave utility function for which leisure and consumption are superior.

[14]Source CPS Series P60 for the year 1975. I am grateful to Nancy Williamson for providing these figures.

[15]It will be seen to be a simple version of that discussed by Zeckhauser (1971), Garfinkel and Schlenker (1971), for example.

[16]A budget constraint similar to (17) is used by Hartley and Revankar (1974), and by Sjoquist (1976). Raniv (1979) uses a constraint similar to (16).

[17]This can be established by noting that for positive, finite ε, the sign of z_x depends on the magnitude of $x^{\varepsilon}/q\varepsilon$--if this ratio is sufficiently large then $z_x<0$. Since χ and q may have any positive value we can see that z_x may indeed be negative for $0<\varepsilon<\infty$.

[18]The result in this limiting case concurs with the standard result regarding levels of work incentive under certainty established by, for example, Kesselman (1971, 1973).

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