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REVEALED PREFERENCES, FUNCTIONAL FORM, AND LABOR SUPPLY

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Revealed Preferences, Functional Form, and Labor Supply

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ABSTRACT

This study is an investigation of the extended properties of income-leisure preference structures. The methodology centers on preference maps composed of income consumption curves. These maps are both the basis for a flexible estimation form and a device for direct interpretation of the preference structure revealed by the estimates. The properties of preference maps are categorized into three basic dimensions that are useful in differentiating among the preference structures implied by previously estimated labor supply models and that provide a framework for statistical testing of hypotheses about functional form.

Estimates are obtained for a subpopulation of prime-age married males who were free to reveal their preferences through unconstrained choices on well-defined opportunity sets. Marginal wage rates, including overtime premiums, were directly reported and were adjusted for increasing marginal income tax rates. The estimates indicate that representative preferences for this subpopulation may be closely approximated by a parallel preference structure similar to those implicit in models estimated by Ashenfelter and Heckman (1973, 1974). The estimates also suggest satiety or near-satiety with leisure at a level of work hours not far below standard fulltime work.

Revealed Preferences, Functional Form, and Labor Supply

1. INTRODUCTION

The field of individual labor supply has provided fertile ground for the development of models based on the classical model of static utility maximization. The basic economic theory underlying most studies of labor supply has been very general, assuming only that individuals choose those combinations of income and leisure that maximize a well-behaved utility function subject to the constraints of their opportunity sets. With few exceptions, however, empirical studies of labor supply have significantly restricted the generality of the basic model through a priori choices of specific functional forms.

Choices of functional form are of minor consequence if one is interested only in evaluating parameters of the labor supply function in a small range around an initial equilibrium. However, the restrictions imposed by a particular functional form may become progressively more serious as the range of behavior described by a supply function is increased. In many applications, both in estimation and in policy analysis, the relevant range is indeed large. In cross-sectional observations of labor supply, wage rates often vary by a factor of five or more and observed levels of labor supply range from zero to 3000 or more hours per year. Projections of labor supply responses to income maintenance programs involve estimated responses to net wage cuts of the order of 50% and to income subsidies of several thousand dollars. Simulations of optimal income tax policies are based on utility functions that are implicitly assumed to generate the labor supply behavior of the entire working population.

The extent of the demands placed on our empirical models of labor supply makes the adequacy of their functional form an important empirical problem. In this paper I employ an estimation form that is significantly less restrictive than those employed in most previous studies. I also seek to interpret the estimates in a way that provide a clear intuitive picture of the extended properties of a preference structure.

The methodology is based on preference maps constructed from sets of income consumption curves (ICCs) in the leisure--market-goods plane. Income consumption curves may be directly estimated and the construction of a preference map from these estimates provide for immediate interpretation of the underlying preference structure. The translation from estimated ICCs to a preference map relies on revealed preference analysis and may be arbitrarily precise, given unlimited data. In practical application, estimates of a limited number of ICCs translate into an approximate preference map which reveals the basic structure of underlying preferences over the range of the available data.

For most purposes, the general estimates of labor supply preferences will be more useful if they can be represented in reasonably simple parametric functional form. To this end, ICC preference maps also provide a common basis for comparison of the implied preference structures of the diverse functional forms previously employed in labor supply analysis. The structures for a number of familiar forms will be seen to differ significantly when compared over an extended behavioral range.

A systematic comparison of the characteristics of flexible estimates of a structure with those implied by different parametric forms can then suggest those forms that are suitable for modeling labor supply behavior with minimal distortion. Our objective is thus not simply to utilize a more flexible estimation form but also to represent and interpret our estimates as simply as is possible, consistent with the demands of a given problem.

The organization of the remainder of the paper is as follows. In section II I briefly review the general model of utility-maximizing labor supply and then characterize its extended characteristics in terms of ICC preference maps. Section III presents a comparison of the extended characteristics of familiar empirical forms. The flexible empirical specification, estimates, and inferences are then presented in section IV. Section V includes a brief summary and concluding remarks.

2. CLASSICAL LABOR SUPPLY MODELS AND PREFERENCE MAPS

The discussion in this section is in four parts: (a) a brief review of the familiar elements of the classical labor supply model that serves to establish notation; (b) a discussion of the general properties of ICC preference maps as graphical representations of preference structures and of the corresponding optimizing behavior; (c) a categorization of the properties of preference maps into three basic dimensions that are useful in differentiating among alternative functional forms and that provide a framework for interpretation of our empirical results; and (d) a discussion of the rationale for limiting our empirical investigation to models with linear ICC's. The interpretation of our results in the presence of interpersonal differences in preferences is also discussed.

A. The Classical Model of Labor Supply

The formal theory of individual labor supply is developed in the form of a classical model of demand for two goods, market consumption, M, and nonmarket time or "leisure," L.[1] The basic elements of the model are (1) a well-behaved preference or utility function, U; (2) an identity defining leisure as that portion of total time, T, not allocated to work hours, H; (3) the budget constraint defined by the wage rate, w, and the level of nonwage income, I, both normalized by the price of market goods; (4a, b) the functions describing levels of leisure demand or labor supply that maximize utility on any given opportunity set; and (5) the corresponding demand function for market consumption.

(1) $U = u(M, L)$

(2) $L \equiv T - H$

(3) $M = wH + I$

(4a) $L = \ell(w, I)$

(4b) $H = h(w, I) = T - \ell(w, I)$

(5) $M = m(w, I)$

In the fully general model, the only restrictions on the preference function are that it is increasing, continuous, and quasi-concave. The corresponding labor supply and market demand functions are similarly general, restricted only to satisfy the budget constraint and the familiar restrictions on the compensated wage derivatives.[2] In this paper we focus on the directly observable labor supply function rather than on the complementary leisure demand function, so that the directions of responses are reversed from those in usual demand models.

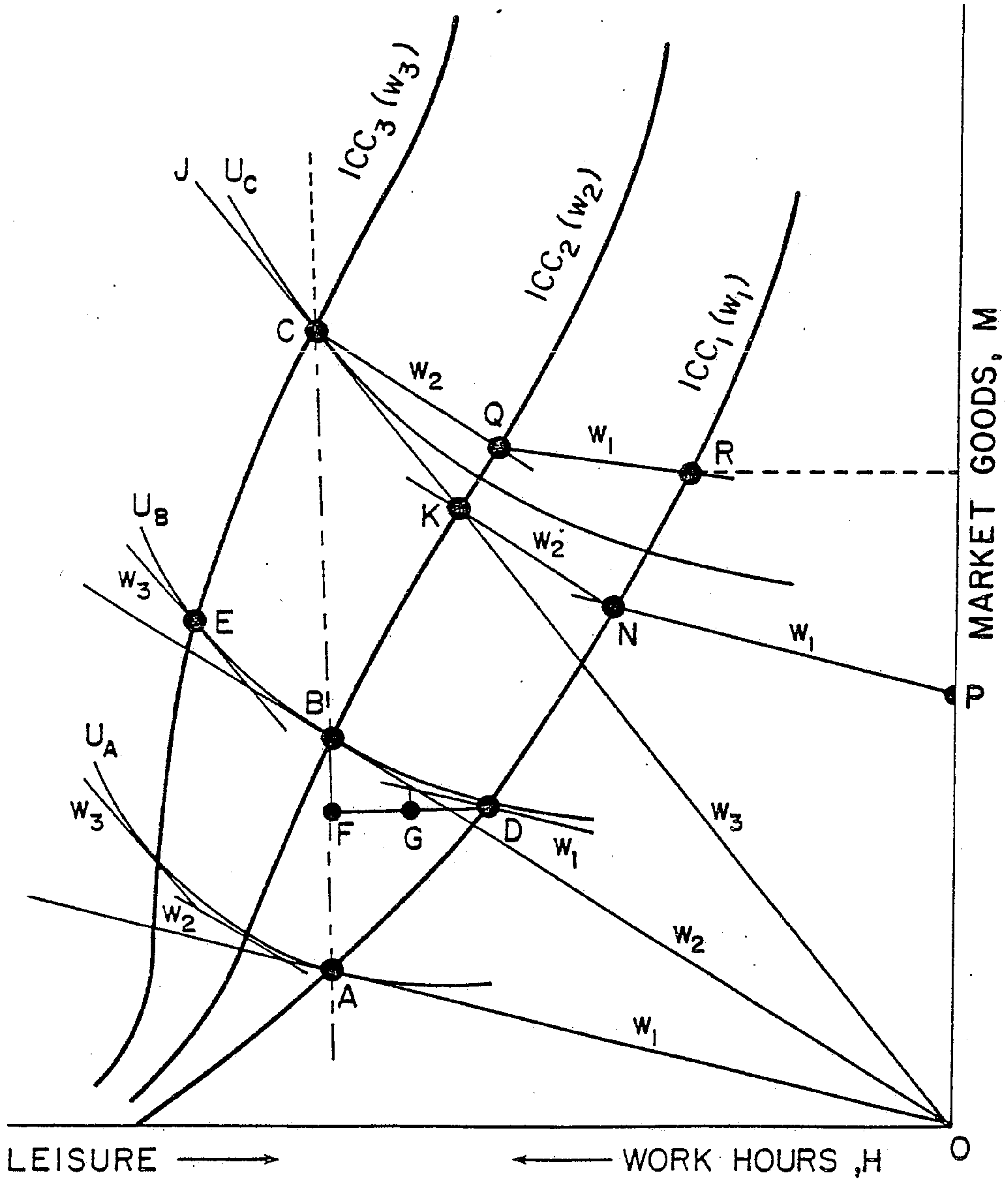
B. Preference Maps Based on Income Consumption Curves

Operational empirical models are usually based on specific functional forms that are regarded as approximations to a general supply function or to the underlying preference function. We turn first to the characterization of a general supply or preference function in terms of an ICC preference map, and then return to a consideration of the properties of specific analytical forms in section III.

An example of an approximate preference map based on income consumption curves is shown in Figure 1. The three curves rising to the northeast represent ICCs at each of the three marginal wage rates, w_1, w_2, and w_3. In principle, any such locus may be measured directly by observing equilibria at a given marginal wage rate and varying levels of nonwage income, I. The full ranges of the ICC's shown correspond to ranges of nonwage income variation, both positive and negative, that are larger than are commonly observed.[3] The equilibria for zero nonwage income at each wage rate are shown as points A, B, and C at the intersections of the respective budget lines with corresponding ICCs. Budget lines at a variety of other income levels that are useful for interpretation of the preference map are shown as segments labeled by the appropriate marginal wage.

The set of income consumption curves may be formally translated to an approximate indifference map using simple revealed-preference techniques. The intersections of any budget line with its corresponding ICC represent

Figure 1: An Approxmate Preference Map
Based on Income Consumption Curves



an equilibrium, revealed preferred to other points on the budget line. Appropriately selected budget segments thus define limits for an indifference curve through any consumption point. The segmented curve J C K N P in Figure 1 defines the lower limit for the indifference curve through C, and CQR, with dashed extensions based on nonsatiety, defines the upper limit. As shown by Houthakker (1950), the region of uncertainty may be made arbitrarily small if additional information about intervening ICCs is obtained.

In most practical applications, we are interested not in exact measurement of the preference structure but in a relatively simple approximation. Graphically, such a representation may be obtained by interpolation, such as that illustrated by indifference curve U_B in Figure 1. The method shown assumes linear changes in the marginal rate of substitution or, equivalently, a constant substitution effect over each interval between ICCs. The slope of the indifference curve as it cuts an ICC is determined by the tangent budget line. Adjacent tangents for a given interpolated curve are than positioned so that their intersection bisects the horizontal distance between the tangency points.[4] In the figure, point G bisects the horizontal segment DF. The length of segment DF represents the total substitution response over the discrete wage interval w_1 to w_2 and the approximate substitution effect is the total response divided by the wage change. This interpolation method may easily be modified if the intervals between successive ICC's indicate systematic changes in the substitution effect. Curves may be interpolated at any real income level spanned by the ICCs, thus yielding a flexible graphical approximation of the underlying structure over an extended range.

C. Income, Substitution, and Divergence Dimensions of Preference Maps

The pattern of a small number of ICCs provides significant guidance for the selection of a functional form for a parametric approximation to a preference function. For this purpose it is useful to characterize the properties of an ICC map with respect to three interrelated dimensions. We shall refer to these as the income, substitution, and divergence dimensions of the preference structure. The income properties are reflected in the slope and curvature of ICCs and are closely related to income effects and higher-order income derivatives.[5] The substitution properties refer to the substitution effect and higher-order compensated wage derivatives along any given indifference curve. As described earlier, the substitution properties are reflected in the sequence of lateral intervals between ICCs which translate into approximate values of the substitution effect along an indifference curve. The divergence properties refer to the interrelationship between income and substitution effects and can be expressed in terms of cross-derivatives of labor supply with respect to income and marginal wage rates.

The divergence properties are of particular interest because they have received relatively little attention in the literature.[6] These properties refer literally to the divergence (or convergence) of ICCs in a preference map. The resulting interrelationship may be viewed either as a change in income properties as the marginal wage changes or as a change in substitution properties as real income changes. In a divergent structure, for instance, increasing marginal wage rates lead to more nearly vertical ICCs or, alternatively, increasing real income leads to stronger substitution effects. The illustrative structure in Figure 1 is shown as divergent in the lower range of real income and roughly neutral or parallel in the upper ranges.

The expression for the divergence properties in terms of derivatives is:

$$(6) \qquad \frac{\partial S}{\partial I} = \left.\frac{\partial B}{\partial w}\right|_u - B^2$$

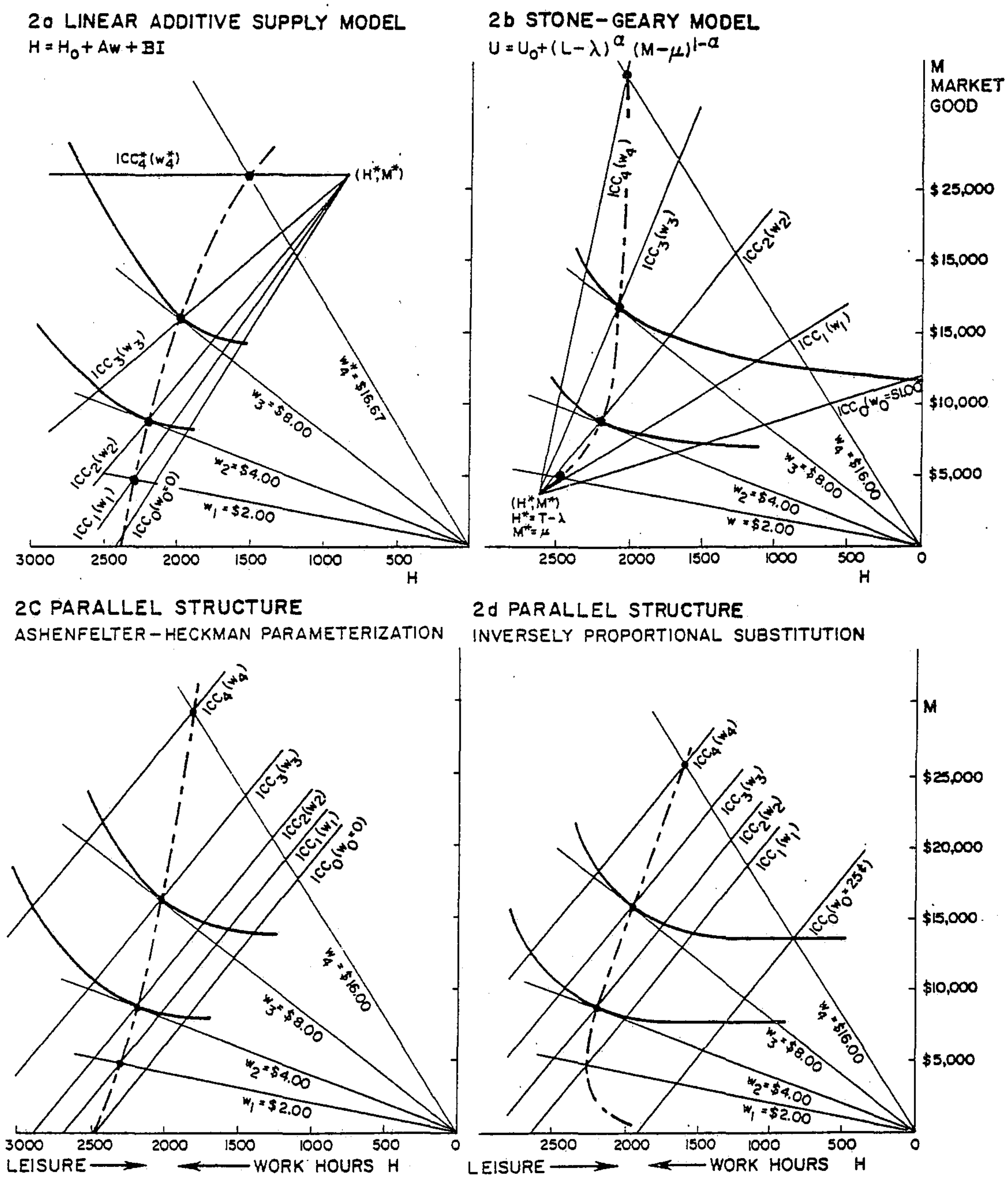
where $S = \left.\frac{\partial H}{\partial w}\right|_u$ is the substitution effect, and $B = \frac{\partial H}{\partial I}$ is the income effect.[7] The relationship in equation 6 is essentially a compensated version of Young's theorem. The important point to note is that the zero point of $\left.\frac{\partial B}{\partial w}\right|_u$ does not mark the boundary between divergent and convergent forms. A constant income effect (not zero) yields convergent ICCs with the degree of convergence proportional to B^2. This occurs because the constant work-hours response to nonwage income implies larger foregone earnings at higher wage rates and thus smaller marginal expenditures on market goods. A specific example is discussed in section III and illustrated in Figure 2a below.

The analytics of substitution properties are more familiar. One aspect that has not received much specific attention, however, is the behavior of the substitution effect at low marginal wage rates. A specification that does not provide for strong substitution effects in this range may have unintended implications of leisure satiation. Specific examples are also discussed below.

D. Restriction to Linear Income Effects

In this paper I shall treat the income dimension of preference structures less generally than the other two dimensions. Specifically, I shall concentrate on the slopes of ICCs, with only brief consideration of curvature. This restriction of the analysis is based on a combination of empirical and theoretical factors. Empirically, the range of nonwage income at any given

Figure 2: Extended Properties of
Simple Labor Supply Models



wage rate is limited, providing little prospect of good curvature estimates. Theoretically, there is very limited literature on consistent parameterizations with independent flexibility in the income dimension to serve as a basis for interpretation of curvilinear estimates.[8] Models with linear income effects have been employed in a large number of recent labor supply studies so that there is a rich body of literature for comparison despite this restriction.

The models we consider explicitly, having linear income effects but general substitution and divergence properties, belong to a class first characterized by Gorman (1953, 1961). This class, the Gorman Polar Form (GPF), accommodates a wide variety of parameterizations, but there is no single general parameterization that includes others as restricted versions.[9] The GPF thus provides an analytical structure for interpretation and extension of our research, particularly to models with more dimensions. Our current discussion is focused on more familiar analytical forms that are specific cases of the GPF.

Gorman's work also provides a basis for interpretation of a "representative" preference structure in the context of interpersonal differences in preferences. He has shown that if diverse preferences are well behaved, in the sense that all individuals have parallel linear ICCs at any given wage rate, then average behavior will be consistent with a preference function in the same class (the GPF). These conditions amount to identical income and divergence properties between individuals but allow for differences in position parameters and substitution properties which are averaged in the representative function. Elsewhere, I have investigated (Dickinson, 1976) whether income properties are identical; I found limited evidence of differences. These differences have, however, relatively minor consequences for the present estimates. For this study we maintain the assumption of well-behaved interpersonal preference differences and interpret estimates as representative preference functions in the Gorman sense.

3. THE EXTENDED PROPERTIES OF REPRESENTATIVE LABOR SUPPLY MODELS

Labor supply models that are parameterized at the level of the utility function, direct or indirect, incorporate explicit assumptions about the underlying preference structure. Models parameterized at the level of the supply function also incorporate such assumptions, though less explicitly.[10] ICC preference maps provide a common basis for comparison of the preference structures underlying different labor supply models, whatever the mode of parameterization. The maps shown in Figures 2 and 3 illustrate the range of characteristics implied by a representative selection of such models. To facilitate comparison of extended properties, all forms shown have the same first-order properties at the equilibrium for w_2 = \$4/hr, I_o = \$0. These values, chosen to be reasonably representative of recent estimates for working men, are $H(w_2, I_o)$ = 2200 hrs/year, $S(w_2, I_o)$ = 80 hrs^2/\$, and $B(w_2, I_o)$ = -.06 hrs/\$.

Contrasting divergence properties are illustrated by the models in Figures 2a and 2b. The linear additive supply model in 2a is the most familiar example of a convergent form.[11] At high wage rates, the constant income effect on work hours implies large expenditures on leisure out of nonwage income, at the expense of market consumption. At wage rates exceeding the negative inverse of the income effect (\$16.67/hr in the example) market consumption becomes an inferior good. The convergence point of the ICCs, (H^*, M^*),[12] may be interpreted as a "bliss point," inasmuch as the implied indifference curves take on reverse curvature in the region above and to the right. Additive supply models with curvilinear income and/or wage responses also imply convergent structures but do not result in a single "bliss point."[13]

The Stone-Geary or Translated Cobb-Douglas utility model shown in 2b is a well-known divergent form.[14] In this model the ICCs diverge from a point (λ, μ). These coordinates are often interpreted as minimum subsistence levels of leisure and market consumption.[15] The degree of divergence is reflected in the relative strengths of income effects at different wage rates. In the Stone-Geary model the magnitudes of the income effects on work hours are inversely proportional to the wage rate. Greater or lesser relative difference would be provided by translated CES models with elasticities greater or less than one respectively. Variable-elasticity divergent forms may also be parameterized.[16] Our inferences about the divergence properties of the income-leisure preference structure will be based on the relative magnitudes of freely estimated income effects at different wage rates, as compared with those implied by these representative models.

The two examples of parallel preference structures shown in figures 2c and 2d represent the intermediate case with respect to divergence properties. The absence of divergence or convergence in these models allows us to focus on comparative substitution properties. Extended substitution properties may be flexibly parameterized but, for a given model, they are the same at all utility levels.

The structure in figure 2c is that implicit in the empirical models estimated by Ashenfelter and Heckman (1973, 1974). Their model, translated from deviation form, may be expressed as the linear function (7).[17]

$$(7) \qquad H = \xi(M, w) = H_o + B^{\dagger}M + S^{\dagger}w$$

Superscripts have been added to the parameter notation to indicate that the authors' interpretations need slight revision. The parameter $B^{\dagger}$ represents the slope of the parallel ICCs rather than the income effect on work hours, and $S^{\dagger}$ represents the unit distance between ICCs at constant M rather than at constant utility. The conventional income and substitution effects for the model are

$$\partial H/\partial I = B(w) = B^{\dagger}/(1 - B^{\dagger}w) \quad \text{and} \quad \partial H/\partial w\big|_u = S(w) = S^{\dagger}/(1 - B^{\dagger}w).$$ [18]

The form of the income effect is characteristic of parallel models. The form of the substitution effect is specific to this parameterization and carries the implication of leisure satiation unless $S^{\dagger}$ is very large. The substitution effect has a maximum value of $S^{\dagger}$ if leisure is a normal good (negative $B^{\dagger}$). A compensated reduction in the wage rate from w^* to zero thus implies a labor supply reduction of less than $w^*S^{\dagger}$. The zero-wage ICC in the illustrated case indicates the onset of satiety at labor supply levels not far below standard, full-time work hours.

The parallel model shown in 2d does not imply leisure satiation because the substitution effect is parameterized as a simple inverse function of the wage rate $S(w) = \gamma/w$. Even in this model, however, typical parameter values imply a very low shadow price of leisure over a large range. A technical discussion of the properties of parallel models in a variety of parameterizations is presented in another paper by this author (1979) and a brief exposition is given in the appendix to this paper.

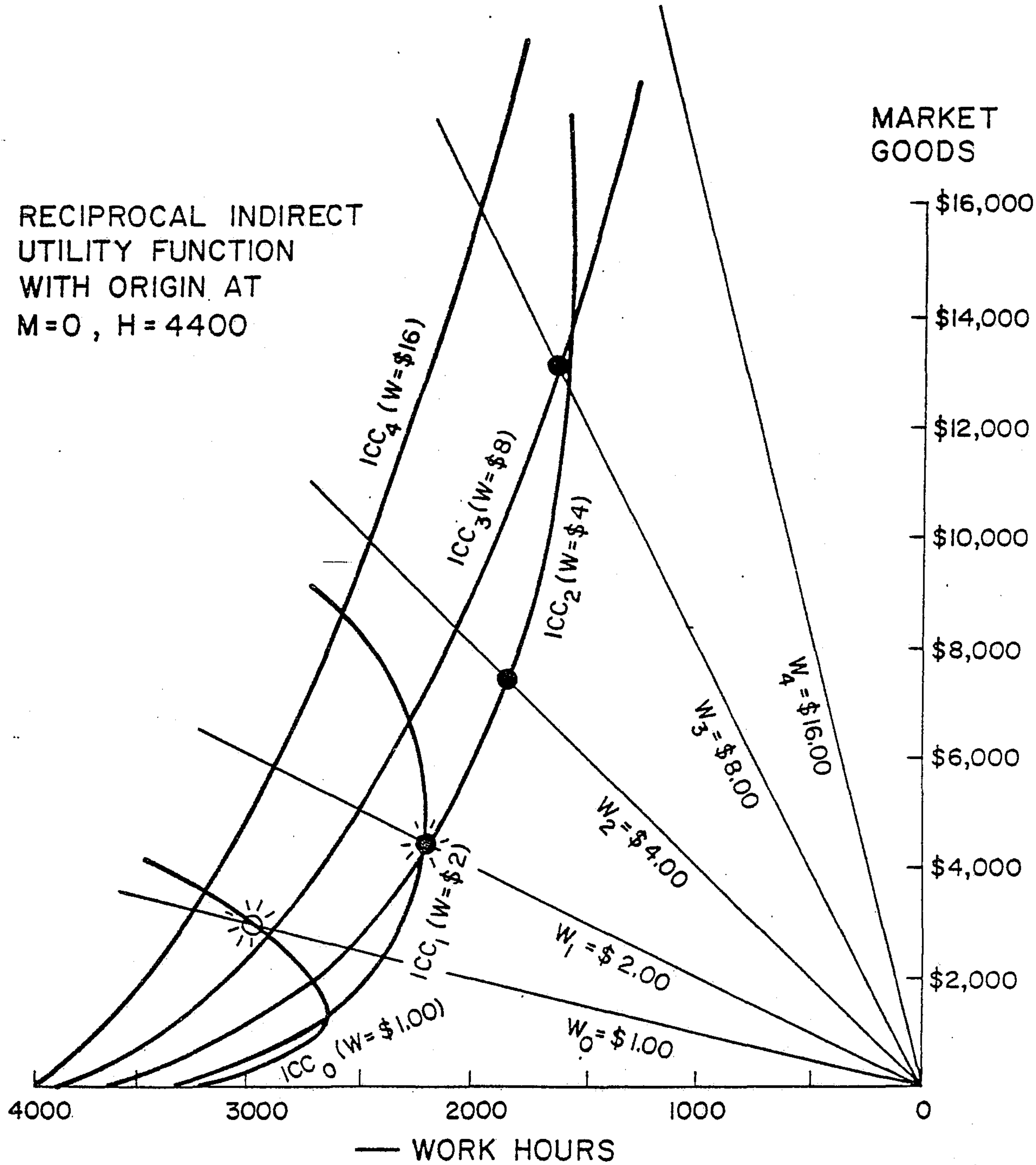
The contrasting substitution properties of the two parallel models are also present in the examples in 2a and 2b, though substitution and divergence properties are intertwined in those cases. At a central utility level the linear additive supply model implies a more rapid onset of satiety than the Ashenfelter-Heckman model. On the other hand, any CES utility model (with nonzero σ) implies absolute substitution responses that go to infinity as the marginal wage rate approaches zero, and thus no satiation effects are present. As in the parallel model, more flexible substitution parameterizations that alter these implications may be incorporated in either divergent or convergent models.[19] With respect to substitution properties, I shall attempt to distinguish empirically those parameterizations that are least likely to distort policy inferences in the low-wage range.

Most empirical labor supply models do not place restrictions on the basic income properties of the preference structure, other than the linearity restriction that I have also imposed. In a few models, further restrictions are imposed through the assumption of a homothetic form with the point of origin fixed a priori. Such restrictions risk distortions of both income and divergence properties. The potential biases are intuitively analogous to those in a simple regression model with a constrained intercept. Given that there is relatively limited variation in nonwage income in most labor supply data, estimated income effects, absolute and relative, will be dominated by the position of a constrained origin relative to mean-labor-supply/market-consumption positions at different wage rates. If some forms of translated homothetic models prove to be consistent with our flexible estimates, it will be possible to construct approximate confidence limits for the point of origin. These will be based on the simplifying approximation that ICCs at different wage rates pass through the corresponding observed mean equilibrium points.

The form shown in Figure 3 is an example of the flexible, duality-based models that have gained wide currency in recent production and demand literature.[20] It is presented for perspective on a major genre of models not otherwise considered in this paper because of our restriction to linear income effects. The illustrated structure is derived from a "translog," reciprocal, indirect utility function with parameters chosen to match those in Figure 2 with respect to first order properties at w_2 = \$4.00/hr and I = \$0. The extended properties depend on the relative positions of the origin and the approximation point. Once the origin and the first-order properties of the model are fixed, the divergence, income curvature, and extended substitution properties are also fully determined.

The indirect translog structure, with the given origin and first-order properties, is not well behaved in the low-wage range. Low-wage ICCs that cross to the left of higher wage loci are indicative of negative substitution effects. The illustrated equilibria at w_o = \$1/hr and w_1 = \$2/hr fall in this unacceptable range. The illustrated structure is also not defined for most levels of work hours below 2000 hours per year. The extended properties of the indirect translog model improve as the origin is moved to lower work hours, so that the model is more nearly homothetic.[21] Presumably, greater regularity of the model would be

Figure 3: Translog Model

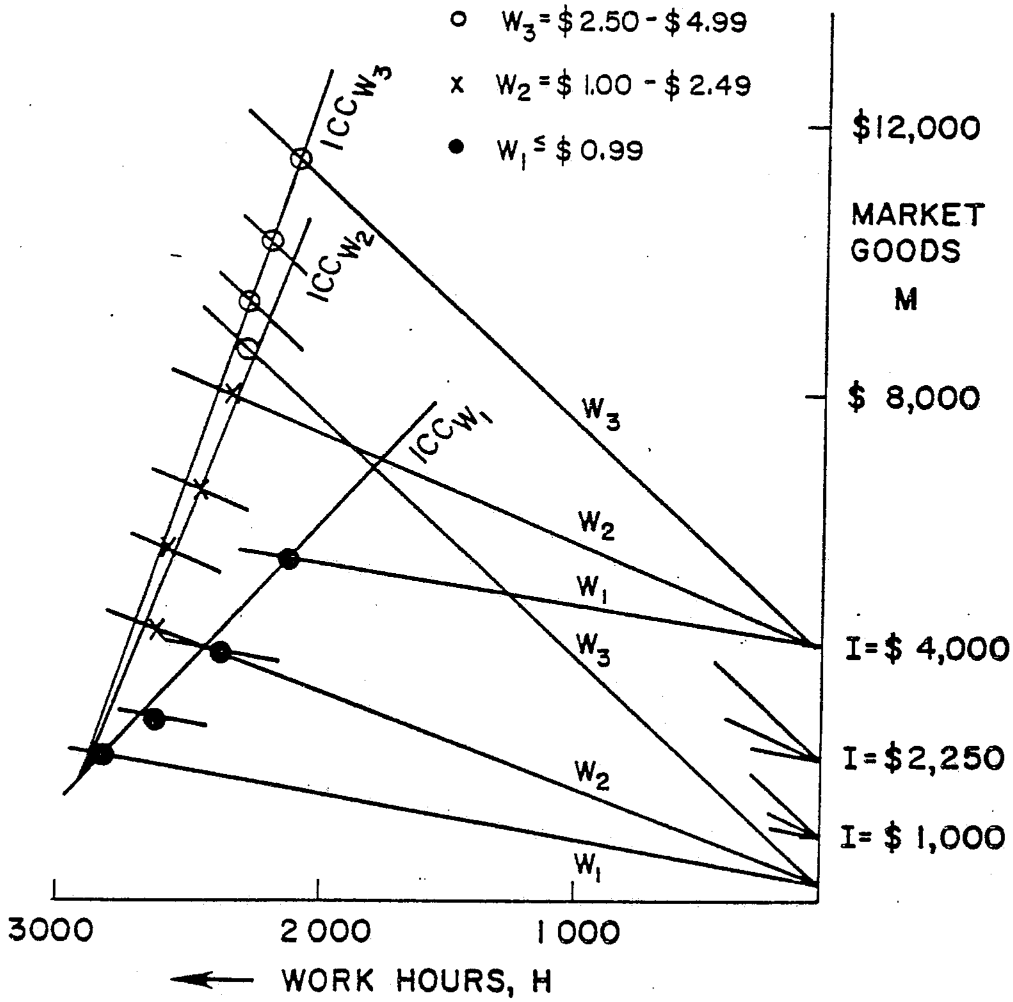


assured if parameters defining the point of origin were freely estimated.[22] Even in this case, however, the curvature, substitution, and divergence properties are not independently flexible. Models of this sort may thus be viewed as possible alternatives to those that we consider explicitly, but they do not represent an inherently more general class.

All of the models discussed above are based, at one stage or another, on specific analytical forms. A more fully flexible empirical model of labor supply was employed by Cohen, Rea, and Lerman (1970) and elaborated in Rea (1971). In their work, wage rates and nonwage income are specified as categorical variables, complete with full interactions between the two. Their presentation and interpretation of results is in terms of a matrix of point estimates of income and substitution effects.

The results of Cohen, Rea and Lerman may easily be interpreted within the preference map methodology. An apt example is provided by Rea's estimates for married males, shown in Figure 4. The step-function specification together with the ample number of observations provided by their CPS sample creates the potential for estimates of structural characteristics in all dimensions, including curvature of ICCs. Little consistent evidence of curvature is apparent in Rea's estimates, however. The income-consumption curves in the figure represent my own freehand fit of a regular linear structure to Rea's flexible estimates. With the exception of one point on the high-wage ICC and a suggestion of curvature on the low-wage curve, for which data are sparse, the estimates appear to be remarkably consistent with a displaced homothetic structure having an origin in the vicinity of (H = 2900 hrs, M = \$1700) and an elasticity of substitution inversely related to wage rate.

Figure 4: Preference Map Based on Estimates by Samuel Rea (1973)



The empirical specification employed in this paper is based on the Cohen, Rea, Lerman model with the restriction to linear ICCs noted earlier. Our estimation procedures differ in important respects, most notably in data source, sample selection, and budget specification. Nonetheless, the Rea results, free of potential distortion arising from analytical restrictions, provide an important benchmark for subsequent comparison.

IV. THE EMPIRICAL MODEL, ESTIMATES AND INFERENCES

The data for our empirical analysis of preference structures are drawn from The Panel Study of Income Dynamics for the years 1969-1972.[23] The analysis is carried out for a sample of prime-age, white, married males whose labor supply decisions were judged to reveal their income-leisure preferences as fully and directly as possible. In particular, the estimates apply to workers who reported freedom to vary their work hours on their primary jobs at known, nonzero marginal wage rates.[24] The labor supply positions of these workers are believed to correspond closely to the marginal equilibria presumed by the theory of consumer choice.

One consequence of the stringent selection criteria for our sample is that the estimates are directly representative of only a select sub-population of employed males.[25] It is hoped, of course, that the basic structure of the estimates will be generalizable and can serve as the basis for a consistent model for a much broader population.[26] For the present analysis, the restriction to the subsample permits the use of straight-forward regression techniques and facilitates a direct focus on the extended properties of the estimated preference structures.

The workers in the subsample gave direct reports of both regular or "straight-time" wage rates and marginal wage rates. In 85% of the sample, the marginal wage included a 50% overtime premium with the remainder distributed between "straight-time" pay and higher overtime premiums. In our analysis, the gross marginal wage for all workers was imputed at the modal value of 150% of the straight-time wage in order to avoid problems of endogenous self-selection onto higher or lower wage segments of the opportunity set. The marginal wage line, assumed to intersect the "straight-time" budget line at 2000 hours per year, was then extrapolated to zero to yield the effective level of gross nonwage income. As noted earlier, the resulting values of income were predominantly negative. The wage and income variables were then further adjusted to account for imputed progressive marginal income tax rates and total tax liabilities. The tax adjustment lowered the marginal wage rate but generally increased the effective level of nonwage income because the graduated total tax liability is less than if the marginal rate were applied to all earnings.[27]

The budget variables, w_{net} and I_{net}, resulting from the combined tax and overtime transformations are formally defined by equations (8a, b).

(8a) $$w_{net} = 1.5(1 - \hat{t})w_{st}$$

(8b) $$I_{net} = I - 1000(1 - \hat{t})w_{st} + \hat{t} \cdot \hat{E} - \hat{T}$$

where w_{st} is the gross straight-time wage, $\hat{t}$ and $\hat{T}$ are the imputed tax rate and total tax, and $\hat{E}$ is predicted earnings, given w_{st} and personal characteristics.

The specification for estimation of sample analogs of income consumption curves is a linear adaptation of the Cohen, Rea, and Lerman (1970) empirical model. Independent linear income effects are estimated for each of the three wage intervals shown in column 1 of Table 1. Our comparisons between intervals and inferences about the extended properties of the preference structure are based on the ICCs at the mean wage of each interval. The response to the limited wage variation within each interval is accommodated by a linear spline on the wage variable, allowing separate uncompensated wage slopes within intervals, while maintaining continuity at the boundaries. The small number of intervals chosen represents a compromise between the objective of revealing the higher-order properties of the structure and the need for reasonable confidence in the comparative estimates.

The estimated parameters of representative, linear, income-consumption curves for the three wage intervals are given in columns 2 through 4 of Table 1.[28] The corresponding ICC preference map is shown in Figure 5.

The divergence properties of the estimated ICCs suggest either a parallel model or a model with a linear, additive, income effect. The essential equality of the estimated income effects in the lower two wage intervals is clearly consistent with a simple, linear, income model with its implied convergence of ICCs. The moderate weakening of the income effect in the upper wage interval is consistent with a parallel model in that the slope of the upper ICC is approximately equal to the mean slope of the lower two.

For more formal inference about the divergence properties of the underlying preference structure, we interpret the statistical evidence in the light of the theoretical discussion of parts II and III. The

Table 1

Estimated Parameters of Income Consumption Curves

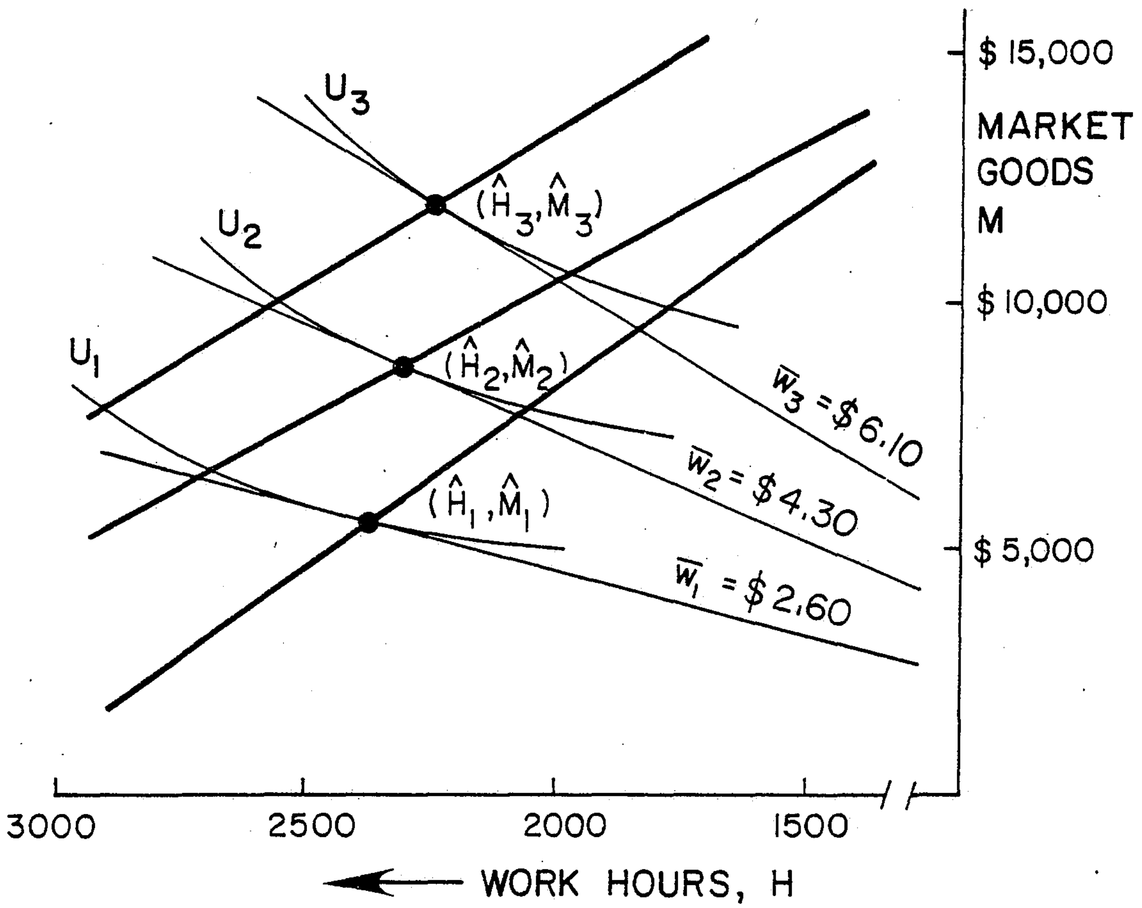
Wage Interval ($\bar{w}_j^*$) ($\bar{I}_j^*$)	Interval Means[a] $\hat{H}(\bar{w}_j, \bar{I}_j)$ $\hat{M}(\bar{w}_j, \bar{I}_j)$	Estimated Income Effect[b] $B_j = \left(\frac{\delta H}{\delta I}\right)_j$	ICC Slope[c] at $\bar{w}_j$ $D_j = \frac{B_j}{1 - B_j \bar{w}_j}$
1. under \$3.25			
($\bar{w}_1^*$ = \$2.60	$\hat{H}_1$ = 2374	−.1013	−.137
($\bar{I}_1^*$ = −\$669)	$\hat{M}_1$ = \$5,503	(.042)	(−.070−−.288)
2. \$3.25 −4.99			
($\bar{w}_2^*$ = \$4.30)	$\hat{H}_2$ = 2312	−.1009	−.178
(I_2^* = −\$1280)	$\hat{M}_2$ = \$8661	(.035)	(−.092−−.327)
3. \$5.00 −9.99			
($\bar{w}_3$ = \$6.10)	$\hat{H}_3$ = 2250	−.080	−.157
(I_3^* = −\$1806)	$\hat{M}_3$ = \$11,923	(.033)	(−.066−−.364)

a) Assumes the following standard characteristics: age 40, 4-9 years on current job, 1 week sick time, children present, mean values of other variables.

b) Estimated standard errors shown in parentheses.

c) range shown in parentheses is translated from $\hat{B} \pm \hat{\sigma}_{\hat{B}}$.

Figure 5: Estimated Preference Map



testable differences implied by the contrasting divergence properties illustrated in part II are relatively small when compared to the sampling errors of the estimates. The limited power of hypothesis tests under these circumstances places a substantial premium on the selection of a maintained hypothesis. Fortunately our consideration of the extended properties of alternative models provides a partial basis for such a choice. Specifically, we argue that the properties of the divergent Stone-Geary model on the one hand, and the convergent, linear, additive supply model on the other, render these models inappropriate as bases for maintained hypotheses.

The extended properties of the linear, additive, supply model are implausible in their implication that market consumption is an inferior good for high-wage workers. If, for instance, the estimated income effect for the lower wage intervals (-.101 hr/$) remained constant at higher wage rates it would imply a horizontal ICC and zero marginal expenditure on market goods at a $10 hourly wage rate. This is an implication that might be accepted on the basis of strong affirmative evidence but is not a hypothesis we wish to maintain unless conclusively rejected. The estimates do not, in fact, affirm strong convergence but instead suggest the absence of convergence. The estimated decline in the magnitude of the income effect is .021 and its standard error, including covariance terms, is .023. A t-statistic of 0.91 would not be grounds for rejection of an appropriate maintained hypothesis but is sufficient basis for a decision not to maintain an implausible hypothesis. The empirical evidence, reinforced by theoretical considerations, thus recommends against the linear additive supply model or similarly convergent structures for applications involving the high wage properties of the model.

Our evaluation of the properties of functions on the divergent side of the spectrum are based on considerations of conservatism in extrapolation properties. Strongly divergent forms, such as the Stone-Geary model, imply progressively stronger income effects at low wage rates and stronger substitution effects at higher utility levels resulting from high nonwage income. Simulations of responses to income maintenance programs project responses in both of these directions to the limits of the range of experimental observations and well beyond those in nonexperimental data. It is thus reasonable and prudent to seek confirmatory evidence of increasing magnitudes of responses in these ranges before employing a functional form that carries those implications. These estimates provide no such evidence. The Stone-Geary form implies income effects inversely proportional to the wage rate which, for our low-wage ranges, translates to the restriction $4.3\ B_2 = 2.6\ B_1$. This restriction is rejected at a 20% level ($t_{577} = 1.31$) which is modest by conventional standards but is sufficient to indicate that extended implications of a Stone-Geary model should not be exploited in studies of income maintenance unless further evidence controverts the present estimates.

In the intermediate range between strong convergence and strong divergence, the selection of an appropriate model may be based on criteria of simplicity and empirical tractability. The parallel model satisfies these criteria. It is particularly attractive because it can incorporate very flexible substitution properties and (potentially) may be generalized to a self-consistent specification with curvilinear income effects.

Moderately divergent homothetic models or similarly moderate convergent models would also provide approximations that are statistically consistent with our flexible estimates of divergence properties. Since homotheticity is frequently assumed in labor supply models, confidence limits on parameters, conditional on homotheticity, are of some interest. Our flexible estimates of absolute and relative income effects, together with the observed mean positions of labor supply and market consumption, may be combined to construct approximate conditional limits for the parameters (H*, M*) that define the point of origin of a homothetic model. Negative values of M* would not ordinarily be imposed as a priori constraints, and an unconstrained estimate of the origin would evidently fall in this range. If M* is constrained to zero, the upper limit of a 95% interval on H* falls at approximately 5100 hours per year. At M* = $3000 the limit is H* = 3500 hrs.[29] The evidence is thus quite strong that if a homothetic model is assumed, it is not appropriate to constrain the origin to the zero point of total time (H* = 8760 hrs) or of total waking time (H* = 5840 hrs) and even more moderate constraints should be employed with caution.[30]

The basic substitution properties of the estimated preference structure are visually evident in the lateral intervals between ICCs in Figure 5. The estimated substitution effects are presented more formally in Table 2. The point estimates at interval means are based on the within-interval estimates of income and uncompensated wage responses. The between-interval estimates are based on horizontal distances between ICCs along interpolated indifference curves at each of three utility levels. The estimates represent the average response over the discrete between-interval wage change.[31]

Table 2

Extended Substitution Properties:
Estimates of the Substitution Effect for
Different Regions of the Preference Structure

Real Income[1] (U_j)	Point Estimate[2] $\hat{S}(\bar{w}_1, \bar{I}_1)$	Between-Interval Estimate[3] $\hat{S}(\bar{w}_2-\bar{w}_1, U_j)$	Point Estimate $\hat{S}(\bar{w}_2, \bar{I}_2)$	Between-Interval Estimate $\hat{\hat{S}}(\bar{w}_3-\bar{w}_2, U_j)$	Point Estimate $\hat{S}(\bar{w}_3, \bar{I}_3)$
U_1	131.1 (113.2)	185 (85.8)	---	129 (117)	---
U_2	---	147 (75.8)	182.8 (84.8)	143.5 (78)	---
U_3	---	119 (118)	---	152 (63.9)	141.2 (69.0)

[1]The three real-income levels correspond to the interpolated indifference curves through the mean equilibrium points of each interval. See Figure 5.

[2]Based on within-interval estimates of the uncompensated wage effect and the income effect. $\hat{H}_i$ is treated as a constant in the calculation of the standard error.

[3]Calculated as the integral response to the given uncompensated wage change plus the compensating income response. Compensating income, determined as described in section II, is treated as a constant in the calculation of approximate standard errors.

The estimates do not provide conclusive evidence of a consistent relationship between the substitution effect and the marginal wage rate. Given the relatively broad confidence intervals, any of the parameterizations that imply stable or slowly varying substitution responses within the data range would be statistically consistent with our estimates. This means that we do not currently have evidence that the leisure-satiety implications of the linear additive model or the Ashenfelter-Heckman model are distortions resulting from restrictive forms. Neither does the evidence rule out moderate inverse substitution relationships (e.g., the model in Figure 2d) that avoid strict satiety implications. Even if the true substitution properties are of the latter sort, however, the estimates suggest that the shadow price of leisure falls at least close to zero if labor supply drops significantly below full-time work. This implication may indeed be reasonable, in light of the importance of the work ethic in American culture. The question is certainly open to additional evidence from samples of workers in equilibrium at low marginal wage rates.

All of the above inferences about the extended properties of the income-leisure preference structure are drawn from estimates using the net overtime wage specification. In order to explore the sensitivity of the results to budget specification, comparative estimates were obtained for two alternative models, gross and net of taxes, based on the straight time wage with the overtime premium ignored. The overall results were not substantively affected by the tax adjustments and the first order properties of the estimates were insensitive to the presence or absence of the overtime premium. Dropping overtime premium from the specification did, however, affect the divergence properties of the estimates. In estimates from the straight time specifications, the upper two ICCs were essentially parallel but the lower ICC diverged significantly.[32] If it were deemed appropraite to ignore reported overtime premiums, these point estimates would suggest a mixed preference structure, parallel in the range above the mean wage rate and homothetic with elasticity somewhat greater than unity in the lower range. However, given that more than 90% of the workers in our select sample reported overtime premiums, our estimates from the overtime specification are the ones in which we place the most confidence.

The fact that our estimates for a homogenous select sample are demonstrably sensitive to incomplete specification of budget variables suggests that the question of appropriate budget specifications for a broader population of workers warrants direct empirical attention. In many studies, the only available wage variable has been the average wage rate. For workers who in fact receive overtime premiums and are free to optimize against them, a specification based on the average wage would be a misspecification similar to our straight-time model. For constrained workers or those on fixed salaries, much less is known about marginal earnings opportunities. Fragmentary evidence suggests that the effective marginal wage is less than the average.[33] The consequences of ignoring such differentials may be quite different.

The problems of budget specification impart a degree of uncertainty to any comparison of our results with those from earlier studies. We do note that there is some qualitative similarity between our straight-time estimates for a select sample and Rea's estimates using the average wage rate for a broad sample. Both sets of estimates indicate increased divergence in the lower wage range, though the precise ranges differ between the two. We would expect Rea's estimates for the broad population to be different, given a more complete budget specification. Whether such estimates would be more similar to our select-sample estimates from the overtime specification is an open question.

5. SUMMARY AND CONCLUSION

The purpose of this study has been to investigate the extended properties of income-leisure preference structures and to evaluate simple models of individual labor supply with respect to those properties. The methodology has centered on preference maps composed of income consumption curves. These maps are both the basis for our flexible estimation form and a device for comparing the properties of our estimates with those implied by a variety of parametric models previously used in the analysis of individual labor supply.

Estimates were obtained for a subpopulation of prime-age males who were free to reveal their preferences through unconstrained choices on well-defined opportunity sets characterized by premiums for overtime work and by increasing marginal income tax rates.

Our interpretation of the estimation results focuses on the divergence and substitution properties of the preference structure. In the divergence dimension, the estimates suggest a structure whose extended properties may be closely approximated by a parallel preference model similar to those estimated by Ashenfelter and Heckman (1973, 1974). A number of familiar models are found to be inappropriate if applied over an extended range, though they may be satisfactory as local approximations. The Stone-Geary model implies strong differentials in income effect at different wage rates which were not substantiated by our estimates. This form is also non-conservative in its extrapolation properties in ranges important to the analysis of responses to income maintenance programs. The linear additive supply model provides a good and empirically tractable approximation over the lower and middle ranges of wage rates but implies that market consumption becomes an inferior good at high wage rates. The model is thus deemed inappropriate for applications in the high wage range, a judgment supported by suggestive, though not conclusive, statistical evidence.

Models with more moderate divergence than the Stone-Geary model or more moderate convergence than the linear additive model would be acceptable on the basis of the current evidence. For homothetic (divergent) models, it was found to be inappropriate to constrain the point of origin in the hours dimension to values more than the annual equivalent of 14 hours per day.

The substitution properties of a number of previously estimated models imply leisure satiation if labor supply falls significantly below full-time work. Our flexible estimates of substitution properties are not inconsistent with this implication. Our estimates would also be consistent with models that avoid strict satiation implications but the estimated marginal rate of substitution is at least close to zero over a large range of labor supply.

The low-wage properties of the preference structure invite further investigation, as does the question of possible curvature of income consumption curves. Both questions require a significantly richer body of data than that available for this study.

Our estimates of extended properties of the preference structure were found to be sensitive to misspecification of the opportunity set. If the overtime premium was ignored, the estimates showed significant divergence of ICCs in the low-wage range. The appropriate budget specification and the consequences of misspecification are reasonably simple in our homogeneous select sample. These questions are less simple for broader populations that include workers with constrained work hours and those with fixed annual or monthly salaries. The question of whether our estimates for a convenient subpopulation generalize to broader populations remains open to further research.

APPENDIX

Parallel Preference Structures

Parallel preference structures are characterized by indifference surfaces that have identical shapes at all utility levels. The Hicksian demand functions (or compensated labor supply functions) for a parallel structure thus have a particularly simple form with price responses separable from responses to real income. This form is shown in A1 for our two dimensional labor supply model with the price of market goods explicitly denoted as numeraire.

(A1a) $H = \Psi^H(U, W/P) = f^H(U) + \Psi^H(W/P)$

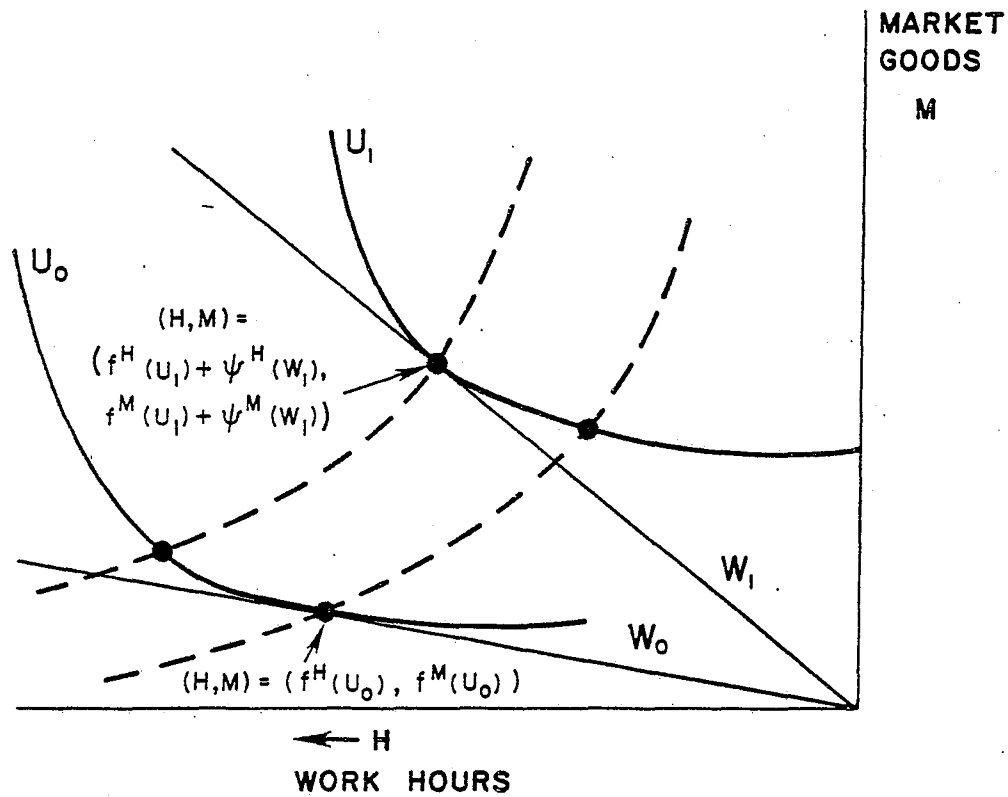
(A1b) $M = \Psi^M(U, W/P) = f^M(U) + \Psi^M(W/P)$

The functions Ψ^H and Ψ^M together provide a parametric representation of an indifference curve with W/P, the marginal rate of substitution as parameter. The functions f^H and f^M define the position of a reference point on any given indifference curve corresponding to the marginal rate of substitution for which $\Psi^H = \Psi^M = 0$. As utility varies the f^H, f^M functions trace out a reference income consumption curve, ICC_o, that determines the positions of all indifference curves for the structure. An illustrative two-dimensional structure is shown in Figure A1.

The general theoretical restrictions on the Hicksian functions (A1) may be expressed in terms of the expenditure or cost function (A2) which must be a concave, positively linear homogeneous function in prices and must be monotonically increasing in U.

(A2) $C(U, W/P) = W\Psi^H(U, W/P) + P\,\Psi^M(U, W/P)$

Figure A1: A Parallel Structure



Alternatively, the price derivatives of the Hicksian functions (the pure substitution effects) must obey the usual symmetry, definiteness, and aggregation restrictions (Goldberger, 1967) and the price-weighted sum of utility derivatives must be positive. In the two-dimensional case these restrictions mean that only one of the functions Ψ^H, Ψ^M is freely parameterized.

The utility index is arbitrary, up to a positive monotonic transformation. If the numeraire good, M, is normal, $f^M(U)$ is such a transformation and we may conveniently redefine the utility index as $U^* = f^M(U)$. The index U^* orders successive indifference curves by the level of the numeraire good at which they intersect ICC_o. This index is directly calculable if prices and the parameters of the Hicksian functions are known or have been estimated. This calculation follows from the simple inversion of (A1b) shown in (A3).

(A3) $$U^* = u^*(M, W/P) = M - \Psi^M(W/P)$$

Equation (A3) is the basis of our generalized interpretation of the Ashenfelter-Heckman (A-H) estimation form. Given linear ICCs, we may write the compensated labor supply relationship as shown in (A4).

(A4) $$H = \Psi^* (U^*, w) = H_o + D\ U^* + \Psi^H(w)$$

where $w = W/P$ and D is the slope of an ICC. Substituting the expression (A3) we obtain the generalized form of the A-H model

(A5) $$H = \xi(M, w) = H_o + DM - D\ \Psi^M(w) + \Psi^H(w)$$

In the A-H model, the expression $\Psi^H(w) - D\ \Psi^M(w)$ is parameterized as $S^\dagger w$ which implies somewhat complicated forms of Ψ^H and Ψ^M individually.

Like most direct parameterizations of a supply relationship this model is strictly self-consistent over a finite range for a one-worker model but only at a point for models with more dimensions.

The form (A5), solved jointly with the budget constraint (M = wH + I), converts to the conventional supply function, (A 6).[1]

$$\text{(A6)} \qquad H = h(w, I) = \frac{H_o + DI - D\,\Psi^M(w) + \Psi^H(w)}{1 - Dw}$$

The conventional income and substitution effects for the A-H model follow from this form. For the inversely proportional substitution model shown in Figure 2d, the price response functions are $\Psi^H = \gamma \ln w$ and $\Psi^M = \gamma w$. A model with a constant substitution effect, S, would have $\Psi^H = Sw$ and $\Psi^M = \frac{1}{2} Sw^2$. Other parameterizations and technical attributes of the model are discussed in Dickinson (1979).

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[1]Alternatively, the supply function for a linear parallel model may be obtained by inverting the expenditure function (A2) to obtain an indirect utility function and using Roy's identity. In general, explicit conventional supply functions cannot be obtained for nonlinear parallel models.

NOTES

[1]Simple one-worker models are used, for ease of exposition and consistency with the empirical work. For the most part, the methodology may be extended to models with more dimensions treated as a set of two-dimensional projections. Properties unique to the one-worker model will be noted.

[2]In the simple two-worker model the substitution effect is restricted to be positive for work hours or negative for leisure. In models with more dimensions, there are additional symmetry and definiteness restrictions on the Slutsky matrix of substitution effects. See, for instance, Ashenfelter and Heckman (1974).

[3]Note that negative effective levels of I may result from premium wage rates for overtime work. Our empirical specification accounts for such premiums (see section IV).

[4]It is easily confirmed that the indifference curves implied by a constant substitution effect are parabolic and that the intersection of tangents to a parabola bisects the horizontal distance between the points of tangency.

[5]The slope of an ICC is the ratio of income effects. $\partial H / \partial M = \dfrac{\partial H / \partial I}{\partial M / \partial I}$, rather than an income effect per se.

[6]Extended substitution properties have been estimated by Heckman (1974), Hall (1973) and recently by Dunn (1978) using a rather different methodology. A number of studies have allowed for curvature of income responses--see for instance Hall (1973) and Kalachek and Raines (1970). The specification of

Cohen, Rea, and Lerman (1970) and Rea (1971) is the only one known to this author that maintains flexibility in all dimensions (see discussion at end of section III.)

[7]Differentiation of the Slutsky equation, $S = \partial H/\partial w - H \cdot B$, with respect to income yields $\partial S/\partial I = \partial/\partial I(\partial H/\partial w) - H \cdot \partial B/\partial I - B^2 = \partial B/\partial w\big|_u - B^2$ since $\partial/\partial I(\partial H/\partial w) = \partial B/\partial w$ by Young's theorem and the definition of B, and $\partial B/\partial w\big|_u = \partial B/\partial w - H\,\partial B/\partial I$. This expression also has simple analogs for cross-substitution effects in models with more dimensions. Note that equation 6 implies that the income effect and the substitution effect cannot both be constant over a range of income and wage rates unless the income effect is zero.

[8]Recent work in demand theory by Pollak and Wales (1978) and by Howe (1974) has introduced consistent parameterizations with quadratic income effects. The extended properties of this model appear to be rather complex in the substitution dimension, however. Dickinson (1979) has introduced a curvilinear parameterization for a class of models, briefly discussed in the appendix, whose global properties remain relatively simple because the ICCs are constrained to be parallel. Such a constraint appears to be a reasonable one for some labor supply models, so these models may hold promise for future empirical work on data with richer income variation. Some models based on flexible parameterizations of the indirect utility function have curvilinear ICCs but the slope and curvature are not independently flexible. An example is discussed in section III.

[9]The Gorman Polar Form is discussed in some detail by Blackorby, Boyce and Russell (1978). The GPF may be parameterized as an expenditure function $C(p, u) = h(u)\pi(p) + \Lambda(p)$, where $h(u)$ is a monotonic utility index and $\pi(p)$ and $\Lambda(p)$ are any concave positive linearly homogeneous functions. In terms

of our characterization of functions $\Lambda(p)$ determines the substitution properties at a base level of utility and $\pi(p)$ determines the divergence properties. Blackorby et al. discuss alternative forms of Λ and π and Dickinson (1979) discusses the extended properties of several parameterizations of $\Lambda(p)$.

[10]In one-worker models, supply-function parameterizations usually imply self-consistent preference structures over some finite range. In models with more dimensions, such parameterizations usually satisfy the symmetry conditions exactly at a single point only and approximately over some range. This approximation needs to be considered along with income, substitution, and divergence properties in assessing the extended characteristics of such models.

[11]See, for instance, Dickinson (1974), Greenberg and Kosters (1973), Kosters (1966), Rosen and Welch (1971). The recent application by Keeley et al, (1978) is basically a linear additive model but the parameters differ, depending on the initial equilibrium position for an individual.

[12]For $H(w, I) = H_o + Aw + BI$, $H^* = A/B$, $M^* = -1/B(H_o - A/B)$. H^* is in the positive range of work hours if both A and B are negative, but outside the observable range if the simple supply curve is rising and leisure is a normal good.

[13]Such models are estimated in Hall (1973), Kalachek and Raines (1970), and Rosen and Welch (1971). These and most other models referred to in this section are discussed in more detail in Dickinson (1975a, 1976). The only other simple convergent model known to the author is based on a quadratic

direct utility function. It has concentric elliptical indifference curves and a cumbersome estimation form. Wald (1940), in a neglected article pointed out by James Heckman, suggested a method for approximating a general utility function by quadratic segments.

[14]See Stone (1954). Labor supply applications are found in Christensen (1971), Dickinson (1970), Horner (1973), and Leuthold (1968).

[15]λ or μ may be negative, in which case they are interpreted simply as parameters that improve the fit of the model within the relevant range.

[16]A translated CES model has been estimated by Wales (1973). Relative income effects in that model are given by $\partial H / \partial I = -w^{-\sigma} / (k^{\sigma} + w^{1-\sigma})$ where k is a parameter and σ is the elasticity of substitution. Parameterizations of variable elasticity models are discussed in Barzel and McDonald (1973), or in Dickinson (1975a, 1976).

[17]Note that the function $\xi(M, w)$ is not a conventional supply function because the endogenous variable M appears as an argument in place of the exogenous variable I. The model shown is a one-worker version of the Ashenfelter and Heckman two-worker model (1974). Their one-worker model (1973) combines aspects of the parallel model and a linear additive supply model because M is averaged with an exogenous measure of income.

[18]See the appendix for derivations.

[19]In the additive supply model, for instance, the addition of a logarithmic wage term with a positive coefficient would eliminate the satiety implications. A homothetic model with σ approaching zero in the limit of low wage rates would incorporate satiety. Within the framework of the GPF

it is also possible to specify divergent models that are not homothetic to a single point and thus have further flexibility of substitution properties.

[20]See, for instance, Christensen et al. (1975), Christensen and Manser (1977) and Diewert (1974). The form shown (not the parameter values) is a one-worker version for a model employed by Wales and Woodland (1976).

[21]Wales (1977) shows that a homothetic version of the indirect translog model is not necessarily well behaved over a wide range. Caves and Christensen (1978) show that the model has desirable global properties if the elasticity of substitution is close to unity.

[22]Manser (1975) considers translog models with subsistence parameters.

[23]The study is conducted by the Survey Research Center of the Institue for Social Research at the University of Michigan.

[24]The analysis applies to white married males, aged 25-60 in 1972, employed in each year 1969-1972, not farmers or proprietors, who did not moonlight and whose wives did not work. Workers were selected if they directly reported a nonzero marginal wage rate and if they reported freedom to increase and decrease work hours or freedom to vary in one direction and no dissatisfaction with the constraint in the other direction. Workers who were fully constrained were excluded from the analysis, whether or not they expressed dissatisfaction. The workers in the roughly equal groups that had one-sided freedom of choice were retained for reasons of sample size. The variables employed in the analysis for each individual are averages over those years that he satisfied the selection criteria during the period 1969-1972. Heteroskedasticity corrections were made for differential reductions in year-to-year variance.

An extensive discussion of the rationale for the sampling criteria is presented in Dickinson (1975b, 1976). The criteria are similar to those used by Wales and Woodland (1976), who used one year's data from the same study, except that they excluded neither those who were fully constrained but satisfied nor the large number of workers with zero or indeterminate marginal wage rates.

[25]The sample includes about 20% of those otherwise eligible. The cumulative, equilibrium-related reasons for exclusion were: 46% had zero or indeterminant marginal wage rates (these were primarily salaried workers, 85% of whom were satisfied with work hours), 17% wanted more work, 3% wanted less work, 9% were fully constrained but satisfied and 5%, not otherwise excluded, held second jobs. Factors related to the presence or absence of marginal wages are discussed in Dickinson (1974).

[26]Such a model would need to integrate the simple equilibria considered here with corner solutions and discrete choices on discontinuous, endogenous opportunity sets. It would require careful analysis of both differential preferences and differential opportunities. One aspect of the problem, the moonlighting decision, has been treated with partial success by Shisko and Rostker (1976), within the context of a relatively simple analytic and stochastic parameterization.

There is a potential for truncation bias in our estimates from the select equilibrium sample which could be tested for in a satisfactory general model. We acknowledge the possibility of such bias but do not believe it to be serious because the exclusion criteria are not systematically related to the income and wage variables. See Heckman (1977) for a discussion of truncation problems.

[27]Similar methods of budget specification have been employed in other studies, e.g., Hall (1973), Keeley et al. (1978), Rosen (1976), and Wales and Woodland (1976). (See Burtless and Hausman, 1978, for a more elaborate specification.) Ours differs in taking explicit account of the overtime premium and, less importantly, in the method of tax imputation. The progressive marginal tax schedule was approximated by two linear segments and total tax liability by corresponding quadratics. Tax imputations were then based on predicted taxable income given the worker's gross regular wage rate, nonwage income, personal characteristics, personal exemptions, and the greater of 15% or the minimum standard deduction.

[28]The control variables in the analysis are age, age squared, tenure on present job, whether children, an index of achievement motivation, and an analog of the Greenberg-Kosters (1973) preference variable. Education was initially included but was insignificant. Unemployment time was included as a control variable following the specification of Rea (1971, 1974). Illness time and commuting time were specified analogously. Inclusion of the various control variables improved the precision of the estimates but did not substantively affect the estimates of economic parameters. The actual regression analysis also included nonwhite workers, workers whose wives were employed, and moonlighters. These observations were isolated from the reported estimates for the select sample of white males by dummy variables and interactions on the wage and income variables. Estimates of the economic parameters for these groups, as well as the control variable estimates are presented in Dickinson (1976).

[29]In our approximation, F-statistics were calculated for the joint values of the income effects that yield ICCs from a given origin through the mean points of each wage interval. The estimation errors in the mean points were assumed to be negligible relative to the errors in the income coefficients. The approximate limits were then determined by trial and error.

[30]See, for instance, the values assumed in Horner (1973) or Morgan and Smith (1969). The origin point imposed by Wales (1973) is also outside our estimated confidence region, but his sample of self-employed businessmen differs from ours in numerous respects.

[31]The between-interval distance along a given curve was calculated as the integral of hours changes due to the combination of uncompensated wage changes along the splined wage curve and the compensating income variations. The expression is nonlinear because the amount of compensating income is itself a function of the estimates. The standard errors were approximated by treating compensating income as a constant and should thus be taken with a grain of salt. The large standard errors for responses at utility levels well removed from observed mean portions (e.g., the $w_1 - w_2$ response at U_3) provide a useful reminder that those estimates apply to a range where the data are thin at best.

[32]Details of the comparative estimates are given in Dickinson (1976).

[33]See, for instance the Shisko and Rostker (1976) estimates for accepted moonlighting opportunities and similar evidence in Dickinson (1975b, 1976). A qualitative analysis of the earnings tradeoffs between salaried, constrained hourly, and unconstrained hourly employment in Dickinson (1974) also suggests unfavorable long-run marginal opportunities.

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