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ABSTRACT

This paper examines the hypothesis that short run changes in the U.S. size distribution of personal income have been systematically related to aggregate levels of output, personal income, and employment during the postwar period.

Part one proposes a procedure for describing the size distribution of income. The observed distribution is approximated by a displaced lognormal distribution which corresponds formally to the three parameter lognormal distribution. Quantile methods of curve fitting are used.

Part two examines postwar movements in measures of the size distribution. Separate patterns are established for each of three family groups: families with a male head and a wife in the labor force, those with a male head and a wife not in the labor force, and those with a female head.

In part three these patterns are related to changes in aggregate personal income components and in aggregate leveles of employment and labor force participation. For each group a three equation system is used to estimate mean income and two relative quantile measures. Distributional responses of the three groups are compared. Some tentative conclusions are drawn in part four.

THE SIZE DISTRIBUTION OF PERSONAL INCOME DURING THE BUSINESS CYCLE

As econometric models have grown in size and complexity in an attempt to mirror the structure of the American economy, a determination of the size distribution of personal income has never been included. The Brookings model, for instance, limits its consideration of the income distribution to a determination of factor shares.[1] Issues relating directly to how families or income units are distributed by size of income were never raised.

Two reasons can be cited for this omission. First, data relating to the distribution of income lag far behind other economic data in accuracy and detail, and are not consistent with the National Income Account data which prevade most econometric models. Second, it is generally presumed that the relative shape of the income distribution has been roughly constant since World War Two.[2] A recent study by T.P. Schultz failed to establish a significant cyclical pattern in the concentration of personal income.[3] Yet whenever the issue of price stability is discussed, the "adverse" effects of inflation on the distribution of income are persistently cited.[4]

This paper examines the hypothesis that short run changes in the U. S. size distribution of personal income have been systematically related to aggregate levels of output, personal income, and employment during the post-war period. Elsewhere the relationships presented in this paper are incorporated into an econometric model of the United States.[5]

Part one proposes a procedure for describing the size distribution of income, while part two examines postwar movements in measures of the size distribution. Separate patterns are established for each of three family groups: families with a male head and a wife in the labor force, those with a male head and a wife not in the labor force, and those with a female head. In part three these patterns are related to changes in aggregate personal income components and in aggregate levels of employment and labor force participation. Some tentative conclusions are drawn in part four.

I. Measuring Changes in the Size Distribution of Income

A convenient mechanism must be found to describe the size distribution. Such a mechanism may reflect a particular feature of the distribution, or it may approximate the entire distribution by a functional form.

Traditionally the important feature of the size distribution of personal income has been the degree of income inequality. Various measures of income inequality have been developed and discussed in the literature.[6] More recently, attention has been directed specifically to the lower tail of the income distribution; measures of income inequality have been replaced by measures of the proportion of the cumulative distribution lying below some absolute income level or conforming to some definition of poverty.[7]

If the distribution is described by a functional form, measures may in general be derived to reflect a variety of specific features. Available functional approximation include Pareto-Levy distributions, Pearson curves, and normal transformations.[8] Two criteria appear to be important in making a selection. First, the chosen function should describe the available information about the distribution of income with a reasonable degree of accuracy. Second, it should be analytically tractable.

Given the positive skewness of the observed distributional data and the analytical convenience of utilizing the normal distribution, one alternative is to find a transformation of the data which is approximately normal. The lognormal distribution, a common choice in this regard,[9] is inadequate in two respects. First, it overcorrects for the positive skewness of the data; after logarithmic transformation the observed distribution exhibits negative skewness. Second, it forces a symmetric treatment of movements in the two tails of the distribution. This property severely restricts the ability of the function to characterize cyclical movements in the observed income distribution.[10]

The transformation utilized in this paper is the displaced lognormal distribution, which corresponds formally to the three parameter lognormal distribution.[11] Given the empirical assertion that f (Y) is positively skewed and f(ln Y) is negatively skewed, there clearly exists some value of $C > 0$ such that the transformation $\ln(Y+C)$ has zero skewness, with the range of Y being $-C < Y < \infty$.[12]

With the assumption that $\ln(Y+C)$ is normally distributed, it is possible to find a value of C such that the distribution possesses the desired degree of skewness. Observed movements in the skewness of the actual distribution over time may be reflected by changing the value of C, independently of changes in the variance of the distribution. If $\ln(Y+C)$ is normally distributed, the variable $(Y+C)$ has all the properties of a two parameter lognormal distribution, although it is not directly observable.

Given the availability of data grouped into open-ended and unequally sized cells,[13] quantile methods of curve fitting provide a simple and reasonably efficient procedure for estimating the displaced lognormal distribution empirically.[14] Let D be the observed median of the distribution, Y10 the income level below which ten percent[15] of the group population lie, and Y90 a symmetrically defined income level for the uppermost decile. With the relative quantile measures defined as $H = (Y10/D)$ and $J = (Y90/D)$, the solutions for the mean (m), standard deviation (S) and constant of displacement (C) for the transformed distribution are:

(1) $M = \ln(D+C)$,

(2) $S = \frac{1.}{G} \ln \left[\frac{(D+C)}{(HD+C)}\right] = \frac{1.}{G} \ln \left[\frac{(JD+C)}{(D+C)}\right]$, and

(3) $C = D.\left[\frac{(HJ-1)}{(2-h-J)}\right]$,

where G is the number of standard normal deviations from the mean appropriate for the two decile income cut-offs.[16] The mean of the nontransformed distribution is:

(4) $B = \left[\exp(M+.5S^2)\right] - C.$

The relative quantile measures H and J indicate the relative positions of the two tails of the income distribution: their interpretation does not depend on the appropriateness of the displaced lognormal distribution. If the displaced lognormal distribution is assumed, observed values of H, J, and either the mean or the median are sufficient to charactize changes in the features of the entire distribution.[17] For instance, the proportion of the population having incomes below some arbitrary poverty level YPOV can be derived from the transformation

(5) $GPOV = \frac{1.}{G} \ln\left[\frac{(YPOV+C)}{(D+C)}\right]$,

where GPOV, the number of standard deviations between the transformed mean and the poverty cut-off, can be converted into an incidence of poverty by using tables of the normal distribution.

II. Postwar Changes in the U. S. Income Distribution

Table one reports constructed values of the H and J distributional variables for three major family groups during the period 1949-1965. Since families differ substantially in the source and variability of their income, a sharper view of distributional patterns can be obtained by examining different subgroups in the population than by observing the aggregate population.

The groups considered here are families with a male head and a wife in the paid labor force; families with a male head and a wife not in the labor force; and families with a female head. Elsewhere[18] similar statistics have been reported for families with a male head, "other" marital status; unrelated individuals who were earners in the previous year; and unrelated individuals who were not earners. The three groups included in this paper receive about 88 percent of all personal income, and almost 98 percent of all personal income going to families.[19] The H and J values were constructed by interpolation from Current Population Survey data.[20]

The labor force orientation of families with a male head has changed substantially during the postwar period. The number of such families with the wife in the labor force grew from 7.3 million in 1949 to 14.2 million in 1965, while the number of families with the wife not in the labor force has remained roughly constant at 28 million.

The increase in the proportion of families with a wife in the labor force has an important impact on the aggregate size distribution for two reasons. First, since the two group distributions are shaped differently, a change in their relative proportions will appear as a change in the aggregate distribution even if there is no change in the shapes of the individual distributions.[21] Second, the two groups appear to show different responses to cyclical movements in the economy.

Families at the lower decile cut-off for the wife-in-labor-force (MWL) group have incomes in the vicinity of forty percent (36.3%-44.4%) of the group median, while the corresponding level for the wife-not-in-labor-force (MWN) group is about thirty percent (28.2%-33.0%) of the respective median. The lower decile statistic for the MWL group shows a fairly systematic cyclical

TABLE I

H AND J DISTRIBUTIONAL MEASURES FOR THREE U.S. FAMILY GROUPS

Prefixes: H__ = income level at bottom decile cut-off divided by median income for families with a male head, income level at 15.87% quantile cut-off for families with a female head.

J__ = income level at top decile cut-off divided by median income for families with a male head, income level at 84.13% quantile cut-off for families with a female head.

Suffixes: _MWL = families with male head, wife in paid labor force

_MWN = families with male head, wife not in labor force

_FEM = families with female head

	(1) HMWL	(2) JMWL	(3) HMWN	(4) JMWN	(5) HFEM	(6) JFEM
MEAN	.4090	1.7401	.3096	2.0027	.3037	2.2051
1965	.4443	1.7448	.3224	2.0391	.3746	2.1492
1964	.4410	1.7247	.3247	2.0134	.3681	2.1923
1963	.4202	1.7518	.3269	2.0336	.3223	2.2192
1962	.4251	1.7636	.3296	2.0404	.3223	2.1910
1961	.4098	1.7634	.3097	2.0815	.3167	2.1988
1960	.4052	1.7545	.3092	1.9918	.3342	2.2106
1959	.4158	1.7181	.3178	1.9882	.3267	2.2685
1958	.4115	1.7494	.3277	1.9663	.3141	2.2050
1957	.4118	1.7131	.3097	1.9030	.2961	2.1408
1956	.4172	1.6941	.3072	1.9720	.2709	2.1402
1955	.4143	1.6791	.2950	1.9577	.2970	2.2246
1954	.4097	1.7091	.2822	2.0195	.2659	2.3021
1953	.4099	1.6967	.2856	1.9630	.2701	2.3241
1952	.3631	1.7429	.3279	1.9591	.2725	2.1083
1951	.4157	1.7605	.3261	1.9351	.2347	2.1739
1950	.3726	1.7979	.2836	2.0851	.2753	2.2340
1949	.3664	1.8187	.2790	2.0978	.3017	2.2032

Source: Metcalf, op. cit., Tables IV-1, IV-2, IV-4.

pattern with peak values observed in 1951, 1953, 1956, and 1959, and an upward trend since a 1960 trough. The corresponding statistic for the MWN group follows a different pattern; it rises substantially in 1951, falls sharply in 1953, builds to a peak in 1958 and then reaches a trough in 1960.

The upper tail parameter J has a range of 1.68-1.82 for the MWL group and 1.90-2.10 for the MWN group. The MWN group has a lower median than the MWL group, but it has an upper distributional tail which is considerably more extended. This pattern suggests that high income families in the MWL group deserve their position largely from the presence of multiple labor force participants, while the wives in families having large concentrations of non-wage income or a head with a relatively high salary tend to stay out of the labor force. The value of J for both groups shows a countercyclical tendency, rising in recession years. This implies that family incomes are more sharply affected by the business cycle at the median than above the median.

For families with a female head, the value of H fell until 1951 before assuming an upward trend for the remaining years. The value of J showed no discernable trend.

III. Toward an Econometric Model of the Income Distribution

An attempt is made here to characterize a reduced form relationship between the size distribution of personal income and the business cycle. Implicitly this will be done by relating changes in each group distribution to fluctuations in alternative sources of personal income received by that group. Levels of alternative income sources are endogenously determined in the econometric model cited earlier.

For each of the groups discussed above a three equation system will be estimated to determine the levels of B (the group mean), H, and J.[22] Together with parallel equation systems for the three groups excluded from this paper, transformations parallel to equations (2) - (5) in part one for each group, and two participation functions to allocate households into the appropriate groups, the equations presented in this section constitute the distribution sector of the econometric model. Since it was noted in part one that the H and J measures are directly interpretable without reference to an underlying distributional form, the scope of this section will be limited to a discussion of the estimated equations.

Group Mean Income Equations

The construction of a mean income specification for group i begins with an identity such as

(6) B_i =[(Annual Wage Rate]$_i$ x (Employment Rate)$_i$ x (Labor Force Participation Rate)$_i$] + [(Unemployment Benefit Rate)$_i$ x (Unemployment Rate)$_i$ x (Labor Force Participation Rate)$_i$] + ("Other" Transfer Benefits)$_i$ + ("Other" Personal Income)$_i$,

where the subscripted components are mean levels for the group in question.[23] Since detailed data about the sources of group assumed relationships between the above components and corresponding aggregate data reported in the National Income Accounts or by the Bureau of Labor Statistics.

The equation to be estimated is then a relationship between an estimated group mean [24] and aggregate personal income and employment data. Coefficients on individual components are interpretable as implicit weights determining the sources of the group's income; the weights need not add to one because the group may have a different mean income than the population at large, and because the distributional data are not consistent with the data sources used elsewhere in the model.

Wives in the MWL group are in the labor force by definition; it is further assumed that the proportion of group family heads in the labor force is also constant, and that other secondary participants are not of quantitative significance. Because the structure of the MWL group may change as it becomes a larger proportion of total families, the relationship between the group and the aggregate wage rates is permitted to vary as a function of time and the proportion of wives in the labor force.[25] All other components in equation (6) are assumed to be proportional to the comparable global variables. When these assumed relationships are substituted into equation (6) the specification for BMWL becomes

$$(7)\quad BMWL = (a_1 + a_2 + a_3\ PAR1).W.E + a_4\ UBR.U + a_5 Y_{TR} + a_6\ (Y_{ID} + Y_o) + U_1\ ,$$

the variables being defined in Table Two. Since the MWL group presumably depends primarily upon wage income, it is not surprising to find that the coefficients associated with non-wage income sources turn out to be insignificant and often to have the wrong sign; they are therefore deleted from the equation.

TABLE II

VARIABLES INCLUDED IN DISTRIBUTION EQUATIONS

(Mean values in parenthesis. Per capita figures deflated by the non-institutional population aged 14 or over; dollar figures in real terms, deflated by the GNP implicit price deflator).

1. HMWL, HMWN, HFEM, JMWL, JMWL, JFEM: See Table One
2. BMWL (6638), BMWN (5431), BFEM (3550): mean incomes for the three family groups. Derived from CPS data observations using equation (4), then deflated to 1958 dollars.
3. W(4369): Private wage and salary disbursements per private wage and salary employee (including agricultural wage and salary employees), 1958 dollars per year.
4. UBR (639): unemployment benefits per unemployed person, 1958 dollars per year.
5. U (.051): unemployment rate.
6. E (.949) = 1 - U: employment rate.
7. E^*(.531): employment as fraction of population 144.
8. Y (2919): personal income per capita, 1958 dollars, excluding rental income of persons.
9. Y_w (2064): wage disbursements per capita, 1958 dollars.
10. Y_{TR} (169): government transfer payments per capita, 1958 dollars, excluding unemployment benefits.
11. Y_{UB} (18): unemployment benefits per capita, 1958 dollars.
12. Y_{ID} (270): interest and dividend benefits per capita 1958 dollars.
13. Y_o (398) = $Y - Y_W - Y_{TR} - Y_{UB} - Y_{ID}$: other personal income per capita, 1958 dollars.

(TABLE II CONT'D)

14. PAR1 (.277): families with male head, wife in labor force as fraction of total families with male head, wife present. (CPS data)

15. PAR2 (.3955): labor force participation rate, females other marital status, March of year following income period. BLS data.

16. PROF (.173): corporate profits and capital consumption allowances as a share of gross private product.

17. T: time trend, 1947 = 1, 1948 = 2, etc.

18. CWEN (1.0366) = $(W_t/W_{t-1}) \cdot (PGNP_t/PGNP_{t-1})$: rate of change of annual wage rate, current dollars.

(Source: annual income data from National Income Accounts, Office of Business Economics; employment data from Bureau of Labor Statistics).

Table three provides estimates of the mean income equations for each of the group distributions. A two stage least squares estimation procedure was used, with each nonlinear cluster of variables being treated as a single variable for estimation purposes.[26]

In the MWL equation (III-1) the wage coefficient has a mean value of about 1.6, a sensible result given the typical presence of two labor force participants in such a family. Abstracting from trend (which is positive) the annual wage rage is lower than the proportion of families in the MWL group is high. The coefficient on unemployment benefits is of no statistical value.

Mean income for the MWN group (III-2) depends quite strongly on transfer income as well as wage income. In fact the transfer coefficient of 6.2 is surprisingly large. Other sources of income do not have a significant effect on the estimate of the mean. Whether this is so because interest and dividend income is more likely to be underreported in the C.P.S. data than other types of income or because it is too collinear with other income sources is unclear.

Estimating the mean for families with a female head is considerably more difficult, both because of the smaller sample size underlying the data source and because of substantial fluctuation in the group mean during the early postwar and Korean War periods. With the simplifying restriction that wage income goes entirely to group members with a participant in the labor force and that transfer income goes exclusively to the remaining members, equation III-3 is obtained.

H and J Equation Estimates

Obtaining empirical estimates for the H and J variables involves a more complex procedure. While a group mean is a linear sum of mean income components,

TABLE III

Mean income equations, 1949-1965.

(Variables defined in Table II. Equations estimated by two stage least squares, by treating each non-linear group of variables as a single variable for estimation features. See Metcalf, op. cit., Ch. IV-V. Coefficient standard errors appear in parentheses).

(III-1) $BMWL = (\underset{(.139)}{1.855} + \underset{(.0054)}{.0313T} - \underset{(.7051)}{2.1705}\ PAR1) \times W \times E + \underset{(1.374)}{1.343}\ UBR \times U$

$R^2 = .997$ $F(3,13) = 1296$

$D-W = 1.86$ $s.e. = 56.9$

(III-2) $BMWN = \underset{(231.5)}{-712.6} + \underset{(.078)}{1.222} W \times E + \underset{(.639)}{6.225} (Y_{TR} + Y_{UB})$

$R^2 = .994$ $F(2,14) = 1101$

$D-W = 2.25$ $s.e. = 58.9$

(III-3) $BFEM = \underset{(565.6)}{1533.1} + \underset{(.4731)}{.8787}\ W \times E \times PAR2 + \underset{(2.928)}{6.007}\ Y_{TR} \times (1 - PAR2)$

$R^2 = .742$ $F(2,14) = 20.1$

$D-W = 2.25$ $s.e. = 170.5$

quantile distributional variables are non-linearly related to the group mean. Instead, a quantile income total equals the sum of component incomes at that quantile, each of which is at best non-linearly related to the mean level of that income source. Since it is not possible to aggregate displaced log-normal functions, income components will not be distributed according to the same functional form as the distribution of total group income. As a result, some approximation must be utilized.

When group decile income $Y10_i$ is expressed as a sum of components in a manner parallel to the construction of equation (6) for the mean, such as

(8) $Y10_i$ = (decile wage income)$_i$ + (decile transfer income)$_i$ + (decile "other" personal income)$_i$,

a critical problem is immediately encountered: no order statistics of the required sort are available. The decile income components must be approximated by functions of mean levels of the appropriate aggregate variables. One approximation of equation (8) which can be estimated from available data is:

(9) $Y10_i \sim$ [b_1 (mean wage income)$_i$ + b_2 (mean transfer income)$_i$ + b_3(mean "other" personal income)$_i$]. [Di/Bi],

where quantile components are replaced by linear functions of group mean income components deflated by the ratio of the median to the mean. The deflation is assumed to linearize the relationship, although such an assumption is true only in the limit as the quantile being observed on the lefthand side approaches the median. At that point equation (9) degenerates into a trivial identity. Empirically, the inclusion of [Di/Bi] improves the goodness of fit substantially in most H and J equations.[27]

If both sides of equation (9) are divided by D_i, the value of H is specified to be a linear function of group income shares, with the sum of the shares theoretically adding to one. The inclusion of an intercept would be redundant under such circumstances. In fact, however, variables such as (mean wage income)$_i$; are approximated as functions of global income data, while B_i is derived from the CPS distributional data. The implicit shares in the specification will therefore not add to one, and an intercept could be included. All equations in this section were tried both with and without intercept terms; where the incercept was not significant it was deleted.

While equation (9) serves as a stylistic model for the H and J specifications, the statement that H or J is estimated as a linear function of group income shares is not strictly true. Before estimation the income components in equation (9) are further broken down in the manner shown by equation (6). Assumed relationships between individual components and their global counterparts are not necessarily the same in different equations.

Consider the specification for the lower tail of the MWL group:

$$\text{(10)} \quad HMWL = \left\{(b_1 + b_2 U).\ UBR + (b_3 + b_4 T + b_5\ PAR1).\ W.\ (b_6 + b_7 E) + b_8\ Y_{TR} + b_9\ (Y_{ID} + Y_o)\right\} \Big/ BMWL$$

The specification of the numerator differs in form from the specification of the mean [equation (7)], in that the group employment and unemployment rates are linear functions of their global counterparts but not strictly proportional.[28] The form of the group wage rate is the same as in equation (7), but the coefficients are permitted to take on different values.

An unrestricted estimation procedure for equation (10) would require ten degrees of freedom. Because the MWL group is primarily dependent upon wage income, however, coefficients b_8 and b_9 were not significant. Furthermore, because of paucity of available degrees of freedom and because of collineary among the multiplicative terms involving b_3 through b_6, the equation was modified by the omission of the cross product terms involving $b_4 . b_6$ and $b_5 . b_6$.

Estimated equations for the above specification and for similar relationships for the upper tail of the distribution appear for the three groups in Table IV. It was not possible to find a specification which could significantly determine the upper tail of the FEM group. Both upper tail equations and the lower tail equation for the FEM group include an intercept term. In addition, corporate profit as a share of gross private product was found to have a strong negative effect on the upper tail of the MWL group.

The coefficients in equations (IV-1) through (IV-5) do not indicate the total effect of global variables on the tails of the group distributions, because the same variables often enter the mean income equation as well; the mean, in turn, is an argument of the H and J equations. Total elasticities of estimated variables with respect to all global variables appear in Table V. The elasticity of an H (or J) equation with respect to any argument X is defined as follows:

$$(11) \quad \frac{E(H)}{E(X)} = \left[\frac{\partial H}{\partial X} + \frac{\partial B}{\partial X}\right] \cdot \frac{X}{H}$$

According to equation (IV-1), the lower tail of the MWL group distribution responds positively to increase both in the real wage rate and in the employment rate. There is a positive time trend, while the sign on PAR1, the wife labor force participation rate, is negative. Since PAR1 has increased over time these effects are largely offsetting. An increase unemployment benefits per unemployed person improves the lower tail relative to the median at unemployment rates in excess of 5.8 percent.

TABLE IV

Equations for Distributional Variables, 1949-1965

(See note to Table III. The prefix H refers to the lower tail of the group distribution; J to the upper tail)

(IV-1)
$$HMWL = \Big\{ \underset{(1.300)}{(-3.755} + \underset{(22.70)}{64.86U)} \times UBR + W\,[\underset{(3.098)}{-9.682} + (\underset{(3.26)}{11.43} + \underset{(.0047)}{.0379T} - \underset{(.601)}{3.365\ PAR1}) \times E] \Big\} \Big/ BMWL$$

$R^2 = .928$ $F(5,11) = 28.4$

$D - W = 2.08$ s.e. = .00728

(IV-2)
$$JMWL = \underset{(.596)}{2.618} - \underset{(.639)}{2.383\ PROF} + \Big\{ (\underset{(1.169)}{-3.010} - \underset{(.0226)}{.0531\ T} + \underset{(.302)}{1.172\ CWEN} + \underset{(1.886)}{1.943\ PAR1}) \times W \times E + \underset{(2.151)}{7.746\ Y_{TR}} + \underset{(3.141)}{12.604\ Y_{ID}} \Big\} \Big/ BMWL$$

$R^2 = .952$ $F(7,9) = 25.0$

$D - W = 2.19$ s.e. = .0108

(IV-3)
$$HMWN = \Big\{ \underset{(590.1)}{829.6} + (\underset{(.2249)}{-.9230} + \underset{(.1588)}{.7151\ CWEN} + \underset{(.343)}{1.327\ PAR1}) \times W \times E + \underset{(1.652)}{2.450\ UBR \times U} \Big\} \Big/ BMWN$$

$R^2 = .845$ $F(4,12) = 16.4$

$D - W = 2.59$ s.e. = .00824

(TABLE IV cont'd)

$$\text{(IV-4)}\quad JMWN = \underset{(.530)}{4.500} + \left\{\underset{(2239.2)}{-7452.6} + (\underset{(.0261)}{-.0979}\,T - \underset{(.4813)}{4.3715}\,CWEN) \times W \times E^{*} + \underset{(1.816)}{10.186}\,(Y_{ID} + Y_{o})\right\} / BMWN$$

$R^2 = .882$ $F(4,12) = 22.4$

D - W 2.33 s.e. = .0219

$$\text{(IV-5)}\quad HFEM = \underset{(.2115)}{.3384} + \left\{\underset{(524.5)}{-1087.8} + [\underset{(.3290)}{-.5263} + (\underset{(1.1988)}{3.0402} - \underset{(1.048)}{2.325}\,CWEN) \times E \times PAR2]\,W + \underset{(2.369)}{2.997}\,Y2 \times (1-PAR2) + \underset{(1.481)}{2.858}\,(Y_{ID} + Y_{o})\right\} / BFEM$$

$R^2 = .842$ $F(6,10) = 8.88$

D - W 2.46 s.e. = 0186

TABLE V

Partial Elasticities of Distributional Variables with respect to to Global Variables.

With respect to:	Elasticity of: BMWL	HMWL	JMWL	BMWN	HMWN	JMWN	BFEM	HFEM
W	.998	.089	.042	.933	1.354	-1.000	.406	-4.794
E	.875	1.305	-.427	.544	-.969	-.518	.406	1.005
UBR	.066	-.112	.002	.021	.217	.026	-	-
T^*	130	.0157	-.0240	-	-	-.0418	-	-
PAR1	-.376	-1.0485	.093	-	.907	-	-	-
PAR2	-	-	-	-	-	-	.293	.806
PROF	-	-	-.237	-	-	-	-	-
PGNP	-	-	.436	-	1.828	-.967	-	-3.665
Y_{TR}	-	-	.113	.194	-.194	.242	.173	.304
Y_{ID}^{**} or $(Y_{ID} + Y_o)^{***}$	-	-	.294**	-	-	.626***	-	1.771***

*Partial derivative not elasticity

Source: Equation numbers III-1 through III-3 and IV-1 through IV-5.

if B = F(... x, ...), then $E(B)/E(x) = \frac{\partial B}{\partial X} \cdot \frac{X}{B}$

if H or J = F(... X ..., B), then $\frac{E(H)}{E(X)} = \left[\frac{\partial H}{\partial X} + \frac{\partial M}{\partial B} \cdot \frac{\partial B}{\partial X}\right] \cdot \frac{X}{H}$

When the effects of these same variables on the mean are also accounted for, an increase in the real wage rate has a negligible effect ($E < .1$) on the relative position of the lower tail of the distribution. Roughly speaking, a one percent increase in real wages shifts the entire MWL distribution upward by one percent. A change in the employment rate has a substantial effect on both the position and the shape of the distribution, with the elasticities of the mean (BMWL), HMWL, and JMWL with respect to the employment rate being .9, 1.3, and -.4 respectively. An increase in the employment rate raises the lower tail of the distribution relative to the median by a substantial amount, while the relative (but not the absolute) position of the upper tail of the distribution tends to fall.

Changes in the participation rate of wives have a noticeable effect on the shape of the MWL distribution. Abstracting from trend, marginal entrants into the group have the effect of lowering the mean of the distributional ($E = -1.0$). Although PAR1 enters the JMWL equation directly with a positive sign, the total effect of that variable is negligible.

A number of factors tend to affect the upper tail of the MWL distribution exclusively. Both transfer payments (excluding unemployment benefits) and interest and dividend payments have a positive effect on the upper tail of the distribution, with the elasticity with respect to interest and dividend payments being .3. At the same time, an increase in corporate profits (including depreciation allowances) as a share of gross private product tends to depress the upper tail of the distribution. The reason for this negative effect may be that the corporate share of GPP, except for the dividend component, represents the portion of GPP which is retained from personal income. While upper income families ultimately have an interest in the increased corporate share, in the short run their share of measured personal income declines.

Finally, the upper tail of the MWL group improves relative to the remainder of the distribution when current dollar wage rates are changing rapidly, given the real level of all income components (including wage income). Since an increase in nominal wages given the real wage rate is equivalent to a change in prices, there is an implicit positive elasticity of response to the GNP price deflator of .4.[29] This finding is unique for the upper tail of a group distribution; it is probably attributable to the labor-force-oriented character of the group.

The response of families with a male head and a wife not in the labor force differs substantially from the group just observed. The MWN group family typically has one earner rather than two, and is considerably more dependent upon non-wage income sources than is the MWL group family. The mean of the MWN group has an elasticity of .93 with respect to W and .54 with respect to E, compared to corresponding elasticities of 1.00 and .88 for the MWL group. An increase in real wages has ~~a~~ positive effect on the lower tail of the distribution (E = 1.35) and a negative effect on the upper tail (E = 1.0). An increase in the employment rate affects both tails of the distribution negatively, but the opposite signs on W and E in the lower tail equation may be due to a collinearity problem. Given that W and E tend to move together during the business cycle, the net effect appears to be that the lower tail of the distribution rises and the upper tail fails during a period of tight employment. This conclusion is reinforced by the elasticity of response to a change in prices. A one percent increase in the GNP price deflator would bring about a 1.8 percent increase in HMWN and a 1.0 percent decline in JMWN. It should be kept in mind, however, that real, not nominal, levels of income sources are held fixed when these elasticities are calculated.

Non-wage income sources have a significantly positive effect on the level of JMWN, a much stronger effect than what is observed for JMWL. This finding is consistent with the presumption that the MWN group is more dependent of the two upon non-wage income sources. What is surprising is that except for unemployment benefits, non-wage income sources have no significant impact on the lower tail of the MWN distribution.

Taken as a group, families with a female head respond positively to an increase in real wages or employment, with the elasticities of .4 being close to the labor force participation rate of these heads. The elasticity of response to an increase in real transfer levels (excluding unemployment benefits) is positive but rather small. As one might expect, an increase in the labor force participation rate (or females, "other marital status") has a positive effect on the mean of the distribution, and an even stronger effect (E = .8) on the lower tail of the distribution.

Families at the bottom end of the female head income distribution respond in a radically different way from other low income families. While an increase in the employment rate does have a positive effect (E = 1.0), increases in W and PGNP have an overwhelmingly negative effect on the level of HFEM, with elasticities of -4.8 and -3.7 respectively. The dependence of such families upon non-wage income sources is apparent, with an elasticity with respect to real transfer levels of 0.3 and with respect to other non-wage income sources of almost 1.8.

IV. Conclusions

Despite major problems in coordinating current Population Survey and National Income Account data, a number of significant and plausible relationships between the size distribution of personal income and changes in aggregate economic activity have been uncovered. Increases in real wages and employment rates tend to improve the relative position of low-income families which are labor-force oriented, and to lower the relative but not the absolute, position of high income families. With the exception noted in the text, increases in the price level have a parallel effect. The relative position of the upper tail of the distribution is positively correlated with the level of non-wage income sources.

Families with a female head respond less elastically to employment and real wage changes than do families with a male head. The lower tail of the female-head distribution shows a strongly negative response to increases in prices and real wages, although the effect of an increase in employment is positive. This same group also shows a stronger positive response to increases in non-wage income sources than any other observed group. In short, low-income households cannot be viewed as a homogenous group when changes in the size distribution of income are examined.

Despite the crudity of the specifications tested in this paper, the results are encouraging for further research. The heterogeneity of response of group size distributions to economic phenomena can be predicted to a significant extent by differences in sources of income. Knowledge about the behavior of group distributions can be combined with information about changes in the relative sizes of the group to provide a picture of the aggregate size distribution of personal income.

Finally, the notion of incorporating a block of distributional equations into an econometric model appears to be feasible, since the arguments of the presented equations are typically endogenous to such models. Hypotheses relating the distribution of income to consumption behavior, labor force participation, and tax receipts can also be tested. While any conclusions reached would be highly tentative, simulation experiments with such a model could be used to examine the effect of government economic policies upon the size distribution of income. Efforts are being made in this direction.

NOTES

[1]See James S. Duesenberry, et al, ed., The Brookings Quarterly Econometric Model of the United States, (1965, Rand McNally & Company, Chicago), Chapter 8.

[2]See Herman P. Miller, Income Distribution in the United States, (1960 Census Monograph, U.S. Department of Commerce), p. 22: "All available evidence presented in this chapter points to stability in the distribution of family income during the fifties, following a period of rather rapid change during World War II." This conclusion was also reached by T. Paul Schultz in "Secular Trends and Cyclical Behavior of Income Distribution in the United States: 1944-1965" in Six Papers on the Size Distribution of Wealth and Income, Lee Soltow, ed., Studies in Income and Wealth, No. 33, National Bureau of Economic Research (New York, 1969).

[3]Schultz, op. cit., p. 87

[4]See, for instance, The Annual Report of the Council of Economic Advisers, particularly 1964, pp. 116-117; 1968, pp. 100-102; and 1969, p. 63.

[5]See C.E. Metcalf, The Size Distribution of Personal Income in an Econometric Model of the United States, unpublished doctoral dissertation, Massachusetts Institute of Technology, 1968, for a preliminary form of the complete model.

[6]See T. Paul Schultz, The Distribution of Personal Income: Case Study of the Netherlands, unpublished doctoral dissertation, Massachusetts Institute of Technology, 1965, pp. 167-196; see also his "Secular Trends...", op. cit.

[7]See L. Galloway, "The Foundations of the 'War on Poverty'", American Economic Review, March 1965, Vol. 55, pp. 122-131; and communications on the same subject by H. Aaron and Galloway, American Economic Review, December 1967, Vol. 57, pp. 1229-1243.

[8] See B. Mandelbrot, "The Pareto-Levy Law and the Distribution of Income," International Economic Review, No. 2, May 1960, pp. 79-106; see also his "Stable Paretian Random Functions and the Multiplicative Variation of Income," Econometrica, Vol. 29, No. 4, October 1961, pp. 517-543; Miller, op. cit., pp. 213-221; M.G. Kendall, The Advanced Theory of Statistics, ed. 5, (Hafner Publishing Co., New York, 1952) Vol. I, pp. 137-145, and Vol.II, pp. 43-44; W.P. Elderton, Frequency Curves and Correlation, ed. 3, (Cambridge University Press, 1938); G.E.P.Box and D.R. Cox, "An Analysis of Transformation." Journal of the Royal Statistical Society, Series B, 1964, No. 2, pp. 211-243; J. Aitchinson and J.A.C. Brown, The Lognornal Distribution, (Cambridge, at the University Press, 1957).

[9] See Aitchinson and Brown, op. cit., for a thorough discussion of the lognormal distribution.

[10] See Metcalf, op. cit., pp. 19-22 and 50-67.

[11] See Aitchinson and Brown, op. cit., pp. 4, 14-16, and 55-63.

[12] The constant of displacement discussed by Aitchinson and Brown for the three parameter lognormal distribution was restricted to be opposite in sign from the constant used here.

[13] The data used in this study was obtained from United States Bureau of the Census, Current Population Reports, Series P-60, Consumer Income, Numbers 1 through 53, (Washington: U.S. Government Printing Office, 1948-1967).

[14] See Aitchinson and Brown, op. cit., pp. 37-64.

[15] Any symmetrically defined quantile observations may be used.

[16] In the normal transformation one expects both Y10 and Y90 to be G standard deviations from the transformation mean. The value of C which equalizes these distances can be algebraically solved for. See Metcalf, op. cit., pp. 37-46.

[17] So long as $H + J > 2$, which implies positive skewness of the absolute

distribution, there will exist values of C and S which make the observed H and J consistent with an assumed displaced lognormal distribution. Since the two parameter lognormal distribution restricts C to zero, it will in general not be consistent with arbitrary values of H and J.

[18] Metcalf, op. cit.

[19] These estimates were obtained by weighting the derived group means by the proportion of households falling into each group.

[20] See note 13 for the data reference. See Metcalf, op. cit., Chapter III for a description of the interpolation procedure.

[21] An attempt is made to make the participation rate of wives endogenous to the econometric model. See Metcalf, op. cit., pp. 260-261.

[22] No equation was specified for JFEM.

[23] The definition includes all personal income sources covered in the Current Population Survey. Generally, imputed income sources are omitted.

[24] Given direct measures of the median, H, and J , the value of B was constructed from equation (4). No direct stochastic relationship was specified between the median and aggregate mean data because of the non-linearities implied by the displaced lognormal distribution in relationships between quantiles and moments.

[25] That is, the proportion of families with male head, wife present which constitutes the MWL group.

[26] The instruments used in the first stage of the estimation procedure were based upon principle components of all exogenous variables in the econometric model. Least squares estimates of the presented equations had similar coefficients and a slightly higher goodness of fit. See Metcalf, op. cit., pp. 181-184

[27] See Metcalf, op. cit., Ch. V.

[28] Since $U = 1 - E$, a strict specification would require that $a_6 = (1 - a_1 - a_2)$ and $a_2 = a_7$.

[29] It should be noted that if the rate of change in nominal wages is interpreted as consisting of a real wage change and a price change, the major source of variation is the price component. The explicit use of a price change variable produces virtually equivalent empirical results.