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TAXATION OF EXTERNALITIES: DIRECT VERSUS INDIRECT

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ABSTRACT

First, a model with separable utilities in the externalities is presented to analyze the impact of direct and indirect taxes on the correction of externalities that are caused by the consumption of one good. It is shown that the sign of the indirect tax is based not on the complementarity (or lack thereof) between the two taxed goods, but on the link between complementarity of the two goods for each consumer and the size of the marginal impact of his consumption on the externality. Second, a model of externality aggregate is further developed along the previous lines and alternative definitions of complementarity are considered.

Taxation of Externalities: Direct versus Indirect

In their article on "Direct versus Indirect Remedies for Externalities," Green and Sheshinski (1976) show that, when the marginal impact of consumption on the externality varies among individuals, a mix of direct and indirect taxes is superior to direct taxes alone. The purpose of this note is to give some insights into their various results. If direct taxes affect everyone's consumption without discrimination, while a discriminating instrument is required to obtain the social optimum, it appears as less of a surprise that any additional instrument not totally correlated with the direct tax (in a sense that will be defined later) will increase the range of achievable control of the externalities.

This paper follows the same line of approach as Diamond's (1973) "Externalities and Imperfect Correction Pricing," which studies only direct taxes. Whenever possible, I relate my results to his and show under which conditions they are identical. In the first section, I analyze a model with utility separable in externalities. In the second section, I analyze a model of an externality aggregate.

1. SEPARABLE UTILITY

Each individual maximizes his utility function, given as

$$U^h(\alpha^h, \beta^h) + \overline{U}^h(\alpha^1, \ldots, \alpha^{h-1}, \alpha^{h+1}, \ldots, \alpha^N) + \mu^h,$$

subject to the following budget constraint:

$$(p + t)\alpha^h + (q + z)\beta^h + \mu^h = m^h,$$

where α^h denotes the consumption level of the externality-causing good, β^h the consumption level of another good, μ^h (assumed to be positive) the income spent on all goods but on α^h and β^h, m^h the total income and t(z) the direct (indirect) tax on $\alpha(\beta)$. Also, from the individual f.o.c., we have

$$U_1^h = p + t, \qquad U_2^h = q + z.$$

Finally, taking derivatives of the f.o.c. with respect to t and z, it follows that

$$\partial\alpha/\partial t = U_{22}/(U_{11}U_{22} - U_{12}^2), \qquad \partial\alpha/\partial z = -U_{12}/(\text{idem}),$$

$$\partial\beta/\partial t = -U_{12}/(\text{idem}), \qquad \partial\beta/\partial z = U_{11}/(\text{idem}) \tag{1}$$

Since we are concerned only with remedies to externalities, we can assume that the tax revenues are given back to the individuals in a lump-sum manner. We find the optimal level of taxes t and z by maximizing the following welfare function:

$$W(t, z) = \sum_{h=1}^{N} U^h(\alpha^h, \beta^h) + \sum_{h=1}^{N} \bar{U}^h(\alpha^1, \ldots, \alpha^{h-1}, \alpha^{h+1}, \ldots, \alpha^N) - p\sum_{h=1}^{N} \alpha^h - q\sum_{h=1}^{N} \beta^h.$$

Let $A_t = \sum_{h=1}^{N} \partial\alpha^h/\partial t$, $B_t = \sum_{h=1}^{N} \partial\beta^h/\partial t$, $\tilde{U}_i = \sum_{h\neq i} \partial\bar{U}^h/\partial\alpha^i$

and $\tilde{U} \circ A_t = \sum_{h=1}^{N} \tilde{U}_h\, \partial\alpha^h/\partial t$ (same is true for z). Taking the partial

derivatives of W(t, z), setting them equal to zero and replacing the individual f.o.c., we obtain

$$t\, A_t + z\, B_t + \tilde{U} \circ A_t = 0,$$

$$t\, A_z + z\, B_z + \tilde{U} \circ A_z = 0.$$

Solving for t and z, we get

$$t^* = \frac{-\tilde{U} \circ A_t}{A_t} \left[\frac{B_z - B_t\left(\tilde{U} \circ A_z \,/\, \tilde{U} \circ A_t\right)}{B_z - B_t A_z / A_t} \right] \tag{2}$$

$$z^* = + \frac{\tilde{U} \circ A_t}{A_t} \left[\frac{A_z - A_t\left(\tilde{U} \circ A_z \,/\, \tilde{U} \circ A_t\right)}{B_z - B_t A_z / A_t} \right] \tag{3}$$

In particular, if $\tilde{U} \circ A_z / A_z = \tilde{U} \circ A_t / A_t$, then $z^* = 0$ and $t^* = t_D = -\tilde{U} \circ A_t / A_t$, as shown in Diamond (eq. 10). This condition holds, in particular if $\tilde{U}_i$ are equal for all i or if U^i_{22}/U^i_{12} are equal for all i. The latter condition will be satisfied in the present context if everyone has the same utility function for goods α and β, but this assumption is highly unrealistic. The former, on the other hand, implies that everyone's consumption has an equal marginal effect on the sum of the utility functions of everyone else; this holds in the case of an "atmosphere" externality (see Meade, 1952).

For simplicity, I shall assume that $\tilde{U}_i$ is negative for all i in the sequel[1], i.e., the consumption of good α causes a negative externality. Since both $\tilde{U} \circ A_t / A_t$ and $B_z - B_t A_z / A_t$ (by concavity of utility functions)[2] are negative,

$$z^{*} \lessgtr 0 \quad \text{as} \quad A_z/A_t \lessgtr \tilde{U} \circ A_z \,/\, \tilde{U} \circ A_t. \tag{4}$$

Rewriting

$$t^{*} = t_D \left[1 + \frac{B_t\left(A_z/A_t - \tilde{U} \circ A_z \,/\, \tilde{U} \circ A_t\right)}{B_z - B_t\, A_z/A_t} \right], \tag{5}$$

we obtain

$$t^{*} \gtrless t_D \quad \text{as} \quad z^{*} \lessgtr (\gtrless)\, 0 \text{ if } B_t < (>)\ 0. \tag{6}$$

Note that for $B_t = 0$, $t^{*} = t_D$ though z^{*} can be negative or positive. Interpreting B_t as an index of aggregate complementarity between goods α and β in particular, if goods α and β are complements (substitutes) i.e., $U^{i}_{12} > (<)\ 0$, for all individuals, $B_t < (>)\ 0$, one may be tempted to conclude that conventional wisdom stating that the tax on the complement of an externality-causing good ought to be positive is incorrect. Since $B_t = A_z$, B_t also measures the response of the aggregate demand for good α to the indirect tax z.

To understand the relationship between the taxes and aggregate complementarity (or lack of it), we must first determine under what circumstances the second inequality in (4) goes one way or the other. Using the definitions in (1), we see that the problem is mathematically equivalent to

$$\Sigma x_i / \Sigma y_i \gtrless \Sigma \lambda_i x_i / \Sigma \lambda_i y_i \tag{7}$$

such that λ_i, $y_i < 0$. If x_i/y_i and λ_i $(-\lambda_i)$ are increasing in i, the left (right) hand side is greater. Therefore, z^{*} is positive (negative) when the largest externality-causing individuals at the margin have a utility with the greatest relative level of complementarity (substitution) between goods α and β (i.e., with the smallest [greatest] U_{12}/U_{22}). As the direct tax t similarly affects individuals whose consumption of good α causes a different level of externality at the margin, it is no surprise that an additional instrument, the indirect tax z, should improve the situation if one can link differentially the large externality-causing individuals with its impact on the consumption of good α. As expected, if the large externality-causing individuals view goods α and β as complements (substitutes), good β should also be taxed (subsidized).

Condition (6) can be interpreted in the following manner: t_D (the direct tax in absence of indirect taxes) represents the desired level of taxation to reduce consumption of good α, which causes the externality; as an indirect tax or subsidy is levied on good α to reach differentially the worst offenders, this indirect tax z affects everyone--and thus the consumption of good α--because $B_t = A_z$. So, if goods α and β are complements in the aggregate ($B_t < 0$) and $z^* < 0$, the consumption of good α is increased by the indirect subsidy, necessitating additional direct tax to maintain aggregate consumption of good α at the desired level. The reader can interpret the other possibilities along those lines. Thus, conventional wisdom is proven correct.

As a matter of curiosity, a similar analysis could be performed for $t^* = 0$ when $B_z/B_t = \tilde{U} \circ A_z \,/\, \tilde{U} \circ A_t$. In that case, $z^* = -\tilde{U} \circ A_t \,/\, B_t$, thus we tax (subsidize) good β when goods α and β are complements (substitutes) in the aggregate. Once again, conventional wisdom is correct.

2. EXTERNALITY AGGREGATE

Each individual maximizes a utility function given by

$$U^h(\alpha^h, \beta^h, \gamma) + \mu^h$$

subject to the following budget constraint:

$$(p+t)\alpha^h + (q+z)\beta^h + \mu^h = m^h,$$

with the convention defined in section 1. The externality level is defined as $\gamma(\alpha^1, \ldots, \alpha^N)$, which depends on the consumption of everyone. We also assume that the individual choice of a consumption bundle is independent of its own effect on the aggregate level of externality; this implies that the individual f.o.c.'s are

$$U_1^h = p + t, \qquad U_2^h = q + z.$$

The impact of both taxes on the externalities can be signed, as

$$\frac{d\gamma}{dt} = \frac{\Sigma\gamma^h \partial\alpha^h/\partial t}{1 - \Sigma\gamma^h \partial\alpha^h/\partial\gamma}, \qquad \frac{d\gamma}{dz} = \frac{\Sigma\, \gamma^h \partial\alpha^h/\partial z}{1 - \Sigma\gamma^h \partial\alpha^h/\partial\gamma},$$

if we assume that the consumption of everyone increases externality ($\gamma^h > 0$) and the externality causes a reduction in consumption ($\partial\alpha^h/\partial\gamma < 0$).

As in the previous section, the taxes, t and z, are chosen so as to maximize the following social welfare function:

$$W(t, z) = \sum_{h=1}^{N} U^h(\alpha^h, \beta^h, \gamma) - p \sum_{h=1}^{N} \alpha^h - q \sum_{h=1}^{N} \beta^h.$$

Let $dA/dt = A_t + A_\gamma\, d\gamma/dt$ and $U_t = U_\gamma\, d\gamma/dt$, where $A_\gamma = \sum_{h=1}^{N} \partial\alpha^h/\partial\gamma$ and the notation of the previous section is used (the same is true for B and z). Taking the f.o.c. of $W(t,z)$, we obtain

$$t\, dA/dt + z\, dB/dt + U_t = 0,$$

$$t\, dA/dz + z\, dB/dz + U_z = 0.$$

Solving for t and z, we get

$$t^* = \frac{-U_t}{dA/dt}\left[\frac{dB/dz - (U_z/U_t)\; dB/dt}{dB/dz - [(dA/dz)\;/\;(dA/dt)]\; dB/dt}\right]$$

$$z^* = \frac{U_t}{dA/dt}\left[\frac{dA/dz - (U_z/U_t)\; dA/dt}{dB/dz - [(dA/dz)\;/(dA/dt)]\; dB/dt}\right]$$

From this point on, an analysis similar to the one done in section 1 can be performed; I shall only point out the highlights and explain what happened in Green and Sheshinski.

If $(dA/dz) \;/\; (dA/dt) = U_z/U_t$, $z^* = 0$ and $t = t_D = -U_t/(dA/dt)$, which is the result obtained by Diamond (see his equation (22)). That condition is equivalent to $A_z/A_t = (\partial\gamma/\partial z) \;/\; (\partial\gamma/\partial t) = (\Sigma\gamma^h \partial\alpha^h/\partial z) \;/\; (\Sigma\gamma^h \partial\alpha^h/\partial t)$, i.e., the ratio of the marginal impact of the two taxes on aggregate demand (either partial A_t or total dA/dt) of good α is equal to the ratio of the marginal impact of the two taxes on the externality aggregate. Also, if $(dB/dz) \;/\; (dB/dt) = U_z/U_t$, $t^* = 0$ and $z^* = -\, U_t/(dB/dt)$; in this case, $z^* \lesseqgtr 0$ as $dB/dt \gtreqless 0$.

If one interprets aggregate complementarity in an operational way, the positive response of demand for good 1 when the price of good 2 is increased should indicate that goods 1 and 2 are substitutes. However, in the presence of externalities, there is no guarantee that the response of demand for good 1 to a price increase in good 2 should be in the same direction as the response of demand for good 2 to a price increase in good 1. With that caution, the conventional wisdom holds for the above operational definition of complementarity.

Even though the example of Green and Sheshinski does not exactly fit the two models solved here, the apparent contradiction lies in the fact that dA/dz (dB/dt) can be of a different sign than A_z (A_t). In particular, in their example they have $dA/dz = \partial\alpha_1/\partial z + (\partial\alpha_2/\partial\gamma) \cdot (d\gamma/dz)$, where $\partial\alpha_2/\partial\gamma$ is extremely large; and since $\partial\alpha_1/\partial z$ and $d\gamma/dz$ are of the same sign, the operational complementarity dA/dz is of a different sign than the simple definition they used, which employs only the cross-partial derivative of the utility function.

3. CONCLUSION

This paper shows that there is no simple relation between the signs of indirect taxation and complementarity as defined in the usual way, but that a detailed analysis enables us to explain the sign taken by the indirect tax. This additional instrument is used to get at individuals who are the largest sources of externality if as a group they are different from the whole of society, in terms of the complementarity of the goods in their consumption bundle.

NOTES

[1]The argument could be carried as easily without that assumption, but the interpretation would have to be adjusted accordingly.

[2]If $a_i < 0$, $b_i < 0$ and $a_i b_i > c_i^2$ for all i, then $(\Sigma a_i)(\Sigma b_i) > (\Sigma c_i)^2$. By (1), $B_z - B_t A_z / A_t$ has that structure.

[3]If we solve in extenso, we obtain

$$t^* = \frac{-U_\gamma \, d\gamma/dt \, (B_t \, d\gamma/dz - B_z \, d\gamma/dt)}{dA/dt \, (B_t \, d\gamma/dz - B_z \, d\gamma/dt) + dB/dt \, (A_z \, d\gamma/dt - A_t \, d\gamma/dz)}$$

$$z = \frac{U_\gamma \, d\gamma/dt \, (A_z \, d\gamma/dt - A_t \, d\gamma/dz)}{(\text{idem})},$$

where the coefficients $B_t \, d\gamma/dz - B_z \, d\gamma/dt$ and $A_z \, d\gamma/dt - A_t \, d\gamma/dz$ are exactly the conditions which determine if t and/or z are zero. Diamond's result is $t^* = -U_\gamma \, (d\gamma/dt) \, / \, (dA/dt)$.

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