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PROBABILISTIC MODELS FOR PENSION BENEFITS WITH AN APPLICATION TO THE CANADIAN LABOR FORCE

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Probabilistic Models for Pension Benefits with an Application to the Canadian Labor Force

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ABSTRACT

First, a basic probabilistic model is presented for the computation of the expected value and the distribution of pensionable service under an arbitrary service-age vesting rule. This model is then extended to allow for graded vesting, minimum age for participation, maximum age of eligibility, partial coverage, optional vesting, and portability. Also discussed through further extensions of the basic model are the pension benefits, using different benefit formulas, with or without wage and/or inflation indexing. All the models proposed in the paper are applied to the Canadian labor force using real data and the results are discussed.

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1. INTRODUCTION

One of the main functions of a pension plan is to provide vested termination benefits to its members in their retirement.[1] An important measure or proxy of these benefits is the length of <u>pensionable service</u> (or <u>qualifying service</u>) that meets the vesting provisions of the plan. These provisions are usually expressed in terms of a minimum age and/or minimum years of service with the employer, both at time of termination.[2]

The central actuarial problem in the design of a pension plan is related to the estimation of annual contributions required to develop a satisfactory reserve fund to meet future liabilities.[3] In this context, vesting provisions are considered as instruments for the firm--the age requirement reduces the short term administrative costs by deferring the membership of new employees, while the service requirement decreases employee turnover in the longer term by postponing the accrual of non-forfeitable benefits for an additional period. From the point of view of an employee, however, both of these requirements have the effect of prolonging his obligations to the firm and thus increasing the risk to him. Such considerations have made the private pension industry the center of much public debate in North America in recent years. In 1974, a pension reform law, the Employee Retirement Income Security Act (ERISA), was enacted in the United States which allows the sponsors of a pension plan to select one of three vesting provisions in satisfaction of the minimum vesting requirements and limits the service requirement to 15 years.[4]

Six provinces in Canada have now enacted legislation regarding vesting provisions. More recently, the indexing of pension benefits for the Canadian Civil Service and its possible extension to the private sector is being passionately debated in that country.

These developments call for a systematic examination of the effects of vesting provisions and termination rates on the accumulation of pensionable service and of pension benefits. In order to be realistic, such an examination should also incorporate the influences of other plan parameters and external variables such as coverage, portability, indexing, optional vesting, etc. In this paper, we shall concentrate on the characterization of cumulative pensionable service and of pension benefits as functions of vesting rules, termination rates, and other pension and institutional parameters, from the point of view of a typical employee.

Suppose, for example, that a person starts his working life at age 20, changes employment at age 40, then at 48, again at 60, and retires at 65. Accumulation of his pensionable service from his career membership in pension plans (assuming full coverage, no transferability and a uniform vesting rule) can be illustrated graphically as in Figure 1, where the horizontal axis represents the cumulative tenure and age of the employee and the vertical axis the tenure in his current employment. His lifetime pensionable years of service are the sum of those peaks that meet the vesting provisions. Suppose that the service provision of the vesting rule is 8 years and the age provision requires a cumulative tenure of 25 years (i.e., minimum qualifying age is 45), as depicted in the figure. (In the sequel, such a vesting rule will be referred to as the service-age vesting rule (8,25), symbolized in general by (s,a), although the second entry corresponds to cumulative tenure rather than the calendar age). As seen in Figure 1, the total qualifying service under the vesting rule (8,25) is 20 years. If the vesting rule were (10,20) or (5,20),

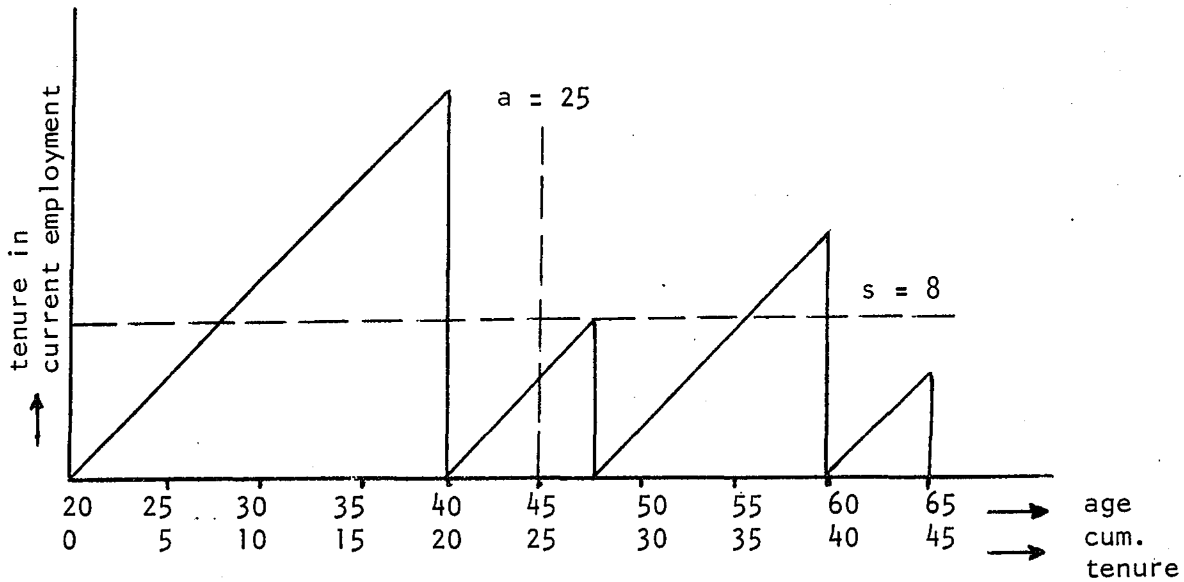


Figure 1

the employee would have been entitled to 32 or 45 years of pensionable service, respectively.

In what follows, we shall first present in section 2 a basic model for the computations of the expected value and distribution of pensionable service under an arbitrary but uniforn service-age vesting rule. This model is formulated in discrete time using annual select rates of termination as input. Similar models were considered in Sahin and Balcer (1976) using continuous completed lengths of service distributions and some renewal theoretic developments. The basic model is then extended in section 3 to allow for graded vesting, minimum age of participation, and maximum age of eligibility. In section 4 pension benefits are discussed through benefit formulas involving career average and last year's wage, with or without wage and/or inflation indexing. In section 5, the model is further extended to incorporate partial coverage, optional vesting and portability. A continuous-time theoretical model using semi-Markov processes which allows for partial coverage, portability and coverage-dependent termination rates was also considered in Sahin (1977). All the results obtained in the paper are computed in section 6 using real data from a study of Prefontaine and Balcer (1977) involving Canadian labor force surveys; the results obtained the basic assumptions of the models are discussed in sections 6 and 7.

2. THE BASIC MODEL

Let X_n denote the number of years of service for an individual in his current employment at the end of his n-th year of working life. The length of the working life is taken to be N years. For simplicity, we ignore unemployment and assume that a job change can occur only at the end of a year; accordingly $\{X_n;\ n = 1,\ldots, N\}$ forms a sequence of random variables with possible values ranging from 1 to N.[5] An individual who currently has j years of service at his job will have a year later either 1 year of service at a new job or j+1 years at his present job. If, during the course of his working life, he holds t jobs (with different employers) and job changes occuf at the end of years $n_1, n_2, \ldots, n_t$, the total number of pensionable years under the service-age vesting rule (s,a) will be:

$$X(s,a) = \sum_{k=1}^{t} X_{n_k} \text{ such that } n_k \geq a,\ X_{n_k} \geq s,$$

assuming that all the jobs have the same vesting rule. Note, for example, that in the preceding illustration (Figure 1) we have $N = 45$, $t = 4$ job changes occur at $n_1 = 20$, $n_2 = 28$, $n_3 = 40$ and $n_4 = 45$, so that $X_{n_1} = 20$, $X_{n_2} = 8$, $X_{n_3} = 12$ and $X_{n_4} = 5$. The vesting rule is $(s,a) = (8,25)$ which is met by X_{n_2} and X_{n_3}, resulting in a total pensionable service of 20 years.

Since we do not know ahead of time what the employment pattern of an individual will be, $X(s,a)$ is a random variable. We shall first be concerned with its expected value $\mu(s,a)$. To compute this quantity, it is sufficient to note that an addition of j years to pensionable service can be represented by an event of the type $\{X_n = j,\ X_{n+1} = 1\}$, provided that $n \geq a$ and $j \geq s$. Occurence of such an event implies a "peak," to the right of (a) and above (s), in a realization such as the one depicted in Figure 1.[6] It follows that

$$\mu(s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} j\, P[X_{n+1} = 1,\ X_n = j] \tag{1}$$

where P [...] denotes the probability of [...]. Under the conditions of the model, the event $\{X_{n+1} = 1,\ X_n = j\}$ is equivalent to the event $\{X_{n+1} = 1,\ X_n = j, \ldots, X_{n+1-j} = 1\}$, for the employment that terminates at the end of the n-th year must have started at the beginning of the $n + 1 - j$-th year of employment. In view of this observation, probability of the event $\{X_{n+1} = 1,\ X_n = j\}$ can be decomposed in terms of conditional probabilities as:

$$P[X_{n+1} = 1, X_n = j] = P[X_{n+1} = 1 | X_n = j] P[X_n = j | X_{n+1-j} = 1] P[X_{n+1-j} = 1]. \tag{2}$$

The conditional probabilities on the right hand side of (3) are directly obtained from the data (see section 6 and Appendix 2), while the last term can be derived inductively, for n = 1,2,...,N, from:

$$P[X_n = 1] = \sum_{j=1}^{n-1} P[X_n = 1 | X_{n-1} = j]\ P[X_{n-1} = j | X_{n-j} = 1]\ P[X_{n-j} = 1]. \tag{3}$$

This formula relates the start of a new employment to the start, duration and termination of the previous employment and is illustrated in Appendix 1.

Based on these developments, an algorithm for the computation of the expected pensionable service is presented in Appendix 2. This algorithm was first developed and applied to Canadian Public Service data in Sahin and Balcer (1976).

The expectation computed above provides a partial characterization of the pensionable service. Since X(s,a) is a random variable, we can characterize it fully only by constructing its distribution function. Additional information provided by this distribution, although not crucial as to the financial stability of the fund, should be highly useful for public policy purposes. It would be required, for example, in an examination of the impact of vesting legislation on the income distribution of the elderly. A recursive algorithm for the distribution of pensionable service is also presented in Appendix 2; this algorithm was first applied in Sahin and Balcer (1976) to Canadian Public Service data.

3. EXTENSIONS: GRADED VESTING MINIMUM, QUALIFYING AGE, AND MAXIMUM AGE FOR PARTICIPATION

In this section, we extend the basic model to approximate better the realities of a pension system. The extensions will lead to certain modifications of equation (1) without changing the basic events $\{X_{n+1} = i,\ X_n = j\}$. Extensions which require the analyses of different events are discussed in section 5.

First, we consider <u>graded vesting</u>. Suppose, for example, that the plan provides 10 percent vesting after 1 year of service, 20 percent after 2 years, and so on, resulting in full vesting after 10 years of service. Let $k_j = 0.10\, j$ for $j = 1,2,\ldots, 10$ and $k_j = 1$ for $j > 10$, and denote by (1-10,a) the graded vesting described above. Equation (2) now takes the form

$$\mu(1\text{-}10,a) = \sum_{n=a}^{N} \sum_{j=1}^{n} jk_j P[X_n = j,\ X_{n+1} = 1]. \tag{4}$$

Evidently, <u>any</u> graded vesting rule can be incorporated in (4) by suitably adjusting the percentages k_j.

In addition to the usual vesting provisions, some pension plans also stipulate a minimum qualifying age for participation. As a result, only the years of service after the qualifying age can be counted in satisfaction of the vesting requirements. If the qualifying age is $\bar{a}$ (measured from the start of working life) we have, for the expected pensionable years under the service-age provisions (s,a), provided $a \geq s+\bar{a}$, that:

$$\mu(s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} \min\,(j,n-\bar{a})\ P[X_n = j,\ X_{n+1} = 1] \tag{5}$$

where min $(j, n-\bar{a})$ represents the smaller of j and $n-\bar{a}$. This is in reflection of the fact that any qualifying tenure cannot exceed $n-\bar{a}$. Expression (5) can be modified further to exclude only the services rendered prior to a certain age in determining an employee's vesting status.[7]

In noncontributory pension schemes, a maximum age $\tilde{a}$ for participation is often stipulated, denying membership to new entrants past this age. (Again, we measure $\tilde{a}$ from the beginning of working life.) To account for this threshold, the basic model can be modified as:

$$\mu(s,a) = \sum_{n=a}^{N} \sum_{j=\max(s,n-\tilde{a})}^{n} j \, P[X_n = j, X_{n+1} = 1], \tag{6}$$

which requires a length of service to exceed the larger (both) of s and $n-\tilde{a}$ to qualify. By virtue of this representation, an employment that meets the service-age provisions of the plan and terminates in the interval $(\tilde{a},N)$ is counted only if it started before $\tilde{a}$, thus discounting in effect any length of service that commences after $\tilde{a}$.

4. WAGE GROWTH, INFLATION INDEXING, AND BENEFITS

So far, we have investigated the accumulation of pensionable service as a proxy measure of pension benefits. Vesting in itself, however, does not provide preservation of the replacement ratio of the wage at the time of retirement, except under the rare circumstances of wage indexing of pension benefits and of relative constancy of the wages as a function of age. In this section, we shall extend the basic model to allow for wage growth, inflation indexing and wage profiles.

Let $w_n = (1+g)^{n-N}$ be the unindexed relative wage at time n such that $w_N = 1$ and g is the growth rate of the average wage in the economy. Also, let $w_n^i = w_n(1+i)^{N-n}$ be the value at time N of the relative wage paid at time n, if it were indexed by the inflation rate i from time n to time N. The expected values of the pension benefits in terms of the wage at time N can then be expressed for unindexed and indexed plans, respectively, as

$$\mu\ (s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} w_n j\ P[X_n = j,\ X_{n+1} = 1] \tag{7}$$

$$\mu\ (s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} w_n^i\ j\ P[X_n = j,\ X_{n+1} = 1]. \tag{8}$$

These expressions are written down by using the last year's wage as a basis for establishing pension benefits. In (7), for example, an employment that terminates at the end of year n with j years of pensionable service is regarded as having generated the total wage $w_n xj$, the last year's wage w_n times the length j of the qualifying service. Even if a plan is wage-indexed, however, there will be differences between benefits based on last year's salary and benefits based on the career average, due to wage differences in different ages. It has been shown in Diamond, Anderson and Balcer (1976) for the United States, and in Marcotte and Balcer (1977) for Canada that important differences exist in the relative wages of workers of different ages. To maintain a certain level of comparability with the other extension, those wage profiles were normalized such that $w_n^C = w_n^{US} = 1$. These findings are summarized in Table 1.

Age	20	25	30	35	40	45	50	55	60	65
Canada	.714	1.368	1.694	1.863	1.919	1.863	1.845	1.657	1.393	1.000
U.S.A.	.278	5.93	.843	.917	1.056	1.068	1.068	1.034	.995	1.000

TABLE 1: Relative Wages as a Function of Age

Using these wage profiles for Canada (w_n^C) and the United States (w_n^{US}), we have, for the expected values on the basis of the last year's wage:

$$\mu(s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} w_n^C \, j \, P[X_n = j, X_{n+1} = 1] \tag{9}$$

$$\mu(s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} w_n^{US} \, j \, P[X_n = j, X_{n+1} = 1]. \tag{10}$$

Using career averages, on the other hand, we obtain again for Canada and the United States respectively:

$$\mu(\bar{s},a) = \sum_{n=a}^{N} \sum_{j=s}^{n} \sum_{m=n-j+1}^{n} w_m^C \, P[X_n = j, X_{n+1} = j] \tag{11}$$

$$\mu(s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} \sum_{m=n-j+1}^{n} w_m^{US} \, P[X_n = j, X_{n+1} = j]. \tag{12}$$

The impact of career average versus last year's wage can be investigated through these results. Note that the last two formulas are written down by observing (in (11), for example) that an employment that terminates at the end of the n-th year with j years of pensionable service would have yielded the total wage $\sum_{m=n-j+1}^{n} w_m^C$.

5. FURTHER EXTENSIONS: OPTIONAL VESTING, COVERAGE AND PORTABILITY

In this section, we shall extend further the basic model of section 2. First we allow optional vesting. That is, if an individual terminates an employment without meeting the compulsory vesting rule (s,a), he may opt to vest his pension as he sees fit provided that a weaker vesting rule (s',a') is satisfied. This particular feature is available to federal government employees in Canada. We denote by $v_{n,j}$ the probability that an individual elects vesting if available at the end of a job that terminates during his n-th year of employment and has lasted j years; and by Y(s',a') the total number of pensionable years when (s',a') is the only vesting rule. We have:

$$Y(s',a') = \sum_{n=a'}^{N} \sum_{j=s'}^{n} j\, P[X_{n+1} = 1,\ X_n = j]\, v_{n,j}. \tag{13}$$

Also, let X(s,a; s',a') be the number of pensionable years, throughout the working life of an individual, where (s,a) and (s',a') are the compulsory and optional vesting rules, respectively. It is easy to see that

$$X(s,a;s',a') = X(s,a) + Y(s',a') - Y(s,a), \tag{14}$$

where Y(s,a) is substracted to avoid double counting. The same relationship holds among expectations. Note that this rule makes sense provided that (s',a') is less stringent than (s,a). (That is, $s' \leq s$ and $a' \leq a$.)

Next, if only a fraction c of jobs offer pension plans, so that the probability of an employee being in a "covered" employment is c, the expected number of pensionable years will simply be c times the expectation under full coverage. Thus we have

$$\mu(s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} j\, P[X_{n+1} = 1,\ X_n = j]\ c. \tag{15}$$

Obviously, this assumes that an individual chooses his jobs completely at random, or that c is typical of the particular jobs an individual is likely to encounter (i.e., jobs in similar firms carry similar benefits because of cross-firm unionization and collective bargaining).

Finally, we may allow for tansferability from one employment to another. Let π denote the probability of portability (i.e., percentage of covered employments from which any number of pensionable years of service can be transferred to another employment). First, note that there will be nothing to transfer from an employment if it is not covered, or it is covered but not transferable, or it is covered and transferable but the next job is not covered. Also, note that there may be sequential transferability; i.e., benefits may be transferred more than once provided that the jobs are covered and the benefits are transferable. Next, let Z_n be the number of transferable years of service and Y_n the number of pensionable years already accumulated by time n. We are interested in the probabilities of the joint events $\{Z_n = j,\ X_{n+1} = 1\}$ and $\{Y_n = j,\ X_{n+1} = 1\}$ as the random variables Z_n and Y_n are recorded only at times of resignation. The first of these events can be decomposed into the union of j disjoint events:

$\bigcup_{k=1}^{j} \{$job is covered, pension is transferable, $X_{n+1} = 1,\ X_n = k,$

$Z_{n-k} = j - k,\ X_{n-k+1} = 1\}$. The probability of this event can in turn be developed recursively to obtain

$$P[Z_n = j, X_{n+1} = 1]$$

$$= c\pi \sum_{k=1}^{j} P[X_{n+1} = 1, X_n = k | Z_{n-k} = j-k, X_{n-k+1} = 1] \cdot P[Z_{n-k} = j-k, X_{n-k+1} = 1]$$

$$= c\pi \sum_{k=1}^{j} P[X_{n+1} = 1, X_n = k | X_{n-k+1} = 1] \cdot P[Z_{n-k} = j-k, X_{n-k+1} = 1] \qquad (16)$$

for $j = 1,\dots, n$ and $n = 1,\dots, N$ with $P[Z_0 = 0, X_1 = 1] = 1$, where we assumed that the events $\{X_{n+1} = 1, X_n = k\}$ and $\{Z_{n-k} = j-k\}$ are independent. The independence assumption means that the number of transferable years acquired as of the beginning of an employment (which is covered and transferable) does not influence the probabilities with which this employment is terminated after a number of years; the number of years employed (n), and the number of years in the current employment (k) are the only relevant factors.[8]

Next we consider the event $\{Z_n = 0, X_{n+1} = 1\}$ that a nontransferable or noncovered employment is terminated at time n. This event can be decomposed as $\bigcup_{j=1}^{n} \bigcup_{k=1}^{n} \{$job is not covered or pension is not transferable, $X_{n+1} = 1, X_n = k, Z_{n-k} = j-k, X_{n-k+1} = 1\}$ and leads to, after similar developments as above,

$$P[Z_n = 0, X_{n+1} = 1] = (1-c\pi)/c\pi \sum_{j=1}^{n} P[Z_n = j, X_{n+1} = 1]. \qquad (17)$$

(16) determines $P[Z_n = j, X_{n+1} = 1]$ recursively for $0 < j \leq n$; these probabilities could then be used in (17) to complete the sequence.

The pensionable years acquired at time n would come from a terminating covered job that is either nontransferable or transferable but followed by a noncovered employment. Through a similar argument as above we find, for the distribution of Y_n, that

$$P[Y_n = j] = c(1-c\pi) \sum_{k=1}^{j} P[X_{n+1} = 1,\ X_n = k]\ P[Z_{n-k} = j-k] \tag{18}$$

for $n = 1,2,\ldots,N-1$; for $n = N$, $c(1-c\pi)$ is replaced by c. Note that $c(1-c\pi)$ is the probability that a terminating employment is covered and nontransferable (which has probability $c(1-\pi)$) or covered and transferable but followed by a noncovered job (which has probability $c\pi(1-c)$). The expected pensionable years of service can now be expressed as:

$$\mu(s,a) = \sum_{n=a}^{N} \sum_{j=s}^{n} j\ P[Y_n = j] \tag{19}$$

Based on the above results, an algorithm for the computation of the expected cumulative pensionable service with coverage and transferability is outlined in Appendix 2.

6. AN APPLICATION OF THE MODELS USING CANADIAN LABOR FORCE DATA

In this section, we shall report the results of an application of the various models proposed using Canadian data. The select termination rates used in this application, taken from a study of Balcer and Prefontaine (1977), are summarized in Table 2 in the form of probabilities of remaining in the same employment for one year.

AGE \ TENURE	1	2	3-5	6-10	11-15	16-25	26-35	36-45
20-24	.3293	.6425	.7660					
25-29	.3706	.7095	.7876	.8651				
30-34	.3559	.7086	.8170	.8918	.9315			
35-39	.3888	.7165	.8150	.8240	.9348	.9371		
40-44	.3242	.6777	.7620	.8454	.8547	.9315		
45-49	.3018	.6282	.7610	.8291	.8983	.9158	.9225	
50-54	.2721	.7564	.7536	.8372	.9125	.8950	.9192	
55-59	.1922	.6196	.6727	.7857	.8853	.8516	.9125	.9507
60-64	.1801	.5282	.5570	.7060	.7068	.7695	.7389	.7272

TABLE 2: Probabilities of Remaining in the Same Employment for 1 Year As a Function of Age and Tenure

This table was constructed from monthly labor force surveys in Canada covering 60,000 people, of whom approximately 20,000 are male workers of prime age employed in the private sector. Thirteen surveys from March 1976 to March 1977 were used. An examination of the table would seem to indicate that tenure has a major influence on the termination rates, while the effect of age is relatively unimportant. Evidently, ultimate rates may lead to very serious biases depending on the tenure composition of the particular age group.

All the results are tabulated in Appendix 3. Table A1 gives the expected lengths of pensionable service under the vesting rule (s,a) with full coverage and no portability, as computed from formula (1) using the algorithm presented in Appendix 2, part 1. The last column of this table refers to graded vesting as introduced in section 3 (i.e., equation (4)). It should be noted that in this and the subsequent tables, (a) refers to calendar age, rather than to cumulative tenure as in the text. A comparison of the fifth and the last columns of Table A1 indicates clearly that, for the same age requirement, graded vesting

(10 percent incremental vesting from 1 to 10 years of service in our case) is more advantageous to the workers than a "comparable" simple vesting (i.e., full vesting after 5 years of service). This is a direct consequence of the decrease, as a function of tenure, in the termination rates.[9] Although several other conclusions can be drawn from Table A1, its use is mainly comparative. In particular, the effects of a minimum qualifying age of 30 and a maximum age of participation of 50 can be seen by examining Tables A2 and A3, which are computed from equations 5 and 6, respectively. Useful comparative conclusions can also be drawn from Tables A4, A5 and A6 (computed from equation (19) and the algorithm given in Part 2 of Appendix 2) in view of the results in Table A1. As expected, the influences of portability and coverage are quite drastic.

Distribution of pensionable service, as computed through the algorithm of Appendix 2, Part 3, is presented in Table A7, both in density and cumulative forms, for some of the more common vesting rules. Rather high probabilities associated with zero pensionable service should be noted. This probability ranges from 0.082 for the most liberal vesting rule (5,40) to 0.509 for the most stringent rule (10,45). The latter result means, evidently, that if the vesting rules in a segment of the economy were to be comparable on the average to (10,45), better than 50 percent of the workers would not have <u>any</u> vested pension benefits. The results indicate that, as a measure of the ultimate benefit (or lack of it) derived from a career membership in pension plans, or, as a measure of risk, "no pensionable service" is highly sensitive to the vesting provisions, at least in the Canadian context. Note that the vesting rules being compared also imply expected pensionable years of 16.67 and 10.04, respectively; the degree of sensitivity is not, therefore, so pronounced if the measure used is merely the expected value. This is just one example of the kinds of additional information that can be obtained from the distribution of pensionable service. Table A7 also contains the related means (also listed in Table A1 for a variety of vesting rules) and standard deviations. Other measures of interest, such as higher moments and percentiles, can readily be computed from the distributions. We note, as a last comment on the distributions, the high degree of skewness to the right (i.e., higher probabilities are associated with fewer years of pensionable service). In a different application of the basic model (not reported here) to Canadian Public Service data, we arrived at a contrary trend-left skewness.

The rest of the results tabulated in Appendix 3 refer to pension benefits. All the benefits are expressed as a percentage of the wage at retirement, i.e., $w_{44} = 1$. Note that with this convention, and as remarked in the text, Tables A1 through A6 can also be interpreted as expected benefits when the pension is wage indexed (i.e., the pension is accrued at the same rate as the rate of growth of the average wage). Table A8 is computed from equation (7), which allows no indexing, while in Table A9 a 5 percent inflation indexing is assumed (equation 8). In both tables a wage growth rate of 7 percent was used. Benefits in Tables A10 through A13 are again wage indexed but now the wage profiles given in Table 1 are used in computations, thus correlating age and wage. In Tables A10 and A11 the last year's wage, and in Tables

A12 and A13 the career average wage, are used as bases for establishing benefits. Since the main data is Canadian, utilization of the U.S. wage profile will be meaningful to the extent that the termination rates are also representative of the U.S. labor force which we do not claim. In spite of this, however, a comparison of the results gives useful information as to the effects of the wage profile.

An examination of the Tables A8 and A9 shows, for example, that under the vesting rule (40,10) an inflation indexed pension is 40 percent larger than the corresponding unindexed pension. This is perhaps misleading, as by age 75, for example, the difference between the two would have been much larger, since the nominal value of the indexed pension would have been multiplied by $(1.05)^{10} \cong 1.7$. Also, care must be taken in comparing Tables A10 and A11 (A12 and A13). The apparently larger numbers for Canada are essentially due to the decline of relative wages in Canada after age 55.

We leave other comparisons and conclusions of similar nature to the reader.

7. CONCLUSIONS

The basic model and its extensions presented above provide convenient analytical tools to assess the impacts of institutional parameters and of structural features of pension plans on the accumulation of pensionable service and pension benefits. Any number of the extensions we have considered could have been brought together, but we treated them separately for clarity and to isolate better, in the application presented, the consequences of additional features involved.

An apparent weakness of the models lies in the underlying independence assumptions. First, it may appear that, in view of one of the basic assumptions, the models cannot allow any correlation that might exist between termination rates and vesting provisions and/or total qualifying service to date. As noted in the paper, such dependencies can, however, be incorporated into the models without much difficulty. Actually, if it is taught that the vesting rules have a marked influence on termination rates, and _if_ there is data, all one has to do would be to use the rate schedule appropriate to the vesting rule being used in computations. Unfortunately, there is little empirical work done in the area due to a lack of adequate data. It is generally accepted that the existence of an employer-based pension plan influences an employee's decision to quit--hence our emphasis on select termination rates. The influences of the structural characteristics of pension plans on firm attachment, however, are less clear. This point has recently been examined by Howard (1976) and Shiller and Weiss (1976). Howard states, without giving any empirical evidence, that the vesting schedule adopted will influence the termination rates. In Shiller and Weiss, it is concluded empirically that "the implicit loss bound up in an unvested exit [strongly] restrains the quit decision" for young entrants, but, also that the "stringent vesting requirements markedly increase quit probabilities among younger workers." These somewhat contradictory findings must also be reconciled with the well-known tendency for the probability of termination to decline with age and tenure. Not enough is known even as to the specific structure of this tendency. Most termination schedules used in applications have a select period of only three to five years and ultimate schedules are not uncommon. Models that call for additional dimensions in the termination rate schedules such as vesting status, employment level, etc., are not, therefore, likely to find data, except in a few exceptional cases at the firm level. And, as pointed out earlier, without any formal modification, the assumption of the independence of termination rates and vesting rules can be removed from the above models by using a proper termination rate schedule for every vesting rule of interest--if such data is available.

The second independence assumption we used in the models is related to the coverage and transfer probabilities. Specifically, we assumed that upon termination of an employment, an individual moves to a covered employment with probability c or to a noncovered employment with probability 1-c, independent of his work history. A similar assumption was made on portability. In practice, it is unlikely that c is constant over time. Depending on the accumulation of pension benefits, this probability may increase or decrease as an individual approaches retirement. Insufficient qualifying service, for example, would place a strong restraint on moves to noncovered jobs near the age of retirement, while satisfactory pension might have a contrary effect. To say that such influences would cancel out each other, since we are dealing with the statistical behavior of a group of individuals, would be an oversimplification. It is not, however, very difficult to make the coverage probability depend on the qualifying service to date in the above models; the algorithms would be more complicated, but still computable. The real problem again is a lack of adequate data and empirical evidence of reasonable scope. It might also be argued that an applied model should be free of burdensome detail so that the consequences of its important features could be better investigated. We certainly feel that the operationally restrictive assumptions of the above models are not stronger (perhaps even weaker) than the well-established actuarial assumptions made in pension mathematics.

In relation to the application of the models to Canadian labor force, in addition to the points raised in section 6, we would like to draw the attention of the reader to the substantial overall impacts of coverage and portability, and of minimum qualifying age and maximum age of participation when the service-age requirements are less stringent. In the distribution of pensionable service, surprisingly high probability accumulations at no qualifying service for high service requirements must be noted. Also interesting are the somewhat low coefficients of variations (ratios of mean to standard deviation) in Table A7. It looks as though the densities can be graduated by exponential curves. Here and elsewhere, it should be kept in mind, however, that the findings are extremely sensitive to data used. As we mentioned before, we obtained unimodal left-skewed densities in an application of the basic model to Canadian Public Service data (see Sahin and Balcer, 1976).

APPENDIX 1

Illustration of Equation (3)

To illustrate equation (3) of section 2 for n = 4, for example, we have:

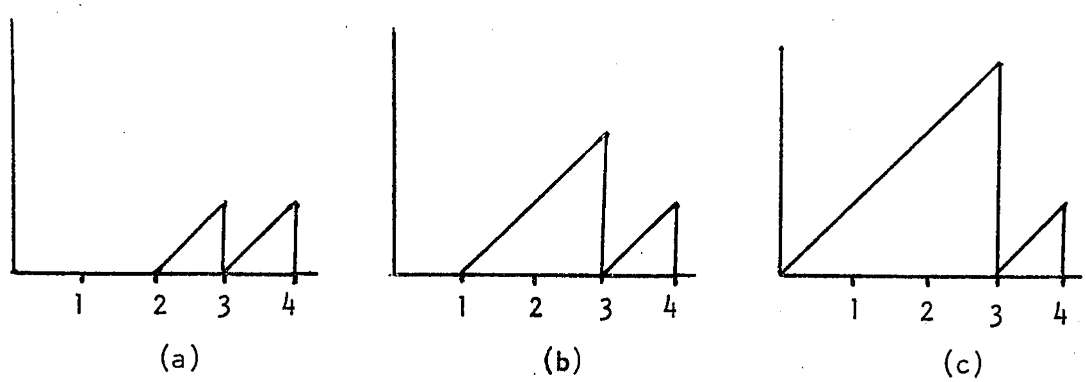
$$P\,[X_4 = 1] = \sum_{j=1}^{3} P[X_4 = 1 | X_3 = j]\; P[X_3 = j | X_{4-j} = 1]\; P[X_{4-j} = 1]$$

(a) $\quad = P[X_4 = 1 | X_3 = 1]\; P[X_3 = 1 | X_3 = 1]\; P[X_3 = 1]$

(b) $\quad + P[X_4 = 1 | X_3 = 2]\; P[X_3 = 2 | X_2 = 1]\; P[X_2 = 1]$

(c) $\quad + P[X_4 = 1 | X_3 = 3]\; P[X_3 = 3 | X_1 = 1]\; P[X_1 = 1]$

The events underlying lines (a), (b) and (c) can be depicted by using the convention of Figure 1 as below.



Numerically, if the probability of keeping one's job for one year were to be 2/3, regardless of age and tenure, we would have

$$P[X_4 = 1] = \frac{1}{3} P[X_3 = 1] + \frac{2}{9} P[X_2 = 1] + \frac{4}{27} P[X_1 = 1]$$

where we used

$$P[X_4 = 1 | X_3 = 1] = P[X_4 = 2] = P[X_4 = 1 | X_3 = 3] =$$

P [there is a job termination in year 3] $= \frac{1}{3}$,

$$P[X_3 = 2 | X_2 = 1] = P \text{ [the job is kept for another year]} = \frac{2}{3} ,$$

and $\quad P[X_3 = 3 | X_1 = 1] = P$ [the job is kept for two more years] $= \frac{4}{9}$.

We know $P[X_1 = 1] = 1$, as we are recording job changes at year ends. Therefore, if we know $P[X_2 = 1]$ and $P[X_3 = 1]$, we can determine $P[X_4 = 1]$. If we repeat the computations for $n = 2$ and $n = 3$, we can relate $P[X_2 = 1]$ to $P[X_1 = 1]$, and $P[X_3 = 1]$ to $P[X_2 = 1]$ and $P[X_1 = 1] = 1$, as above. We finally obtain that $P[X_2 = 1] = P[X_3 = 1] = P[X_4 = 1] = \frac{1}{3}$ for this example.

APPENDIX 2

Algorithms

1. EXPECTED PENSIONABLE YEARS--THE BASIC MODEL

Recall that a job termination is recorded only at the end of the year, that pension credit for the full year is given, and that a new job is obtained at that time. We shall modify slightly the notation used in Jordan (1967). Let $q^{(w)}_{[n]+j}$ denote the probability that an employment taken at the end of the n-th year which has lasted j-1 years will terminate one year later. Let ${}_{j|}q^{(w)}_{[n]}$ denote the probability that an employment taken as of the end of the n-th working year will be terminated with exactly j years of service. These probabilities can be constructed from the select withdrawal (termination) rates as follows:

$${}_{j|}q^{(w)}_{[n]} = q^{(w)}_{[n]+j}(1 - q^{(w)}_{[n]+j-1})(1 - q^{(w)}_{[n]+j-2})\dots(1 - q^{(w)}_{[n]+1})$$

$$= P[X_{n+j+1} = 1 | X_{n+j} = j]P[X_{n+j} = j | X_{n+1} = 1],$$

where $1 - q^{(w)}_{[n]+j}$ is obtained from **Table 2 such** that j denotes the tenure and [n] +j + 19 the age at time of termination. $P[X_n = 1]$ as defined by (3) can now be computed recursively by

$$P[X_n = 1] = \sum_{j=1}^{n-1} P[X_{n-j} = 1]\ {}_{j|}q^{(w)}_{[n-j-1]}, \qquad n = 2,\dots,N$$

with $P[X_1 = 1] = 1$. Also by (2) , $P[X_{n+1} = 1, X_n = j]$, denoted by W^n_j, is given by

$$W^n_j = {}_{j|}q^{(w)}_{[n-j]}\ P[X_{n+1-j} = 1], \qquad n = 1,\dots,N; \quad j = 1,\dots,n.$$

W_j^n is the probability that at his n-th year of employment an individual terminates j years of service. We can now construct

$$R_s^n = \sum_{j=s}^{n} jW_j^n, \qquad s = 1,\ldots,n, \qquad n = 1,\ldots,N, \tag{A1}$$

and

$$\mu(s,a) = \sum_{n=a}^{N} R_s^n, \qquad s = 1,\ldots,a, \quad a = 1,\ldots,N,$$

where R_s^n is the expected number of years in a job that terminates at the n-th year of employment with at least s years of service and $\mu(s,a)$ is the expected number of pensionable years for an individual, accumulated under the vesting rule of minimum age a and minimum years of service s. This establishes the expectation in question for all possible vesting rules.

All the extensions in sections 3 and 4 can be computed directly by modifying equation (A1). In most cases, it suffices to replace j by the appropriate functions as defined in these sections.

2. EXPECTED PENSIONABLE YEARS--OPTIONAL VESTING, COVERAGE AND PORTABILITY

The above algorithm can also be used in the presence of optional vesting by repeating the computations with R_s^n, replaced by $R_s^n \cdot v_n$ where v_n is the probability that an individual chooses to vest his pension, provided that a weaker rule is satisfied, in a job that terminates at the end of his n-th year of employment.

In the case of coverage (with probability c) and portability (with probability π), through similar arguments as in section 1 of this Appendix and against the background provided in section 5, we have

$$P[Z_n = j, X_{n+1} = 1] = A_j^n\,, \qquad j = 1,2,\ldots,n, \quad n = 1,2,\ldots,N$$

$$P[Z_n = 0, X_{n+1} = 1] = (1 - c\pi)/c\pi \sum_{j=1}^{n} A_j^n\,, \qquad n = 1,\ldots,N$$

$$P[Y_n = j, X_{n+1} = 1] = (1 - c\pi)/\pi\, A_j^n, \qquad j = 1,2,\ldots,n, \quad n = 1,2,\ldots,N-1$$

$$P[Y_N = j, X_{N+1} = 1] = 1/\pi\, A_j^N\,, \qquad j = 1,2,\ldots,N$$

where A_j^n are given recursively by

$$A_j^n = c\pi \sum_{k=1}^{j} {}_{k|}q_{[n-k]}^{(w)}\, A_{j-k}^{n-k}\,, \qquad j = 1,2,\ldots,n, \quad n = 1,2,\ldots,N$$

$$A_0^n = (1 - c\pi)/c\pi \sum_{j=1}^{n} A_j^n\,, \qquad n = 1,\ldots,N$$

and $A_0^0 = 1$.

3. DISTRIBUTION OF PENSIONABLE SERVICE

Let $W^n(i,k)$ denote the probability that by his n-th year of employment, an individual has accumulated k years of pensionable service in his previous jobs and has currently i years of service in his present employment, under the uniform service-age vesting rule (s,a). For given n, i and k, $W^n(i,k)$ contains all the information that might be required as to the number of pensionable years and/or the years of service in the current employment at a given time. In particular, the distribution $Q_n(k)$, $k = 0,s,s+1,\ldots,n$, of the number of pensionable years at year n is given by

$$Q_n(k) = \sum_{j=0}^{s-1} W^n(j,k) + \sum_{j=s}^{n} W^n(j,k-j).$$

The probabilities $W^n(i,k)$ can be computed recursively from the following with the initial condition $W^1(1,0) = 1$

$$W^{n+1}(i+1,k) = W^n(i,k)(1 - q_{[n-i]+i}), \quad a \le n \le N-1, \; s \le k \le n, \; 1 \le i \le n-k \text{ or } 1 \le n \le a-1, \; k = 0, \; 1 \le i \le n$$

$$W^{n+1}(1,0) = \sum_{j=1}^{n} W^n(j,0)q_{[n-j]+j}, \quad 1 \le n \le a-1$$

$$W^{n+1}(1,k) = \sum_{j=1}^{\min(s-1,n)} W^n(j,k)q_{[n-j]+j} + \sum_{j=s}^{\max(n,s-1)} W^n(j,k-j)q_{[n-j]+j}, \quad a \le n \le N-1, \; k = 0 \text{ or } s \le k \le n+1$$

Note that j must be no larger than k in the last term and all the undefined $W^n(i,k)$ are assumed to be 0.

APPENDIX 3

Results of the Application

a \ s	1	2	3	5	8	10	12	15	20	Graded Vesting
21	45.00	30.93	27.09	21.59	16.11	13.33	11.33	9.19	6.39	27.94
25	42.23	30.00	26.68	21.59	16.11	13.33	11.33	9.19	6.39	25.86
30	38.56	27.68	24.85	20.54	15.91	13.33	11.33	9.19	6.39	23.48
35	34.94	25.15	22.69	18.95	15.01	12.83	11.15	9.19	6.39	21.06
40	30.90	22.06	19.93	16.67	13.46	11.76	10.43	8.81	6.39	18.52
45	26.14	18.39	16.60	13.88	11.36	10.04	9.02	7.83	6.01	15.78
50	21.12	14.67	13.30	11.11	9.18	8.19	7.44	6.55	5.09	12.85
55	16.04	11.04	9.97	8.37	7.02	6.32	5.76	5.07	3.99	9.82
60	10.18	7.02	6.30	5.32	4.66	4.31	3.98	3.52	2.84	6.18
65	2.88	2.20	1.96	1.64	1.45	1.35	1.27	1.14	.93	1.62

TABLE A1: Expected Pensionable Service Under Different Service (s) - Age (a) Rules and Graded Vesting

a \ s	1	2	3	5	8	10	12	15	20
30	35.00	24.35	21.64	17.60	13.46	11.28	9.66	7.85	5.45
35	33.17	23.39	20.92	17.18	13.27	11.18	9.62	7.85	5.45
40	29.73	20.89	18.76	15.49	12.28	10.58	9.26	7.68	5.45
45	25.34	17.58	15.79	13.07	10.56	9.24	8.22	7.02	5.22
50	20.58	14.14	12.77	10.58	8.65	7.65	6.91	6.02	4.55
55	15.69	10.69	9.63	8.03	6.67	5.97	5.42	4.72	3.65
60	9.94	6.78	6.06	5.08	4.42	4.07	3.74	3.28	2.61
65	2.82	2.14	1.89	1.58	1.39	1.29	1.21	1.08	.87

TABLE A2: Expected Pensionable Service Under Different Vesting Rules and Minimum Qualifying Age 30

a \ s	1	2	3	5	8	10	12	15	20
21	34.38	26.18	23.51	19.68	15.29	12.89	11.14	9.19	6.39
25	31.61	25.25	23.11	19.68	15.29	12.89	11.14	9.19	6.39
30	27.94	22.94	21.27	18.62	15.09	12.89	11.14	9.19	6.39
35	24.31	20.41	19.11	17.03	14.19	12.39	10.97	9.19	6.39
40	20.28	17.32	16.36	14.75	12.63	11.32	10.25	8.81	6.39
45	15.52	13.64	13.02	11.96	10.54	9.61	8.84	7.83	6.01
50	10.49	9.92	9.72	9.20	8.36	7.75	7.26	6.55	5.09
55	6.46	6.46	6.46	6.46	6.20	5.88	5.58	5.07	3.99
60	3.88	3.88	3.88	3.88	3.88	3.88	3,80	3.52	2.84
65	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14	.93

TABLE A3: Expected Pensionable Service Under Different Vesting Rules and Maximum Participation Age 50

a \ s	1	2	3	5	8	10	12	15	20
21	22.50	20.57	18.70	15.62	12.15	10.38	8.90	7.18	5.03
25	21.56	19.95	18.35	15.62	12.15	10.38	8.90	7.18	5.03
30	20.00	18.56	17.14	14.76	11.94	10.38	8.90	7.18	5.03
35	18.35	17.04	15.75	13.62	11.17	9.84	8.67	7.18	5.03
40	16.51	15.32	14.15	12.23	10.08	8.97	8.01	6.78	5.03
45	14.25	13.20	12.16	10.45	8.63	7.71	6.94	5.98	4.65
50	11.76	10.87	9.99	8.56	7.05	6.32	5.71	4.97	3.95
55	9.24	8.53	7.82	6.67	5.51	4.95	4.49	3.92	3.15
60	6.24	5.76	5.26	4.45	3.72	3.39	3.12	2.74	2.22
65	2.55	2.38	2.18	1.84	1.53	1.40	1.31	1.17	.96

TABLE A4: Expected Pensionable Service with Full Transferability (π= 1) and Partial Coverage (c = 0.5)

a \ s	1	2	3	5	8	10	12	15	20
21	22.50	18.42	15.88	12.66	9.47	7.92	6.70	5.40	3.75
25	21.30	17.79	15.59	12.66	9.47	7.92	6.70	5.40	3.75
30	19.55	16.43	14.49	11.98	9.33	7.92	6.70	5.40	3.75
35	17.79	14.97	13.24	11.03	8.76	7.57	6.56	5.40	3.75
40	15.83	13.28	11.73	9.77	7.86	6.90	6.10	5.14	3.75
45	13.48	11.24	9.88	8.19	6.64	5.89	5.26	4.54	3.50
50	10.97	9.09	7.97	6.60	5.37	4.79	4.32	3.78	2.97
55	8.43	6.96	6.08	5.02	4.13	3.71	3.36	2.95	2.34
60	5.47	4.52	3.92	3.22	2.73	2.51	2.32	2.04	1.65
65	1.80	1.54	1.35	1.10	.94	.87	.82	.73	.60

TABLE A5: Expected Pensionable Service with Partial Transferability (π= 0.5) and Partial Coverage (c = 0.50)

a \ s	1	2	3	5	8	10	12	15	20
21	45.00	41.15	37.39	31.24	24.30	20.76	17.79	14.37	10.07
25	43.12	39.89	36.70	31.24	24.30	20.76	17.79	14.37	10.07
30	40.00	37.11	34.27	29.52	23.87	20.76	17.79	14.37	10.07
35	36.69	34.08	31.51	27.25	22.34	19.69	17.33	14.37	10.07
40	33.03	30.65	28.31	24.46	20.16	17.94	16.01	13.56	10.07
45	28.50	26.39	24.31	20.91	17.25	15.43	13.88	11.96	9.29
50	23.52	21.74	19.99	17.12	14.11	12.64	11.43	9.95	7.90
55	18.48	17.06	15.63	13.34	11.02	9.91	8.99	7.84	6.29
60	12.49	11.53	10.53	8.91	7.44	6.78	6.23	5.48	4.44
65	5.10	4.76	4.36	3.68	3.07	2.81	2.62	2.34	1.92

TABLE A6: Expected Pensionable Service with Partial Transferability (π= 0.5) and Full Coverage (c = 1)

TABLE A7: Distribution of Pensionable Service Under Different Vesting Rules with full Coverage and no Transferability

(s,a) / No. of Pensionable Years	(5,40)		(5,45)		(10,40)		(10,45)	
	Prob.	Cum. Prob.	Prob.	Cum. Prob.	Prob.	Cum. Prob.	Prob.	Cum. Prob.
.000	.082	.082	.156	.156	.422	.422	.509	.509
1.000	.000	.082	.000	.156	.000	.422	.000	.509
2.000	.000	.082	.000	.156	.000	.422	.000	.509
3.000	.000	.082	.000	.156	.000	.422	.000	.509
4.000	.000	.082	.000	.156	.000	.422	.000	.509
5.000	.062	.144	.085	.241	.000	.422	.000	.509
6.000	.035	.180	.046	.287	.000	.422	.000	.509
7.000	.033	.212	.042	.330	.000	.422	.000	.509
8.000	.030	.242	.038	.368	.000	.422	.000	.509
9.000	.028	.270	.035	.403	.000	.422	.000	.509
10.000	.042	.311	.046	.449	.059	.481	.052	.561
11.000	.033	.345	.033	.482	.036	.517	.031	.592
12.000	.036	.380	.035	.517	.034	.551	.029	.620
13.000	.037	.417	.034	.551	.032	.583	.027	.647
14.000	.037	.454	.034	.585	.030	.613	.025	.672
15.000	.039	.493	.033	.618	.029	.642	.023	.694
16.000	.035	.528	.030	.648	.024	.666	.021	.715
17.000	.035	.563	.028	.677	.023	.688	.020	.735
18.000	.034	.597	.027	.704	.022	.710	.019	.753
19.000	.033	.630	.025	.729	.021	.732	.018	.771
20.000	.033	.663	.024	.753	.023	.754	.018	.789
21.000	.031	.694	.022	.774	.022	.776	.018	.807
22.000	.029	.723	.020	.795	.021	.796	.017	.824
23.000	.027	.750	.018	.813	.019	.816	.016	.840
24.000	.026	.775	.017	.830	.018	.834	.016	.856
25.000	.024	.799	.016	.846	.017	.851	.015	.871
26.000	.021	.821	.014	.860	.015	.866	.013	.884
27.000	.020	.840	.013	.873	.013	.879	.011	.895
28.000	.018	.858	.012	.885	.012	.892	.010	.906
29.000	.017	.875	.012	.896	.012	.903	.010	.915
30.000	.015	.890	.011	.908	.011	.914	.009	.924
31.000	.014	.904	.010	.918	.010	.924	.008	.932
32.000	.013	.917	.010	.927	.009	.933	.008	.940
33.000	.012	.928	.009	.937	.009	.941	.007	.947
34.000	.011	.939	.009	.945	.008	.949	.007	.954
35.000	.010	.949	.008	.953	.007	.957	.006	.960
36.000	.008	.957	.007	.961	.006	.963	.006	.966
37.000	.008	.965	.007	.967	.006	.969	.005	.971
38.000	.007	.972	.006	.974	.006	.975	.005	.976
39.000	.006	.978	.006	.979	.005	.980	.005	.981
40.000	.006	.984	.006	.985	.005	.985	.005	.986
41.000	.005	.990	.005	.990	.005	.990	.005	.991
42.000	.004	.994	.004	.994	.004	.994	.003	.994
43.000	.003	.996	.003	.996	.003	.996	.002	.997
44.000	.002	.998	.002	.998	.002	.998	.002	.998
45.000	.002	1.000	.002	1.000	.002	1.000	.002	1.000
Mean	16.665263		13.875871		11.758070		10.044941	
St.Dev.	10.309260		10.693926		12.032725		11.962092	

a \ s	1	2	3	5	8	10	12	15	20
21	18.37	12.86	11.53	9.58	7.79	6.86	6.13	5.26	3.98
25	18.21	12.80	11.51	9.58	7.79	6.86	6.13	5.26	3.98
30	17.93	12.62	11.37	9.50	7.78	6.86	6.13	5.26	3.98
35	17.54	12.35	11.14	9.32	7.68	6.80	6.11	5.26	3.98
40	16.93	11.88	10.72	8.98	7.44	6.64	6.00	5.20	3.98
45	15.92	11.10	10.01	8.39	6.99	6.27	5.70	4.99	3.90
50	14.42	10.00	9.03	7.56	6.35	5.72	5.23	4.61	3.62
55	12.30	8.48	7.64	6.42	5.45	4.94	4.53	3.99	3.17
60	8.87	6.14	5.50	4.64	4.07	3.77	3.49	3.08	2.50
65	2.88	2.20	1.96	1.64	1.45	1.35	1.27	1.14	.93

TABLE A8: Expected Benefits as a Percentage of Wage at 64 Under Different Vesting Rules (EB) with 7% Annual Wage Growth.

a \ s	1	2	3	5	8	10	12	15	20
21	32.83	22.82	20.20	16.38	12.62	10.68	9.25	7.66	5.49
25	31.59	22.40	20.02	16.38	12.62	10.68	9.25	7.66	5.49
30	29.80	21.26	19.12	15.86	12.52	10.68	9.25	7.66	5.49
35	27.85	19.91	17.96	15.01	12.04	10.41	9.15	7.66	5.49
40	25.47	18.08	16.33	13.66	11.11	9.77	8.72	7.43	5.49
45	22.38	15.70	14.17	11.85	9.75	8.66	7.81	6.79	5.24
50	18.80	13.05	11.82	9.88	8.20	7.34	6.68	5.88	4.58
55	14.83	10.21	9.21	7.74	6.51	5.87	5.36	4.72	3.73
60	9.78	6.75	6.05	5.12	4.48	4.15	3.83	3.38	2.74
65	2.88	2.20	1.96	1.64	1.45	1.35	1.27	1.14	.93

TABLE A9: EB with 7% Annual Wage Growth and 5% Inflation Indexing from Time of Termination

a \ s	1	2	3	5	8	10	12	15	20
21	70.81	49.74	43.91	35.25	26.14	21.39	17.98	14.37	9.68
25	68.23	48.80	43.47	35.25	26.14	21.39	17.98	14.37	9.68
30	63.07	45.52	40.87	33.74	25.83	21.39	17.98	14.37	9.68
35	56.85	41.18	37.16	31.00	24.29	20.53	17.66	14.37	9.68
40	49.31	35.39	32.00	26.73	21.37	18.52	16.32	13.65	9.68
45	40.24	28.39	25.66	21.43	17.38	15.26	13.64	11.78	8.95
50	30.89	21.48	19.52	16.28	13.32	11.81	10.69	9.40	7.24
55	21.76	14.94	13.53	11.35	9.43	8.44	7.67	6.74	5.27
60	12.41	8.52	7.65	6.48	5.67	5.23	4.82	4.25	3.43
65	2.88	2.20	1.96	1.64	1.45	1.35	1.27	1.14	.93

TABLE A10: EB with Canadian Wage Profile

a \ s	1	2	3	5	8	10	12	15	20
21	42.29	29.86	26.51	21.52	16.34	13.61	11.62	9.45	6.56
25	41.30	29.50	26.35	21.52	16.34	13.61	11.62	9.45	6.56
30	38.99	28.02	25.17	20.84	16.20	13.61	11.62	9.45	6.56
35	35.88	25.85	23.32	19.47	15.42	13.18	11.46	9.45	6.56
40	31.92	22.81	20.61	17.23	13.89	12.12	10.75	9.07	6.56
45	26.89	18.93	17.09	14.28	11.68	10.31	9.26	8.03	6.15
50	21.52	14.96	13.57	11.33	9.35	8.33	7.57	6.66	5.17
55	16.15	11.11	10.04	8.43	7.06	6.35	5.79	5.10	4.01
60	10.14	6.99	6.27	5.30	4.64	4.29	3.96	3.50	2.83
65	2.88	2.20	1.96	1.64	1.45	1.35	1.27	1.14	.93

TABLE A11: EB With U. S. Wage Profile

a \ s	1	2	3	5	8	10	12	15	20
21	72.02	50.95	45.13	36.53	27.54	22.87	19.49	15.84	11.00
25	69.53	50.10	44.76	36.53	27.54	22.87	19.49	15.84	11.00
30	64.75	47.20	42.52	35.28	27.30	22.87	19.49	15.84	11.00
35	58.95	43.27	39.21	32.91	26.03	22.18	19.24	15.84	11.00
40	51.80	37.89	34.45	29.03	23.45	20.45	18.11	15.26	11.00
45	43.03	31.18	28.39	24.01	19.74	17.46	15.68	13.60	10.39
50	33.77	24.36	22.36	18.97	15.80	14.13	12.88	11.36	8.80
55	24.57	17.75	16.30	13.99	11.88	10.75	9.85	8.71	6.87
60	14.68	10.79	9.88	8.61	7.69	7.17	6.66	5.93	4.83
65	3.84	3.16	2.90	2.54	2.30	2.17	2.06	1.87	1.54

TABLE A12: EB With Canadian Wage Profile
Using Career Average

a \ s	1	2	3	5	8	10	12	15	20
21	40.79	28.36	25.05	20.19	15.25	12.69	10.82	8.80	6.11
25	39.85	28.04	24.91	20.19	15.25	12.69	10.82	8.80	6.11
30	37.77	26.80	23.96	19.67	15.14	12.69	10.82	8.80	6.11
35	34.96	24.93	22.40	18.56	14.56	12.37	10.71	8.80	6.11
40	31.31	22.21	20.00	16.62	13.28	11.53	10.17	8.52	6.11
45	26.61	18.65	16.81	13.99	11.37	9.99	8.93	7.69	5.81
50	21.46	14.90	13.50	11.26	9.25	8.22	7.45	6.51	4.99
55	16.21	11.17	10.10	8.48	7.09	6.37	5.80	5.08	3.95
60	10.19	7.04	6.32	5.35	4.68	4.33	3.99	3.51	2.82
65	2.88	2.20	1.96	1.65	1.45	1.36	1.28	1.14	.93

TABLE A13: EB With U. S. Wage Profile
Using Career Average

NOTES

[1]An employee is said to be vested in his or her accrued benefit when its payment at retirement is no longer contingent upon the employee remaining in the service of the employer. When an employee with vested benefits terminates employment, that employee is entitled to a benefit commencing at his or her early or normal retirement age in the amount of his or her vested accruals (cf. Winklevoss, 1977, p.5).

[2]The service requirement is more predominant in private pension plans in North America. In 1969, for example, in over 99% of the plans in the United States, a worker had to make at least a five-year commitment to a firm in order to qualify for a pension; in addition, almost half the covered workers had to fulfill an age requirement (Bell, 1975). More complicated vesting rules, such as graded vesting, are discussed in section 3. See also Note 4 below for minimum vesting provisions under the Employee Retirement Income Security Act (ERISA) of 1974.

[3]The primary objective has been to ensure that the plans are adequately funded or at least to know the added liabilities as the plan matures.

[4]ERISA allows the sponsors of a pension plan to select one of three vesting provisions in satisfaction of the minimum vesting requirements. The first one is full vesting after 10 years of service. The second involves graded vesting, providing 25 percent vesting after 5 years of service, increasing by 5 percent per year for the next five years; this schedule results in full vesting after 15 years of service. The

third minimum vesting provision is known as the Rule of 45. This method provides for 50 percent vesting when the participant's age and years of service total 45, and an additional 10 percent for each of the 5 subsequent years.

[5]The discrete approximation being used to the employment termination process overestimates the length of an employment, as a termination that occurs during a year is recorded (regarded as having taken place) at the end of the year. On the other hand, the length of the next employment is underestimated by being regarded as having started at the end of the year in which it actually commences. Quarterly or monthly versions of the models can easily be designed, if data is available, to count only full years (i.e., 1000-hour work year). It is sufficient for this purpose to divide the index j in equation (1) by 4 or 12 and take its integral part.

[6]Note that when $n = N$, the "peak" is characterized only by $\left[X_N = j\right]$ For notational uniformity, however, we shall represent this event by $\left[X_{N+1} = 1,\ X_N = j\right]$ with the formal convention that $P[X_{N+1} = 1] = 1$.

[7]Under ERISA, if the maximum eligibility requirements for plan membership (age 25 and one year of service) are used, only service rendered prior to age 22 can be excluded.

[8]Actually, our models can be extended to incorporate the effects of the number of transferable years and of vesting provisions, if termination rate schedules can be constructed to take these variables into account. We shall return to this issue in section 7.

[9]Because of the same reason, the first minimal ERISA alternative of full vesting after 10 years is likely to be inferior, from the employee's point of view, to the second minimal provision of graded vesting (see Note 4) irrespective of the age at entry. ERISA does not, of course, prevent the employers from offering more liberal vesting provisions than the minimal ones.

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