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ON OPTIMAL SEARCH STRATEGIES AND LAYOFFS

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A B S T R A C T

In previous studies on worker job search, as with most labor supply literature, it has been assumed that workers are informed only about the wage rate offered by hiring firms. In practice, they often have additional information about these firms. In particular, workers generally have some idea of the probability a given hiring firm will lay off workers at some future date, or alternatively fail to renew workers' contracts. The purpose of this paper is to specify a worker's optimal search strategy in a labor market where firms offer different wage rates and different layoff (or contract renewal) probabilities. At first blush, the results appear counterintuitive; they may therefore add some insight into the nature of a worker's labor supply strategy under conditions of uncertainty. To add spice to the discussion it will be embedded in a context all too familiar to many economists.

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Consider the problems faced by Dougal, a bright young economist. After completing his thesis he waits for some of the better universities to offer him a job. As Dougal was a well respected graduate student, many universities want his services. Indeed, he will receive one offer a week until an acceptable offer is found. Universities that make him an offer, in an attempt to pressure him, insist on a definite answer before Dougal receives the next offer. When a university makes Dougal an offer, the department chairman tells him the salary he will receive and the likelihood his contract will be renewed each period if the job is accepted. He goes on to explain that as the department is fully aware of Dougal's undoubted talents, the university will only fail to renew his contract in some future period because of variations in student enrollment. The chairman assures him that the variations in student enrollment are such that the probability his contract will be renewed is the same in each future period. It will come as no surprise that different universities offer different salary/renewal probability combinations.

Dougal spent much time studying the labor market for academic economists and has learned the distribution of salary/renewal probability combinations in the market for someone of his talents. He confidently predicts this distribution will be the same in all future periods. Finally, he rightly believes that each offer can be envisaged as a random draw from the joint distribution of salaries and renewal probabilities. Let F indicate this joint distribution function. Hence, $F(y,s)$ denotes the probability an offer made to Dougal has (a) a salary per period no greater than y, and (b) a contract renewal probability no greater than s. Possessing little sense of history and caring nought for prestige or location, Dougal wants to discover the search strategy that maximizes his expected discounted lifetime income. Dougal's expected lifetime is quite long. In fact, he expects to live forever.

Having taken a graduate course in labor economics, Dougal recognizes that his problem is similar to that faced by unemployed workers in sequential job search models.[1] Indeed, the only difference is that in the job search literature it is assumed that all firms in the market offer the same contract renewal probability.[2] Nevertheless, Dougal feels that results from this literature can be used to determine his optimal strategy. Having been trained as an economic theorist he, of course, disdains to work out the precise numerical solution. He makes instead two conjectures:

Conjecture 1: For any offer with a particular renewal probability s, the optimal strategy involves the use of a reservation salary $y^*(s)$. Any offer with renewal probability s should be accepted if and only if the salary offered is at least as great as $y^*(s)$.

Conjecture 2: The reservation salary $y^*(s)$ is a strictly decreasing function of s.

Dougal is confident conjecture 1 is true but is unsure of the validity of the second conjecture. Ultimately, the following argument persuades him it is correct. Suppose two offers are received that have the same salary attached.[3] Suppose one of the offers implies a greater contract renewal probability. If at least one of these offers is acceptable, it can be shown that the one with the greater contract renewal probability should be accepted. Indeed, it is possible to show that even if the offer with the greater renewal probability implies a smaller salary, it might still be preferred. Feeling he has solved his problem, Dougal waits for the offers to roll in.

It is relatively easy to show that Dougal's first conjecture is true. Let $\Omega(y,s)$ denote the expected discounted lifetime income if a job with salary y and renewal probability s is accepted. The expected payoff to rejecting an offer depends on the future search strategy, i.e., the salary/renewal probability combinations that will be accepted if offered. Let V denote the expected payoff to the search strategy that yields the greatest expected discounted income. Note that assumptions have been made that guarantee that V will also denote the maximum expected payoff if Dougal's contract is not renewed in any period. Given the assumptions stated above, and letting r denote the discount factor, we have

$$\Omega(y,s) = y + \frac{(1-s)V}{(1+r)} + \frac{s\ (y,s)}{(1+r)}$$

$$= \frac{y(1+r)}{1+r-s} + \frac{(1-s)V}{1+r-s}. \qquad (1)$$

Note that the expected payoff to a job depends on the salary offered, y, and the expected payoff if the contract is not renewed at any time, V. It follows immediately that Dougal should accept an offer if and only if $\Omega(y,s) \geq V$. Defining $y^*(s)$ by $\Omega(y^*(s),s) = V$, and noting $\Omega(y,s)$ increases with y, establishes the validity of conjecture 1.

Surprisingly, Dougal's conjecture 2 is false. To demonstrate this claim note

$$\Omega(y^*(s),s) = V = \frac{y^*(s)(1+r)}{1+r-s} + \frac{(1-s)V}{1+r-s},$$

which implies

$$y^*(s) = \frac{r}{(1+r)} V. \tag{2}$$

Hence the optimal search strategy for Dougal is such that the contract renewal probability offered by a firm can be ignored when evaluating an offer.

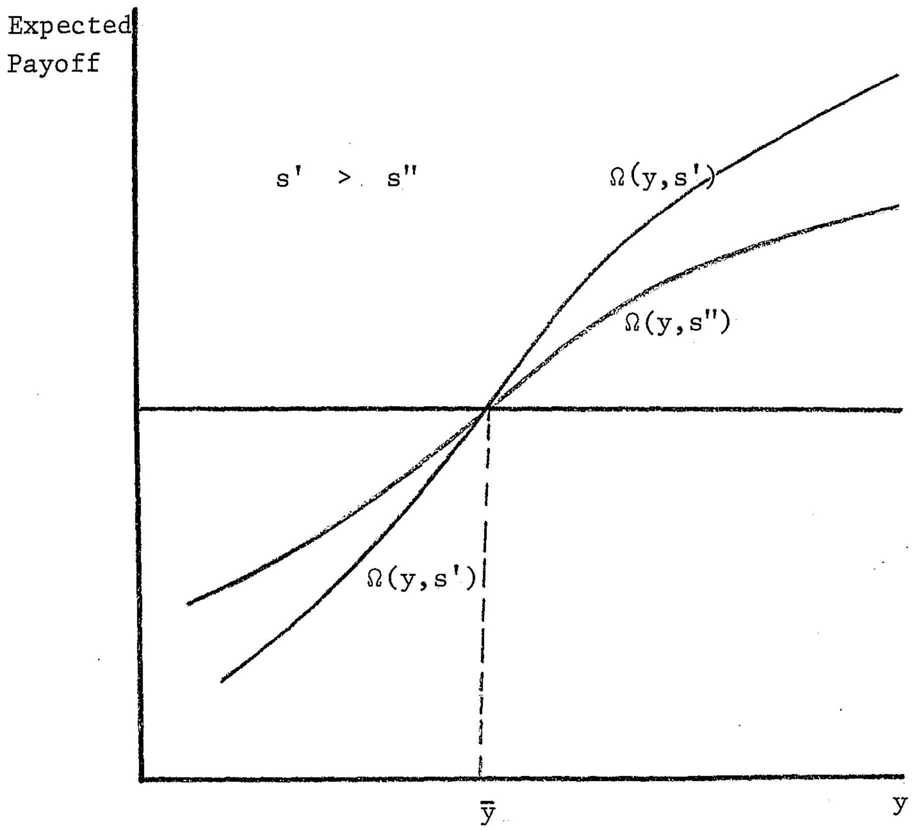
Dougal's mistake stems from confusing a nonsequential model of job search with a sequential model. **To understand this point more clearly, consider the** derivative of (1) with respect to s. When (2) is used we have

$$\frac{\partial\Omega(y,s)}{\partial s} = \frac{(1+r)}{(1+r-s)^2}\left[y - y^*(s)\right].$$

This implies $\partial\Omega(y,s)/\partial s \gtrless 0$ as $y \gtrless \bar{y}$, where $\bar{y} = y^*(s)$ for any s. This is illustrated in Figure 1. Note that the expected payoff to job offers with the same salary is a decreasing function of the contract renewal probability if and only if the salary offered is at least as great as $\bar{y}$. At the salary $\bar{y}$ the expected payoff to a job is independent of the renewal probability offered.

A critical assumption in the above analysis is that Dougal only fails to get his contract renewed in a job because of variations in student enrollment. This assumption, and the restriction that Dougal's expected life is infinite,

FIGURE 1



implies V indicates not only the expected return to search before a job is found, but also the return to search if Dougal fails to get his contract renewed. Suppose this assumption is dropped. Instead, assume Dougal only fails to get his contract renewed in a period because the department feels he is incompetent In this case failure to get his contract renewed may well have an adverse effect on Dougal's future prospects. Specifically, assume $V_1 > V_2$, where V_1 denotes the expected return to search before an acceptable job is found, and V_2 the expected return to search if Dougal fails to get his contract renewed in any period. Given the new assumption made above, the expected return to a job offer with salary y and contract renewal probability s, $\Gamma(y,s)$, can be written as

$$\Gamma(y,s) = \frac{y(1+r)}{1+r-s} + \frac{V_2(1-s)}{1+r-s}.$$

Letting $y^{**}(s)$ be defined by $\Gamma(y^{**}(s),s) = V_1$ for any given s, it follows that

$$y^{**}(s) = \frac{(1-s)}{(1+r)}[V_1 - V_2] + \frac{r}{(1+r)}V_1. \tag{3}$$

This implies that the optimal reservation salary $y^{**}(s)$ is a strictly decreasing function of s as

$$\frac{dy^{**}(s)}{ds} = \frac{-s}{(1+r)}[V_1 - V_2].$$

The conclusions reached above can be applied to a wide variety of job search models. Suppose (1-s) is now interpreted as the probability a firm will lay off an employee in a period. From the assumptions made above the following claim can be stated. The proof follows directly from the above analysis.

<u>Proposition 1</u>

If the expected return to search if a worker is laid off is the same as (less than) the expected return to search before an acceptable job is found, the optimal reservation salary defined in (2) (in (3)) is independent of s (a decreasing function of s).

Suppose Dougal decides to restrict his job search to those universities that want economic theorists. All universities that want to hire theorists offer the same contract renewal probability. Dougal's friend Alice is looking for a job as an economic historian. All universities wanting economic historians offer the same contract renewal probability, which is greater than that offered to theorists. Further assume the distributions of salary offers faced by Dougal and Alice are the same.[4] Let G denote this distribution function. Which of them will have the greater reservation salary? It can be shown in this case that the one facing the greater renewal probability will have the greater optimal reservation salary.[5] Let $\bar{y}(s)$ denote the optimal reservation salary of an individual looking for a job in a labor market where all firms offer contract renewal probability s. The expected return to search V(s) can be written as

$$V(s) = \frac{1}{(1+r)} \int_{\bar{y}(s)}^{\infty} \Phi(y,s)\, dG(y) + \frac{G(\bar{y}(s))}{(1+r)} V(s),$$

where

$$\Phi(y,s) = \frac{y(1+r)}{1+r-s} + \frac{(1-s)V(s)}{1+r-s}$$

indicates the expected return to accepting a job with salary y in a market where all firms offer the same contract renewal probability s. Manipulating the above yields

$$\bar{y}(s) = \frac{1}{(1+r-s)} \int_{\bar{y}(s)}^{\infty} (y - \bar{y}(s))\, dG(y).$$

Differentiating with respect to s gives the desired result:

$$\frac{d\bar{y}(s)}{ds} = \frac{1}{(1+r)(2+r-s+G(\bar{y}(s)))} \int_{\bar{y}(s)}^{\infty} (y - \bar{y}(s))\, dG(y) > 0.$$

NOTES

[1] A good survey of the job search literature is presented by Lippman and McCall (1976).

[2] Most contributions in this area have assumed that there is zero probability a worker will fail to get his contract renewed.

[3] Note that it is impossible in the model specified to receive two offers at the same time.

[4] In the following analysis it is assumed that individuals cannot change markets. Hence we are ruling out any compensating differential effects.

[5] This result implies that the expected duration of search is greater for Alice. Alice can, however, expect to obtain a greater acceptable salary.

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