#417-77

INSTITUTE FOR RESEARCH ON POVERTY DISCUSSION PAPERS

CHANGES IN URBAN HOUSING SUPPLIES THROUGH CONVERSION OR RETIREMENT

Katharine Bradbury

UNIVERSITY OF WISCONSIN - MADISON

Changes in Urban Housing Supplies through Conversion or Retirement

Katharine Bradbury

May 1977

The research reported herein was supported in part by funds granted to the Institute for Research on Poverty at the University of Wisconsin-Madison, by the Department of Health, Education, and Welfare pursuant to the provisions of the Economic Opportunity Act of 1964. The author wishes to thank Jerome Rothenberg and Robert F. Engle for helpful comments on earlier work on which this paper is based.

ABSTRACT

The housing stock available in any area can change through two modes: new housing units can be built and changes can be made in existing units. This paper examines the conversion-retirement process in urban housing markets in order to gain an understanding of how and why changes in the number of housing units in any locality may occur through alterations in or withdrawals of existing housing units. The discussion begins with a briefly sketched model of new construction to clarify the market forces that may encourage changes in housing units once built. Then a model of conversion-retirement is proposed and econometrically estimated using data on decade changes in the number of housing units by structure type for eighty-nine geographic zones in the Boston metropolitan area. The determinants of conversion-retirement decisions in a zone are seen to be housing price changes, vacancy rate changes, contemporaneous new construction per vacant acre, zoning constraints, and most importantly, the age and structure type composition of the beginning-of-decade stock. The first three variables are treated as endogenous and interact with the origin stocks in the nonlinear two stage least squares estimation process.

Interactions among the structure types implied by the estimates are examined, as are the implied magnitudes of demolition to make land available for new construction. Also, the price elasticity of housing supply through the conversion-retirement mode is derived.

Changes in Urban Housing Supplies through Conversion or Retirement

The housing stock available in any area can change through two modes: new housing units can be built and changes can be made in existing units. Most analysis of housing supply concentrates on the former, partly because it is the most obvious, and partly because it may account for a larger share of overall changes. For example, in the Boston metropolitan area during the two decades between 1950 and 1970, changes in existing units and retirement affected 10 percent of each of the two beginning of decade stocks, as compared to a 15 percent increase in units through new construction each decade. These figures suggest that while new construction is indeed greater in magnitude, alterations in the existing stock are also a major component of change. The importance of the two sources of supply varies geographically, as well. Within the Boston metropolitan area, new construction increased the stock by over three-quarters in one jurisdiction and almost not at all in others between 1960 and 1970, while the net change in 1960 stock caused by alterations and retirement ranged from a net loss of half the housing units in one town to a 6.5 percent net increase in units over the decade in another.

This paper examines the conversion-retirement process in urban housing markets in order to gain an understanding of how and why changes in the number of housing units in any locality may occur through alterations in or withdrawals of existing housing units. The discussion begins with a briefly sketched model of new construction to clarify the market forces that may encourage changes in housing units once built. Then a model of conversion-retirement is proposed and econometrically estimated using data on decade changes in the number of housing units by structure type for eighty-nine geographic zones in the Boston metropolitan area. The measure of housing is the number of housing units of three types: single family, multifamily (in structures with two to four units), and apartment units (in structures with five or more units). The unit of observation is the geographic zone, considered to be a housing submarket area within the metropolitan housing market.

1. NEW CONSTRUCTION OVER TIME

Consider a competitive industry producing new housing units with a constant returns to scale production function

$$Q = Q(L, N), \tag{1}$$

where Q is the number of housing units produced and L and N are the amount of land and nonland inputs, respectively. From the production function, a relationship between output price and factor prices can be derived

$$p = p(r, n), \tag{2}$$

where r and n are the price of land and nonland inputs, respectively, and p is the price of a unit of housing.

If the elasticity of substitution between land and nonland inputs is not zero, producers will use less of a factor where its price is higher. Thus the land input per housing unit depends on the relative factor prices

$$L/Q = m = m(r/n), \tag{3}$$

where m is the land per unit or lot size. Equations (2) and (3) taken together imply that associated with each output price for new housing units there is a price of land and a land per unit. Therefore, if structure types could be defined in terms of land per unit or land to nonland input ratios, all housing produced for a given output price range would be of a certain structure type.

Equation (3) directly implies that the total derived demand for land by housing producers, L, is a function of housing output and land per unit

$$L = Q \cdot m(r/n). \tag{4}$$

If the price of land is an increasing function of its scarcity, then the price of land in any location can be expressed as a function of the fraction developed

$$r = r(v) = r(V/T) = r((T - L - J)/T), \tag{5}$$

where T is total land area of a geographic zone, V is vacant acres, and J the other already-developed land area. This equation implies that as more land is used up in housing production, the price of land facing housing developers and other land users is higher. The same result would hold for other inputs in limited supply; but within zones in a metropolitan area, each of which is a small part of the total, both labor and capital can be assumed to be perfectly elastically supplied.

If all these functions are "well-behaved," then we can derive from (2), (3), and (5) a supply function for housing in each zone which relates quantity produced to output price, incorporating the feedback effect of land development on the factor input price:

$$p = f(Q/T) \text{ or } Q/T = s(p). \tag{6}$$

Housing supply in a zone is an increasing function of price, in spite of constant returns to scale in production, because the local housing industry faces a rising supply schedule for land.[1]

We can use this set of new construction supply relationships to examine the characteristics of the housing stock resulting from a sequential development process. Since units are durable, as development occurs units of different density (lot size) will appear side by side.

Figure 1 shows a supply curve S, of the form of equation (6), and the underlying equations (2) and (3). In equation (6) quantity is not expressed per acre of land (T) because the diagrams represent the situation of a given town, with T fixed. The supply curve tells what quantity would be produced by an industry faced with each output price. If producers are faced with demand D_0, they will produce Q_0 housing units in period one. If demand then grows to D_1, however, they will not produce Q_1 units the next period (as they would if Q_0 units had been consumed entirely in the previous period). They also do not produce $(Q_1 - Q_0)$ units the next period because the supply curve S is no longer relevant once the initial Q_0 units are in place. The curve S traces out the quantity of housing that would be produced at each output price, taking into consideration the effect on land price of the demand for land derived from that production. Each point on S, thus assumes that all units built will have the same lot size, m(r), where $r = r((T - L - J)/T)$, and $L = Q \cdot m$. But once units are put in place, their factor input ratio cannot be changed except through conversion,

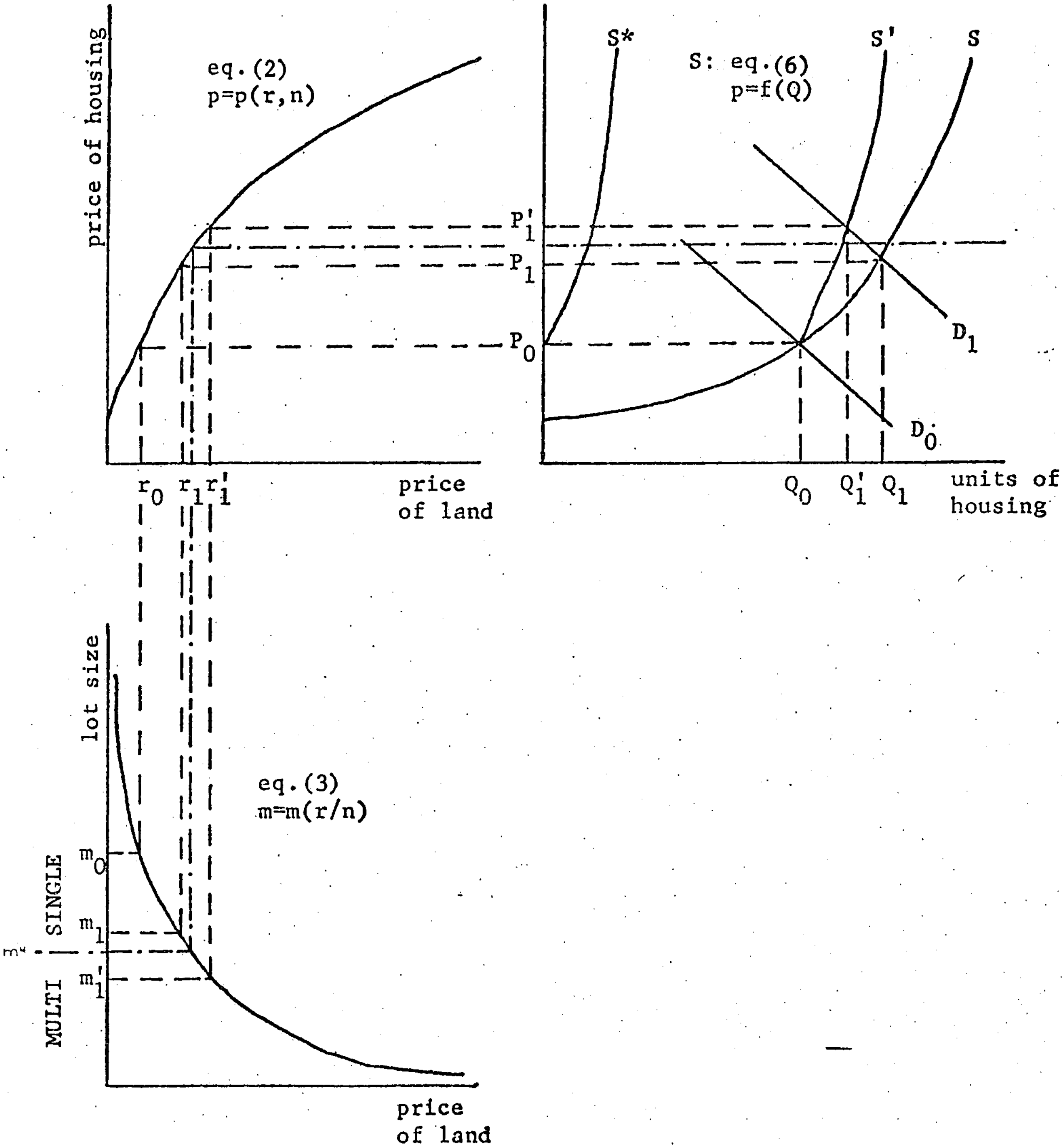

FIGURE 1.

New construction housing supply in two periods so the points further out the S curve past Q_0 do not represent the actual cost-minimizing possibilities open to new housing producers. The appropriate supply curve is S', which is steeper than S at every price greater than P_0 because the Q_0 units in place are using more land per unit than would be desired if they were not durable. This curve S' is a horizontal transposition of the curve S*, which represents new housing supply possibilities in a town in which the same number of acres ($J^* = Q_0 \cdot m_0$) has been used up by nonresidential activities. The difference, of course, is that in one town the Q_0 existing housing units are available to help meet the next period's demand, whereas in the other town there are no housing units available; this is why S' is anchored at Q_0 rather than zero.

When demand increases to D_1, it is expected that $(Q_1' - Q_0)$ new units will be produced, with lot size m_1'. If we make the arbitrary assumption that all units with lot size smaller than m* are multifamily units, then these units are multifamily. Thus, at the end of the second period, in the absence of conversion and retirement, the housing stock consists of Q_1' units, Q_0 of which have lot size m_0 (single family) and $(Q_1' - Q_0)$ with lot size m_1' (multifamily). This is in contrast to what the stock would be if all old units disappeared each period. In that case, with demand D_1, the stock at the end of period two would be Q_1 units, all with lot size m_1 (single family). Note that m_1 is larger than m_1' but smaller than m_0. Thus the inappropriately large lots of the first Q_0 units (inappropriate to the later higher land price) distort the land supply curve facing suppliers later and result in new units more dense (on smaller lots) than would be built if all housing were used up each period. In this context, the importance of history in determining current development is obvious. Fewer units are supplied in total ($Q_1' < Q_1$), total residential land use is greater ($Q_0m_0 + (Q_1' - Q_0) \cdot m_1' > Q_1m_1$), and the price of housing is higher ($p_1' > p_1$) as a result of the durability of previously supplied units. (The relative size of these adjustments of price and quantity depends on the slope of demand curve D_1.) The smaller total quantity of housing means that the gross and net residential densities of a zone are lower when development occurs in stages than when it occurs all at once: the metropolitan area spreads out more than it would if "inappropriate" buildings were not durable.

2. CONVERSION SUPPLY

As we have just seen, if the existing units built during the first period wore out and fell down, they would be replaced with units that economized more on land. But this is unlikely, for structures generally depreciate slowly. However, existing housing units can be converted to other types; they can be considerably changed without being entirely demolished. While both forms of supply combine land, labor, capital, and materials inputs, conversion supply differs from new construction in that certain of the inputs--land and some of the capital--are already in place in a given quantity and form. Owners of existing units, if they perceive the increasing demand for housing in their location, may convert a structure by adding more capital to the given lot and structure to produce more units of housing. If the capital input into housing production were entirely malleable once in place, the same production function would be applied to conversion supply as to new construction. For an existing unit, the amount of land is given, as is the existing (depreciated) nonland input. With known factor prices and output price, the most profitable Q for the given L could be derived, implying a certain incremental N.

However, such a calculation ignores the important feature of conversion supply alluded to above, that owners are in fact dealing with existing structures which have a given form (external architecture, inside layout, existing walls, plumbing, wiring, and fixtures); that is, the capital is not entirely malleable. A conversion thus involves not only incremental nonland inputs, but also costs of modification of the existing N. These costs depend on how much of a change is required, how old the structure is, and how many times it has previously been altered. If these modification costs are extremely high, demolition and new construction may be a lower-cost supply option. But this points out the other half of the difference between new construction and conversion suppliers, alluded to previously. In addition to having capital that is not entirely malleable, converters do not deal in the land market. Demolition costs keep a gap between the market value of the entire property as a prospective site and what its value would be if it were a vacant lot. Thus converters respond to a set of signals different from those faced by new suppliers, whose capital is entirely uncommitted as to location and use. Owners of structure-and-land real estate parcels compare the operating costs and revenues of the current use with the incremental capital (and demolition) costs, operating costs, and revenues of uses to which the property could be converted (or uses with which it could be replaced after demolition). If the incremental revenue of the best of the conversion-demolition options exceeds the incremental costs, the conversion will occur; that is, when output price times the new Q minus annual incremental capital (and demolition) and operating costs is greater than output price times the old Q. Because incremental costs are smaller for conversion than for demolition-new construction, a smaller disequilibrium revenue gap is required to elicit the former than the latter supply response.

Referring back to Figure 1, we can add conversion activity to the new construction supply curve, given the existing units available for conversion, Q_0. The total housing supply curve would be to the right of S': more units of housing could be provided without using more land. To the degree that output is increased through conversion without using more land, increases in the land input price are avoided. However, the supply price will rise because of the modification costs (another input cost). Conversion activity cannot shift the supply curve as far to the right as S; that is, conversion supply is more costly than new construction, in total, to supply a given unit; although the incremental costs, given the original unit as input, are generally smaller than new construction supply.

When the incremental costs of conversion to produce a given unit type from a different type are greater than new construction costs for the desired type, then demolition-new construction may be the preferred option. However, the difference between conversion costs and new construction costs must be large enough to cover demolition costs. And new construction of the same structure on an already vacant lot would be cheaper (by the amount of demolition costs). Thus "radical conversion," that is, demolition expressly for replacement with a very different residential structure type, would be undertaken only when the current structure is very different from the desired one (high conversion costs) and there is very little vacant land available in the zone.[2] Such activity would also appear in Figure 1 as a total supply curve between S' and S, occurring when S' is virtually vertical from its anchoring point because the existing stock (and nonresidential land uses) consumes all the vacant land. The demolition-new construction activity, like conversion, allows for more units to be provided with the same total land use, hence constant land (input) price. But given that land use, total input costs are higher than with S (which would apply if all units disappeared each period) because of the costs of demolition to undo the past construction.

3. THE EMPIRICAL MODEL

This formulation of conversion-retirement activity as it relates to new construction suggests some of the influences which act on the conversion-retirement decisions of the owners of residential land-and-structure properties.

Prices give a variety of signals to current owners considering conversion or demolition. It is through prices that the changes in demand for housing in the area are transmitted to owners. If the demand for housing in a location has risen significantly since some existing units were put in place, there is considerable pressure to use the existing residential land area more intensively. Therefore, it is through output price changes that changes in intensity of land use are encouraged, as Figure 1 shows. But the relevant price change for the conversion-demolition judgment is the change in output price since a given unit was built or last altered. This relevant price change may not be related to the current (endogenous) rate of price change, because the current rate may not be indicative of price changes over the whole life of the unit to date.

The average direction and magnitude of price changes is the same for all units in a zone if consumer and/or producer arbitrage across types keeps relative prices of different types within the jurisdiction in a known pattern, and geographic submarkets "clear" somewhat independently. This implies that the current rate of price change is an indicator of current revenues for both the origin and the destination unit type in a conversion decision. Thus the current rate of change may give no indication to an owner of what the appropriate land use intensity is. For example, the choice between currently supplying a single family unit and adding capital to supply instead one and a half units (two units of a duplex, each of which sells for three-quarters of what the original single family unit sold for), is not illuminated by housing price increases of 1% per year versus 2% per year. The current rate of price change applies to either use of the property. On the other hand, for an owner considering retirement or demolition with no particular residential replacement type in mind, the current rate of price change is the appropriate indicator of revenues which would be foregone if demolition were undertaken, or the structure converted to other nonresidential uses.

When price change since the unit was built or last altered is not directly observable, one useful indicator of the likely magnitude is the age of the unit, for old units are less likely to be appropriate to current demand conditions. However, in zones where the demand for housing has not significantly changed over time, old units may be just as useful in their unconverted state as units built more recently.

In addition to its age, the condition of a unit may affect its owner's decisions about its future disposition. Demolition activity reflects the need for normal replacement as well as radical conversion. Worn out units are therefore most likely to be retired or demolished, since current returns may not be as high and serious deterioration may also make conversion costs prohibitive.

Other indicators of local revenues also influence conversion supply decisions. Vacancy changes as well as price changes help clear the market. High housing unit vacancy rates act as a signal of excess supply in the market. More directly, for the individual supplier with a vacant unit, there is no current revenue foregone by withdrawing the unit from the stock or changing the type of property use. Thus high or increasing vacancy rates should encourage withdrawals, while having the same uncertain effects as current prices on conversion activity.

Shifting from the revenue side to the cost side, the most crucial determinant of conversion costs is how different the desired "destination" type is from the existing unit type. For the individual owner, the set of feasible alternatives may be fairly limited. However, in each zone a large array of origin housing types and destination property uses exists, making the calculation of aggregate conversion possibilities problematic. The most straightforward approach to modeling empirically the conversion-retirement process would be to estimate the pairwise probabilities directly as a function of the market variables just described, using the observed origin-destination transition frequencies as the dependent variables. The hypothesis is that the owner of an existing housing unit compares the costs and revenues of a set of alternative property uses and either maintains the unit in its current status, adds capital to convert it to more or fewer units, or withdraws the unit from the stock.

Conceptually there is an equation for each origin-destination pair, similar to the following group of equations that catalogue the possible pairs in which multifamily units are the origin or destination type:

$$PMS = CMS/M = f_1(X_j)$$
$$PMA = CMA/M = f_2(X_j)$$
$$PMM^* = CMM^*/M = f_3(X_j)$$
$$PMN = CMN/M = f_4(X_j)$$
$$PAM = CAM/A = f_5(X_j)$$
$$PSM = CSM/S = f_6(X_j)$$
$$PNM = CMN/N = f_7(X_j), \qquad (7)$$

where M, A, and S are the initial number of multifamily, apartment, and single family units, respectively, and C<u>ab</u> is the number of units of (origin) type <u>a</u> that are changed to (destination) type <u>b</u>, with types S, M, A as above, and type N referring to structures in nonresidential use or demolition. Structure type M* refers to multifamily units produced by increasing or decreasing the number of units in a multifamily structure while remaining within the definitional range of multi-family (two to four units in structure). Thus each of the ratios, P_{ab}, measures the proportion of initial units of the origin type a that are converted to the destination type b, and hence reflects the average probability of such a conversion for units of each origin type. The probabilities in each zone are a function of zonal housing market supply and demand conditions represented by the vector x_j, including prices, vacancies, initial housing stock characteristics, and zoning constraints on type or intensity of use.

If subcategories of the various structure types have different probabilities of conversion-retirement, then even more probability functions like (7) should be specified. If the difference between subcategories is simply an additive shift term, it can be modeled by including the appropriate terms that define subcategories as "initial housing stock characteristics" in the vector of market variables, X_j. However, if the pairwise probability of conversion is a different function of the endogenous and exogenous variables for different subcategories of the origin type, then the subcategories should be treated as different origin types. For example, old units, it was argued earlier, are riper for conversion than new units, because they are more likely to be out of factor-proportion equilibrium with regard to current input and output prices. If this means that old units are more responsive to current price changes, then the category of multifamily units should be divided into more categories, one of which is old multifamily units. Thus there would be separate equations for conversion-retirement probabilities for each of the more narrowly defined origin types:

$$PM1S = CM1S/M1 = f_{11}(X_j)$$
$$PM2S = CM2S/M2 = f_{21}(X_j)$$
$$PM1A = CM1A/M1 = f_{12}(X_j)$$
$$PM2A = CM2A/M2 = f_{22}(X_j)$$
$$PM1M^* = CM1M^*/M1 = f_{13}(X_j)$$
$$PM2M^* = CM2M^*/M2 = f_{23}(X_j)$$
$$PM1N = CM1N/M1 = f_{14}(X_j)$$
$$PM2N = CM2N/M2 = f_{24}(X_j), \qquad (7a)$$

where M1 and M2 are two subcategories of multifamily units. These eight equations would replace f_1 through f_4 of (7).

However, the aggregate zonal data available on conversion-retirement activity make such detailed examination of pairwise transition probabilities impossible. The available data are derived from Census measures of the housing stock in 1960 and 1970. The process of change is not documented. Only the 1960 and 1970 stocks are known, as well as what part of the final stock was added through new construction during the decade. As a consequence, the measure of conversion-retirement activity used here is the net decade change in units by structure type not accounted for by decade new construction. For example, the measure of multifamily conversion-retirement activity, CONV MULTI, equals the number of multifamily units in 1970 minus the number of multifamily units built between 1960 and 1970 minus the number of multifamily units in 1960, for each zone. It is not documented what part of this change in the multifamily stock is due to conversion of singles or apartments into multis, conversion of multis into fewer or more multifamily units, singles or apartments, or withdrawals of multifamily units from the stock through conversion to nonresidential use or demolition.

Thus the direct measure of overall conversion-retirement activity combines the parts sketched above. Continuing with the multifamily unit example, the empirical measure of conversion-retirement of multifamily units can be conceptualized as the sum of the following terms taken from equations (7) and (7a):

$$\begin{aligned} \text{CONV MULTI} = {} & \text{CM1S} - \text{CM2S} - \text{CM1A} - \text{CM2A} - \text{CM1N} \\ & - \text{CM2N} + (m_{m1} \cdot \text{CM1M*}) + (m_{m2} \cdot \text{CM2M*}) \\ & + (m_s \cdot \text{CSM}) + (m_a \cdot \text{CAM}) + (m_n \cdot \text{CNM}), \end{aligned} \tag{8}$$

where the C--terms are as described previously for expressions (7) and (7a), and the m-factors convert the number of origin units into the number of multifamily (destination) units resulting from the conversion. Thus,

m_s is the average number of multifamily units created out of each single family unit converted to multifamily use ($2 \leq m_s \leq 4$);

m_a is the average number of multifamily units created out of each apartment unit converted to multifamily use ($m_a \leq 4/5$);

m_{m1} and m_{m2} are the average net change in the number of multifamily units resulting from conversion of (type 1 and type 2) multis to more or fewer multis, per multi so converted ($-1/2 \leq m_{m1} \lesseqgtr m_{m2} \leq 1$); and

m_n is the average number of multifamily units created out of each nonresidential structure converted ($2 \leq m_n \leq 4$). N is measured in structures, not housing units, since it is not residential.

(The numerical limits on the "m" terms are derived from the definition of a multifamily unit, as being in a structure containing two to four units.) Because "conversion out" subtracts one unit from the origin stock, the "m" terms on conversion out of multifamily use (CM1S, CM2S, CM1A, CM2A, CM1N, CM2N) are implicitly equal to -1. The subscripted "m" factors may be constant across zones or may be assumed to vary systematically with local conditions.

If expression (8) is rewritten using the individual probabilities, it becomes

$$\begin{aligned} \text{CONV MULTI} = {} & M1 \cdot (-PM1S - PM1A - PM1N + m_{m1} \cdot PM1M^*) \\ & + M2 \cdot (-PM2S - PM2A - PM2N + m_{m2} \cdot PM2M^*) \\ & + S \cdot m_s \cdot PSM + A \cdot m_a \cdot PAM + N \cdot m_n \cdot PNM. \end{aligned} \tag{9}$$

With full substitution from (7) and (7a), the equation finally becomes

$$\begin{aligned} \text{CONV MULTI} = {} & M1 \cdot (- f_{11} - f_{12} - f_{14} + m_{m1} \cdot f_{13}) \\ & + M2 \cdot (- f_{21} - f_{22} - f_{24} + m_{m2} \cdot f_{23}) + S \cdot m_s \cdot f_6 \\ & + A \cdot m_a \cdot f_5 + N \cdot m_n \cdot f_7, \end{aligned} \tag{10}$$

where we recall that the f's are functions of zonal housing stock characteristics, price change, vacancy rate change, and zoning restrictions. Starting with equation (10) (and its counterparts relating to single family and apartment units), it should be possible to use a priori information to eliminate some of the elements of (10), and then to estimate an aggregate form of the remaining elements. For example, if it is known that the contribution of nonresidential structures and apartments to multifamily stock is negligible, it might be possible to estimate the parameters of an equation such as the following:

$$\text{CONV MULTI} = \text{M1} \cdot (h_1\ (X_j)) + \text{M2} \cdot (h_2\ (X_j)) + \text{S} \cdot (h_3\ (X_j)) + \varepsilon, \qquad (11)$$

where the X_j are the housing market variables of which the f's are functions, and where

$$h_1 = -\ f_{11} - f_{12} - f_{14} + m_{m1} \cdot f_{13}$$
$$h_2 = -\ f_{21} - f_{22} - f_{24} + m_{m2} \cdot f_{23}$$
$$h_3 = m_s \cdot f_6$$

and ε is an additive random error term affecting the combination of all these elements into CONV MULTI.

If the f's are linear functions of their arguments, and m_s, m_{m1} and m_{m2} are constants or a linear function of S, M1 and M2, respectively, then the h's are linear functions of their arguments.

This derivation, described above for the case of CONV MULTI, applies also to CONV SINGLE and CONV APART (apartment), with appropriate choice of origin and destination types. For each of the three structure type equations, the measures of origin stock are chosen to reflect the probable degree of disequilibrium of stock as well as the likelihood of the initial unit structure type playing a part in the particular net conversion-retirement activity being estimated. All those origin types for which the probability of conversion-retirement activity affecting the dependent variable is non-negligible should be included. At the same time, we wish to distinguish among origin types for which the probability is a different function of the multiplicatively-included market variables.

The likelihood of any pairwise conversion depends both on the age of the existing unit and on its structure type. Old units are more likely to be out of equilibrium with respect to current demand conditions than new, so the stocks included in each equation are limited to those that in 1960 were more than twenty years old. In addition, the general historical trend is one of demand growth. This implies increasing densities at each location over time unless transportation costs decrease just in proportion to income and population growth. Thus old stocks with a higher average ratio of land to nonland inputs than the dependent variable structure type are included in each equation as sources of conversion into the particular structure type. The old units of the same structure type are also included because they may be the origin for retirements or conversions out or (with the definitional exception of single family units) the basis for subdividing or aggregating units into more or fewer of the same structure type.

Units which are in poor condition are generally more likely to be subject to conversion-retirement activity because the returns fore-gone by the owner in undertaking any change are likely to be lower than the market average for units of that type. Thus, although deteriorating units are, in general, a subset of the old stock, they are treated separately for each structure type to reflect (and estimate) the degree of difference in market responsiveness.

Withdrawals of units from the housing stock are likely to occur if a very different property use is significantly more profitable than the current use, and so different as to make direct conversion not feasible. The measures of conversion-retirement are such that in the case of radical conversion of property use, only the demolition is counted as conversion-retirement, the replacement structure being a part of new construction. To capture this effect of new construction in vacant-land-scarce areas, one of the variables affecting the probability of conversion-retirement of each structure type is NEW TIGHT, defined as the number of new housing units built in the zone during the decade per acre of vacant land initially available. NEW TIGHT is not included in the apartments equation because apartments are less often torn down to make room for new residential units. This is both because apartments are already more intensive uses of land (hence they are less likely to be inappropriate as land prices rise over time) and because demolition costs are higher for such structures.

The other variables expected to affect the probabilities of conversion-retirement activity are of two types. The first are the market signals of endogenous price change and vacancy rate change. The second are zoning regulations, which may restrict the range of conversion-retirement options open to the owner of a unit.

Since both price change and vacancy rate change refer to the housing stock as a whole, it was argued earlier that they have no clear a priori effect on conversion activity. As housing prices rise, an owner may be inclined to stay with his or her current use, as a proven and improving thing, or he or she may convert the structure to provide more units at the increasing prices (what would be considered the usual supply response for most commodities). Movements in either prices or vacancy rates, however, which indicate rising demand are expected to discourage retirements, holding constant the intensity of pressures toward demolition activity as a means of making land available to new construction (captured by the inclusion of the variable NEW TIGHT).

A measure of the fraction of the locality zoned for minimum lot sizes of greater than 25,000 square feet (PZ) is included to test the effect of zoning. If such zoning impedes the conversion of singles to multifamily or apartment units, then the variable should have a positive effect on CONV SINGLE and a negative effect on CONV MULTI. In the single family conversion-retirement equation, the zoning variable PZ is included additively in the terms multiplied by the two origin stock types, and the coefficient is expected to show a probability of conversion out that is lower where PZ is higher. In the multifamily equation, the zoning variable (UZ = 1 - PZ) is included multiplicatively under the assumption that only unzoned singles are available for conversion to multis.

Thus the three equations to be estimated are of the following form:

$$\text{CONV SINGLE} = \text{OLD SINGLE}_{60} \cdot h_{11}\ (\Delta P,\ \Delta V,\ PZ,\ \text{NEW TIGHT}) + \text{DETER SINGLE}_{60} \cdot h_{12}\ (\Delta P,\ \Delta V,\ PZ) + \varepsilon_1,$$

$$\text{CONV MULTI} = \text{OLD MULTI}_{60} \cdot h_{21}\ (\Delta P,\ \Delta V,\ \text{NEW TIGHT}) + \text{DETER MULTI}_{60} \cdot h_{22}\ (\Delta P,\ \Delta V) + UZ \cdot \text{OLD SINGLE}_{60} \cdot h_{23}\ (\Delta P,\ \Delta V) + \varepsilon_2 \qquad (12)$$

$$\text{CONV APART} = \text{OLD APART}_{60} \cdot h_{31}\ (\Delta P,\ \Delta V) + \text{DETER APART}_{60} \cdot h_{32}\ (\Delta P,\ \Delta V) + \text{OLD MULTI}_{60} \cdot h_{33}\ (\Delta P,\ \Delta V) + \varepsilon_3,$$

where ΔP and ΔV represent measures of price change and vacancy rate change, respectively, and DETER refers to deteriorating housing.

If the aggregated probabilities (the h's) are linear in their arguments, for example, if

$$h_{21} = b_{21} + b_{22} \cdot \Delta P + b_{23} \cdot \Delta V + b_{24} \cdot \text{NEW TIGHT},$$

then the initial housing stock measures can be multiplied through the h's, yielding specifications of the three equations that are linear functions of the variables (housing stocks, price change, vacancy rate change, and zoning). For example, if the h's are linear functions of their arguments, after multiplying through, the CONV SINGLE equation in (12) becomes

$$\text{CONV SINGLE} = \text{OLD SINGLE}_{60}\ (b_{111} + b_{112}\ \Delta P + b_{113}\ \Delta V + b_{114}PZ + + b_{115}\ \text{NEW TIGHT}) + \text{DETER SINGLE}_{60}\ (b_{121} + b_{122}\ \Delta P + b_{123}\ \Delta V + b_{124}PZ) + \varepsilon_1 \qquad (13)$$

$$\begin{aligned}\text{CONV SINGLE} = {} & b_{111}\ \text{OLD SINGLE}_{60} + b_{112}\ \Delta P \cdot \text{OLD SINGLE}_{60} \\ & + b_{113}\ \Delta V \cdot \text{OLD SINGLE}_{60} + b_{114}PZ \cdot \text{OLD SINGLE}_{60} \\ & + b_{115}\ \text{NEW TIGHT} \cdot \text{OLD SINGLE}_{60} \\ & + b_{121}\ \text{DETER SINGLE}_{60} + b_{122}\Delta P \cdot \text{DETER SINGLE}_{60} \\ & + b_{123}\ \Delta V \cdot \text{DETER SINGLE}_{60} + b_{124}PZ \cdot \text{DETER SINGLE}_{60} \\ & + \varepsilon_1. \end{aligned} \qquad (14)$$

Kelejian (1971) has shown that in such a context, the two stage least squares estimation technique is consistent if certain conditions are met, although it has not been shown to be efficient (Amemiya, 1974). Kelejian's conditions are that polynomials in the exogenous variables be used as instruments, that polynomials of the same degree be used as instruments for all the endogenous functions, and that the endogenous function be the regressand in the first stage, not the endogenous variable itself. All of these conditions are met through the use of the usual two stage least squares computer algorithm, if the variables which result from multiplying the stocks through the linear h functions are used as the right hand variables in each equation (for example, the variables as written in (14)) and polynomials in the exogenous variables are added to the instruments list. Then the multiplied stock times endogenous price or vacancy appears as the endogenous variable and the instruments list is the same for all endogenous functions.

Therefore, the three equations are estimated using two stage least squares in the manner just described. The price variable, the vacancy rate variable, and the new construction per vacant acre are all multiplied by a stock and treated as endogenous each time they appear in an equation. The instruments, in addition to exogenous supply variables from these equations and a new construction equation, are taken from a list of exogenous demand variables and market adjustment variables, and include some higher powers and products of exogenous variables, as well. The observations are a cross-section of eighty-nine geographic zones in the Boston metropolitan area, consisting of seventy-five cities and towns surrounding Boston and fourteen districts within the city of Boston. (See appendix for variable definitions.)

4. THE RESULTS

The results for the equation modelling coversion-retirement activity affecting the single family stock are as follows (asymptotic standard errors in parentheses below estimated coefficients):

$$\begin{aligned}
\text{CONV SINGLE} = \underset{(72.1)}{-7.23} + \text{OLD SINGLE}_{60} \cdot \Bigg[& \underset{(.112)}{-.221} + \underset{(.181)}{.180} \frac{\Delta\text{PRICE}}{\text{PRICE}_{60}} + \underset{(.0747)}{.223} \frac{\Delta\text{VAC RATE}}{\text{VAC RATE}_{60}} + \underset{(.149)}{.193}\ \text{PZ} \\
& \underset{(.000162)}{-.000286}\ \text{NEW TIGHT} \Bigg] \\
+ \text{DETER SINGLE}_{60} \cdot \Bigg[& \underset{(.578)}{-.509} \underset{(.766)}{-.537} \frac{\Delta\text{PRICE}}{\text{PRICE}_{60}} \\
& \underset{(.534)}{-1.54} \frac{\Delta\text{VAC RATE}}{\text{VAC RATE}_{60}} \underset{(.907)}{-1.50}\ \text{PZ} \Bigg] + e.
\end{aligned} \qquad (15)$$

$R^2 = .7549$

Standard error of the regression = 331.

$F(9, 79) = 27.0$

The constant term is included in the regression to make interpretation of the summary statistics (R^2 and F) unambiguous, and is, as the specification might lead one to expect (see equation (13)), not significantly different from zero. Published data for the Boston metropolitan area as a whole, based on a sample of units actually followed from 1960 to 1970, indicates that about three-quarters of the activity measured by

CONV SINGLE is retirements or demolitions (U.S. Bureau of the Census 1973, table 3). It is useful to interpret the estimated coefficients in this light. The coefficients suggest that while, in general, old single units are withdrawn at a significant rate, price or vacancy rate increases tend to discourage such action. New construction where there is little vacant land has the expected effect of encouraging retirements, presumably to make land available for (denser) new construction. Minimum lot size zoning weakly discourages conversion of old singles to other structure types or their retirement. For single units deteriorating in 1960, the effects of price and vacancy rate changes in offsetting the general tendency to withdrawal are reversed; decreases in either variable discourage reduction of the stock. Minimum lot size zoning has the opposite (still weak) effect on deteriorating stock as well; towns with more minimum lot size zoning experience more retirement of their deteriorating single family stock. If one hypothesizes that the sign pattern indicates the effect of zoning on old singles is the prevention of conversion out while the effect on deteriorating singles is the encouragement of retirement, then zoning can be understood as a means of quality enforcement, as indeed it is often intended. It is hard to imagine much profit in offering a deteriorating unit on a large lot (greater than 25,000 square feet) in a nonrural area such as the fringes of Boston where large lot size is generally an aspect of high quality.

The direction of the overall effect of price and vacancy rate changes on single family conversion-withdrawal activity can be evaluated by collecting the appropriate terms in equation (15). For example, the total coefficient on the price variable is

$$(.180 \text{ OLD SINGLE}_{60} - .537 \text{ DETER SINGLE}_{60}).$$

This represents the increase in the number of single family units resulting from conversion-retirement, which is associated with a one unit increase in the price change variable. This coefficient, by construction, varies across the zones in proportion to the single family stocks available for conversion and withdrawal. Evaluated at the sample mean values of OLD $SINGLE_{60}$ and DETER $SINGLE_{60}$, this expression yields a positive overall effect of price change on the single family stock as affected by conversion-retirement of singles. The analogous term for vacancy rate change also implies a net positive effect of vacancy rate increases on single family conversion-retirement activity at the mean. The positive price sign is what would be expected if price change is the usual market signal to suppliers. Since the vacancy rate would be expected to act as a signal in the opposite direction, the sign on vacancy rate seems wrong by this argument. An asymptotic t-test, constructed by evaluating the asymptotic standard error of the total coefficient at the mean,[3] implies that the hypothesis that the coefficient is zero cannot be rejected with 95% confidence for either the price or vacancy rate variable just described. However, this does not necessarily imply that the contribution of price (or vacancy rate) to the explanatory power of the equation is not significantly different from zero. To make this judgment, one might also want to examine the asymptotic t-statistics of the individual coefficients, and an asymptotic F-test of the hypothesis that both price coefficients are zero or that both vacancy rate coefficients are zero.

Comparing coefficients with their reported standard errors, it is clear that individual t-tests asymptotically reject the hypothesis of zero coefficient for both the vacancy variables at the 5% level, but fail to reject for the two price coefficients. An F-test of the hypothesis of jointly zero coefficients asymptotically fails to reject for either the price pair or the vacancy pair at the 5% level.

It is also possible to collect terms and evaluate, at the mean, the net conversion-retirement rates (frequencies, probabilities) of each of the base stocks implied by the estimated coefficients. The expression in the first brackets in equation (15), when the sample mean values of $\frac{\Delta PRICE}{PRICE_{60}}$, $\frac{\Delta VAC\ RATE}{VAC\ RATE_{60}}$, PZ, and NEW TIGHT are inserted, yields the value -.09. This implies that for the average zone, one out of eleven old single family housing units are retired or converted out during the decade. A similar calculation for the second bracketed expression in equation (15) suggests a higher average probability of withdrawal for deteriorating single units, of 22%. The results thus seem to support what one would expect a priori, that the deteriorating stock is even more out of equilibrium than the stock that is more than twenty years old.

The estimated equation for conversion-retirement of multifamily units is as follows (asymptotic standard errors in parentheses below estimated coefficients):

$$\begin{aligned} \text{CONV MULTI} = &\underset{(41.9)}{88.1} + \text{OLD MULTI}_{60}\Bigg[\underset{(.0766)}{.277} - \underset{(.101)}{.399} \\ &\frac{\Delta \text{PRICE}}{\text{PRICE}_{60}} - \underset{(.0456)}{.0279}\frac{\Delta \text{VAC RATE}}{\text{VAC RATE}_{60}} - \underset{(.0000821)}{.000153}\ \text{NEW TIGHT}\Bigg] \qquad (16) \\ &+ \text{DETER MULTI}_{60}\Bigg[\underset{(.400)}{-.921} + \underset{(.409)}{.304}\frac{\Delta \text{PRICE}}{\text{PRICE}_{60}} \\ &\underset{(.305)}{-.574}\frac{\Delta \text{VAC RATE}}{\text{VAC RATE}_{60}}\Bigg] + \text{UNZONED OLD SINGLE}_{60}. \end{aligned}$$

$$\left[\underset{(.0810)}{-.0247} + \underset{(.146)}{.209} \frac{\Delta PRICE}{PRICE_{60}} + \underset{(.0458)}{.0795} \frac{\Delta VAC\ RATE}{VAC\ RATE_{60}} \right] + e.$$

$R^2 = .9094$

Standard error of the regression = 268.

$F(10, 78) = 78.3$

In this case there are three origin stock types thought to contribute to net conversion-retirement activity affecting multifamily units. Old multis, ceteris paribus, appear to be a source of partitioning to produce additional multifamily units, but as zonal prices or vacancy rates rise, this tendency is reversed. As expected, new construction in the zone is also associated with retirements of old multifamily units. Deteriorating multifamily units at base have a high rate of retirement or conversion out, but changes in price or vacancy rate offset this in the expected directions. Old single family units not subject to minimum lot size zoning show only a weak association with multifamily conversion-retirement. The inclusion of the zoning variable multiplicatively precludes the usual statistical test for nonzero effects of a single variable (asymptotic t-test). However, the equation was run with zoning not included, and with zoning included additively in the $OLD\ SINGLE_{60}$ brackets, and both the sum of squared residuals and the standard error of the regression show improvement in the version presented (16) over both the alternatives considered.

As discussed for the last equation, there are several ways to evaluate the overall effect of the endogenous price change and vacancy rate change variables on the conversion and retirement of multifamily units. The total coefficient on price or vacancy rate is the sum of three terms, each term consisting of an estimated coefficient times its corresponding origin stock type. Evaluated at the sample mean values of OLD $MULTI_{60}$, DETER $MULTI_{60}$, and UZ • OLD $SINGLE_{60}$, both the price and vacancy rate coefficients are negative, and less than twice their calculated asymptotic standard errors in absolute value. Thus for conversion-retirement of multifamily units, the price sign is wrong, but the vacancy rate sign agrees with a priori expectations at the mean. Looking at the asymptotic t-statistics for individual estimated coefficients, only one of the six is large enough to reject the null hypothesis of a zero coefficient. However, an F-test of the hypothesis that the three price coefficients or three vacancy rate coefficients are jointly zero, valid only asymptotically, rejects the hypothesis with 99% confidence for each set of coefficients.

Collecting terms for each origin structure type and evaluating at the mean yields net conversion-retirement rates or probabilities for each base type. The rate for old multifamily units is .06, suggesting that, on average, old multis are a source of additional multis, through conversion. The calculated total coefficient might mean, for example, that six duplexes out of every two hundred old multifamily units are divided into four-unit structures during the decade, in the average zone. The calculated withdrawal rate for deteriorating multifamily units is a substantial 69% at the mean, and unzoned single family units have an implied coefficient of .08 at the mean. This latter value can be interpreted with reference to equation (9) which makes clear that the estimated coefficient on "S" (here, UNZONED OLD $SINGLE_{60}$) is made up of two parts, the probability of a single family unit being subject to a single-to-multi conversion, PSM, and the yield of such a conversion in terms of multifamily units, m_s. The calculated mean coefficient value of .08 implies (approximately) that for every twelve unzoned old single units, one multi was added to the stock through conversion. This could occur if PSM were equal to .04 and m_s were 2; that is, if one out of twenty-five unzoned old singles were converted to a duplex. It could also occur if PSM were .02 and m_s = 4; one out of fifty unzoned old singles were converted to a four-unit structure. These possibilities cannot be distinguished on the basis of the available information.

The results of estimating the final conversion-retirement equation for units in the apartment structure type are as follows (with asymptotic standard errors in parentheses below estimated coefficients):

$$\begin{aligned} \text{CONV APART} = {} & \underset{(30.4)}{81.0} + \text{OLD APART}_{60} \cdot \left[\underset{(.111)}{-.297} + \underset{(.134)}{.548} \frac{\Delta\text{PRICE}}{\text{PRICE}_{60}} - \underset{(.0982)}{.315} \frac{\Delta\text{VAC RATE}}{\text{VAC RATE}_{60}} \right] + \text{DETER APART}_{60} \cdot \\ & \left[\underset{(.479)}{2.43} - \underset{(.417)}{3.67} \frac{\Delta\text{PRICE}}{\text{PRICE}_{60}} - \underset{(.460)}{.204} \frac{\Delta\text{VAC RATE}}{\text{VAC RATE}_{60}} \right] \\ & + \text{OLD MULTI}_{60} \cdot \left[\underset{(.0374)}{-.0253} + \underset{(.0547)}{.0978} \frac{\Delta\text{PRICE}}{\text{PRICE}_{60}} + \underset{(.0187)}{.0678} \frac{\Delta\text{VAC RATE}}{\text{VAC RATE}_{60}} \right] + e. \end{aligned} \tag{17}$$

R^2 = .9324

Standard error of the regression = 238.

$F(9, 79)$ = 121.

The coefficients relating to old apartment units are generally quite strong and of the expected signs. Price increases or vacancy rate declines offset a tendency for old apartment units to be retired or converted out. Judging from the sign on the price variable (significantly different from zero according to an asymptotic t-test), owners of deteriorating apartments units seem to behave in a way theory does not predict, unless it is argued that the response to poor market conditions is to hold onto deteriorating apartments units that one might replace under better conditions. Old multifamily units appear to be a weak source of apartment units through conversion.

The total coefficient on the price variable, evaluated at the sample mean values of the origin stock types, is positive, but smaller than its corresponding calculated asymptotic standard error. The total coefficient on the vacancy rate change variable is negative, and greater in absolute value than twice its calculated asymptotic standard error. It may be noticed that asymptotic t-tests for four of the six estimated coefficients under discussion allow rejection of the null hypothesis of zero coefficient. The hypothesis of jointly zero price or vacancy rate coefficients can also be rejected at the 1% level for both sets of coefficients, according to an asymptotic F test. Thus for apartment units it seems clear that the net response to market signals is to increase the supply of units in a tight market. Apartment units would be expected to be more unambiguous on this score than the other structure types since they are the top end of the density scale. Multifamily units, for example, may respond negatively to a price increase because an increase in supply (number of units) by the owner turns his or her multifamily units into apartments, decreasing the multifamily supply.

Collecting terms for each structure type and evaluating at the sample mean values of the variables yields the coefficient values of .04, .42, and .02 for OLD $APART_{60}$, DETER $APART_{60}$, and OLD $MULTI_{60}$, respectively. It is interesting to note that in contrast to the results for singles and multis, the deteriorating stock on average is not a source of withdrawals, but rather a strong source of subdivision into more apartment units. Old multifamily units contribute on the order of one small apartment structure (six units) per three hundred old multifamily units.

5. FURTHER IMPLICATIONS

The three equations (15), (16), and (17), discussed in turn above are not independent. The three processes interact by definition, and it is interesting at this point to examine the patterns of signs across the three conversion-retirement equations for additional information about subcategories of the conversion and retirement processes. The conversion of old single family units to multifamily units, and the conversion of old multifamily units to apartments each enter two of the three equations, being a subtraction from one dependent variable and an addition to another. We can examine the estimated coefficients for the expressions in each of the two equations that include that origin-destination pair. Such examination should allow us to hypothesize about some of the pairwise rates and likely components of the overall changes measured by the dependent variable.

An example of the interaction is that the conversion of a single family unit into a duplex involves a (-1) in CONV SINGLE and a (+2) in CONV MULTI. References to equation (8) makes clear how this interaction is incorporated into the model. The term CSM would include that particular pairwise change (or actually, CS1M, where S1 is old single family units) as an increment of one, in the case where m_s is 2. The term CS1M also appears in the CONV SINGLE equation that corresponds to (8) with an implicit multiplier of minus one.

That is,

$$\text{CONV SINGLE} = -\text{CS1M} - \text{CS2M} - \text{CS1A} - \text{CS2A} - \text{CS1N} - \text{CS2N} + (s_m\text{CMS}) + (s_a\text{CAS}) + (s_n\text{CNS}), \tag{18}$$

and, as before, substituting the probabilities and stocks for the frequencies,

$$\text{CONV SINGLE} = \text{S1} \cdot (-\text{PS1M} - \text{PS1A} - \text{PS1N}) + \text{S2} \cdot (-\text{PS2M} - \text{PS2A} - \text{PS2N}) + \text{M} \cdot (s_m \cdot \text{PMS}) + \text{A} \cdot (s_a \cdot \text{PAS}) + \text{N} \cdot (s_n \cdot \text{PNS}). \tag{19}$$

In the multifamily equation (9), the only term multiplied by the initial stock of single family units is $m_s \cdot$ PS1M, so the estimated coefficients in the UNZONED OLD SINGLE_{60} brackets of equation (16) represent UZ times m_s times the coefficients in the (theoretical) equation like those of (7) and (7a) for PS1M. However, in the single family equation (19) there are several terms multiplied by "S1" (namely, PS1M, PS1A, and PS1N), only one of which is PS1M. So we can learn something about the other subelements by using a priori knowledge about the limiting values of m_s and comparing signs across equations.

The estimated multifamily equation suggests that the conversion of unzoned old singles to multis is encouraged by price increases and vacancy rate increases. The estimated single family equation, on the other hand, implies that the conversion of old singles out (that is, to multis, apartments, or nonresidential use) or demolition of old singles is discouraged by price or vacancy increases. These two pieces of information taken together seem to imply that the conversion of old singles to apartments or nonresidential use, or the demolition of old singles is even more strongly discouraged by price and vacancy rate increases than the overall OLD $SINGLE_{60}$ coefficients directly suggest. Since it has been assumed (in the specification of the CONV APART equation) that the conversion of singles to apartment units is negligible, and since unzoned old singles are only a subset of all old singles, this result condenses to the notion that residential price and vacancy rate decreases encourage the demolition of old single family housing units or the conversion of them to nonresidential uses. The price effect is certainly what one would expect a priori.

Similarly, the average conversion rates calculated for each origin type in each equation can be compared. The CONV SINGLE equation estimates implied that on average the rate of retirement and conversion out of old singles was 9%. The CONV MULTI equation results, it may be recalled, could be interpreted to mean a single-to-multi conversion of 2-4% of old singles, depending on the average number of multifamily units resulting from such a conversion, m_s. Thus retirement and conversion to nonresidential use takes 5-7% of old singles in the average zone.

The same type of analysis allows the pairwise conversion of multifamily units to apartment units and multis to other destinations to be examined, by looking at the coefficients within the OLD $MULTI_{60}$ brackets in the CONV MULTI and CONV APART equations. By construction similar to equations (8), (9), (18), and (19), the apartment conversion activity can be disaggregated into

$$\begin{aligned} \text{CONV APART} = &- \text{CA1S} - \text{CA2S} - \text{CA1M} - \text{CA2M} - \text{CA1N} \\ &- \text{CA2N} + (a_{a1} \cdot \text{CA1A*}) + (a_{a2} \cdot \text{CA2A*}) + (a_{m1} \cdot \text{CM1A}) \\ &+ (a_{s1} \cdot \text{CS1A}) + (a_n \cdot \text{CNA}) \end{aligned} \qquad (20)$$

$$\begin{aligned} \text{CONV APART} = &\ \text{A1} \ (- \text{PA1S} - \text{PA1M} - \text{PA1N} + a_{a1} \cdot \text{PA1A*}) \\ &+ \text{A2} \ (- \text{PA2S} - \text{PA2M} - \text{PA2N} + a_{a2} \cdot \text{PA2A*}) + \text{M1} \cdot \\ &(a_{m1} \cdot \text{PM1A}) + \text{S1} \ (a_{s1} \cdot \text{PS1A}) + \text{N} \ (a_n \cdot \text{PNA}). \end{aligned} \qquad (21)$$

The coefficients within the OLD $MULTI_{60}$ brackets ("M1") in equation (17) thus indicate the impacts of price and vacancy on the probability of conversion of a multifamily unit to apartments, times the likely outcome of such a conversion in terms of number of apartment units. The coefficients within the OLD $MULTI_{60}$ brackets in the CONV MULTI equation (16) include, as well, the price and vacancy effects on conversion of multis to more or fewer multis or to nonresidential use, and the demolition of old multis.

The estimates of the CONV APART equation (17) imply that, on average, every old multifamily unit becomes .02 apartment units. If, for example,

a_{m1} is equal to 2 (each old multifamily unit converted on average becomes two apartment units), then this implies that 1% of old multifamily units are converted to apartment units. In contrast, the CONV MULTI equation (16) implies a net positive output of six additional multifamily units per one hundred old multis. Suppose m_{m1} is equal to one-half; that is, the average conversion of multis to multis takes a duplex and turns it into a three-unit structure. Then with 1% conversion to apartments, the net additional .06 requires the frequency of multi-to-multi conversion to be 14%. If there is, in addition, a nonzero retirement (or conversion to nonresidential use) rate, the multi-to-multi rate must be even higher.

Taking the equations together also allows us to examine the estimated magnitude of the impact of new construction on demolition as modelled in the single family and multifamily equations. The measure of units supplied through new construction divided by acres of vacant land, NEW TIGHT, was included as an endogenous right-hand variable. The less vacant land there is initially, the more demolitions per unit of initial stock are expected in order to make land available for any given amount of new construction; and given the vacant land area, the more new construction activity there is, the greater the expected rate of such demolitions. It was argued that apartment demolitions are not expected for new construction purposes because apartments are more capital-intensive uses of land and are more costly to tear down.

The estimated equations showed a negative coefficient on the new construction per vacant acre variable NEW TIGHT (entered multiplicatively with initial old stock) for both single and multifamily conversion-retirement. The estimated coefficient on NEW TIGHT times old stock measures the number of single (or multifamily) units per old single (or multifamily) unit initially available, which are destroyed for each new unit constructed per vacant acre. The variable specification assumes that this rate of demolition (units destroyed as a fraction of old units) is proportional to the gross density of new units (new construction per acre), and inversely proportional to the fraction of land which is vacant.

At the mean sample values of new construction per vacant acre, the demolition rate attributable to new construction is .0061 for old single family units and .0033 for old multis. The overall demolition rate as a fraction of all old units due to new construction can be calculated by using a weighted average of the estimated coefficients from the separate structure-type equations. The two estimated coefficients are weighted by the importance of the stock to which they refer as a fraction of all old stock. This estimated rate is .0015, at the mean. The range of values is also interesting, from virtually zero in the open suburbs, to a rate of .43 and .23 for old single and old multifamily units, respectively, in the Back Bay-Beacon Hill district in the heart of the city of Boston.

One interesting question is what this translates into in terms of units demolished per unit built. The demolition of units of the ith structure type per unit of new construction can be calculated as the estimated coefficient times the gross density of old units divided by the fraction of land vacant. Since the fraction of land vacant and the availability (as measured by gross density) of likely-to-be-demolished units varies considerably over the sample zones, so does the predicted number of demolitions per newly-built unit.

At the mean value of old single family (or multifamily) stock per acre divided by fraction of land vacant, there are .0075 single family units destroyed per new unit built, and .0083 multifamily units demolished. Thus, in total, at the mean, each sixty-three new units built account for about one unit's destruction. The range of values is again from virtually zero in areas with almost no old units and much vacant land to fairly significant demolitions in denser areas. The Back Bay-Beacon Hill district with the highest ratio of old singles and old multis to vacant land, has the greatest number of predicted demolitions per unit of new construction (.39 and .33 for singles and multis, respectively, implying that for every seven new units built, five are demolished).

Another interesting derivative of the set of estimated conversion-retirement equations is an estimate of the price elasticity of housing supply through the conversion-retirement mode.[4] Since the price variable appears more than once in each equation, first a total coefficient on price is calculated (as already described), and then translated into an elasticity. For the single and multi conversion-retirement equations, there is both a direct effect of price, through the explicit appearance of the price variables in the equation, and an indirect effect of price through its effect on the endogenous variable NEW TIGHT (new construction per vacant acre). We want to calculate a value for

$$\frac{\Delta Ci/Si60}{\Delta P/P60},$$

where ΔCi is the change in number of units of structure type i resulting from conversion-retirement activity, Si60 is the 1960 stock of structure type i, and $\Delta P/P60$ is the decade percentage change in price.

Temporarily setting aside the indirect effect of price on demolitions to make room for new construction, the price elasticity of conversion-retirement of structure type i is a weighted average of the estimated price coefficients, the weights being the importance of the origin stock type relative to the total stock of structure type i. The intuitive reason for this is clear: if, for example, the price coefficient in the UZ·OLD SINGLE_{60} brackets of the CONV MULTI equation is positive, this implies that the probability of a single-to-multi conversion is an increasing function of price change; and a given probability will cause a larger increment to the multi stock, the larger the number of unzoned old single family units to which the probability is applied.

For single family and multifamily units, the indirect effect of price must be included as well. Since price increases encourage new construction of housing units, and new construction is seen as a cause of demolitions of single and multi units in zones with little vacant land, the indirect effect of price is negative. As prices rise, demolitions are encouraged. The strength of this indirect effect varies over the metropolitan area as the new construction price elasticity and the amount of vacant land vary.[5] By construction, the indirect effects of price on the demolition rate are stronger where old units are a greater share of the relevant stock and the net residential density is higher.

The complete effect of price on conversion-retirement supply is calculated by combining the direct and indirect elasticities. Evaluated at the mean (excluding those zones where the starting stock of that structure type is zero and hence the elasticity is infinite), these full price elasticities of conversion-retirement supply of single family and multifamily units are .068 and .507, respectively. The full price elasticity of conversion-retirement supply of apartments does not include an indirect (negative) term, and is equal to .649 at the mean (excluding those zones with no apartment units in 1960).

The price coefficients from the three structure type conversion-retirement equations can be combined to calculate the overall price elasticity of conversion-retirement supply. This elasticity is calculated as the change in the number of housing units resulting from conversion-retirement relative to the total beginning stock attributable to a 1% change in prices over the decade. This price elasticity of the conversion-retirement supply mode varies with the importance of the several origin stock types in the total housing stock, and the net residential density of old singles and multifamily units. The former occurs because the more origin units there are, the more outcome units there will be, given the impact of price change on the conversion-retirement probability. The latter reflects the fact that the more old singles or multis there are in any area, the more likely they are to be hit by the need to make room for new construction. The price elasticity of conversion-retirement supply is equal to .044 evaluated at the mean. It is worth noting that these estimates imply that conversion-retirememt supply is generally less responsive to price changes than is new construction supply.[6] But its positive value implies that the conversion-retirement supply mode does augment the new construction responsiveness to price, as the discussion of Figure 1 suggested.

APPENDIX

Definition of Empirical Variables

CONV SINGLE	Difference between 1970 and 1960 number of single family housing units minus new single family units built between 1960 and 1970.
CONV MULTI	Difference between 1970 and 1960 number of multifamily housing units minus new multifamily units built between 1960 and 1970. A multifamily unit is in a structure containing two to four units.
CONV APART	Difference between 1970 and 1960 number of apartment units minus new apartment units built between 1960 and 1970. An apartment unit is in a structure containing five or more units.
OLD $SINGLE_{60}$	Estimate of number of 1960 single family units built before 1940.
OLD $MULTI_{60}$	Estimate of the number of 1960 multifamily housing units built before 1940.
OLD $APART_{60}$	Estimate of the number of 1960 apartment units built before 1940.
DETER $SINGLE_{60}$	Estimate of the number of 1960 single family units deteriorating.
DETER $MULTI_{60}$	Estimate of the number of 1960 multifamily units deteriorating.
DETER $APART_{60}$	Estimate of the number of 1960 apartment units deteriorating.

$\text{UNZONED OLD SINGLE}_{60} = \text{UZ} \cdot \text{OLD SINGLE}_{60}$	Estimate of the number of 1960 single family units built before 1940 which are not subject to minimum lot size zoning restrictions.
PZ	Fraction of residential and vacant land zoned for minimum lot sizes greater than 25,000 square feet.
UZ	= 1. - PZ
$\frac{\Delta \text{PRICE}}{\text{PRICE}_{60}}$	Percentage change in housing unit price 1960 to 1970, for unchanged housing units existing in both 1960 and 1970.
$\frac{\Delta \text{VAC RATE}}{\text{VAC RATE}_{60}}$	Percentage change in housing unit vacancy rate, 1960 to 1970.
NEW TIGHT	Number of housing units built between 1960 and 1970 per acre of vacant land initially available.

NOTES

[1]This model of new housing construction, including more detailed treatment of the urban land market, is presented in Bradbury (1976) and summarized in Bradbury et al. (1975).

[2]An interesting analysis of this process is contained in Hufbauer and Severn (1974). They ignore the actual costs of demolition and nonetheless find that the price per unit of housing service has to increase by three to five times before a structure is demolished and replaced by one with the current cost-minimizing factor proportions.

[3]If price enters the equation twice, multiplied by two stocks, then a "standard error" corresponding to the total coefficient derived above can be calculated as the square root of $(T1^2 s_1^2 + T2^2 s_2^2 + T1 \cdot T2 \cdot s_{12})$, where T1 and T2 are the sample means of the two stock types and s_1^2, s_2^2, and s_{12} are the estimated variances and covariance, respectively, of the estimated coefficients.

[4]It is important to recall that the measure of housing supply is the number of housing units. Hence all the elasticities to be derived represent the responsiveness of the stock of housing units to changes in price. These measures are not production elasticities; they do not reflect the responsiveness of the supply of housing services, nor do they control for quality or size of housing unit in any way. Also, because vacancy rate changes are part of the market clearance process, these price elasticities do not reflect the total responsiveness of housing supply to demand conditions.

[5]The price elasticity of new construction housing supply used in the calculation of these indirect effects of price on conversion-retirement activity is derived from an equation for new construction estimated over the same data set (see Bradbury 1976 or Bradbury et al., 1975). The estimated price elasticity of the stock through new construction is .152 evaluated at the sample mean of geographic zones in the Boston metropolitan area.

[6]See preceding note for comparable new construction price elasticity estimate. This result is consistent with Richard Muth's analytically derived prediction that conversion supply involving parcels of land with existing capital improvements is less price elastic than new construction (1969, ch. 3).

REFERENCES

Amemiya, Takeshi. 1974. The non-linear two-stage least squares estimator. Journal of Econometrics, 2, 2:105-110.

Bradbury, Katharine. 1976. Housing supply in a metropolitan area. Ph.D. dissertation, Massachusetts Institute of Technology.

Bradbury, Katharine; Engle, Robert; Irvine, Owen; and Rothenberg, Jerome. Forthcoming. Simultaneous estimation of the supply and demand for household location in a multi-zoned metropolitan area. In NBER studies in income and wealth, ed. Gregory K. Ingram. Conference on Economics of Residential Location and Urban Housing Markets, Cambridge, Massachusetts, May 15-16, 1975.

Hufbauer, G.C., and Severn, B.W. 1974. The economic demolition of old buildings. Urban Studies, 2:349-51.

Kelejian, Harry H. 1971. Two-stage least squares and econometric systems linear in parameters but nonlinear in the endogenous variables. Journal of the American Statistical Association, 66, 334:373-74.

Muth, Richard F. 1969. Cities and housing. Chicago: University of Chicago Press.

U.S. Bureau of the Census. 1973. Components of inventory change. Census of Housing 1970, Boston Massachusetts Standard Metropolitan Statistical Area, HC(4)-3. Washington, D.C.: U.S. Government Printing Office.

ADDITIONAL DATA SOURCES

Metropolitan Area Planning Council (MAPC). 1972. Residential zoning as of March 1972. Boston.

U.S. Bureau of the Census. 1960. Census Tract Summary Tape. Boston area. Census of Housing and Population 1960.

U.S. Bureau of the Census. 1963. States and small areas. Part 4: Iowa-Massachusetts. Census of Housing 1960, vol. 1. Washington, D.C.: U.S. Government Printing Office.

U.S. Bureau of the Census. 1970. Fifth Count Tape. Minor Civil Divisions, Census Tracts. Census of Housing 1970.

U.S. Bureau of the Census. 1972. Housing characteristics for states, cities, and counties. Part 23: Massachusetts. Census of Housing 1970, vol. 1. Washington, D.C.: U.S. Government Printing Office.

Vogt, Ivers, and Associates. 1964. 1963 comprehensive land use inventory report. Boston.