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A DYNAMIC THEORY OF INDIVIDUAL LABOR SUPPLY

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## ABSTRACT

The purpose of this study is to make some progress in integrating the standard analysis of an individual worker's labor supply decisions with the job search approach to this topic. This will be achieved by allowing workers to make allocation of time decisions within the context of a labor market with imperfect information about wages available, instead of the usual perfect information market structure. Alternatively, the model constructed can be perceived as a job search model in which workers can vary their search intensity. It will be shown that this approach leads to new, and empirically relevant, insights into the labor supply decision. In this paper only the short-run labor supply decision will be considered. Nevertheless, the framework developed appears general enough to contemplate analyzing long-run labor supply decisions in a later study.

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The most commonly presented theory of an individual's labor supply decision[1] analyzes the percentages of a given period a worker will select to (a) work, and (b) enjoy leisure, given there is a known, fixed wage rate associated with working. The well-known, single-period version of this class of models involves a worker whose utility function is a positive function of leisure time consumed and income earned. The worker is assumed to face a known wage rate, w', such that the worker's income will be w'h if h percent of the period is spent working. The greater the income earned, the smaller the percentage of the period spent enjoying leisure. It has been shown many times that a utility maximizing worker will choose leisure and work times such that the ratio of the marginal utilities of leisure and income equals the wage rate. Hence, given the usual convexity assumptions are made about the individual's preferences, there is a unique utility maximizing choice of labor time for any known, fixed wage rate. The individual labor supply function can now be derived by determining the effects on labor time supplied of changes in the known wage rate.

In this study the assumption that a worker faces a known wage rate is dropped. Instead, it is assumed that the worker may be offered employment at different wage rates by different firms. Further, the worker does not know which firm is offering any particular wage before it is visited. In this environment a worker may turn down a job offer and hence supply no labor time in a period, not because of a preference for leisure but out of a desire to obtain another, better, offer. It is this element of the labor supply decision that is analyzed in the job search models. However, the job search models presented in the literature so far have ignored the allocation of time decision highlighted in the standard theory.

In job search models[2] presented to date, it is assumed that there is a known probability that an unemployed worker will receive a job offer in a given period of time. Associated with a job offer is a fixed wage per period. Different job offers may imply different wages. Hence a job offer is perceived as a random draw from a known distribution of wage offers in the market. If a worker accepts an offer, he/she is assumed to work at the offered wage per period until retirement. If an offer is rejected, the worker remains unemployed until an acceptable offer is found. The problem for a worker in such an environment is the determination of the set of acceptable wage offers in each period of unemployment. It has been shown that the best strategy for an unemployed worker can be characterized by a reservation wage in each period of unemployment. The worker will accept a job offer if and only if the wage offered is at least as great as the relevant reservation wage.

In the present study it is assumed that a worker can increase the probability of obtaining a job offer in a period by sacrificing leisure. Hence an unemployed worker not only determines the set of acceptable wage offers but how much time to spend looking for a job, i.e., not enjoying leisure. In section 1 the utility maximizing strategy of an unemployed worker is characterized given a particular duration of unemployment. This involves the selection of a reservation wage and leisure choice in the period. Further, it is shown how these utility maximizing choices change as the duration of unemployment increases.

With few exceptions, it has been assumed in constructing job search models, that workers do not look for another job while employed. Recently, it has been shown that this assumption can only be justified if the cost of search while employed is great relative to the cost of search while unemployed.[3] In this study employed-worker job search will be considered. In section 2 the problems faced by an employed worker are analyzed. In this case, the worker has to select the percentage of any period to spend (a) working, (b) enjoying leisure, and (c) looking for another job. It will be shown that an employed worker will select to look for another job if and only if the wage rate faced is less than a calculated wage rate, termed the stopping wage. This leads to complete characterization of the strategy of a worker in a market with incomplete information about job opportunities. A job offer will be accepted by an unemployed worker if and only if the wage rate is at least as great as the relevant reservation wage. A wage rate offered less than the stopping wage, but at least as great as the reservation wage, implies the worker will accept the offer but continue to look for a job while employed. An offer with a wage rate at least as great as the stopping wage implies the worker will accept the offer and not look for a job while employed. Employed workers will accept any job offer received if and only if the wage rate offered is greater than their current wage.

In the later sections testable predictions of the model developed in the first two sections are specified. In section 3 predictions about the expected duration of unemployment and the expected post-unemployment are derived when the parameters of the model developed are held constant. In section 4, the sensitivity of these predictions to changes in the unemployment insurance scheme is considered. In the final section the effects of changes in the demand for labor are analyzed. In each of the last three sections the primary objective is to derive testable predictions about unemployed worker behavior. Due to limitations on space, the implications from the model on job turnover is only briefly considered. It should be noted that special assumptions will be made in the model to be developed so that precise predictions can be made. More general restrictions can be made without disturbing the basic results obtained. However, the more restrictive assumptions will be used so that the exposition will not become overly technical.

## 1. THE LEISURE AND SEARCH DECISIONS OF UNEMPLOYED WORKERS

In this section a model of the labor market is described that is similar in most respects to others used in the job search literature.[4]

This framework, plus new assumptions about worker behavior, will be used to analyze the time an unemployed worker assigns to looking for a job and enjoying leisure. Further, it will be shown how these choices change as the duration of unemployment increases.

Consider the problems faced by an unemployed worker looking for a job. Suppose time can be divided into periods of equal length, termed search periods. In each search period an unemployed worker will select to spend a certain percentage of the time looking for a job. The length of each search period is selected so that, no matter how much time the worker allocates to looking for a job, at most one job offer will be received by that worker. Workers have limited information about job opportunities. This implies a worker may be offered a job in a period with a wage that is unacceptable, or recieve no offer at all.

Let us denote the percentage of a period a worker assigns to looking for a job. A worker not looking for a job is assumed to be enjoying leisure. Hence, let $\ell=(1-s)$ denote the percentage of a period a worker enjoys leisure. The probability a worker obtains a job offer depends on leisure time selected. Specifically, let $\pi(s)$ denote this probability and assume

$$\pi'(s) > 0 \text{ and } \pi''(s) \leq 0. \tag{1}$$

Hence the more leisure sacrificed in a period, the greater the probability the worker obtains a job offer. The rate of increase in this probability declines as s increases.

Suppose an unemployed worker obtains a job offer in a particular period. Associated with a job offer is a wage rate per search period. Uncertainty in the market is such that the wage rate offered by different firms may not be the same. Let $F(w)$ indicate the distribution function describing the wage offers in the market. Hence $F(w'')$ denotes the proportion of wage offers in the market less than wage rate $w''$. Workers are assumed not to know which firm is offering a particular wage and therefore do not systematically select the firm to search in a period. It appears reasonable to assume a job offer can be perceived as a random draw from the distribution of wage offers. It follows that $\pi(s)[1-F(w')]$ denotes the probability an unemployed worker, who selects to look for a job for s percent of a period, obtains a job offer with a wage rate at least as great as $w'$. Note that it has been assumed that sacrificing leisure increases the probability of obtaining a job offer but does not influence the likelihood of receiving a "high" wage offer, given an offer is made. This is clearly a special case, but one that much simplifies the exposition. To further simplify the analysis, but in this case without any real loss of generality, it is assumed $F(w)$ is differentiable and let $F'(w) = f(w)$.

A worker is assumed to receive Unemployment Insurance (UI) payment $x_t$ in search period t of a spell of unemployment. The most commonly used (UI) scheme implies a worker receives a fixed amount in each of the first $t''$ periods of a spell of unemployment. No (UI) payments are received by workers who have been unemployed more than $t''$ periods. Formally, let

$$x_t = \begin{cases} u, & \text{if } t \leq t''. \\ 0, & \text{if } t > t''. \end{cases} \qquad (2)$$

To highlight the issues under consideration only the simplest possible preference structure for each worker is assumed. Specifically, assume that the lifetime utility of a worker is the discounted sum of each future single-period utility, and the form of each single-period utility function is the same. Let $v(y,\ell)$ denote the single-period utility accruing to a worker who chooses to spend $\ell$ percent of a period enjoying leisure and obtains income y in a period. This function is assumed to be unique up to a linear combination. Assume

$$v_y > 0,\ v_\ell > 0,\ v_{yy} < 0,\ v_{\ell\ell} < 0, \text{ and} \qquad (3)$$
$$v_{yy}v_{\ell\ell} - (v_{y\ell})^2 > 0.$$

The restrictions placed on the single period utility function in (4) insure the function is strictly concave. The precise results obtained in this section depend on the nature of the utility function assumed. Nevertheless, similar results can be obtained if any separable[5] (by search period) lifetime utility function, with the usual restrictions, is assumed.

Suppose an unemployed worker is offered a job with wage rate $w'$ per period. This offer can be accepted or rejected by that worker. If the offer is rejected, assume the worker cannot return to accept it later on. Let $\psi(w')$ indicate the maximum expected lifetime utility to a worker who accepts an offer with wage rate $w'$. The form and nature of this function will be discussed in some detail in section 2. For the present it is sufficient to make two assumptions. First, suppose the value of this function is positively related to the wage rate offered. This assumption will be justified in the next section. Second, assume the value of this function is independent of how long the worker has been unemployed, or how many periods remain until the worker retires. This assumption appears to be a reasonable approximation to make for all workers except those about to retire. A formal justification can be obtained if it is assumed workers have an infinite life. The generalization to include the situation where the payoff function to accepting a job declines with the age of a worker does not alter the results in any significant way. Including such a complication in the analysis does make the exposition more difficult and will therefore be ignored in the sequel.

Assume in the environment described above that each worker attempts to maximize expected future discounted lifetime utility.[6] Hence, given a worker has been offered a job with wage rate w', the expected payoff to accepting this job has to be compared to the expected payoff to remaining unemployed at least one more period. However, the expected payoff to remaining unemployed depends on which job offers will be accepted, if offered, in future periods, i.e., the search strategy of the worker. The payoff from the search strategy that yields the greatest expected payoff should be compared with the expected payoff from accepting the job. The best search strategy when leisure is not a choice variable is well known. Specifically, it has been shown in this case the search strategy that yields the greatest expected payoff involves the worker selecting a reservation wage in each period of unemployment. Any job offer made in a period will be accepted if and only if the wage rate associated with it is at least as great as the relevant reservation wage. Among the set of possible reservation wages a worker may use in a period there is one that yields a payoff at least as great as all others. This is called the <u>optimal reservation wage</u>.

If leisure is allowed to be a choice variable for a worker, the search strategy that maximizes the expected payoff involves the selection of a doubleton $(w_t^*, \ell_t^*)$ for any $t > 0$, where $w_t^*$ is the optimal reservation wage, and $\ell_t^*$ is the optimal leisure choice for a worker in period t of a spell of unemployment. Suppose an unemployed worker selects to use reservation wage $\bar{w}$ and enjoy leisure $\bar{\ell}$ in period t of a spell of unemployment and then utilizes the relevant optimal reservation wage and leisure choice in any future period of unemployment. Let $\mu_t(\bar{w},\bar{\ell};u,t'')$ denote the expected discounted lifetime utility to a worker that utilizes such a strategy, given the parameters u and t" describe the UI scheme. From the above it follows

$$\mu_t(\bar{w},\bar{\ell};u,t'') = v(x_t,\bar{\ell}) + \frac{\pi(1-\bar{\ell})\Pr(w \geq \bar{w})}{1 + r} E[\Psi(w)|w \geq \bar{w}] + \frac{[1 - \pi(1-\bar{\ell})\Pr(w \geq \bar{w})]}{1 + r} \mu_{t+1}(w_{t+1}^*,\ell_{t+1}^*;u,t''), \tag{4}$$

where r is the discount rate and $w_{t+1}^*$ and $\ell_{t+1}^*$ are the reservation wage and leisure choice that maximize the worker's expected payoff in period t + 1 of unemployment.

Assumptions have been made that guarantee the differentiability of (5) with respect to $\bar{w}$ and $\bar{\ell}$. The first order conditions for the maximization of (5) with respect to $\bar{w}$ and $\bar{\ell}$ are

$$\frac{\partial\mu_t}{\partial\bar{w}} = \frac{\pi(1-\bar{\ell})f(\bar{w})}{1 + r} [\mu_{t+1}(w_{t+1}^*,\ell_{t+1}^*;u,t'') - \Psi(\bar{w})] = 0 \tag{5}$$

and

$$\frac{\partial\mu_t}{\partial\ell} = v\ell - \frac{\pi'(1-\bar{\ell})}{1 + r} \int_{\bar{w}}^{\infty} [\Psi(w) - \mu_{t+1}(w_{t+1}^*,\ell_{t+1}^*;u,t'')] f(w)dw \tag{6}$$

for any $t > 0$, where $\pi'(s) = -\frac{\partial\pi(s)}{\partial\ell}$. Assuming an interior solution, i.e., $(1-\ell_t^*)f(w_t^*) \neq 0$ we have

$$\Psi(w_t^*) = \mu_{t+1}(w_{t+1}^*, \ell_{t+1}^*; u, t") \tag{7}$$

and

$$v_\ell = \pi'(1-\ell_t^*)T[\Psi(w_t^*)], \tag{8}$$

where $T[\Psi(w_t^*)] = \frac{1}{1+r} \int_{w_t^*}^{\infty} [\Psi(w) - \Psi(w_t^*)]f(w)dw$.

Equation (7) implies that, given an interior solution, the expected payoff to accepting an offer with wage $w_t^*$ in period t of a spell of unemployment is equal to the maximum expected payoff to remaining unemployed at least one more period. The second condition (8) implies that, given an interior solution, the worker will select a search time such that the marginal utility of leisure equals the marginal expected gain to search time. If an interior solution does not exist then one of two situations will hold. The worker will either choose a zero search time and hence receive no job offers, or select a reservation wage so any possible job offer in the market would be accepted. It will be assumed in the rest of this study that an interior solution does exist. The results obtained here can easily be extended to include the noninterior case.

From (4), (7) and (8) it follows that

$$\Psi(w_{t-1}^*) = v(x_t, \ell_t^*) + \pi(1-\ell_t^*)T[\Psi(w_t^*)] + \frac{\Psi(w_t^*)}{1+r} \tag{9}$$

for any $t > 0$. Equation (9) demonstrates how the optimal choice of leisure and reservation wage in a period are related to the optimal choices in the previous period. It will now be shown how the optimal choices vary with the duration of unemployment. Specifically, it is demonstrated that the optimal choice of leisure and reservation wage of a worker do not increase as the duration of unemployment increases.

Suppose a worker has been unemployed at least $t''$ periods in a spell of unemployment. This worker is faced with essentially the same problem in each search period he/she remains unemployed as the number of possible job offers is unbounded and no UI payments will be received. Hence the expected payoff to a given search strategy will be the same in each period of unemployment after $t''$ periods. This is not the case if a worker has been unemployed less than $t''$ periods. Here the expected payoff to a given search strategy will depend on the number of periods remaining until UI payments are stopped. The following proposition states the precise relationships that will be important in the sequel.

Proposition 1

(a) $\mu_t(\bar{w},\bar{\ell};u,t'') = \mu_{t+1}(\bar{w},\bar{\ell};u,t'')$ for any $\bar{w}$ and $\bar{\ell}$, if $t > t''$.

(b) $\mu_t(\bar{w},\bar{\ell};u,t'') > \mu_{t+1}(\bar{w},\bar{\ell};u,t'')$ for any $\bar{w}$ and $\bar{\ell}$, if $t \leq t''$ and $\pi(1-\bar{\ell})f(\bar{w}) \neq 0$.

Proof

The proof of this Proposition is given in the Appendix.

The above Proposition states that the expected payoff to a given search strategy, assuming it yields an interior solution, declines with $t$ for the first $t''$ periods of unemployment. After $t''$ periods of unemployment, the expected payoff to any given strategy remains the same. This result is now used to establish the major result of this section.

Proposition 2

(a) $w^*_{t-1} > w^*_t$, if $t \leq t''$.

(b) $w^*_{t-1} = w^*_t$, if $t > t''$.

(c) $\ell^*_{t-1} > \ell^*_t$, if $t \leq t''$.

(d) $\ell^*_{t''} > \ell^*_{t''+1}$, if $v_{y\ell} > 0$.

(e) $\ell^*_{t-1} = \ell^*_t$, if $t > t''+1$.

Proof

The proof of this Proposition is given in the Appendix.

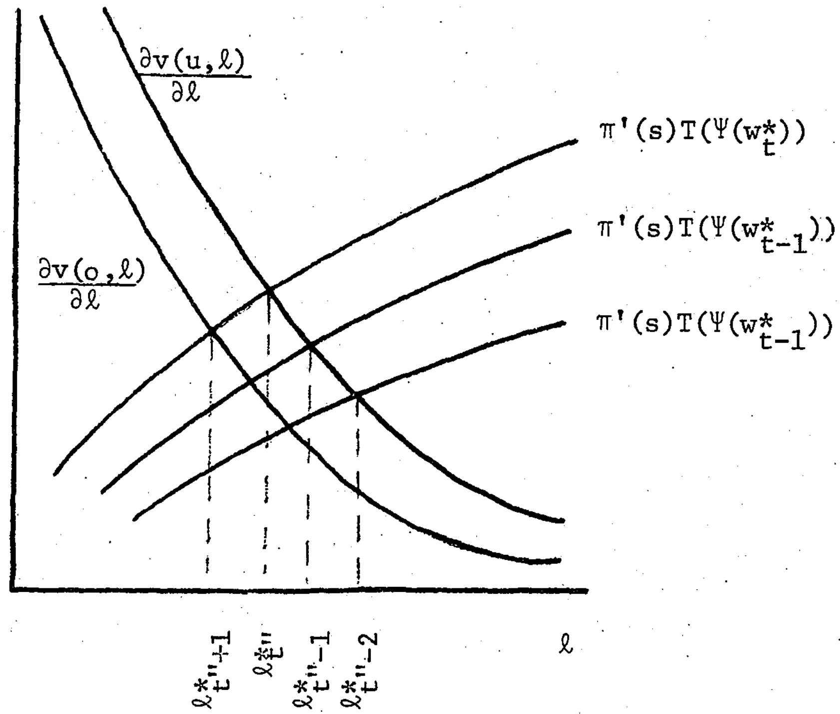
The above Proposition implies that the optimal reservation wage and leisure choice decline in each of the first $t''$ periods of a spell of unemployment. These optimal choices remain constant in each period for a worker who has been unemployed at least $t''+1$ periods. The results with respect to leisure choice in each period are illustrated in Figure 1.

So far only unemployed workers have been considered. In the next section the problems faced by employed workers are analyzed.

## 2. THE WORK-LEISURE-SEARCH DECISIONS OF EMPLOYED WORKERS

In the previous section $\psi(w')$ denoted the expected discounted lifetime utility accruing to a worker who accepts a job offer with wage rate $w'$ per period. Further, it was assumed that this expected payoff function was positively related to the wage rate offered. In this section the nature of this relationship is investigated in some detail. It will be shown how a worker's choice of work, search, and leisure times in each period is related to the wage rate faced.

Figure 1


$\frac{\partial v(u,\ell)}{\partial \ell}$
$\pi'(s)T(\Psi(w^*_t))$
$\pi'(s)T(\Psi(w^*_{t-1}))$
$\frac{\partial v(o,\ell)}{\partial \ell}$
$\pi'(s)T(\Psi(w^*_{t-1}))$
$\ell^*_{t''+1}$
$\ell^*_{t''}$
$\ell^*_{t''-1}$
$\ell^*_{t''-2}$
$\ell$

Suppose a worker is employed at wage rate $w'$ per period. Each period the worker has to select the percentage of the period to spend (a) working, $h$, (b) looking for a job, $s$, and (c) enjoying leisure, $\ell$, subject to $h+s+\ell = 1$. As it has been assumed the worker's working life is unbounded, the choice that maximizes the expected future lifetime utility will remain the same in each search period if the wage rate faced remains the same. Hence $\psi(w')$ can be interpreted as the expected lifetime utility accruing to a worker employed at a firm offering wage rate $w'$, given the optimal choices of work, search, and leisure are made in each future period.

If an employed worker spends a strictly positive percentage of the period looking for a job, it is possible that a job offer will be received. It is assumed there are no fixed costs to changing jobs. Hence a worker employed at wage rate $w'$ will accept another job offer if and only if it implies a wage rate greater than $w'$. Employed workers who look for another job do so to increase their expected future utility at the cost of reducing income and/or leisure in the current period.

Assume a worker, employed at wage rate $w'$, selects leisure, work, and search time percentages $\ell$, $h$, and $s$ respectively in the next period, but then chooses the expected utility maximizing choices in each future period. Let $\phi(y,\ell,s;w')$ denote the expected lifetime utility to a worker who utilizes this strategy, where $y = w'h$ is the income obtained in the current period. This function can be written as

$$\phi(y,\ell,s;w') = v(y,\ell) + \frac{\pi(1-\ell)\Pr(w > w')}{1 + r} E[\psi(w) | w > w'] + \frac{[1 - \pi(1-\ell)\Pr(w > w')]}{1 + r} \psi(w') \tag{10}$$

subject to

$$y = w'(1 - \ell - s). \tag{11}$$

A utility maximizing worker will select y, $\ell$, and s, such that (11) is satisfied and

$$\phi(y,\ell,s;w') \geq \phi(y,\ell,s;w') \text{ for any } y,\ \ell,\ s. \tag{12}$$

Assumptions have been made to guarantee the differentiability of (10) with respect to y, $\ell$, and s. Hence the calculus can be used to obtain the utility maximizing choices. The first order conditions, given strictly positive percentages of work, search, and leisure are optimal, and can be written as

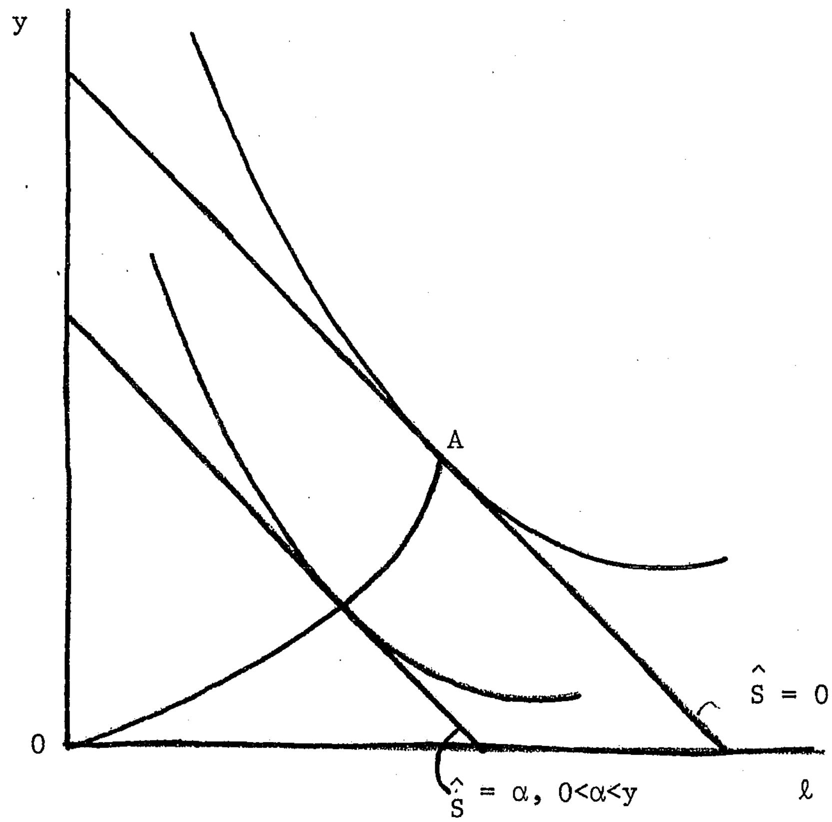
$$v_y - \lambda = 0; \tag{13a}$$

$$\pi'(1-\ell)T(w') - w'\lambda = 0; \tag{13b}$$

$$v - w'\lambda = 0; \tag{13c}$$

and (12), where $\lambda$ is the Lagrangian multiplier associated with the income constraint (12). Hence, given an interior solution, a worker will select leisure and work times such that the ratio of marginal utilities of leisure and income equals the wage rate w'. Further, the worker chooses a search time such that the marginal utility of leisure equals the marginal gain to search time. The second order conditions for a maximum are guaranteed from the assumptions made about the individual period utility function in (4), and the restrictions placed on expected gain to search time in (1). Figure 2 illustrates the leisure-work time percentages given (a) the wage rate w', and (b) any fixed percentage of time allocated to looking for a job. In figure 2, OA is the locus of utility maximizing choices of leisure and work given any search time choice.

Figure 2



When will a worker select to search for another job while employed? To answer this question formally let

$$\eta(\tilde{y},\tilde{\ell};w') = \max_{y,\ell}.\phi(y,\ell,s;w'), \text{ subject to } s = 0.$$

Hence $\eta(\tilde{y},\tilde{\ell};w')$ denotes the maximum expected payoff to an employed worker, employed at wage rate $w'$, given no time is assigned to looking for another job. It follows directly that

$$\eta(\tilde{y},\tilde{\ell};w') \leq \phi(\hat{y},\hat{\ell},\hat{s};w') \text{ for any } w'$$

and

$$\eta(\tilde{y},\tilde{\ell};w') < \phi(\hat{y},\hat{\ell},\hat{s};w') \text{ if and only if } \hat{s} > 0. \tag{14}$$

The next proposition makes some progress in discovering when employed workers will search for another job. It states that there exists a wage rate $\vec{w}$ such that an employed worker facing a wage rate at least as great as $\vec{w}$ will not choose to look for another job. Further, there exists a wage rate, $\overleftarrow{w}$, such that an employed worker facing a wage rate no greater than w will select to look for another job.

<u>Proposition 3</u>

(a) There exists a $\vec{w}$ such that if a worker is employed at any wage rate $w' \geq \vec{w}$, $\eta(\tilde{y},\tilde{\ell};w') = \phi(\hat{y},\hat{\ell},\hat{s};w')$.

(b) There exists a $\overleftarrow{w}$ such that if a worker is employed at any wage rate $w' < \overleftarrow{w}$, $\eta(\tilde{y},\tilde{\ell};w') < \phi(\hat{y},\hat{\ell},\hat{s};w')$.

Proof

Claim (a) can be established if it is noted that a worker employed at a wage rate at least as great as any other wage rate in the market will not select to look for another job. This follows as the probability of obtaining a better job in this case is zero. Of course, $\vec{w}$ may be less than the highest wage offer in the market. As the expected payoff to searching while employed if the wage rate faced declines, if that wage is less than $\vec{w}$, claim (b) can be established. Note that this wage may be less than any wage rate offered in the market.

The Proposition does not rule out the possibility an employed worker chooses to search for another job when employed at a wage less than $\vec{w}$ but at least as great as $w_t^*$. The next proposition states certain relationships that will be utilized in the sequel.

Proposition 4

If $v_{y\ell} \geq 0$ and $\hat{s} > 0$, then

(a) $\dfrac{d\hat{h}}{dw'} \geq 0$ only if $\dfrac{d\hat{s}}{dw'} < 0$, and

(b) $\dfrac{d\hat{s}}{dw'} > 0$ implies $\dfrac{d\hat{h}}{dw'} < 0$ and $\dfrac{d\hat{\ell}}{dw'} > 0$.

Proof

The proof of Proposition 4 is presented in the Appendix.

The above Proposition establishes that, given an employed worker selects to look for another job, an increase in the wage rate faced reduces the amount of time allocated to looking for a job if the hours of work function is positively related to the wage rate. Further, the amount of time allocated to search decreases as the wage rate faced increases, then the hours of work function of the worker is negatively related to the wage rate and leisure time increases with the wage rate faced. The next proposition follows directly from Propositions 3 and 4, and therefore no proof will be presented.

Proposition 5

If an individual's supply of hours of work function has a non-negative slope and $v_{y\ell} \geq 0$, then $\vec{w} = \overleftarrow{w}$, where $\vec{w}$ and $\overleftarrow{w}$ are defined in Proposition 3. Hence, given the above conditions, an employed worker will select a positive search time if and only if the wage rate faced $w' \geq \vec{w}$ $(=\overleftarrow{w})$.

In the remainder of this section it will be assumed the hypotheses made in the above proposition hold true. This guarantees employed workers will only select to look for another job if they are employed in a job with a wage rate less than $\vec{w}$. In this section so far, it has been assumed that the worker is employed. However, from (6), we know an unemployed worker in period t of a spell of unemployment will only select to become employed if a wage offered in that period is at least as great as the relevant optimal reservation wage. In terms of the notation developed in this section an unemployed worker in period t of unemployment will only accept a job offer if the wage rate offered, w', is such that

$$\phi(\hat{y},\hat{\ell},\hat{s};w') \geq \mu_{t+1}(w^*_{[t+1},\ell^*_{t+1};u,t'').$$

However, the above inequality will hold if and only if $w' \geq w_t^*$. Hence, if a worker receives an acceptable offer in period t of unemployment and $w_t^* \geq \vec{w}$, that worker will not look for a job while employed. Figure 3a illustrates this result. If $w_t^* < \vec{w}$, there is a probability a worker, who obtains an acceptable offer in period t of unemployment, receives an offer $w'$ such that $w_t^* \leq w' < \vec{w}$, and hence looks for a job while employed. This situation is illustrated in Figure 3b. The final proposition of this section summarizes the results demonstrated above, and completely characterizes the strategy of workers with respect to wage rates and job search.

Proposition 6

There exists a $\bar{\bar{t}}$ such that $w_t^* \geq \vec{w}$ if and only if $t \leq \bar{\bar{t}}$ ($\bar{\bar{t}}$ may equal zero or infinity).

If an acceptable offer is found in the first $\bar{\bar{t}}$ periods of unemployment, the worker will not look for a job while employed.

If an acceptable offer is found after $\bar{\bar{t}}$ periods of unemployment, the worker will look for another job while employed if and only if $w' < \vec{w}$, where $w'$ is the wage rate offered.

Proof

The claims made above follow from Proposition 2 and the arguments made above. Figure 4 illustrates the claim where $\bar{\bar{t}} = 4$.

The value of $\bar{\bar{t}}$ will depend on (a) the distribution of wage offers, (b) UI payments relative to the wage rates in the market, and (c) the worker's preferences. An implication of Proposition 6 is that workers who suffer a longer duration of unemployment than another group of workers are more likely to search while employed, and hence quit their jobs.

Figure 3a

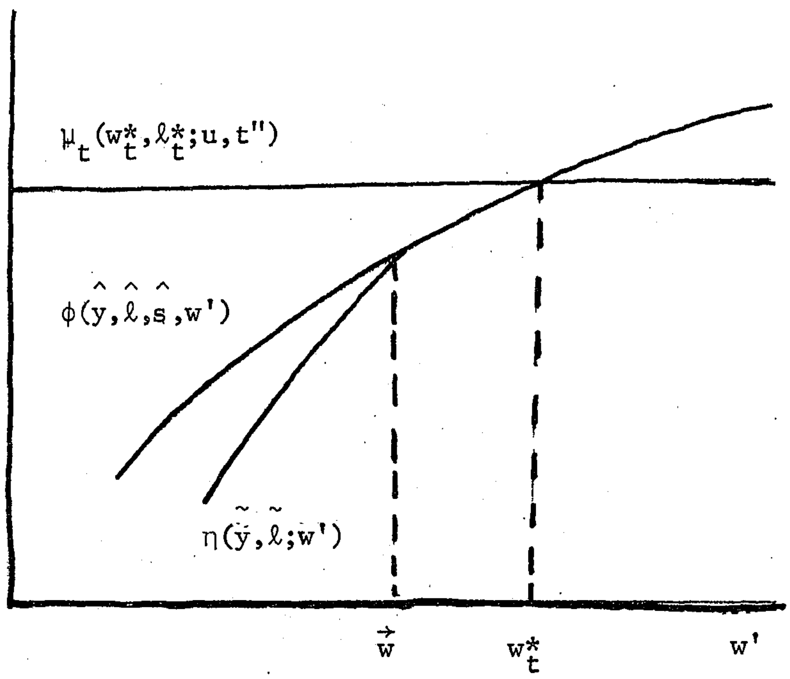



Figure 3b

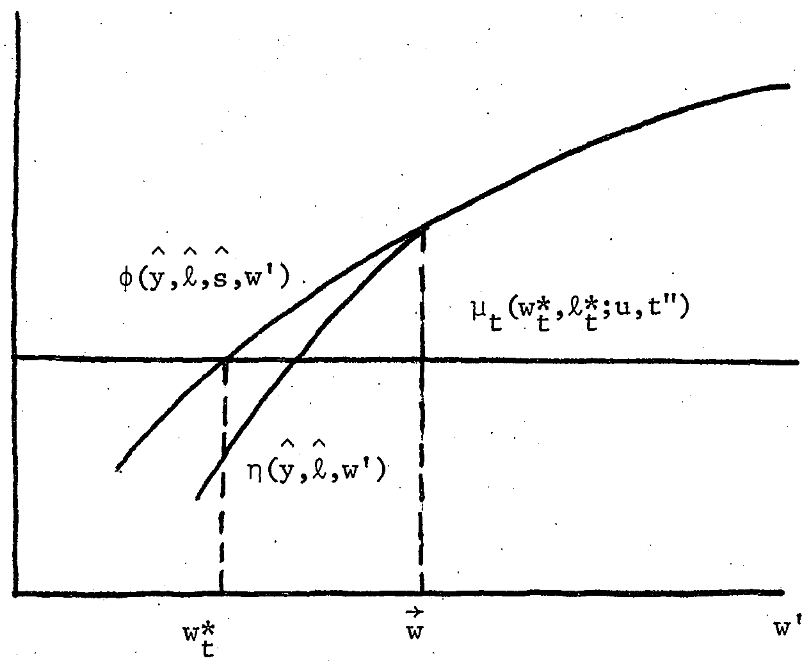
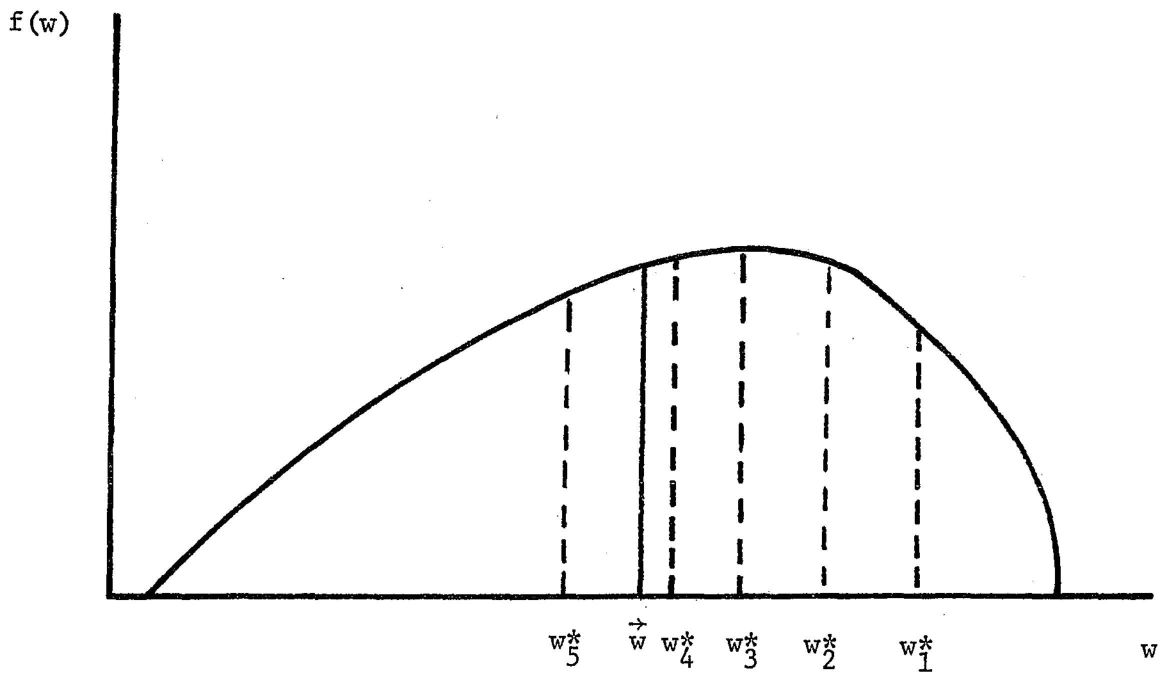


Figure 4



## 3. THE DURATION OF UNEMPLOYMENT AND THE EXPECTED POST-UNEMPLOYMENT WAGE

From (8) and (9) it is possible to write the optimal reservation wage and leisure choice in any search period of a spell of unemployment as a function of the parameters of the model. Specifically,

$$w_t^* = g_t(u,t'') \text{ and} \tag{15a}$$

$$\ell_t^* = h_t(u,t''). \tag{15b}$$

In this short section we consider the probability of a worker suffering various durations of unemployment and his/her expected post-unemployment wage given a spell of unemployment is completed. The model developed implies that even if a worker's strategy is known, only probability statements can be made about when the worker will find an acceptable offer or what wage rate will be accepted. It will be assumed throughout this section that the parameters of the model are fixed. This implies the worker faces the same distribution of wage rate offers each period.

Consider the probability, $\alpha_t$, a worker who has endured t-1 periods of unemployment receives an acceptable offer in period t. From (5) it follows[7]

$$\alpha_t = \Pr(w \geq w_t^*)\pi(1-\ell_t^*). \tag{16}$$

An implication of Proposition 2 is

$$\alpha_{t-1} < \alpha_t, \text{ if } t \leq t''$$

$$\alpha_{t-1} = \alpha_t, \text{ if } t > t''. \tag{17}$$

Therefore the probability of terminating unemployment in a period increases in the first t" periods of a spell of unemployment. The probability of obtaining an acceptable offer in each period after t" periods is constant.

Let $\delta(p')$ denote the probability a given worker who has just become unemployed suffers exactly p' search periods of unemployment before an acceptable offer is found, for any $p' > 0$. From (17) it follows

$$\delta(p') = \alpha_{p'} \prod_{j=1}^{p'-1} (1 - \alpha_j) \text{ for any } p' > 0. \tag{18}$$

Hence the probability a spell of unemployment lasts exactly p' periods declines as p' increases. Further, the rate of decline decreases as p' increases.

The expected wage rate faced by a worker who finds an acceptable job offer after exactly t periods of unemployment, $\beta_t$, can be written as

$$\beta_t = \frac{\int_{w_t^*}^{\infty} wf(w)dw}{\int_{w_t^*}^{\infty} f(w)dw}. \tag{19}$$

From Proposition 2 if follows

$$\begin{aligned} &\beta_{t-1} > \beta_t \text{ if } t \leq t'', \text{ and} \\ &\beta_{t-1} = \beta_t \text{ if } t > t''. \end{aligned} \tag{20}$$

Therefore the expected post-unemployment wage rate, for a given completed spell of unemployment, decreases as the length of unemployment increases if the duration is no more than t" periods. The expected post-unemployment wage rate is the same for any duration of unemployment greater than t" periods. Note that the distribution of post-unemployment wage rates, given a job is found after a particular duration of unemployment, is the distribution of wage offers to the right of the last optimal reservation wage.

As the optimal reservation wage and leisure choice in each period of unemployment can be written as a function of the parameters of the model, we can write

$$\alpha_t = \alpha_t(u, t''), \tag{21}$$

and

$$\beta_t = \beta_t(u, t''). \tag{22}$$

## 4. THE EFFECTS OF CHANGES IN THE UI SCHEME

From the model specified above it is clear the government can influence the future of unemployed workers by changing u or t''. Changes in either u or t'' can effect the optimal reservation wage and leisure choice of an unemployed worker and hence influence the likelihood of that worker becoming employed. Although the explicit forms of (15a), (15b), (21) and (22) are difficult to determine, it is possible to discover the effects of a change in u by taking the total derivative of the first order conditions (5) and (6). The second order conditions can then be used to establish how a change in the UI payments influence an unemployed worker's optimal reservation wage and leisure choice, and hence change the expected duration of unemployment and post-unemployment wage rate. Table 1 presents the results that can be obtained from attempting such a task. E.D., in Table 1, denotes the expected duration of a completed spell of unemployment. An increase in UI payments

(a) increases the reservation wages used by a worker in any of the first $t''$ periods of unemployment, and (b) increases the optimal leisure choices of a worker in the first $t''$ periods of unemployment if $v_{y\ell} \geq 0$. This implies that if $v_{y\ell} \geq 0$ the probability of finding an acceptable job offer in each of the first $t''$ periods of unemployment decreases, but the expected acceptable wage, if found in any of the first $t''$ periods, increases. Changing u does not influence the decisions of workers who have been unemployed more than $t''$ periods.

Suppose the UI duration $t''$ is increased by one period, i.e., $t' = t''+1$, where $t'$ is the new UI duration. This will result in an individual increasing his/her optimal reservation wage and leisure choice in each of the first $t'$ periods of unemployment. This follows from Proposition 1 and by noting from (15a) and (15b) that

$$g_{t-1}(u,t'') = g_t(u,t'),$$

and

$$h_{t-1}(u,t'') = H_t(u,t').$$

Hence an increase in UI duration $t''$ will (a) decrease the probability an unemployed worker, who has been unemployed not more than $t'$ periods, becomes employed, (b) increase the expected acceptable wage rate to a worker who finds a job in the first $t'$ periods of unemployment, and (c) increase the expected duration of unemployment. Note that the decisions of workers who have been unemployed more than $t'$ periods will not be influenced by the increase in UI duration $t''$.

Significantly different results can be obtained from those specified above if it is assumed an employed worker may be laid off. In this case

Table 1

<table>
<tr><th></th><th>$\frac{dw^*_t}{du}$</th><th>$\frac{d\ell^*_t}{du}$</th><th>$\frac{d\alpha_t}{du}$</th><th>$\frac{d\beta_t}{du}$</th><th>$\frac{d(E.D.)}{du}$</th></tr>
<tr><td>$t \leq t''$</td><td>+</td><td>+<br>if $v_{y\ell} \geq 0$</td><td>-<br>if $v_{y\ell} \geq 0$</td><td>+</td><td rowspan="2">+<br>if $v_{y\ell} \geq 0$</td></tr>
<tr><td>$t > t''$</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

a worker who has been unemployed more than t'' periods will lower his/her reservation wage if UI payments are increased. This results from an increase in the expected payoff to accepting a job with a given wage due to the increase in payoff to being laid off.[8]

## 5. FURTHER EXTENSIONS

In this final section two extensions of the original model will be discussed in some detail. First, the basic model is generalized so that the effects of a pre-unemployment income (P.U.Y.) related UI scheme can be analyzed. Second, an attempt at characterizing a change in the demand for labor and its effects is specified.

So far in this study it has been assumed a worker receives UI payment u in the first t' periods of unemployment. Most UI schemes used in modern economies imply that the UI payment received by an unemployed worker depends on his/her P.U.Y. Specifically, a worker's UI is a continuous nondecreasing function of P.U.Y. Formally, let $u = u(z)$, where z is the P.U.Y. and assume

$$\frac{du}{dz} = \begin{cases} \theta > 0 & \text{if } z \leq \hat{z}. \\ 0 & \text{if } z > \hat{z}. \end{cases}$$

Hence the UI payment received by an unemployed worker in each of the first t'' periods of unemployment is an increasing function of P.U.Y., if P.U.Y. is no greater than $\hat{z}$. UI payment in the first t'' periods are independent of P.U.Y. if P.U.Y. is greater than $\hat{z}$. All workers receive no UI payments if they have been unemployed more than t'' periods.

Consider a group of workers looking for a job among the same set of job openings. As they all face the same possible job offer, F(w) denotes the distribution function of possible wage rate offers faced by all workers in the group. These workers are also assumed to have the same preference structure. The only way these workers differ is that they received different P.U.Y.

Consider first all those workers with P.U.Y. no greater than $\hat{z}$. Among this subset of workers the higher a worker's P.U.Y. the greater his/her UI payments in any of the first t" periods of unemployment. But, from Table 1, we know the higher a worker's UI payment the greater his/her optimal reservation wage and leisure choice (if $v_{y\ell} \geq 0$) in each of the first t" periods of unemployment. Therefore, considering only workers with P.U.Y. no greater than $\hat{z}$, the greater the P.U.Y. the smaller the probability of obtaining an acceptable offer, and the greater the expected wage rate, if an acceptable wage is offered in each of the first t" periods of unemployment. Workers with P.U.Y. greater than $\hat{z}$ will act exactly like one with P.U.Y. equal to $\hat{z}$. Hence any worker with P.U.Y. greater than $\hat{z}$ has a smaller probability of obtaining an acceptable job offer in each of the first t" periods of unemployment than a worker with P.U.Y. less than $\hat{z}$. All workers in the group who have been unemployed more than t" periods will have the same reservation wage and leisure choice in any period of unemployment.

The analysis presented above appears to cast doubt on the suitability of a P.U.Y. related UI scheme. It can be argued that workers who are lucky enough to obtain a "high" wage rate job offer are unjustly rewarded with a "high" UI payment, which allows them to find another "high" wage rate job offer. However, two factors should be noted. First, the insurance aspect of a UI scheme can be used to justify a P.U.Y. related scheme. High wage rate workers may have greater obligations to meet than low wage rate workers. Hence a high wage rate worker who becomes unemployed requires a greater UI payment just to survive. High-income workers also contribute more per period to the UI scheme when employed. Secondly, it was assumed the opportunities faced by workers were the same. This is clearly not the case in many labor markets. A brain surgeon does not search the same labor market as a road cleaner.

In the concluding part of this study the effects of changes in the demand for labor are briefly discussed. A change in the demand for labor can be reflected by a change in the number of vacancies in the market and/or changes in the distribution of wage rate offers. In the following discussion, the situation where changes in the demand for labor can be reflected by changes in the number of vacancies is presented. Hence we are considering a fixed wage model where firms can either offer a job at a particular wage rate, or make no offers at all.

Let $\pi(s,k)$ denote the probability that an unemployed worker receives a job offer in a search period when s percentage of the period was spent looking for a job and k denotes the demand for labor parameter. An increase in the parameter k implies that job offers are more plentiful. Assume

$$\pi_s > 0,\ \pi_k > 0,\ \pi_{ss} < 0,\ \pi_{kk} \nless 0,\ \text{and}\ \pi_{ks} \geq 0.$$

Using a similar analysis to that used in establishing the results presented in Table 1, it is possible to determine the effects of a change in the labor demand parameter. The results that can be obtained from performing such a task are presented in Table 2. An increase in k increases the expected duration of unemployment as workers increase their reservation wage and leisure choice in each period of unemployment.

The results presented in the last three sections are merely a sample of the many predictions that flow from the theory of the individual labor supply decision presented. The results appear to be significantly different than those presented in the job search literature, or the human capital literature, to warrant further investigation.

Table 2

| $\frac{dw_t^*}{dk}$ | $\frac{d\ell_t^*}{dk}$ | $\frac{d\alpha_t}{dk}$ | $\frac{d\beta_t}{dk}$ | $\frac{d(E.D.)}{dk}$ |
|---|---|---|---|---|
| + | + | - | + | + |

# APPENDIX

## Proof of Proposition 1

Claim (a) follows from the assumption that a worker has an unbounded life. Details of the formal proof of this claim are not presented as similar proofs have been presented many times in the search literature.

To establish claim (b) let $[1 - \pi(1-\bar{\ell})\Pr(w \geq \bar{w})] \neq 0$. This assumption will hold if $\pi(1-\bar{\ell})f(\bar{w}) \neq 0$. From (4) we have

$$\mu_t(\bar{w},\bar{\ell};u,t'') - \mu_{t+1}(\bar{w},\bar{\ell};u,t'') =$$

$$\frac{[1 - \pi(1-\bar{\ell})\Pr(w \geq \bar{w})][\mu_{t+1}(\bar{w},\bar{\ell};u,t'') - \mu_{t+2}(\bar{w},\bar{\ell};u,t'')]}{1 + r}$$

Hence,

$$\mu_t(\bar{w},\bar{\ell};u,t'') - \mu_{t+1}(\bar{w},\bar{\ell};u,t'') > 0, \text{ if } \mu_{t+1}(\bar{w},\bar{\ell};u,t'') - \mu_{t+2}(\bar{w},\bar{\ell};u,t'') > 0.$$

But

$\mu_{t''}(\bar{w},\bar{\ell};u,t'') - \mu_{t''+1}(\bar{w},\bar{\ell};u,t'') = u > 0$ and the claim follows.

## Proof of Proposition 2

As $\psi(w^*_{t-1}) = \mu_t(w^*_t,\ell^*_t;u,t'')$ and $\psi'(w') > 0$ claims (a) and (b) follow from inspection of Proposition 1.

Taking the total derivative of (8) with respect to $\bar{w}$ yields

$$v_{\ell\ell}\frac{d\bar{\ell}}{d\bar{w}} = -T[\psi(\bar{w})]\pi''(1-\bar{\ell})\,\frac{d\bar{\ell}}{d\bar{w}} + T'(\psi(\bar{w}))\pi'(1-\bar{\ell}).$$

Therefore, as $\pi'(1-\bar{\ell}) > 0$, $v_{\ell\ell} < 0$, and $T(\psi(\bar{w})) > 0$,

$$\frac{d\bar{\ell}}{d\bar{w}} > 0 \text{ if } T'(\psi(\bar{w})) < 0.$$

$$T(\psi(\bar{w})) = \frac{1}{1+r}\int_{\bar{w}}^{\infty} \psi(w)f(w)dw - \frac{1}{1+r}\psi(\bar{w})\int_{\bar{w}}^{\infty} f(w)dw$$

$$= \frac{E[\psi(w)]}{1+r} + \frac{1}{1+r}\int_{\bar{w}}^{\infty} F(w)\psi'(w)dw - \frac{\psi(\bar{w})}{1+r}.$$

Therefore,

$$\frac{dT(\psi(\bar{w}))}{d\bar{w}} = [F(\bar{w}) - 1]\frac{\psi'(\bar{w})}{1+r} < 0,$$ and claims (c) and (e) follow

from claims (a) and (b). It is straightforward to extend the above proof to demonstrate claim (d) and is therefore not presented.

Proof of Proposition 3

Substituting in the equations of (13) yields

$$w'v_y = \pi'(s)T[\psi(w')] \text{ and}$$

$$v_\ell = \pi'(s)T[\psi(w')].$$

Taking the total differential we have

$$w'v_{yy}\frac{dy}{dw'} + w'v_{y\ell}\frac{d\ell}{dw'} - \pi''(s)T[\psi(w')]\frac{ds}{dw'} = \pi'(s)T'[\psi(w')] - v_y$$

$$v_{y\ell}\frac{dy}{dw'} + v_{\ell\ell}\frac{d\ell}{dw'} - \pi''(s)T[\psi(w')]\frac{ds}{dw'} = \pi'(s)T'[\psi(w')].$$

Substituting $\frac{dy}{dw'}$ from the second equation above into the first equation and manipulating yields

$$\frac{d\ell}{dw'} = \frac{\pi'(s)T'[\psi(w')][w'v_{yy} - v_{y\ell}] + v_y v_{y\ell}}{w'[v_{yy}v_{\ell\ell} - (v_{y\ell})^2]} + \frac{\pi''(s)T[\psi(w')][w'v_{yy} - v_{y\ell}]}{w'[v_{yy}v_{\ell\ell} - (v_{y\ell})^2]} \frac{ds}{dw'}$$

Going through the above expression term by term it can be checked that

$$\frac{d\ell}{dw'} = \theta + \tau \frac{ds}{dw'} \text{ where } \theta > 0 \text{ and } \tau > 0.$$

Hence

$$\frac{d\ell}{dw'} > 0 \text{ if } \frac{ds}{dw'} \geq 0.$$

From (11) we have

$$\frac{dy}{dw'} + w'\frac{d\ell}{dw'} + w'\frac{ds}{dw'} = (1-s-\ell).$$

Substituting and using the fact $dy = hdw' + w'dh$ yields

$$\frac{dh}{dw'} = -\theta - (\tau + 1) \frac{ds}{dw'}.$$

Therefore

$\frac{dh}{dw'} > 0$ then $\frac{ds}{dw'} \leq 0$ and the claims made in the proposition are established.

NOTES

1. This theory has been presented numerous times in the literature. Most modern work in this area has flowed from the seminal work of Becker [1964]. Although there are many modifications one can make to the theory, little would be gained by considering the more sophisticated models.

2. Lippman and McCall [1976] have produced a good survey of this literature. The assumption that there is a known distribution is central to all but one contribution in this area. Rothschild [1974] considers the problem when the distribution is unknown.

3. See Burdett [1976].

4. See Lippman and McCall [1976].

5. The real simplification made by the assumption presented is that workers do not save in any period. It is reasonably easy to generalize the model to include saving but only at the cost of significantly complicating the exposition.

6. Mortensen [1976] has presented a model of job search in continuous time which uses similar restrictions to those used in this study.

7. It will be assumed throughout this section that $v_{y\ell} \geq 0$.

8. The author is at present developing a model with this assumption.

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