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THE TESTING AND SORTING FUNCTIONS OF HIGHER EDUCATION

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Abstract

The purpose of this paper is to discuss and clarify some issues raised by Professor K.J. Arrow in his paper 'Higher Education as a Filter' [1973]. The problem that motivates my present interest in this topic can be summarized by the following question: Does higher education, to the extent that it acts as a filter, create new information about the abilities of students; or does it utilize known information to sort students according to their abilities? I term this creation of new information the testing function of higher education and the sorting of students, not surprisingly, the sorting function of higher education. A higher education system that acts as a filter will perform at least one of these functions. The higher education system described by Professor Arrow, in explaining the filter concept, does not create new information, it only performs the sorting function. In the present study the testing function is defined and its relationship to the sorting function considered.

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The purpose of this paper is to discuss and clarify some issues raised by Professor K.J. Arrow in his paper 'Higher Education as a Filter' [1973]. The problem that motivates my present interest in this topic can be summarized by the following question: Does higher education, to the extent that it acts as a filter, create new information about the abilities of students; or does it utilize known information to sort students according to their abilities? I term this creation of new information the testing function of higher education and the sorting of students, not surprisingly, the sorting function of higher education. A higher education system that acts as a filter will perform at least one of these functions. The higher education system described by Professor Arrow, in explaining the filter concept, does not create new information, it only performs the sorting function. In the present study the testing function is defined and its relationship to the sorting function considered.

The role usually ascribed to higher education by economists is that of a human capital accumulator, in the sense that colleges are assumed to improve students' talents. If higher education plays this role it can contribute to the economic performance of society by improving the productivity of those who have attended college. The alternative role proposed by Professor Arrow is that of a filtering device "in that it sorts out individuals of differing abilities, thereby conveying information to the purchasers of labor" [1973, p. 195]. To dramatize his argument a model is developed in which a higher education system is assumed not to improve students' talent, but merely to sort them according to their expected productivity. This system does not create new information about talents;

it performs only the sorting function. In this study Professor Arrow's model is extended to demonstrate conditions under which a higher education system performs the testing function. Although all issues will be discussed within the context of his model, the conclusions reached appear quite general. Indeed, a detailed examination of these two functions does have important implications in the practical area of education policy and the theoretical area of the economic role of filter mechanisms. In the final part of this study the relationship between self-selection devices and the functions of higher education is noted.

1. The Model

Professor Arrow considers a situation where each high school graduate, called here a "school leaver," can be characterized by:

(a) his/her pre-college record, y, and

(b) his/her unobservable productivity, z, where $0 \leq z < \infty$.

Let $f(y,z)$ denote the joint density function of these two variables among school leavers. Suppose each school leaver wants to attend college but there are only a limited number of places available. The college is assumed to be interested only in the probability of a student graduating when deciding whom to admit. To simplify the exposition, Professor Arrow assumes the pre-college record of any school leaver is equal to the probability he/she will graduate if college is attended. Hence for each school leaver $0 \leq y \leq 1$. Students who attend college either graduate or fail; they are not graded.

Suppose the college decides to maximize the expected number of graduates subject to the number of places available. To achieve this

goal the college will select a y_o and only admit school leavers with a pre-college record at least as great as y_o. The y_o chosen will be such that the number of students allowed to attend college is equal to the number of places available. The claim that such an admission policy maximizes the stated objective follows directly from the assumption that pre-college records can be equated with probabilities of graduating.

Utilizing this framework, Professor Arrow makes several claims that can be summarized as follows.

(a) The college can act as a double filter, once in selecting entrants and once in passing or failing students.

(b) The admission procedure specified conveys (positive) information[2] if $E[z|y \geq y_o] > E(z)$, i.e., if the expected productivity of college entrants is greater than that of all school leavers.

(c) The graduation policy specified conveys (positive) information over and above the admission procedure if

$$\frac{E[zy|y \geq y_o]}{E[y|y \geq y_o]} = E[z|y \geq y_o, \text{ grad.}] \geq E[z|y \geq y_o],$$

i.e., if the expected productivity of graduates is greater than that of all college entrants.

(d) If admission and/or graduation policies convey information, the college is said to act as a filter.

2. The Sorting and Testing Functions in Higher Education

Before discussing these claims it will be useful to define more carefully certain terms mentioned earlier. First, higher education will be said to perform a sorting function if known information is used to create a nontrivial partition of a set of individuals. The admission procedure as

specified in Professor Arrow's model is clearly part of this sorting function. In this case a partition of the set of school leavers into two sets (college entrants and non-college-entrants) is created by the admission policy based on known information (the pre-college record). Not all admission policies are of this type: for example, a college may test all applicants and therefore create new information. Alternatively, all school leavers may be admitted in which case only a trivial partition of the set of school leavers is created. Second, higher education will be said to perform a testing function if new information is created about at least one individual's productivity. Within the framework developped by Professor Arrow the graduation policy is part of the testing function if and only if, for at least one $y' \geq y_o$,

$$E[z|y = y', \text{grad.}] > E[z|y = y'], \qquad (1)$$

i.e., the expected productivity of graduates with pre-college record y' is greater than that of all school leavers with record y'.[3] Since the admission policy described by Professor Arrow is part of the sorting function, (1) becomes the test of whether the higher education system specified is performing a testing function or not. It is possible to show that (1) is not satisfied for any $y' \geq y_o$ with the graduation policy presented by him. An example will demonstrate this result.

Suppose there are three equal-sized groups of school leavers having pre-college records 0.4, 0.5, and 0.6 respectively. The relationship between pre-college records and productivity is assumed to be as follows:

$$\Pr(z = y' + 0.1 \mid y = y') = 1/2 \quad \text{and}$$

$$\Pr(z = y' - 0.1 \mid y = y') = 1/2, \; y' = 0.4, 0.5, 0.6.$$

Assume the college selects $y_o = 0.5$. It is straightforward to calculate to three decimal places that

$$E(z) = 1/3[0.400 + 0.500 + 0.600] = 0.500,$$

$$E\{z|y \geq y_o\} = 1/2[0.500] + 1/2[0.600] = 0.500, \text{ and}$$

$$E\{z|y \geq y_o, \text{ grad.}\} = 5/11[0.500] + 6/11[0.600] = 0.556.$$

Hence, according to Professor Arrow's claims the college acts as a filter by its admission and graduation policy. However, the expected productivity of any graduate or nongraduate with a pre-college record 0.600 (0.500) is 0.600 (0.500). The college has graduated 60 percent (50 percent) of college entrants with a pre-college record of 0.600 (0.500) as if they were selected at random from all individuals with a pre-college record of 0.600 (0.500). The reason the expected productivity of graduates is greater than that of all college entrants is that half of the entrants have a pre-college record of 0.600, whereas six-elevenths of the graduates have a pre-college record of 0.600.

Suppose the college randomly selects y' percent of all school leavers with the pre-college record y' for each $y' \geq y_o$. If these selected school leavers are the only ones allowed to attend college, the number and expected productivity of college entrants under this admission policy is equal to t hat of college graduates under Professor Arrow's scheme.

In one sense it can be argued that the college system described by Professor Arrow is the opposite of a filter mechanism. He assumes that firms interested in purchasing labor know only if an individual has graduated or not; information about the pre-college record is assumed to disappear when students attend college. However, firms could presumably

employ individuals directly after they leave high school and calculate expected productivity from pre-college records to reproduce the exact information transmitted by Arrow's higher education system.

Considering the example presented above, it is possible to demonstrate when a college fulfills a testing function. Suppose the college can administer a test such that (a) all individuals with productivity at least as great as 0.600 can be certain of passing, (b) all individuals with productivity less than 0.500 are certain to fail, and (c) 10 percent of individuals with a pre-college record of 0.500 will pass the test, while the others will fail. Further, assume passing the test implies graduation and failure implies nongraduation. It is simple to calculate that

$$E[z|y = 0.600, \text{ grad.}] = 5/6[0.700] + 1/6[0.500] = 0.667,$$

$$E[z|y = 0.600] = 0.600,$$

$$E[z|y = 0.500, \text{ grad.}] = 0.600, \text{ and}$$

$$E[z|y = 0.500] = 0.500.$$

Hence, a college that implements such a test and graduation policy performs a testing function, since (1) is satisfied for all $y' \geq y_o$. Note that this graduation policy also acts as a filter in Professor Arrow's terms.

Can a higher education system which performs only the sorting function be justified in the sense that it contributes to the economic performance of a society? Two situations spring to mind which lead to an affirmative answer. First, suppose there are large sorting costs. For example, assume the information that goes to form an individual's pre-college record is difficult to collect, involving a nontrivial collection cost. The purchasers of labor may prefer paying a higher education system to perform this task on each individual to doing it themselves, if there are economies

of scale in collection. However, there are other institutional arrangements which are often assumed to play this role, e.g., personnel departments, employment agencies, and high school career advisors. Second, suppose the college is the only institution that knows the probabilistic relationship between pre-college records and productivity. This knowledge is a saleable commodity. Indeed, an important function of a college may be to ascertain this relationship.

3. Higher Education as a Self-Selection Device

A concept related to those under discussion is that of a self-selection device. This idea was first discussed within the context of a labor market by Salop and Salop [1972]. Higher education will be said to act as a self-selection device if it motivates a group of individuals to sort themselves so as to create new information about productivities. Hence, if higher education is a self-selection device it performs a testing function. The special feature of a system that acts as a self-selection device is that individuals sort themselves out according to their productivity because of a correlation between productivity and their preferences.

An example will help explain this concept. Suppose there are two groups of school leavers, one group having high productivity and the other low productivity. No one knows which individuals belong to which group. Further, assume there does not exist a test which can determine which of the school leavers have high or low productivity. In this case a higher education system cannot create information about productivities directly. However, suppose it is known that high productivity school leavers prefer a quiet environment to a noisy one, whereas low productivity workers prefer the opposite. Assume that the cost of obtaining a higher education and the wage

rates in the labor market are such that the expected lifetime income net of the cost of higher education to college entrants is equal to the expected lifetime income of non-college-entrants. If college offers a quiet environment and the work situation a noisy one, only high-productivity workers will apply to attend college. Hence, because of a feature (quiet) seemingly unrelated to productivity, the college has acted as a self-selection device, since preference for quiet is correlated with high productivity among school leavers. Other features apart from a quiet environment may play a similar role. Many other factors can complicate the above simpleminded example. For example, the result may still hold even if there is an increase in expected lifetime income from attending college. The information created by such a higher education system as that described above can be of use to the purchasers of labor services in assigning workers to jobs.

For educational policy purposes it is important to determine the relative importance of the possible roles of higher education. For example, if the capital accumulation role is most important, effort should be expended on what to teach students, as in this case students learn skills from faculty. If the testing function is the most important function, effort should be expended on obtaining information about students. Finally, if the self-selection element is most important, the content is relevant only insofar as high-productivity students like it and low-productivity students do not.

Notes

[1]The pre-college record of a school leaver is assumed to be an index of all known information about that individual, including his/her high school record and any other relevant data.

[2]Note that only the first moment of the conditional distribution is assumed to be important.

[3]Again, only the first moment is assumed important, but in general a higher education system will perform a testing function by its graduation policy if

$$F(z|y = y', \text{grad}) \neq F(z|y = y')$$

for at least one y' where F is the conditional distribution function.

References

[1] Arrow, K. J. 1973. "Higher Education as a Filter," Journal of Public Economics 2:193-216.

[2] Salop, J. K. and Salop, S. C. 1972. "Self Selection and Turnover in the Labor Market." Paper presented at Econometric Society meetings, Toronto, Canada, December 28-30, 1972.