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THE STRUCTURE OF INEQUALITY AND THE PROCESS OF ATTAINMENT

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ABSTRACT

This paper proposes a model for the process of attaining occupational status and income, where change in attainment is generated by the creation of vacant positions in social structure. The distribution of attainments, or the structure of inequality, is assumed fixed and described by a simple exponential or geometric distribution function (depending on whether attainment levels are assumed discrete or continuous). Persons leaving the labor force create chains of vacancies in this structure that present mobility opportunities for persons entering the labor force. The implications of the model for the attainment process derived from these considerations for status attainment research and stochastic models for job-mobility are discussed.

Introduction

Research on social mobility, status, and income attainment in sociology has always been heavily oriented toward the methodological problems posed by the subject matter under investigation. Thus the development of indices in mobility research and problems of estimation and measurement in status attainment research have received a great deal of attention. Conceptual issues have been much less of a concern, although they have not been entirely unimportant. The concern for separating structural and exchange mobility in the development of indices of mobility and the concern for the temporal ordering of variables and for causal directions in status attainment research, reflect theoretical assumptions regarding the forces that generate mobility and achievement. Nevertheless, the dominant research strategy has been inductive, rather than deductive: the accumulation of empirical findings from cross-national and cross-temporal studies is believed to produce a pattern from which a sociological theory of attainment and mobility will emerge.

This situation is in sharp contrast to the approach taken in economics to the study of one aspect of the attainment process -- income attainment. Neoclassical economists have applied a powerful conceptual apparatus to income attainment in the form of human capital theory. The attainment of income in this perspective is conceived of as reflecting a person's productivity as determined by his/her ability and skills. Skills are obtained through education and training at a cost primarily in the form of earnings forgone. Returns on the investments in training and education are obtained in a competitive market where earnings are determined by the marginal productivity of labor. A number of empirical predications can be derived from this theory -- the shape of the age-earnings profile, the impact of wage differentials on demand for education, the allocation of training costs for general and specific on-the-job training, etc.[1] Few such predictions can be made from sociological research on attainment processes where there is heavy emphasis on estimating the relationship among observed variables, not on modeling the process that produces the observed outcomes.

Human capital theory provides powerful predictions about the attainment process, but this does not mean that it is the only possible, or necessarily the most useful approach to the study of attainment processes. Some basic predictions from the theory do not square well with reality: from the theory one would predict that changes in the distribution of education would alter the distribution of incomes because of the changed supply at different skill levels. Since the second World War, no such change can be observed in the distribution of income despite a marked shift in the distribution of education [Thurow and Lucas, 1972]. Numerous criticisms of the theory have also been raised because of its apparent failure to account for the processes that are believed characteristic of important segments of the labor markets [Doeringer and Piore, 1971; Thurow, 1975].

Criticisms against a powerful theory, based on the failure of the theory to account for some empirical observations, are often ambiguous. Those who believe in the theory can usually come up with modifications that will save the theory by extending it and altering less important assumptions. Usually human capital theorists are willing to allow for imperfections in the degree to which the real world approximates the neoclassical world that they assume. These imperfections may then be used to excuse the apparent failure of some empirical predications. They can further point with considerable merit to the theory's ability to account for a number of basic features of observed processes, and to the inability of critics to come up with an alternative theory equally parsimonious and with equal explanatory power. Theories are replaced with other theories, not with a set of isolated empirical observations that are subject to different interpretations.

The conception of mobility used in much traditional mobility research could be a point of departure for the formulation of an alternative theory of the attainment process because of the contrast it provides with basic assumptions of human capital theory. In human capital theory changes in attainment are assumed to be brought about exclusively through changes in a person's productivity, i.e., skills and experience. The distribution of skills, in turn, is reflected in the distribution of earnings. In traditional mobility research, change in attainment, in contrast, is assumed to reflect changes in positions in a predetermined structure of inequality, without accompanying changes in personal characteristics. Persons can move only to a slot that is available, i.e., vacant, and while a person's "productivity" (as measured by ability, education, and experience) determines which slots a person gets access to, the distribution of attainments reflects the distribution of slots, not the distribution of personal attributes that are relevant for getting access to slots.

Such a notion would be consistent with the lack of change in income distribution in the face of a marked change in the educational distribution that is contrary to the implication of the neoclassical economic theory. It would also be consistent with the attainment processes that characterize primary labor markets [Doeringer and Piore, 1971] and job competition [Thurow, 1975] in the critiques of the neoclassical theory.

The sociological conception of mobility has, however, never been very well specified. It has been used to justify many attempts at separating structural from exchange mobility in intergenerational mobility tables, but this is a decomposition of the total amount of mobility in society, not a specification of the mechanisms of mobility generated by the creation of vacant positions in social structure. Further, since the objective here is to formulate a theory of change in attainment where mobility rather than change in a person's resources is the source of change, the focus should be on intragenerational mobility rather than on intergenerational mobility as in most traditional mobility research.

Two tasks need to be carried out. It is necessary to specify how the creation of available or vacant positions generate mobility, and it is also necessary to specify how individual characteristics influence a person's utilization of mobility opportunities. Only a few attempts have been made at carrying out these tasks. With respect to the first task, works by Bartholomew [1972] and White [1970] are the main examples. White's [1970] vacancy-chain model is particularly suggestive of how structurally created opportunities generate mobility by generating chains of vacancies. However, the specification of how individual characteristics influence the utilization of vacancies is not attempted in White's work. Some attempts in this direction have been made by Boudon [1974] that resulted, however, in a simulation model and not in a well-specified mathematical model.

The objective of this paper is to suggest a particular solution to the problems of specifying a theory of the attainment process that conceives of structurally induced mobility as the source of change in individual attainment. This will involve (1) specifying a model for the structure of inequality, i.e., the distribution of possible attainments, then (2) specifying how vacancies occur and move in this structure, and finally (3) modeling how change in attainments are brought about by the movement of people along the structurally induced vacancy chains. These are the main tasks of the paper. The final sections of the paper will outline the relationship between the proposed model of the attainment process, status attainment research, and research on intragenerational mobility.

A number of very strong assumptions will be utilized in deriving the model. These assumptions are necessary to simplify an otherwise very complicated problem. The resulting model may to some appear highly unrealistic. That the model provides a very simplified picture of reality will not be denied. However, it does account for important features of observed process, as I shall show.

The Structure of Inequality

The objective is, as mentioned, to formulate a model for the attainment process, where change is brought about by utilizing opportunities for change in position in a predetermined structure of inequality. The positions will be conceived of as jobs, and these jobs may be characterized by the economic, social, and psychological rewards they provide incumbents. Only a change in jobs can provide a change in the level of rewards or in attainment. This is a reasonable assumption with respect to most rewards, but it may appear dubious with respect to earnings. There will be real and inflationary increases in earnings within a job as well as some performance-related variation. These real and inflationary increases will be ignored because they usually do not change a person's relative position. Performance-related variation within jobs will be assumed to be of minor importance. One reason is that major performance differences for people in similar jobs are a source of instability and hence likely to result in differentiation of jobs.

Stated differently, the basic assumption is that different people in the same jobs will obtain the same rewards, while the same person will obtain different rewards in different jobs. With this assumption, the structure of inequality is given as the distribution of jobs with respect to status, income, and other rewards.[2] Jobs may be vacant or filled, and people may be employed or unemployed. Hence, the distribution of jobs will not correspond to the distribution of people, although it will be roughly similar to the distribution of employed people. For the present purposes this distribution will be assumed stable over time.

In the sequel it will be assumed that there exists a measure of attainment level similar to the measures of prestige or socioeconomic status so commonly employed in status attainment research. As argued by Goldthorpe and Hope [1972], these measures reflect the "goodness" of occupations not the "prestige" of occupations in the usual sense of the word where the referent is to deference, that is a relational concept, and not to the distributional concept captured by Duncan SEI, NORC prestige scores, etc. However, the existing measures are ordinal and, though commonly employed as interval scales, this usage does not change their metric properties. The measure of attainment level assumed here is a ratio level measure with a well-defined zero point. In the first derivation of the distribution of jobs according to this measure, it will be assumed mapped onto the set of positive integers, i.e., a discrete distribution will be assumed.

Denote by y the attainment level of a job, where y varies from zero to infinity. The distribution of jobs according to y will be generated from a very simple assumption. It will be assumed that if $n(y)$ denotes the number of jobs at level y (y is assumed an integer), and $n(y + 1)$ the number of jobs at the next higher level, then the following relation holds,

$$\frac{n(y+1)}{n(y)} = s \tag{1}$$

where s is less than one and greater than zero. This means that the number of jobs at level $y + 1$ is a constant proportion of the number of jobs immediately below, for all values of y. Let the total number of jobs be N ; then $f(y) = n(y)/N$ is the density of jobs at level y . It is easily seen that the relation

$$\frac{f(y+m)}{f(y)} = s^m \tag{2}$$

will hold for m , an integer. The distribution of jobs generated this way is the well-known geometric distribution with mean $s/(1-s)$.[3]

In the sequel we shall need the distribution of jobs according to attainment level where this variable is measured as a continuous variable. Assuming therefore now y measured as a continuous variable, the general relation between the density of jobs at level y and at level $y + h$ where h is an interval on y , will be given by (2) with h replacing m . It follows that,

$$\log f(y+h) - \log f(y) = h \log s \tag{3}$$

or

$$\frac{\log f(y+h) - \log f(y)}{h} = \beta \tag{4}$$

where $\beta = \log s$, so that $\beta < 0$. Letting $h \to 0$, equation (4) becomes,

$$\frac{d \log f(y)}{dy} = \beta \quad . \tag{5}$$

Hence for the density $f(y)$, the differential equation

$$\frac{df(y)}{dy} = \frac{df(y)}{d \log f(y)} \; \frac{d \log f(y)}{dy} = \beta f(y) \tag{6}$$

holds. The solution to (6) is,

$$f(y) = f(0)e^{\beta y} \quad . \tag{7}$$

The quantity $f(0)$ is determined from the condition,

$$\int_0^\infty f(y) = \int_0^\infty f(0)e^{\beta y} = 1 , \tag{8}$$

or $f(0) = -\beta$. Hence the distribution of jobs according to y will be,

$$F(y < y') = \int_0^{y'} -\beta u^{\beta u} = 1 - e^{\beta y'} , \tag{9}$$

where $F(y < y')$ is the proportion of jobs providing attainment less than y' . It will be useful to consider the proportion with attainment greater than a certain level y . This proportion will be,

$$P(y) = 1 - F(y) = e^{\beta y} . \tag{10}$$

The distribution of jobs assumed is then simply the exponential distribution when y is considered to be continuous and the geometric distribution when y is considered discrete. The geometric distribution as a representation of the structure of inequality has been suggested by several [Simon, 1957; Bartholomew, 1972; Svalastoga, 1973; Stinchcombe, 1974]. Bartholomew [1972] shows that if the distribution is assumed for an organization, a particular simple promotion schedule will prevail -- a property to be used in this paper too.

The quantity y is a construct. Specifying the relation between y and an observable reward will generate an observable distribution that can be used to evaluate the model (10). Using an argument presented by Lydall (1959), a well-known distribution of incomes may be generated assuming a particular relationship between income and y . The needed assumption is that jobs at level $y + 1$ (returning to the discrete formulation) provide p times the total earnings provided by jobs at level y ; or, if $x(y)$ denotes earnings provided by jobs at level y ;

$$\frac{x(y + 1)}{s \cdot x(y)} = p \, , \tag{11}$$

where p may be less than or greater than 1..

If y alternatively is conceived of as a continuous variable, an argument similar to equations (5) and (6) will show that (11) corresponds to,

$$\frac{dx(y)}{dy} = (\gamma + \beta)x(y) \, , \tag{12}$$

where $\gamma = \log p$ and $x(y)$ is the earnings provided by jobs at level y. The solution to (12) is,

$$\log x(y) - \log x(0) = (\gamma + \beta)y \, , \tag{13}$$

and since $y = 1/\beta \log P(y)$ (cf., equation 10), equation (13) may be written as,

$$\log P(y) = \frac{\beta}{\gamma + \beta} \log x(y) + \text{constant}. \tag{14}$$

If a quantity α is defined as,

$$-\alpha = \frac{\beta}{\gamma + \beta} \, , \tag{15}$$

equation (14) becomes,

$$P(x) = ke^{-\alpha \log x} \, , \tag{16}$$

as $\quad P[x(y) > x(y')] = P(y > y')$.

This is recognized as the model for the income distribution suggested by Pareto. He proposed the model for income distributions that bears his name from inspection of observed income distributions based on tax returns. At that time, no returns were obtained from the lower portion of the distribution, and equation (16) provided an extremely good fit to the upper tail of the distribution. Pareto promoted (16) to a law, but subsequent analysis has shown that it does not fit the lower portion of the income distribution very well, and a number of other distributions will be similar to (16) in their tails. In particular, the log normal distribution first suggested by Gibrat [1931] provides a better overall fit.

The problem is that in observed distributions the density increases with increasing income in the lower portions, contrary to (16). It is well known that persons out of employment or with only marginal attachment to the labor force dominate in this part of the distribution. Equation (16) is here used as a model for the distribution of jobs according to the earnings they provide, and equation (16) may be less unrealistic for this distribution than for the distribution of personal incomes. Further, a conceptual device may be used to argue that (16) indeed is realistic. Only the distribution of filled jobs can be observed, but equation (16) describes the distribution of all jobs whether filled or vacant. Hence it may be argued that the lack of fit is due to the omission of vacant jobs from observed distributions.

An assumption similar to (11) could be used to generate the model for observed prestige distributions. A one-to-one relationship between y and prestige scores would be a reasonable proposal because of the definition of y presented above. However, none of the measures of prestige or socioeconomic status derived from prestige scores (as the SEI index) will result in distributions that can be used to test equation (10). The reason is that prestige scores as mentioned are inherently ordinal. Hence they may be subject to any transformation that preserves rank order. Each transformation will result in a new distribution. The one that is observed using currently used measures is therefore completely arbitrary and cannot be used to validate (10). Only income distributions can be used, but then it is necessary to further assume the validity of equation (11) for the relation between y and income.

Despite the objections that may be raised, equation (10) will be used in the sequel as a model for the distribution of jobs according to y. It leads to a particularly simple and fruitful model for the attainment process and captures basic features of the structure of inequality. These properties are enough rationale for its use as a start.

The Creation of Opportunities for Growth in Attainment

Having formulated a model for the structure of inequality, the task for this section is to formulate a model of how changes in attainments are produced in this structure, that is, how opportunities for change in attainment are created. In the next section, the question of how the characteristics of individuals affect their ability to take advantage of these opportunities will be addressed.

The structure of inequality will be assumed stable over time. People enter and exit the structure when they enter and leave the labor force. When people leave the labor force, they leave vacant jobs. These jobs will be filled either by new recruits or by people moving from other jobs into the job vacated. Following White [1971], two types of moves may be conceived of -- (1) moves by people from filled jobs to vacant jobs, thereby creating new vacancies to be filled by others already in the system or by people entering the system, and (2) moves by vacancies in the opposite direction of the moves by individuals. Chains of moves by persons start when a person enters the labor force and end by retirement (temporary moves out of the labor force will be ignored). Chains of moves by vacancies start with the creation of a vacancy due to retirement (or the creation of a new job) and end by the elimination of a vacancy by a person from outside the system (or by the elimination of a job). Both people and vacancies move among jobs, but the mobility history of a vacancy is something different from the mobility history of a person. The concern in this section is for the mobility of vacancies. In the next section the mobility of people will be linked to the mobility of vacancies.

When a person moves from one attainment level to another, a vacancy moves in the opposite direction. Upward moves by people in the structure are increases in attainment and correspond to moves downward by vacancies. Only such moves will be considered. Although upward moves and horizontal moves by vacancies will take place in empirical systems corresponding to downward and lateral moves by people, they will be ignored here. Assuming persons maximize attainments, this restriction implies that only voluntary moves will be considered. In a later section, the impact of involuntary moves on the attainment process will be briefly considered.

It will be assumed that persons enter and retire at all levels. It is immediately apparent that if voluntary moves are to take place at all, fewer people should enter than leave at some levels; in this way, vacancies will be created for people at lower levels to take advantage of. In work on mobility in organizations, it is often assumed that everyone enters at the bottom and leaves at the top [Bartholomew, 1972]. This is obviously unrealistic for the societal structures of inequality considered here. A more realistic, although very simplifying, assumption will be made here. It will be assumed that a proportion of jobs will be vacated due to retirements in each time period--the same at all attainment levels. Further, it will be assumed that the vacated jobs are not all filled from the outside, and the proportion not filled from the outside constitute a constant proportion at each level. The exception is the bottom level, where all vacancies are filled by persons from the outside.

It is assumed, in other words, that new vacancies are created at a constant rate for each level of attainment. These new vacancies will reflect the addition of new jobs to the economy and/or also that each person enters a promotion ladder that covers some, but not all attainment levels. There is evidence that most job shifts are voluntary [Sørensen, 1975]. Hence, the assumption of new vacancies being created in each time period is reasonable, although the assumption of identical rates of new vacancies at all levels may not be too realistic.

With these assumptions, one may calculate the probability that a vacancy will move from one level to another. Assume y discrete, and denote by $h(y)$ the rate of new vacancies at level y, as measured by the number of new vacancies created at level y in a small time period dt over the number of jobs at level y. Further denote by $q(y)$ the transition rate for a vacancy from $y + 1$ to y, measured as the number of vacancies arriving in y in dt from $y + 1$ over the total number of jobs in y. Vacancies cannot jump levels, and can only move in one direction. Denote as before by $n(y)$ the number of jobs at level y. It must be the case that the number of vacancies arriving in y will equal the number of new vacancies created in $y + 1$ plus the number of vacancies arriving in $y + 1$ from $y + 2$. Hence,

$$q(y) \cdot n(y)dt = h(y + 1)n(y + 1)dt + q(y + 1)n(y + 1)dt \ . \tag{17}$$

As mentioned above, $h(y + 1)$ is assumed constant and equal to h for all y's. It follows from the recursive relationship (17) that,

$$\begin{aligned} q(y)n(y) &= h \sum_{k=y+1}^{\infty} n(k) \qquad (18) \\ &= hN(y + 1) \ , \end{aligned}$$

where $N(y + 1)$ is the total number of jobs at level $y + 1$ or higher. From the model of the structure of inequality proposed in equation (1) it is easily derived that,

$$N(y + 1) = \sum_{k=1}^{\infty} n(y + k)$$

$$= \sum_{k=1}^{\infty} n(y + 1)s^{k-1}$$

$$= \frac{n(y + 1)}{1 - s} \quad . \qquad (19)$$

From (18) and (19), it follows that,

$$q(y) = h\,\frac{N(y + 1)}{n(y)}$$

$$= h\,\frac{n(y + 1)}{n(y)(1 - s)} \qquad (20)$$

$$= h\,\frac{s}{1 - s} \quad .$$

Hence, $q(y)$ is independent of y in a structure of inequality that is described by equation (10). This is an important result for the argument that is presented in the next section. It holds for a structure of inequality that can be described by the geometric distribution. A similar result has been obtained by Bartholomew (1972) for mobility in organizations that may be described by the geometric distributions.

The quantity qdt may be conceived of as a promotion density for persons at a given attainment level. It is important, however, to keep in mind that it is defined on jobs and not on people. While all people at a given attainment level are exposed to the same q, they are not equally likely to take advantage of it. The extent to which they are able to take advantage of the opportunities represented by q will be argued in the next section to be a function of the personal characteristics of individuals (education, ability, and background) and will be linked to the amount of time already spent in the labor force.

The promotion density is a function of h--the rate of new vacancies--and of s that determines the shape of the distribution of inequality. The quantity $s/(1 - s)$ is the mean of y . Hence, q may also be interpreted as the expected number of attainment ladders a vacancy chain will cover in a small interval of time.

The formulation (20) is obtained assuming a discrete distribution of jobs according to attainment levels. The analogue expression for continuous y is easily obtained by noting that $-\beta e^{\beta y}$ represents the density at level y . Hence,

$$q(-\beta e^{\beta y}) = h \int_y^{\infty} e^{\beta u} du \qquad (21)$$

or

$$q = -\frac{h}{\beta} \; . \qquad (22)$$

The expression (22) is to be used in the next section. To avoid a proliferation of symbols, in the sequel, q will be taken as equal to $-\frac{1}{b}$ where $b = \frac{\beta}{h}$ is a function of both the shape of the distribution of jobs and the rate at which new vacancies are created.

The Attainment Process

In a structure of inequality characterized by equation (10), it will be the case that all levels of attainments everyone will be exposed to the same opportunities for increases in attainment as determined by the quantity q of equation (22). The fact that everyone is exposed to the same opportunities does not mean that everyone is equally likely to take advantage of these opportunities. In this section, the question will be addressed of how individual characteristics determine a person's ability to take advantage of the opportunities for growth in attainment given by q.

The individual characteristics relevant for a person's attainment will be said to determine a person's resources. These resources are assumed determined by the time a person enters the labor market, and not subject to further change. This is the exact opposite of the assumptions made in human capital theory where it is assumed that a person's level of resources (as expressed by his productivity) is changing over time due to on-the-job training, experience and the like. Such additions to a person's resources are measured in empirical investigations of human capital theory by time spent in the labor force. Here, time spent in a labor force will be a measure of how long persons have been exposed to the mobility regime formulated in the preceding section. No claims for the universal validity of the assumption of no change in resources over time can be made, but neither can such a universal claim be made for the validity of the assumption that all changes in attainment are due to changes in resources. Empirical analysis does not necessarily confirm the latter assumption when time is used as a proxy for growth in resources.

The higher the attainment level of a job, the higher the level of resources needed to gain access to a job. It will further be assumed that for a given level of personal resources, there is an attainment level that is the best a person can hope to obtain. This is the case because the distribution of jobs according to attainment levels is fixed; hence everyone entering at a certain level has to exit in such a way that the distribution is preserved. A job at the highest attainment level possible for given resources should not be left voluntarily by a person, for there is then no gain to be made. Not all people occupy this level, as voluntary moves are assumed possible in the system as defined above because of the creation of new vacancies at each level of attainment. Some people therefore are in jobs that provide them with lower attainments than they may hope to obtain. Since every move voluntarily undertaken by a person will produce a gain in attainment, those who have just entered the labor force will have the lowest attainment relative to their resources. The longer time a person has spent in the labor force, the more likely it is that the person has the best job (s)he can hope to obtain. Hence a person's ability to take advantage of a vacancy at a higher attainment level will depend on the amount of time spent in the labor force.

Denote by $q(t)dt$ the probability that a person having spent t years in the labor force will change jobs, i.e., take advantage of a vacancy arriving at his/her current attainment level in dt. The probability that a vacancy will arrive at attainment level y in dt is qdt for all values of y. It must be the case that for people at y, the individual rates (that are dependent on the time spent in the labor force) must sum to the overall rate, that is q. Hence

$$\int_0^{\infty} q(t)dt = q = -\frac{1}{b} , \qquad (24)$$

where the integration runs over values of t so that $t \to \infty$ as the rate of leaving the current attainment level approaches zero for people with attainment commensurate with their resources. The specification of $h(t)$ that will satisfy (24) is,

$$q(t) = e^{bt} \qquad b < 0 , \qquad (25)$$

where as before it is understood that b will be a function of both the rate at which new vacancies are created and of the shape of the distribution of jobs according to attainment levels.

The rate of voluntary job shifts integrated over t will give the number of shifts a person has undertaken by time t. Denote this quantity $v(t)$, and define it as,

$$v(t) = \int_0^{t} q(u)du = \frac{1}{b}(e^{bt} - 1) , \qquad (26)$$

with a maximum value $v(\infty) = -\frac{1}{b}$ that is the total number of shifts a person will undertake in his/her lifetime. If y is conceived of as a discrete variable, this quantity will simply be the total growth in attainment a person experiences before he/she achieves the level of y where no further increases are possible. In continuous y, a slight reformulation is useful. Denote by $y(0)$ the level of attainment for a person at entry into the labor force, by $y(t)$ the level obtained by time t, and by $y(e)$ the maximum level of attainment possible. The total growth possible is then $y(e) - y(0)$. At each job shift, a person will realize a fraction Δy of this gain. Since every shift on the average will provide the same gain, it follows that,

$$\Delta y = \frac{y(e) - y(0)}{v(\infty)} . \tag{27}$$

It will be the case that the level of attainment by time t will be equal to the level at entry plus the gain realized up to this point, or,

$$y(t) = y(0) + v(t)\,\Delta y . \tag{28}$$

Substituting equations (26) and (27) in (28) will give,

$$y(t) = y(0) + [\frac{1}{b}(e^{bt} - 1)][-by(e) - y(0)] . \tag{29}$$

Differentiating gives,

$$\frac{dy(t)}{dt} = [-be^{bt}][y(e) - y(0)]$$

$$= -b[y(e) - y(t)] . \tag{30}$$

This is finally the model for change in attainment that obtains in a structure of inequality where mobility takes place in the manner described here.

A person's resources will determine the level $y(e)$ that (s)he eventually will obtain. However, the value of $y(e)$ for the same level of resources will be different in different opportunity structures, i.e., for different values of b. To reflect this, a slight reformulation of

(30) is useful. Define a quantity a through the relation,

$$\frac{da}{dy(e)} = - b \tag{31}$$

Let a be defined as a person's resources. It will vary across people, but for each person be constant over time. From (31) by definition,

$$y(e) = - \frac{a}{b} , \tag{32}$$

that gives

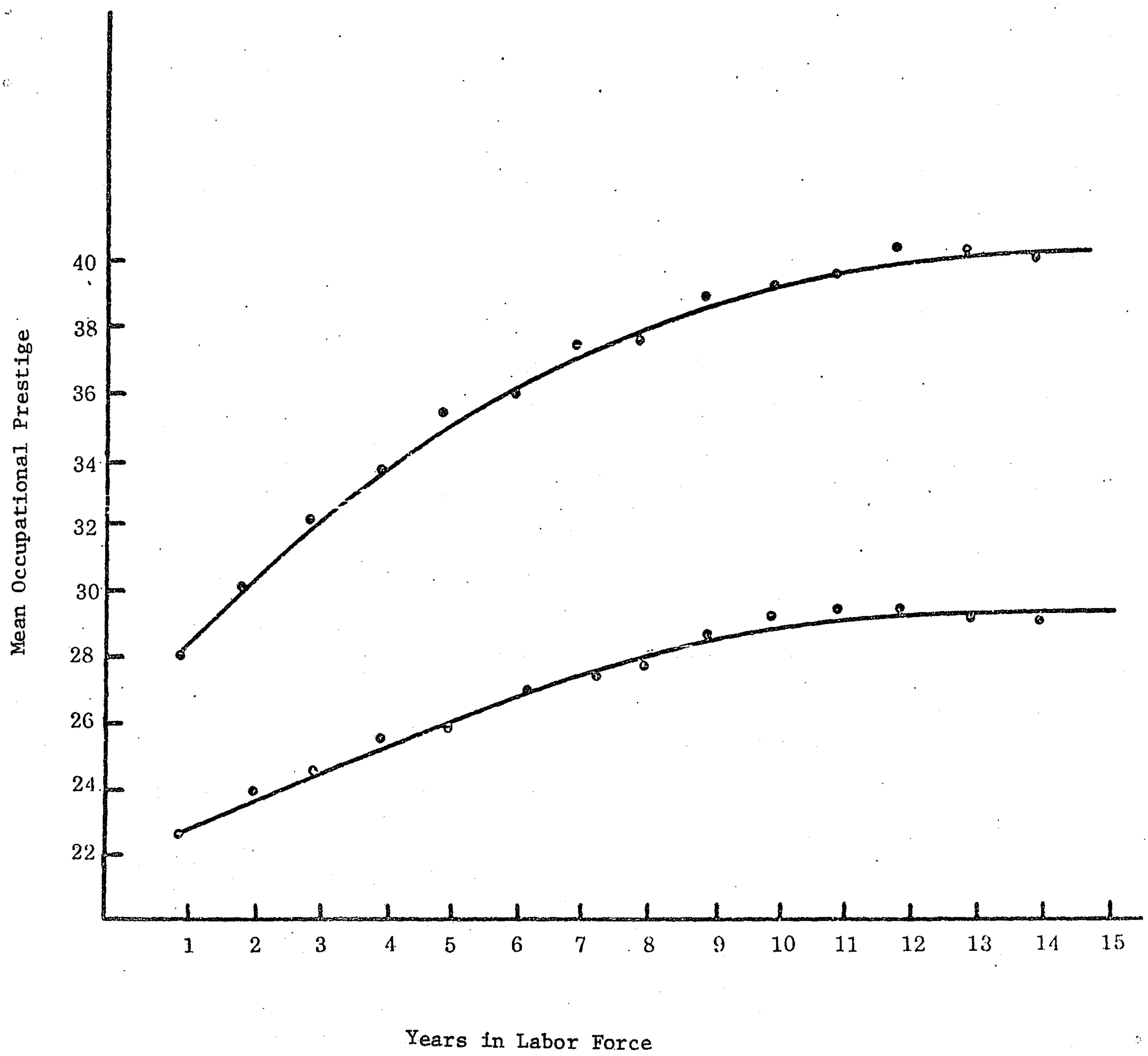
$$\frac{dy(t)}{dt} = a + by(t) . \tag{33}$$

This is the simplest linear differential equation with negative feedback of the dependent variables on itself. The negative feedback has here been shown to be determined by the rate at which new vacancies are created and the shape of the distribution of jobs according to attainment levels.

Equation (33) will describe a career line that is concave to the time axis; that is, there will be rapid growth in attainment in the beginning of the career and slower growth later until the attainment reaches the stable level $y(e) = - a/b$. This pattern is found on observed career curves as Figure 1 shows.

Career lines of whites and blacks are shown separately in Figure 1. The career line for blacks is somewhat flatter than it is for whites reflecting presumably a more unfavorable opportunity structure, that is,

Figure 1. Mean Occupational Prestige by Year in Labor Force



one where q of equation (23) is smaller so that the negative feedback on change in attainment is larger.

The career line observed in Figure 1 and predicted from the model also corresponds to the one predicted from human capital theory. In this theory, the curve is predicted from a pattern of growth in resources where resources grow at a lower rate as people get older, primarily because there is less time left in the labor force in which to recapture costs incurred in acquiring more resources. More specifically, the neoclassical theory assumes that at any point in time the level of attainment is $y(e)$, but the resources, a, change over time in a manner that results in the observed concave career profiles.

Both human capital theory and the theory formulated here predict the same career line. The observed career lines thus do not validate either theory. But the objective here was not to prove human capital theory wrong, but to formulate an alternative theory using assumptions that are the opposite assumptions of those used in the economic theory. It would be a poorer theory if it could not account for the same observed career patterns as the human capital theory.

The theory formulated here readily explains the difference between the career profiles of blacks and whites as reflecting different opportunity structures. This difference is less easily explained by the neoclassical theory which has to resort to devices such as taste for discrimination [Becker, 1957] to account for the persistence of an inefficiency such as discrimination [see also Thurow, 1975].

The model developed in this section is of importance both for the interpretation of status attainment research in the tradition created by Blau and Duncan [1967] and for research on intragenerational mobility. These implications will now be described.

Implications for Status Attainment Research

Research on status attainment usually employs linear algebraic equations where the level of attainment, as measured by SEI or prestige scores, is the dependent variable. Characteristics of the individual are employed as independent variables. Typically, they are measures of respondent's education, father's status/parents' education and other measures of family background. All explanatory variables are then measures of individual characteristics, and no attempt is made to introduce characteristics of the structure of inequality. The model formulated here is derived from consideration of the impact of structural characteristics on growth in attainment, and its parameters are well defined in terms of the various forces that govern attainment processes. The attainment model, therefore, can be used to reinterpret status attainment models and evaluate the appropriateness of the research designs typically employed.

A global measure of resources, a, was used in the derivation of the model above. A formulation of this model that makes it similar to the models employed in status attainment research is obtained by letting a be a linear function of relevant individual characteristics, or,

$$a = c_0 + c_1x_1 + c_2x_2 + \ldots c_nx_n \qquad (34)$$

where the x_i variables stand for education, father's status, parents' education, etc. The coefficients to the x_i variables represent the contribution of these variables to the overall level of resources. In status attainment research, as here, these resources are assumed constant over time, although status attainment research has never been explicit about such assumptions. With this expression for a, the model for the process of attainment becomes,

$$\frac{dy(t)}{dt} = c_0 + by(t) + c_1x_1 + c_2x_2 \ldots c_nx_n \qquad (35)$$

This model has the solution:

$$y(t) = \frac{c_0}{b}(e^{bt} - 1) + e^{bt}y(0) + \frac{c_1}{b}(e^{bt} - 1)x_1 + \frac{c_2}{b}(e^{bt} - 1)x_2 \ldots \frac{c_n}{b}(e^{bt} - 1)x_n \quad . \qquad (36)$$

This is one of the most important equations estimated in status attainment research, as it relates observed states of a respondent to the status of first job and individual resources. Typically, this equation is estimated by pooling all respondents on cross-sectional data. Observed coefficients to the x_i variables will then be,

$$d_i = \frac{c_i}{b}(e^{bt} - 1) \;, \qquad (37)$$

in terms of the parameters that govern the process and time.

This means that the observed coefficients will be a function of (1) the amount of time respondents have spent in the labor force, (2) the quantity of b that measures the opportunities for growth in attainment as determined by both the rate at which vacancies are created and by the shape of the distribution of jobs by attainment level, and (3) of the contribution c_i of the variable in question to a person's overall level of resources.

Equation (35) can be used to estimate the various parameters if applied to over-time data [see Coleman, 1968, and Sørensen, 1976 for details], but when all respondents are pooled in a cross-sectional design such identification is not possible.

It should be noted that the dependency of d_i on both time and b is such that the older the respondent and the more favorable the opportunity structure, the larger the magnitude of the effects of x_i variables. One should therefore expect that the effect of a major determinant of resources such as education should have an observed effect on status that increases with increasing time. Such a pattern can indeed be found on life-history data [Sørensen, 1976]. Further, it is expected that if blacks are assumed to be exposed to a more unfavorable opportunity structure than whites, observed status returns to education should be lower for blacks than for whites. This pattern has been repeatedly found.

Research on the process of stratification and status attainment originated in intergenerational mobility research where the objective of comparing equality of opportunity in different societies and over time has always been a dominant one. Such comparisons could, in the framework of linear models, be carried out by comparing the effect of father's status on son's status observed in different societies or at different time periods. This would amount to estimating the equation,

$$y = d_0 + d_1 x_1 \tag{38}$$

where y is the observed status of sons and x_1 is the status of fathers, and compare d_1 over time or across societies. In the framework of the model proposed here, this means estimating the equilibrium equation,

$$y(e) = -\frac{c_0}{b} - \frac{c_1}{b} x_1 \tag{39}$$

obtained from (36) letting $t \to \infty$, and omitting other x_i variables.

The assumption of equilibrium in the observed attainment processes is clearly not valid when a representative cross-sectional sample is used to estimate d_1, since change in attainment presumably still will be going on for the younger cohorts. More importantly, perhaps, the coefficients d_1 as a measure of equality of opportunity will confound variation in the contribution of father's status to a son's overall level of resources and variation in the opportunity structure. Different implications for our understanding of societies depend on whether the contribution of father's status to resources or the opportunity structure are responsible for the variation. In particular, it can be noted that in two societies where parental status is equally important for a person's resources, the society with the most favorable opportunity structure will show the most inequality of opportunity, because b will be closer to zero and hence d_1 will be larger in absolute magnitude.

Implications for Models of Intragenerational Mobility

Social mobility has always attracted mathematical sociologists as a phenomenon that should lend itself to modeling using stochastic process models. The inherently stochastic nature of the process and the use of discrete occupational categories seem to call for a stochastic process model. Furthermore, mobility tables -- showing the number of persons moving among occupations are readily converted into estimates of transition probabilities of a Markov chain by dividing the row totals into the cell frequencies.

All attempts at testing the simple Markov chain on mobility data has however shown that this model does not account for observed movement. (For an early example, see Blumen, Kogan and McCarthy, 1955.) Numerous reasons have been given for the failure of the model -- heterogeneity in the parameters [McFarland, 1970; Spilerman, 1972], duration specific transitions or cumulative inertia [McGinnis, 1968; Tuma, 1976] and age dependency in the parameters [Mayer, 1972; Sørensen, 1972]. The resulting modifications of the Markov Model usually improve the fit of the model. However, the improved fit does not necessarily indicate the validity of the proposal. Heterogeneity will result in apparent nonstationarity, and vice versa, so that attempts to remedy either problem will improve the fit but not necessarily indicate the true source of failure in the model. Similarly, duration specific rates and age dependency are difficult to tell apart since age and durations in jobs are highly correlated.

Most of the proposals for improving the Markov Model are ad hoc proposals that are not based on an explicit theory of the mobility process. Hence it is not possible to choose among the proposals on theoretical grounds either.

The model for the attainment process proposed here indicates a specific modification of the simple Markov Model. This modification has been described in another paper [Sørensen, 1975], where also an empirical analysis using the model is carried out. The main result shall be briefly summarized here.

The simple Markov Model can be written (cf., Singer and Spilerman, 1974),

$$P(t) = P(0)e^{\lambda(M-I)t} , \tag{40}$$

where $P(t)$ is a vector giving the distribution of people according to job categories (say occupations) by time t . The matrix M has elements m_{ij} that give the probabilities of moving from category i to category j , given that a person is in state i ; and I is the identity matrix.

The parameter λ , a scalar, is the rate of job shift that is assumed constant over time in the simple model. In a system governed by the mobility regime described in this paper, λ will be dependent on time in the labor force, as λ corresponds to the quantity $q(t)$ defined in equation (25). This suggests that a reformulation of equation (40) where λ is dependent on time will be a more adequate representation of the intragenerational mobility process. A particularly simple representation is obtained by redefining time to take into account the decline in $\lambda(t)$ with time.

The desired redefinition of time should be so that in the new time scale the rate of job shift is constant over time; that is, job shifts follow a Poisson process. It still may be the case that the rate of shift will show variation among people; that is, heterogeneity will be present. However, removing the nonstationarity will also remove much of the apparent heterogeneity. In addition, the decline in the rate of job shift by time in the labor force was shown above to be generated by a reduction of the discrepancy between current attainment and the maximum attainment to be obtained. The latter quantity is determined by a person's resources. Hence, the time dependency in the rate indirectly captures important sources of variation among people.

The redefinition of time is easily obtained by defining a new time scale as the number of opportunities for shifts a person has encountered after t years in the labor force. The number of opportunities is captured by the quantity $v(t)$ defined in equation (26) as,

$$v(t) = \frac{1}{b}(e^{bt} - 1) \quad . \tag{41}$$

Assuming the validity of the model, the rate of shift in time scale $v(t)$ will be time independent. Denote this rate of shift λ^*. This quantity will in fact be 1 if it is assumed that people only shift to obtain gains in attainment. If voluntary shifts for other purposes are allowed, a value of λ^* different from 1 will be observed.

The constancy of the rate of shift in $v(t)$ can also be shown by noting that equation (28) is linear in $v(t)$, i.e.,

$$y(t) = y(0) + v(t)[-by(e) - y(0)] \quad . \tag{42}$$

The value of $y(t)$ may be seen as the expected outcome of a Poisson process by time $v(t)$, as each shift contributes a gain in attainment. Hence the rate of shift must be constant in $v(t)$.

With this time transformation, the Markov Model can be written,

$$P(v) = e^{\lambda^*(M-I)v} \quad , \tag{43}$$

assuming $P(0) = I$; and if the time transformation indeed removes time dependency in the rate of shift, a more realistic model is obtained.

A test of the proposed model for the dependency of the rate of job shifts on time in labor force can be obtained using life-history data that give information on the completed durations of each job. The completed durations are the waiting times between events, and if events follow a Poisson process in $v(t)$ waiting times will be exponentially distributed with a mean that will estimate the inverse of the rate. Transforming the completed observed duration into time scale $v(t)$ should therefore produce exponentially distributed durations with means independent of time in labor force. A test of the time transformation using this property was found to be quite satisfactory. A slight departure from the expected pattern could be explained as resulting from a change in the opportunity structure in the period where these job shifts took place. This change in opportunity structure is reflected in a decrease in the parameter b that governs the time transformation. It was further shown that the change in opportunity structure favored whites more than blacks [Sørensen, 1975:458].

The test of the model was carried out on jobs left voluntarily. Involuntary shifts should take place before the occurrence of a voluntary shift, and for this reason the completed durations of such jobs should be shorter than the completed durations of jobs left voluntarily. This can be demonstrated empirically [Sørensen, 1975:459], but on the average blacks were fired when they had held jobs longer than whites had held them when fired or laid-off. Since no one should stay in a job if a better one becomes available, this result also reflects a more unfavorable opportunity structure for blacks.

Involuntary shifts should produce losses in attainments since if a gain is available it should result in a voluntary shift. The impact of involuntary shifts on the career process is explored in another paper [Sørensen, 1974].

The proposed attainment model not only leads to a more empirically adequate stochastic model of mobility but also points to substantively meaningful analysis. The results summarized here, particularly the successful removal of time dependency in rates of shift using the model, in turn lend support to the model proposed in this paper.

Conclusion

This paper has proposed a model for the process of attaining income, status and other occupational rewards. The structure of inequality -- that is, the distribution of jobs according to attainments -- is assumed fixed and not subject to change due to variation in the distribution of personal resources (family background, education, ability) relevant for getting access to jobs. A simple exponential model is assumed for the attainment distribution. In this structure, new vacancies are created in each period of time, and these vacancies represent opportunities for growth in attainment. The mobility regime that prevails in such a structure -- where persons are entering and leaving the labor force at all attainment levels -- was shown to be particularly simple. It is further assumed that individuals' ability to take advantage of the opportunities for attainment gains is dependent on their current attainment relative to the maximum level of attainment they will be able to obtain given their resources. These resources are assumed to remain unchanged after entry into the labor force. From these assumptions, a simple linear differential equation model is derived for change in achievement over time.

The theory proposed here is explicitly derived on assumptions that are contrary to those used in human capital theory. There, change in attainments after entry into the labor market are assumed to reflect changes in personal resources due to on-the-job training, experience and the like. In this theory, a competitive market for skills is assumed to exist with no imperfections that will produce attainment increases without increases in resources (productivity). It is a consequence of this theory that the distribution of attainments will reflect the distribution of people with different levels of resources as the supply of people at various skill levels will affect the returns obtained, assuming a given demand schedule.

The theory formulated here and neoclassical theory give identical predictions regarding the shape of the age-attainment profile -- it will be concave to the age axis showing rapid growth in the beginning that gradually tapers off. In empirical investigations of age-earnings profiles in the human capital tradition, these observed profiles were interpreted as support for the theory as time is assumed to be a proxy for training and experience. But time may as well be interpreted as representing exposure to mobility opportunities as the theory here suggests.

Assuming attainment changes are produced by the creation of vacancies in a predetermined structure of inequality does account for the observed stability of the income distribution since WW II despite a marked change in the distribution of education -- a stability that is contrary to the implications of human capital theory. In the framework proposed here, changes in the distribution of resources do not affect the distribution of attainments. Changes in the distribution of education would presumably change the relative importance of education among the various attributes relevant for attainment, but not the distribution of jobs.

The purpose of the paper has, however, not been to prove human capital theory wrong. Both processes may operate simultaneously, and labor markets may be segmented according to whether one or the other process is dominant. Also, changes in earnings attainments may be more likely to reflect changes in resources than are changes in status attainments and changes in the attainment of psychological rewards from jobs, such as job satisfaction. The empirical identification of which mechanism prevails where and for which type of rewards is a major research task for which the theory proposed here only represents an alternative point of departure to the economic theory.

NOTES

1. A list of other derivations from the theory is presented by Becker [1964:7-8].

2. No attempt will be made here to explain how the distribution of attainments come into being. For the purposes of this paper, it is taken as a given. The assumptions stated here correspond to the one made by Thurow [1975] that marginal productivity resides in jobs, not in persons.

3. The mean of the geometric distribution is usually given as $\frac{1}{s}$. The difference reflects that here the bottom attainment level is obtained for $y = 0$, while the geometric distribution otherwise often is defined with $y = 1$ for the first trial.

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