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ABSTRACT

Allowing for gene-environment correlation, Jensen (1975) reports that under a wide range of assumptions, the twin data show that one-half to three-fourths of IQ variance is accounted for by genetic factors. But that conclusion falls when an arbitrary specification is relaxed.

ON JENSEN'S METHOD FOR TWINS

Introduction

In his latest statement on the heritability of intelligence, Jensen (1975) extends the classical twin method to allow for covariance between genes and environment. His calculations appear to show that for a wide range of assumptions, the twin data yield estimates of heritability in the range .50 - .75.

His results, however, depend on an arbitrary specification.

Jensen's Model

Jensen's specification is contained in his equations (2), (7), (8) for total variance, MZ covariance, and DZ covariance respectively. To simplify notation, we rewrite these in standardized form as

$$1 = h^2 + e^2 + 2rhe + m^2 \quad (1)$$

$$c = h^2 + \rho e^2 + 2rhe \quad (2)$$

$$c' = g'h^2 + \rho'e^2 + 2rhe \quad (3)$$

where

c = observed correlation for MZs

c' = observed correlation for DZs

h^2 = heritability (ratio of genetic variance to total variance)

e^2 = environmentability (ratio of environmental variance to total variance)

m^2 = unreliability (ratio of measurement error variance to total variance)

g' = genetic correlation for DZs

ρ = environmental correlation for MZs

ρ' = environmental correlation for DZs

r = correlation between genotype and environment.

Given observed values of c , c' , and assigned values of g' , ρ , ρ' , r , the three-equation system can be solved for h^2 , e^2 , and m^2 . Thus, Jensen takes $c = .87$, $c' = .56$, sets $\rho = .90$, and alternatively assigns these values:

$$g' = .50, .54, .58$$

$$\rho' = .90, .85, .80, .75, .70$$

$$r = .04, .16, .28, .40.$$

His solutions for h^2 then range between .50 and .75.

Jensen calls particular attention to the case $g' = .58$, $\rho' = .70$, $r = .40$, reporting that it yields $h^2 = .74$. Correcting for attenuation he gets .80, a figure he has made familiar on other occasions.

For future reference, we first obtain the complete solution for this case. Our equations (2), (3) specialize to

$$.87 = h^2 + .90e^2 + .80he \quad (4)$$

$$.56 = .58h^2 + .70e^2 + .80he \quad (5)$$

The relevant solution to these is

$$h^2 = .72 \quad e^2 = .03,$$

whence

$$2rhe = .12,$$

and using also (1):

$$m^2 = .13.$$

Rounding error aside, we confirm Jensen's heritability estimate for this case.

A Plausible Modification

The parameter r --Jensen's ρ_{GE} --was defined simply as "the correlation between genotype and environment." But a moment's thought reveals that it is playing three distinct roles in his model. In (1) it denotes the correlation between an individual's genotype and his own environment, in (2) it denotes the correlation between an individual's genotype and his MZ twin's environment, and in (3) it denotes the correlation between an individual's genotype and his DZ twin's environment. The implicit assumption is that these three correlations are the same.

Since MZs have identical genotypes, there is no objection to using the same r in (1) and (2). On the other hand, since DZ's genotype are far from perfectly correlated, it is not obvious why the correlation of a twin's genotype with his co-twin's environment must be as high for DZs as for MZs. It is true that equality of those two correlations is implied by the causal model provided by Rao, Morton, and Yee (1974), which specifies that the correlation between the twin's environments is entirely due to familial factors. But their model also implies that $\rho' = \rho$.

Let us modify Jensen's model by replacing equation (3) with

$$c' = c = g'h^2 + \rho'e^2 + 2r'he \quad (6)$$

where

r' = correlation between an individual's genotype and his DZ twin's environment.

To what extent do Jensen's results depend on the implicit assumption that $r'/r = 1$?

For present purposes, we need only consider the same particular case which Jensen emphasized, namely

$$\begin{array}{lll} c = .87 & c' = .56 & \rho = .90 \\ g' = .58 & \rho' = .70 & r = .40. \end{array} \quad (7)$$

For given values of r'/r we can proceed to solve the system.

Suppose for example that $r'/r = g'$, i.e. that for DZs, the cross-twin environmental correlation is reduced in the same proportion as the cross-twin genotypic correlation. Then our equations (2), (6) specialize to

$$.87 = h^2 + .90e^2 + .80he \quad (8)$$

$$.56 = .58h^2 + .70e^2 + .464he, \quad (9)$$

which differ from (4)-(5) only in the coefficient on he in the DZ equation. The relevant solution is now

$$h^2 = .33 \quad e^2 = .31,$$

whence

$$2rhe = .26,$$

and using also (1),

$$m^2 = .10.$$

This solution is indeed quite different from that obtained by Jensen for the same particular case. And "the" heritability estimate $h^2 = .33$ falls well outside the range of values spanned by his charts.

Of course, our $r'/r = g'$ is itself arbitrary. Therefore we tabulate the solutions obtained for some alternative values of r'/r :

Solutions to the System (1), (2), (6), (7)

r'/r	h^2	e^2	$2rhe$	m^2
.4	.15	.54	.23	.08
.5	.24	.42	.26	.08
.6	.36	.28	.26	.10
.7	.50	.16	.22	.12
.8	.61	.09	.18	.12
.9	.68	.05	.14	.13
1.0	.72	.03	.12	.13

Note that the estimate of heritability increases monotonically with the assigned value of r'/r .

Remarks:

(i) Our numerical examples should suffice to show how sensitive heritability estimates can be to changes in specification. More generally, it is hard to see how one can hope to obtain meaningful estimates of h^2 from only two piéces of data (c and c') when there are so many unknowns in the system ($g', \rho, \rho', r, r', e^2$).

(ii) Jensen's specification, and his numerical values for c, c', g', ρ, r , are identical to those used in an article previously published

by Hogarth (1974). Hogarth, however, emphasizes the sensitivity of estimates to arbitrary assumptions.

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