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THEORETICAL LABOR SUPPLY MODELS AND  
REAL WORLD COMPLICATIONS

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## Abstract

Theoretical models of individual labor supply behavior are traditionally developed within the context of a highly idealized economic environment. The observable labor market to which economists wish to apply these labor supply models often differ radically from the idealized world assumed in the theory. In this paper we discuss a number of important discrepancies between the two. Our discussion and proposed solutions are focused on the development of an empirical model applicable to data on prime-age males from the Michigan Panel Study of Income Dynamics, but many of the issues discussed are relevant for the construction of models for other groups. The discussion covers five basic areas:

1. The potential limitations of the simple labor supply model, based only on income-leisure tradeoffs, are discussed.
2. Sources of randomness in observed labor supply behavior and its consequences for the estimation of systematic labor supply responses are considered. Measurement error, disequilibrium effects, interpersonal differences, and intertemporal variation are discussed.
3. The model is extended to accommodate earnings opportunities other than a simple constant wage rate. Increasing marginal income tax rates and overtime premiums are the major factors considered.
4. Demand-related factors that prevent workers from achieving marginal equilibrium at their marginal wage rates are explored in some detail. Criteria are suggested for the selection of a sample of workers who are less seriously affected by these problems.
5. The treatment of time lost due to unemployment and illness is discussed in the context of a model developed by Samuel Rea. The model is also applied to time spent commuting to work.

## THEORETICAL LABOR SUPPLY MODELS AND REAL WORLD COMPLICATIONS

The classical economic theory of individual labor supply has been widely employed as the basis for empirical studies of labor supply behavior. The properties of the theoretical model are derived in the context of an idealized and highly simplified economic world. The usefulness of the theoretical model as a basis for inference about workers' behavior in the contemporary labor market depends critically on the successful adaptation of the simple model to the complexities of the real world.

The major simplifying assumptions that are used in deriving the theoretical model are the following:

- a. The preferences on which labor supply decisions are based are adequately represented by a classical static utility function having the leisure time of workers and the total market consumption of the unit as its only arguments.
- b. The labor supply function is an exact function of budget variables and describes the optimal behavior of a single individual or a population of individuals having identical stable preferences.
- c. The opportunity set on which utility is optimized is a simple line<sup>1</sup> defined by a level of nonwage income,  $I$ , and a wage rate,  $w$ , which is constant for all hours of work. The budget variables,  $I$  and  $w$ , are presumed to be exogenously determined.
- d. Workers are presumed to be free to choose any nonnegative level of work hours that maximizes utility subject to the simple budget constraint. In particular, the optimal labor supply points are presumed to be internal solutions as opposed to corner solutions.

In this paper we will review a number of the more important discrepancies between the characteristics of the real economic environment and the assumptions underlying the theoretical model. Much of our data about the complexities of labor market opportunities is drawn from the Michigan Panel Study of Income Dynamics. Accordingly, our discussion of empirical problems with the simple theoretical model and our proposed solutions to those problems will be cast in the context of that data set. The discussion will also focus on employed male heads of households because the data are most complete for that population group. The resulting empirical model, in its full detail, will thus be relatively specialized. Many of the problems discussed arise in other contexts, however, so that elements of our discussion have much broader relevance.

We shall employ two basic strategies in our approach to empirical complexities of the labor supply model. For cases where complicating factors may be quantified and measured, we will modify and expand the theoretical model accordingly. In other cases we will simplify the real world by excluding from the proposed analysis those individuals whose employment opportunities are too complex or too poorly measured to fit within the rubric of the demand theory model. The latter strategy does have the consequence of limiting the scope of our proposed analysis. However, the restriction to workers facing well-measured employment opportunities will facilitate more general inference about the form of the labor supply function and the underlying structure of preferences for income and leisure along the lines discussed in an earlier paper (Dickinson, 1975). The general discussion

will seek to place the strengths and weaknesses of our proposed inference about labor supply behavior in balanced perspective.

The empirical problems discussed in the first four sections of this paper correspond roughly to the major simplifying assumptions outlined above. Some of the more difficult problems result from the simultaneous breakdown of two or more of these assumptions, so the discussions within each section progress from the treatment of a given issue in isolation to a consideration of several simultaneous empirical difficulties.

Section I includes a justification of the use of the simple utility model and a discussion of some of the limitations of this model. The basic model is common to a great majority of labor supply studies and our discussion is brief.

Section II is devoted to an extensive discussion of the stochastic specification of the empirical labor supply function within the context of the utility maximization model. We distinguish four major sources of random variation in observed labor supply behavior: measurement error, minor disequilibrium effects, interpersonal diversity of preferences, and intertemporal variance in preferences. A realistic specification for stochastic effects arising from diversity of preferences is seen to require a random coefficients model. Under this model, the usual cross-sectional estimate of the substitution effect will generally be biased. Section II concludes with a discussion of the problem of the endogeneity of nonwage income due to differential preferences for the accumulation of assets.

The empirical problems caused by segmented budget structures are discussed in Section III. Appropriate transformations are developed to represent the net effects of overtime premiums and progressive marginal income tax rates. The problem that a worker's marginal tax rate or overtime premium depends on his choice of work hours is also discussed. An imputation procedure is developed to combat the resulting biases.

Section IV focuses on the problem of demand-related constraints on work hours. We present tabulations that indicate widespread incidence of such constraints and provide a skeptical review of the thesis that these constraints are not important in the long run. We then discuss the selection criteria for a subsample of workers who are substantially free of demand-related constraints and consider whether estimates based on that subsample will be subject to differential selection bias.

In the last substantive section of the paper, section V, we discuss the empirical treatment of unemployment, illness, and commuting time. The specification developed by Rea (1971, 1974), in which unemployment time is treated as an independent variable, is extended to cover illness time and commuting time as well.

### I. The Simple Utility Function

The first assumption, that labor supply behavior may be modeled on the basis of a simple utility function of leisure and market consumption, is central to our model, as it has been to most labor supply models in the literature. Clearly, many factors other than simple income-leisure choices enter into the labor supply decisions of individual workers. The simple utility model is chosen as a useful

simplification, however, because it is believed to provide a good approximate structure for modeling those aspects of behavior that are systematically related to variations in economic incentives. The effects of the multitude of other, unmeasured, factors that influence labor supply decisions are incorporated into the stochastic specification of the model that will be discussed in some detail in the next section.

The implications of the simple utility approximation bear some consideration, however. Both basic goods are composites of many different elements. Leisure, in the economist's terminology, is simply a shorthand for all time not directly related to market work.<sup>2</sup> The actual components of leisure may include such varied activities as vacation cruises, dinner parties, shaving, mowing the lawn, and repairing the family car. Our assumption that the sum of all such nonmarket time may be treated as a single composite good rests on the further assumption that a worker is relatively free to allocate nonmarket time to suit his preferences. The "time price" of one nonmarket activity relative to others is unity, since time spent at one cannot be spent at another. A worker is thus able to allocate his time so that he derives equal satisfaction from marginal time spent at each activity, which valuation is equal to the marginal utility of the composite good, "leisure." Under the free allocation assumption, the specific allotment of nonmarket time to various activities may vary as the total amount of leisure demanded changes in response to wage rates and income. That allocation need not be of concern for our model, however, so long as it is unimpeded.

The free allocation assumption is not, in fact, entirely realistic. In particular, very few workers are fully free to allocate time

at will between weekdays, weekends, and extended vacations. While our strategy in this paper will be to develop a model to be applied to a sample of workers who have substantial freedom of choice in labor supply decisions, such decisions still must be made within the structure of institutional work schedules. Any marked change in this institutional structure might thus be expected to change the nature of the composite good, "leisure," and might result in significant changes in the parameters of the simple income-leisure utility function.

It is possible that a model could be constructed that would be more robust with respect to the institutional structure that influences demand for the various dimensions of leisure. A potentially fruitful approach might be to construct a model along the lines of those developed by Lancaster (1966) and Becker (1965), in which time is treated as an input to a variety of activities that produce satisfaction for the worker-consumer.<sup>3</sup> For the present study, however, we duly note the limitations of our simpler approach and forge ahead regardless. After accounting for unemployment, illness, and commuting time, which will be discussed below, we define labor supply as the total number of hours worked per year regardless of scheduling. "Leisure" is then the sum of all remaining time.

The treatment of all market consumption goods as a single composite also rests on a number of assumptions. The simplifying approximation is expected to be acceptable so long as the internal price structure of market goods remains reasonably stable, or so long as the

goods whose prices change disproportionately are neither strong complements nor strong substitutes for leisure. For the recent period of the energy crisis, with large changes in prices of gasoline and other travel-related goods, these assumptions might be questionable. The proposed empirical work is based on the four-year period ending in 1972, however, and for that period the assumption of a reasonably stable price structure for most market goods appears to be plausible.

## II. The Stochastic Properties of Labor Supply

The theory of utility maximization from which the properties of the labor supply function are derived abstracts from any random effects. On a given opportunity set there is a single optimal equilibrium point, and the labor supply function describes changes in that optimal equilibrium as the variables defining the opportunity set change.

In virtually all currently available bodies of data, we observe not a single equilibrium position, but a dispersion in the amount of labor supplied for any given values of income and wage rates. The usual practice in previous empirical work has been to graft a single random disturbance term onto the labor supply function with comparatively little direct consideration of the relationships of the random term to the underlying utility maximization model. Substantial attention has been devoted, on a case-by-case basis, to various potential biases that are related to problems with the stochastic specification of labor supply, usually in combination with the failure of one or more

of the other classical assumptions. We will review a number of the more important problems of this sort in succeeding sections, but we turn first to a systematic discussion of the stochastic properties of observed labor supply behavior within the context of the utility maximization model.

If we were to observe the labor supply behavior of an ideal cross-section of workers facing identical budget constraints, we would identify four basic sources of random dispersion in the level of work hours:

- a. simple measurement error;
- b. disequilibrium labor supply by individual workers;
- c. differences between individuals in income-leisure preferences;
- d. temporal variation in income-leisure preferences of individual workers.

Measurement error and some disequilibrium effects are reasonably easily accommodated within the stochastic specification of the standard regression model and do not pose serious estimation problems. Larger variance in the disturbance will reduce the precision of parameter estimates but will not introduce biases, so long as the disturbance is independent of the explanatory variables in the model. The usual presumption of a zero mean for the disturbance term is subject to some question in the cases of measurement error and disequilibrium labor supply, but given independence of the disturbance, this will not result in biases in the directly estimated income and wage effects. There is a slight wrinkle in this model, in that a nonzero mean of the disturbance will result in a bias in the estimate of the expected level of work hours, and

a consequent bias in the estimate of the substitution effect as calculated from the Slutsky equation. A bias of this sort is unlikely to result in an important increase in the mean square error of the substitution effect, however, since the expected level of work hours enters the Slutsky equation as a product with the estimated income effect. The latter estimate has a much larger relative sampling error under virtually all plausible circumstances and thus dominates the mean square error.

The stochastic specification that follows from a diversity of labor supply preferences is somewhat more complex. Again, the direct estimates of income and wage effects are robust, but the usual estimates of the substitution effect may be significantly biased even if the independence and zero mean assumptions hold. We elaborate on these points as we discuss each source of stochastic disturbance in turn.

a. Measurement error. Labor supply information in the Michigan Panel Study was collected by means of personal interviews. While the questions were carefully designed to elicit information about both regular work schedules and extraordinary overtime or layoffs, the resulting measure of total annual work hours is certainly subject to some reporting error. Errors due to rounding or to faulty recall may reasonably be presumed to be uncorrelated with the explanatory variables in the model. There is some evidence from work on "Rotation Group Biases" in the Current Population Survey<sup>4</sup> that reports of weekly work hours in the early months of that survey may be exaggerated relative to reports in later months. The figures suggest a possible upward bias of about 3 percent in annual work hours.

In many studies, the wage rate variable has been constructed by dividing reported earnings by reported work hours. That procedure results in biases from correlated errors in variables. We propose to avoid the problem by using directly reported hourly wage rates. The resultant model is thus not applicable to workers without defined marginal wages, as will be further discussed in Section IV.

b. Disequilibrium work hours. Disequilibrium values of labor supply in a given period may occur because institutional factors prevent a worker from "fine tuning" his hours of work. Examples of displacements from equilibrium are short work weeks or unemployment on the one hand and unwanted compulsory overtime on the other. The magnitude of the displacement may be great enough to create significant dissatisfaction with the constraints on work time or small enough to constitute no more than a minor annoyance. As will be discussed in a later section of this paper, we will find it necessary to distinguish those workers with relatively severe disequilibrium problems and treat them separately in the labor supply analysis. The distribution of such serious displacements from equilibrium is heavily skewed on the negative side, and they cannot be incorporated into the simple stochastic specification without risk of serious biases and/or loss of estimation precision. Below some threshold of utility loss, however, displacements from equilibrium may be expected to have reasonably well-behaved stochastic properties.

The basis for our distinction with respect to the seriousness of labor supply disequilibrium is provided by a series of questions asked each year in the Michigan Panel Study of Income Dynamics. The questions

are designed to ascertain whether a worker had freedom of choice in his hours of work and, if not, whether he was dissatisfied with the limits on his labor supply.<sup>5</sup> The questions about freedom of choice in work hours pertain to reasonably straightforward factual information. Those about satisfaction with existing limits on work hours are more hypothetical. Economists are rightfully skeptical about literal interpretations of responses to such questions. In the present case, however, these responses have a plausible interpretation within the context of the utility maximization model.

Any worker who reports that his work hours were limited in one direction or the other is unlikely to be precisely at his optimal labor supply position. If the disequilibrium displacements are small, they will cause small utility losses which are likely to be ignored or soon forgotten. On the other hand, sufficiently large displacements from equilibrium may be expected to result in tangible utility losses. If the utility loss exceeds a worker's subjective threshold of tolerance, he may be expected to respond that he was dissatisfied with the limits on his work hours. The fact that a large majority of hourly workers faced some limits on their hours but only about half of these expressed dissatisfaction<sup>6</sup> lends credence to this interpretation. In any case, the distinction may be subjected to empirical tests through comparative estimates for workers in approximate equilibrium and for those who expressed dissatisfaction with available work hours.

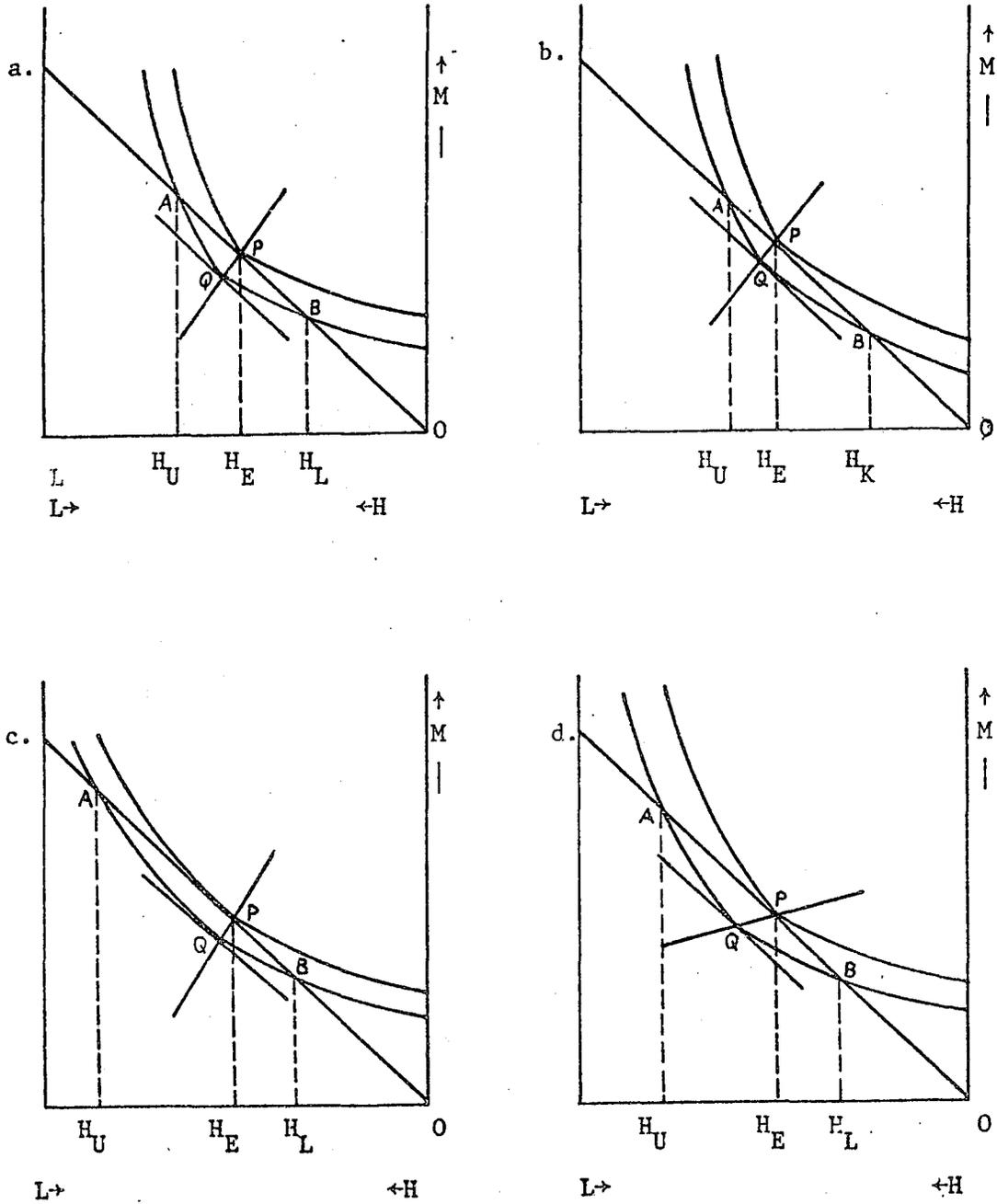
The loss of utility resulting from nonoptimal labor supply will depend on the properties of the underlying income-leisure utility function. The general nature of the dependence of utility loss on the

income and substitution parameters of the supply function is illustrated in Figures 1a-1d. In each of the figures the optimal equilibrium on the budget line OBPA is at the tangency point, P. The indifference curve through points A, Q, and B in each figure represents the hypothetical threshold of utility loss at which a worker expresses dissatisfaction with constraints on work hours. If he faces institutional constraints between work hours  $H_L$  and  $H_U$  corresponding to points B and A on the budget line, he will not express dissatisfaction. If unable to work at least  $H_L$  hours, however, he will report a desire for more work than is available.

In Figure 1a the substitution effect is roughly constant and the limiting values,  $H_L$  and  $H_U$ , are approximately symmetric about the equilibrium value,  $H_E$ .<sup>7</sup> In Figure 1b the income effect is unchanged from 1a, but the substitution effect is changed to a strongly decreasing function of the marginal rate of substitution (or equilibrium marginal wage rate). In Figure 1c the substitution effect varies in the opposite direction. An increase in the substitution effect flattens the curvature of an indifference curve so that it remains close to the budget line for greater distances from the tangency point. Changes in the substitution effect along an indifference curve thus make the interval between the dissatisfaction points asymmetric relative to the optimal equilibrium. Figure 1d is similar to 1a in the shape of the indifference curves but shows a much stronger income effect. The threshold curve is thus shifted to the left, again resulting in asymmetry of the  $H_L$ ,  $H_U$  interval relative to the optimal point. A weaker income effect

FIGURE 1

SYMMETRY PROPERTIES OF REGION DEFINED BY A UTILITY LOSS THRESHOLD



would result in the opposite asymmetry. Clearly the overall symmetry properties of the "acceptable disequilibrium interval" depend on the joint impact of the income and substitution parameters, but only fortuitously would we find perfect symmetry around the optimal labor supply point.

The above analysis indicates that the component of the disturbance term arising from disequilibrium work hours will not, in general, have an expected value of zero. If the asymmetry properties of the acceptable disequilibrium interval are reasonably similar at different levels of income and wage rates, the bias in the disturbance term will not imply biases in the estimated wage and income coefficients. The estimates of the substitution effect will be subject to some bias, but, as in the case of measurement error, the net impact of the bias on the mean square error of the estimate will almost certainly be negligible.<sup>8</sup>

It is possible to construct utility maps in which the asymmetry properties of the acceptable disequilibrium interval differ at different levels of income and wage rates. Such properties could result in biases in the directly estimated wage and income effects. Only comparatively extreme configurations of indifference curves would result in significant biases, however, so it appears reasonable to ignore this issue unless initial estimates indicate that further consideration is necessary. In overview, then, the above discussion provides a useful theoretical structure for our consideration of the stochastic properties of labor supply but does not reveal any substantial empirical problems that have hitherto been neglected. It does, nevertheless, outline a number of

potential problem areas that might be of greater quantitative importance in labor supply analysis for population groups other than prime-age married males.<sup>9</sup>

c. Interpersonal diversity of preferences. In the previous sections we have considered stochastic elements that take the form of differences between observed levels and the true optimal level of work hours. For simplicity in the present discussion we revert to the presumption that the true optimal equilibrium may be directly observed, in order to examine the stochastic properties of diverse preferences. In this model, random dispersion in labor supply arises because different individuals, facing the same budget constraint will choose different optimal work hours. Our concern here is not with those differences in preferences that are associated with observable characteristics such as educational background, race, or family situation. Rather, we wish to abstract from observable differences among individuals and focus on differences in preferences of the sort that give credence to the adage "there is no accounting for tastes." Our discussion will focus on the empirical problems caused by diversity of preferences in the context of cross-sectional analyses. Certain of the problems discussed here may be resolved through the appropriate analysis of panel data on diverse individuals. Issues specifically related to such an analysis are discussed more thoroughly in Dickinson (1976) chapter VI, and will be only briefly introduced in this paper.

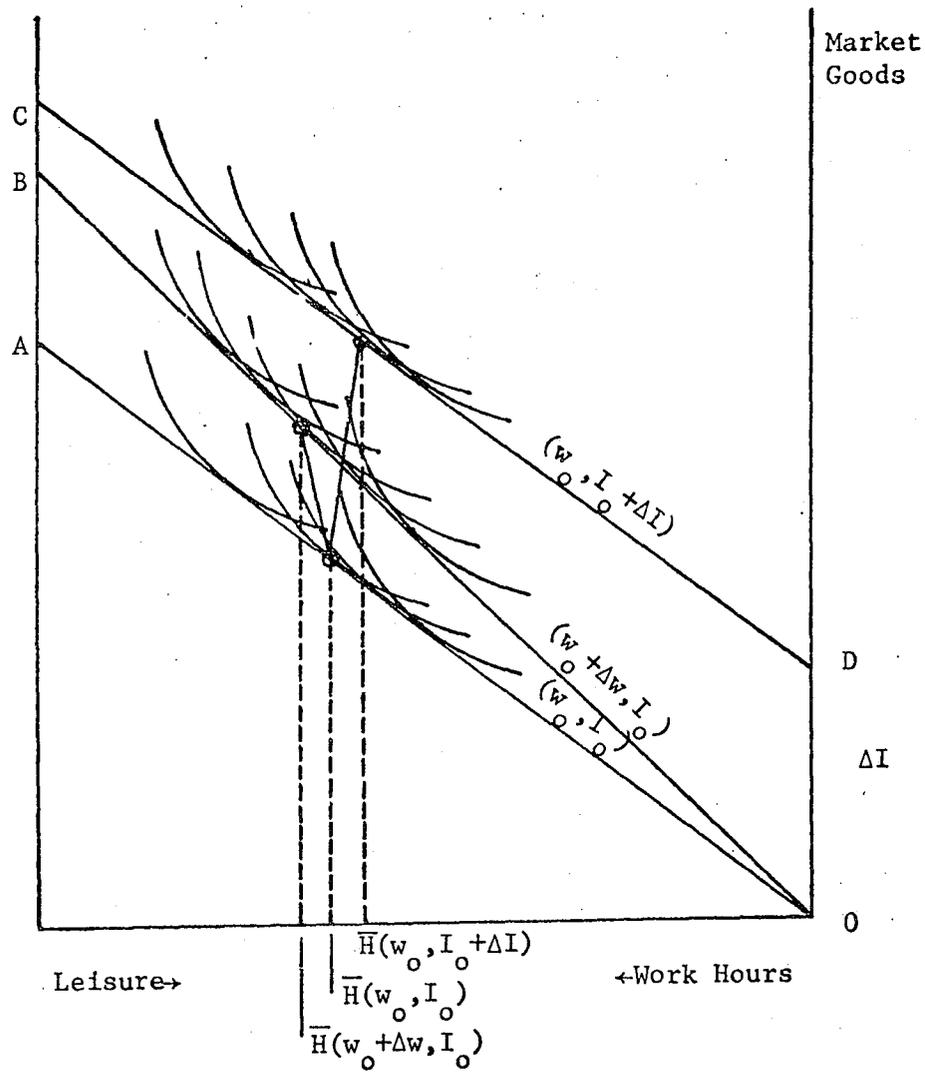
The basic elements of labor supply estimation from a cross section of diverse individuals are illustrated in Figure 2. If the appropriate conditions are satisfied, sample observations of labor supply positions on the three budget lines,  $(w_0, I_0)$ ,  $(w_0 + \Delta w, I_0)$ , and  $(w_0, I_0 + \Delta I)$ , provide sufficient information for unbiased estimates of the mean wage and income effects in the population. A sufficient condition for unbiasedness is that the same distribution of preferences is sampled for each budget line. This assures that the difference between sample mean values on different budget lines will represent an unbiased estimate of the mean of individual responses to the corresponding change in income or wage rates. The necessary conditions are expressed in equations (1a) and (1b). In special cases, such as symmetrical truncation, identical sampling distributions at different budget levels are not strictly necessary for conditions (1) to hold.

$$(1) \quad \begin{aligned} \text{a) } E[H(w_0 + \Delta w, I_0)] - E[H(w_0, I_0)] &= \\ &E[H(w_0 + \Delta w, I_0) - H(w_0, I_0)]. \\ \text{b) } E[H(w_0, I_0 + \Delta I)] - E[H(w_0, I_0)] &= \\ &E[H(w_0, I_0 + \Delta I) - H(w_0, I_0)]. \end{aligned}$$

Given that the conditions of (1) are satisfied, estimates of wage and income effects may be calculated from (2a) and (2b).

FIGURE 2

ELEMENTS OF CROSS SECTIONAL INFERENCE ABOUT LABOR SUPPLY RESPONSES



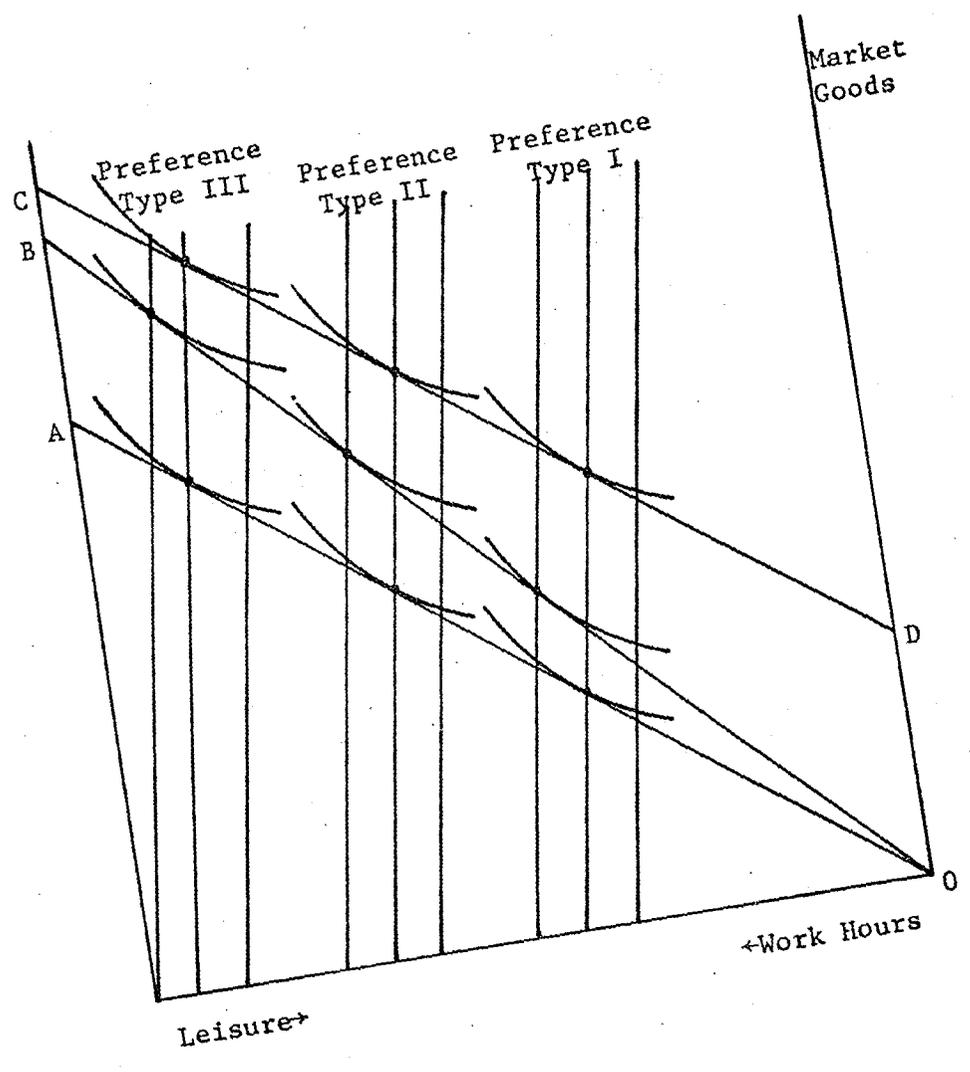
$$(2) \quad \text{a) } \frac{\partial H}{\partial w} (w_0, I_0) = \frac{\bar{H}(w_0 + \Delta w, I_0) - H(w_0, I_0)}{\Delta w} .$$

$$\text{b) } \frac{\partial H}{\partial I} (w_0, I_0) = \frac{\bar{H}(w_0, I_0 + \Delta I) - H(w_0, I_0)}{\Delta I} .$$

In the limit of small  $\Delta w$  and  $\Delta I$  the estimates represent point derivatives at  $w_0$  and  $I_0$ . Otherwise they represent linear approximations for the respective effects over the given range. In actual practice, the estimates will not be derived from a single set of first differences, as in (2), but from a variety of values of income and wage rates using a regression model and particular functional assumptions or approximations.

The extent to which the conditions for unbiasedness are fulfilled cannot be established empirically from a single cross section of observations on diverse individuals. Abstracting from the disequilibrium effects discussed earlier, we may observe the specific equilibrium position of a given individual on a given budget line, but it is not possible to identify individual responses to budget changes. By the equal distribution assumption we presume that individuals with similar preferences are represented in our observations at other wage and income levels, but we do not have the information necessary to establish any direct correspondence between the equilibrium positions of persons with similar preferences at different budget levels. We might assume that the nature of the diversity of preferences is restricted and well behaved, such as that illustrated in Figure 3. The figure shows three different preference functions that differ only in the level of

FIGURE 3  
WELL-BEHAVED DIVERSITY OF PREFERENCES

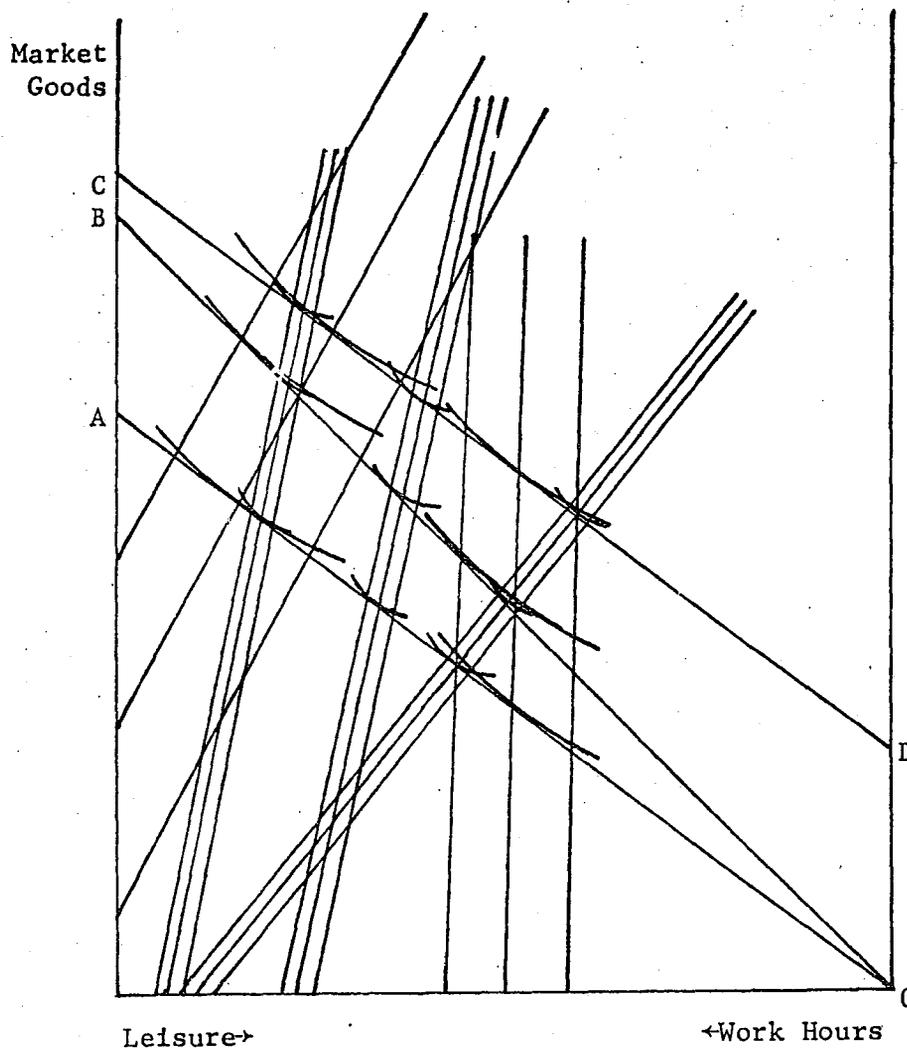


work hours. The parameters of labor supply functions are similar<sup>10</sup> across individuals in such a way that a given individual's labor supply position and individual responses are essentially identical to the population mean response. But even with such a well-behaved set of diverse preferences, generalized to a continuous distribution, it is not possible to verify empirically that the distribution of preferences is randomly sampled at all budget levels. The credibility of our cross-sectional inferences thus rests on our ability to identify and resolve problems that would be expected to result in disproportionate representations of high- or low-preference individuals at different wage and income levels. Almost all of the major problems of this sort arise from a combination of diversity of preferences and other complications of the simple utility model; they are thus appropriately treated in later sections of this paper. We first address an estimation problem that follows solely from diversity of preferences, even in the absence of difficulties with differential sampling.

Diverse preferences of the regular sort shown in Figure 3 could be accommodated, at least to a good approximation,<sup>11</sup> within the stochastic specification of a simple additive disturbance term in the supply function. Under a more general interpretation of diverse preferences we would expect individuals to differ, not only in their expected levels of labor supply, but in their other labor supply parameters as well. An example of such diversity is illustrated in Figure 4. For ease of construction, the degree of diversity is restricted to preference

FIGURE 4

INTERPERSONAL DIVERSITY WITH RESPECT TO ALL  
LABOR SUPPLY PARAMETERS\*



\*Restricted to preference functions in the parallel class  
for ease of illustration.

functions in the parallel class, but the functions shown differ in income, wage, and substitution effects in addition to equilibrium levels of labor supply.

For simplicity of exposition of the properties of the general diversity models, we shall refer to point estimates of labor supply parameters such as would be obtained from idealized cross-sectional observations at the three budget levels illustrated in Figure 4 with sufficiently small wage and income differentials. The stochastic specification that allows for interpersonal diversity of labor supply parameters does not pose serious problems for the direct estimates of wage and income effects. The estimate of the income effect, say, no longer corresponds to a single constant parameter but rather to the expected value of the distribution of income effects across individuals in the population. So long as the conditions of equation (1) are satisfied, we obtain unbiased estimates of that expected value. Furthermore, the interpretation of the expected value, as the expected mean response to an exogenous shift in nonwage income, is virtually indistinguishable for policy purposes from the interpretation of the single constant parameter of the simpler model. The same interpretation clearly holds for the expected value of uncompensated wage effects.

The problems caused by the expanded stochastic specification pertain to the estimate of the substitution effect as it is usually calculated using the form of the Slutsky equation. The usual estimate is shown in equation (3a), and the corresponding population value,

under the current specification, is shown in (3b). As noted above, the equations refer to point values evaluated at some reference budget,  $w_0, I_0$ .

$$(3) \quad \begin{aligned} \text{a) } \hat{S} &= \left( \frac{\partial \hat{H}}{\partial w} \right) - \hat{H} \cdot \left( \frac{\partial \hat{H}}{\partial I} \right). \\ \text{b) } E(\hat{S}) &= E \left( \frac{\partial H}{\partial w} \right) - E(H) \cdot E \left( \frac{\partial H}{\partial I} \right). \end{aligned}$$

In general, the expected value of the conventional estimate, given in (3b), will not equal the expected value of the substitution effect in the population. The difference is easily demonstrated by writing down the true Slutsky relationships for the  $i^{\text{th}}$  individual and then taking the expected value for the population as shown in (4a)-(4c).

$$(4) \quad \begin{aligned} \text{a) } S_i &= \left( \frac{\partial H}{\partial w} \right)_i - H_i \cdot \left( \frac{\partial H}{\partial I} \right)_i. \\ \text{b) } E[S_i] &= E \left[ \left( \frac{\partial H}{\partial w} \right)_i \right] - E[H_i] \cdot \left( \frac{\partial H}{\partial I} \right)_i. \\ \text{c) } E[S_i] &= E \left[ \left( \frac{\partial H}{\partial w} \right)_i \right] - E[H_i] \cdot E \left[ \left( \frac{\partial H}{\partial I} \right)_i \right] - \text{Cov} \left[ H, \left( \frac{\partial H}{\partial I} \right)_i \right]. \end{aligned}$$

The covariance term in (4c) will not generally be zero if there is any variance in the income effect among individuals. Thus, the conventional estimate, which neglects this term, will in general be biased.

The properties of the neglected covariance term and the consequent bias are discussed more fully in Dickinson (1976, chapter VI). In that analysis the time series dimension of data from the Michigan Panel Study

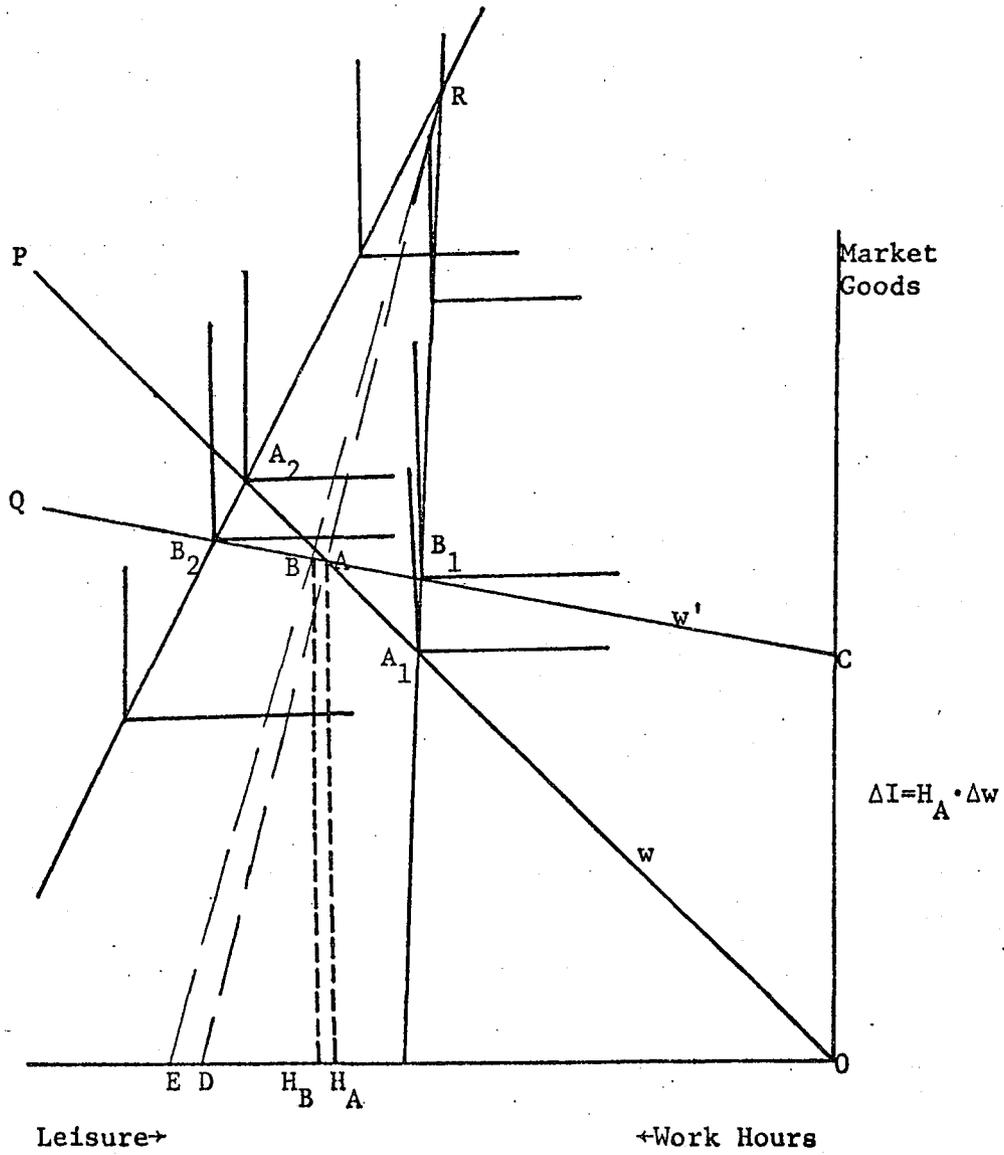
is employed to identify and estimate the covariance term and other parameters related to diverse individual preferences. For the present discussion it will suffice to present the simple concrete example of the bias shown in Figure 5. The population in the example consists of equal numbers of two types of individuals. Both types have zero substitution effects, as indicated by the right-angular indifference curves, but those with higher work hours preferences in the illustrated range have stronger negative income effects at any given wage rate. If the population is originally in equilibrium on the budget line  $OP$ , at wage rate  $w$ , the two types of workers will exhibit equilibria at  $A_1$  and  $A_2$  and the observed population mean will be at point  $A$  with work hours  $H_A$ . If we then consider a compensated decrease in the wage rate, using point  $A$  as the point of compensation, the two types of workers will arrive at new equilibria,  $B_1$  and  $B_2$ , on the new budget line  $CQ$ . The observed population mean work hours,  $H_B$ , will be greater than  $H_A$  despite the lowered marginal wage rate, yielding an apparent negative substitution effect for the population.<sup>12</sup>

The apparently counter-theoretical result would also be obtained if we estimated mean expansion paths for this population at different wage rates following the methodology proposed in Dickinson (1975). The mean expansion path for wage rate  $w$ , is the line  $DAR$ . That for the lower wage rate,  $w'$ , is the line  $EBR$  and lies to the left of  $DAR$ , again implying an apparent negative substitution effect.

The problem is not one of incorrect measurement of variables, nor is there any disproportionate sampling of preference types at different

FIGURE 5

APPARENT NEGATIVE MEAN SUBSTITUTION EFFECTS FROM A CROSS SECTION OF DIVERSE INDIVIDUALS



budget levels. The negative compensated mean wage response is a consistent property of a properly specified model describing the mean labor supply responses of this population. It is simply that the function representing the mean labor supply responses of diverse individuals does not necessarily satisfy the simple restrictions that were derived for the labor supply function of a single individual.

The bias in the estimate of the substitution effect will affect simulations of responses to income maintenance programs, since the most common method of simulating responses relies on the estimated substitution parameter. If all workers at a given wage level are affected by a maintenance program, the unbiased estimates of the uncompensated wage effect and the income effect may be used directly for unbiased simulation of the mean response to the program. The latter simulation method will not be satisfactory for workers at higher wage levels, at which some are affected by the program and others are not. For unbiased simulation of responses for workers in the vicinity of the breakeven level, one needs not only unbiased estimates of the mean substitution effect but also estimates of the distribution of individual values of income and substitution parameters, none of which are available from simple cross-sectional estimates.

The bias in the estimated substitution effect also weakens what has been the basic test of the theoretical acceptability of labor supply estimates. Under most plausible circumstances, however, it is unlikely that the covariance bias alone would be strong enough to yield counter-theoretical estimates of the substitution effect. In our

judgment, it is thus a reasonable strategy to hold the question of diverse preference bias in abeyance for the purposes of cross-sectional estimation. Corrective estimates of the bias may then be obtained from time series analysis of panel data.

d. Intertemporal variations in preferences. Just as different individuals may differ in their income-leisure preferences, a given individual may differ in his preferences at different points in time. The implications of such intertemporal diversity are essentially the same as those of the interpersonal variety discussed above except that repeated observations on the same individuals no longer suffice to identify the diverse parameters. Fortunately we may reasonably presume that the quantitative impact of the instability of individual preferences is markedly smaller. We acknowledge the possibility of such diversity here primarily for the sake of completeness in our discussion of sources of uncertainty in the estimation of labor supply parameters. This diversity is not expected to have any unique consequences and will not be further distinguished as a separate component in the stochastic specification.

### III. Diverse Preferences and Endogenous Nonwage Income

We have focused on the estimation problems associated with various random components of labor supply behavior in isolation. More serious estimation problems arise when these random effects occur in combination with other complications. One such problem that has received attention

in previous studies is that the wage rate and nonwage income variables that are presumed to be exogenous in the basic model may, in fact, depend on unmeasured individual preferences that also affect labor supply. The hypothesis is that workers with unusually high preferences for income relative to leisure will also be likely to have greater than average preferences for saving relative to consumption.

The problem is treated most fully by Greenberg and Kusters (1970). They point out that an unusually high preference for asset accumulation relative to leisure will result in both high income from wealth and high labor supply relative to persons with average preferences. In the language of the previous discussion, there is a disproportionate sample of high-preference individuals at high levels of observed nonwage income. Cross-sectional estimates of the effect of nonwage income on labor supply will thus be positively biased unless the effect of such differential sampling of preferences is controlled for. Greenberg and Kusters constructed a preference variable based on the difference between the observed and expected net worth of a family unit. Including this variable in the model produces estimates of the income effect that are more theoretically plausible than those obtained without it.

The Greenberg-Kusters procedure is compelling in concept, but the interpretation of the preference variable, as actually constructed, may be open to some question. Cain and Watts (1973, p. 357) point out a possible interpretation in which the measure of expected net worth is viewed as a proxy for permanent income. A negative relationship between permanent income and labor supply might then produce their observed results. The constructed preference variable is also highly

correlated with wealth income, which raises potential problems of multicollinearity. Their estimates are remarkably stable across population groups, however, so this estimation problem does not appear to be serious. Potential problems of interpretation notwithstanding, the Greenberg-Kosters method of controlling for differential preferences and asset accumulation appears to be basically correct in concept and to be addressed to an important estimation problem. We thus propose to construct an analog of the preference variable, based on the more limited asset information variable in the Michigan data, and to investigate the sensitivity of our estimates to the inclusion of the preference variable in the estimation model.

#### IV. Segmented Budget Constraints Having Known Structures

Throughout the derivation and discussion of theoretical labor supply functions the opportunity set for optimal labor supply decisions is presumed to be a simple budget line defined by a wage rate,  $w$ , and a level of nonwage income,  $I$ . It is clear that a budget line so simply defined will not accurately represent the net earnings opportunities of contemporary workers who encounter two significant institutional complexities: increasing marginal rates of income taxation and premium wage rates for overtime work. Both of these institutional factors result in budget constraints with linear segments, and with kinks at points where the net marginal wage rate changes. Our basic strategy in dealing with segmented budget constraints will be to represent each segment in terms of the transformed budget

variables that define a locally equivalent simple budget line.<sup>13</sup> The reasoning is that a worker who maximizes his utility at a tangency point on a particular segment of a complicated budget constraint would have chosen exactly the same point if he had faced the simple budget line formed by extending the given segment across the full plane. The equivalence is illustrated in Figures 6a and 6b. In Figure 6a, the worker has income,  $I_0$ , and earns at a net wage of  $w_0$  on the line segment  $I_0A$ . He then encounters a marginal tax rate,  $t$ , so that his net marginal wage rate is reduced to  $(1-t)w_0$  on the segment  $AB$ . His optimal supply choice is at point  $P$  on segment  $AB$ . That choice would have been exactly the same if he had received a level of nonwage income  $I_1^*$  and a constant marginal wage rate,  $(1-t)w_0$ , for all hours of work. In Figure 6b, the worker earns wage  $w_0$  on segment  $I_0C$  and the premium rate  $(1+p)w_0$  on the segment  $CD$ . He would have chosen the same optimal point,  $Q$ , if he had received nonwage income  $I_2^*$  and earned the wage rate  $(1+p)w_0$  for all hours of work.

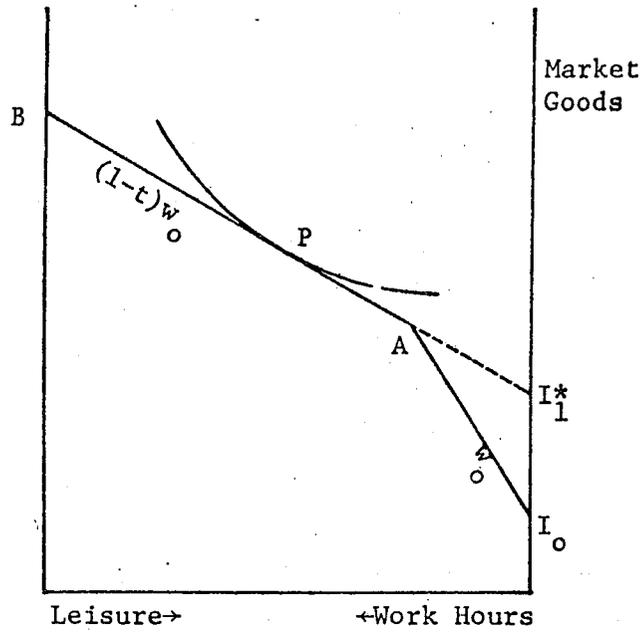
The negative value of transformed nonwage income,  $I_2^*$ , may cause the reader some initial concern, but it creates no particular conceptual problems. At a given wage rate the effect of a small lump-sum tax will be similar, except for sign, to the effect of a similar lump-sum subsidy. Indeed, without negative values of nonwage income, simple budget constraints would never yield equilibria in a large sector to the lower left of the leisure-market consumption plane.

Our treatment of negative income values is markedly simplified by the fact that we have restricted the analysis to the consideration of utility functions with linear expansion paths. Given linearity, the

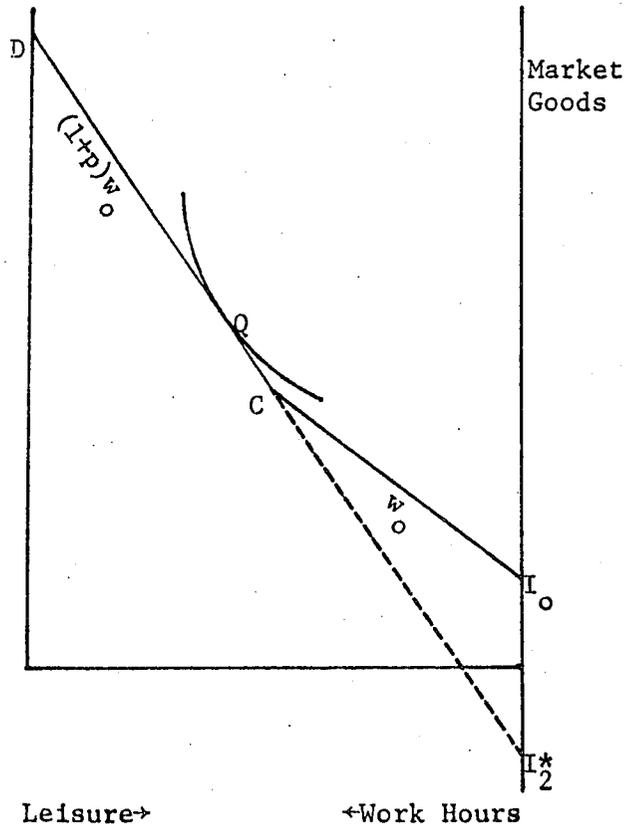
FIGURE 6

TRANSFORMATIONS FOR SEGMENTED BUDGET LINES

a. Progressive Income Tax



b. Overtime Premium



origin of the nonwage income scale is irrelevant, since equal income differentials have equal effect regardless of the initial level. The critical function of the nonwage income variable is to provide a metric for the vertical distance between budget lines at a given marginal wage rate. Hall (1973) chooses to evaluate this vertical distance at 2000 annual work hours, rather than at zero work hours as we do. His method always produces positive income levels but creates problems because he estimates a curvilinear income response without wage interactions. The income effect is thus constrained to be the same on all budget lines that intersect at 2000 hours, regardless of marginal wage rate. Expansion paths at different wage rates will then have a complicated curvilinear structure that may or may not be theoretically acceptable. Hall acknowledges the absence of wage-income interactions but does not discuss the implications. The magnitude of the problem is reduced by his restriction of the range of income and wage rates in his analysis sample and thus may not produce serious distortions in his estimates.

The transformations that yield equivalent simple budget lines are comparatively straightforward for the case of a single individual in equilibrium on a given segment of a multisegment budget line. Life becomes rather more complicated when we consider a sample of diverse individuals in equilibrium on different segments at a variety of budget levels. The problems are also different depending on whether we are engaged in estimation or simulation.

In estimation we begin with observations of individual equilibrium positions on various segments of a complex budget structure and attempt

to estimate the parameters of a supply function that is defined in terms of simple budget variables. If all goes well, the supply function will be continuous and otherwise theoretically plausible and will permit further inference about the structure of the underlying utility function. Utility functions and the simple supply functions are presumed to be stable attributes of individuals in the population so that successful empirical inference should lead to those functions, whatever the particular structure of the observed budget segments.

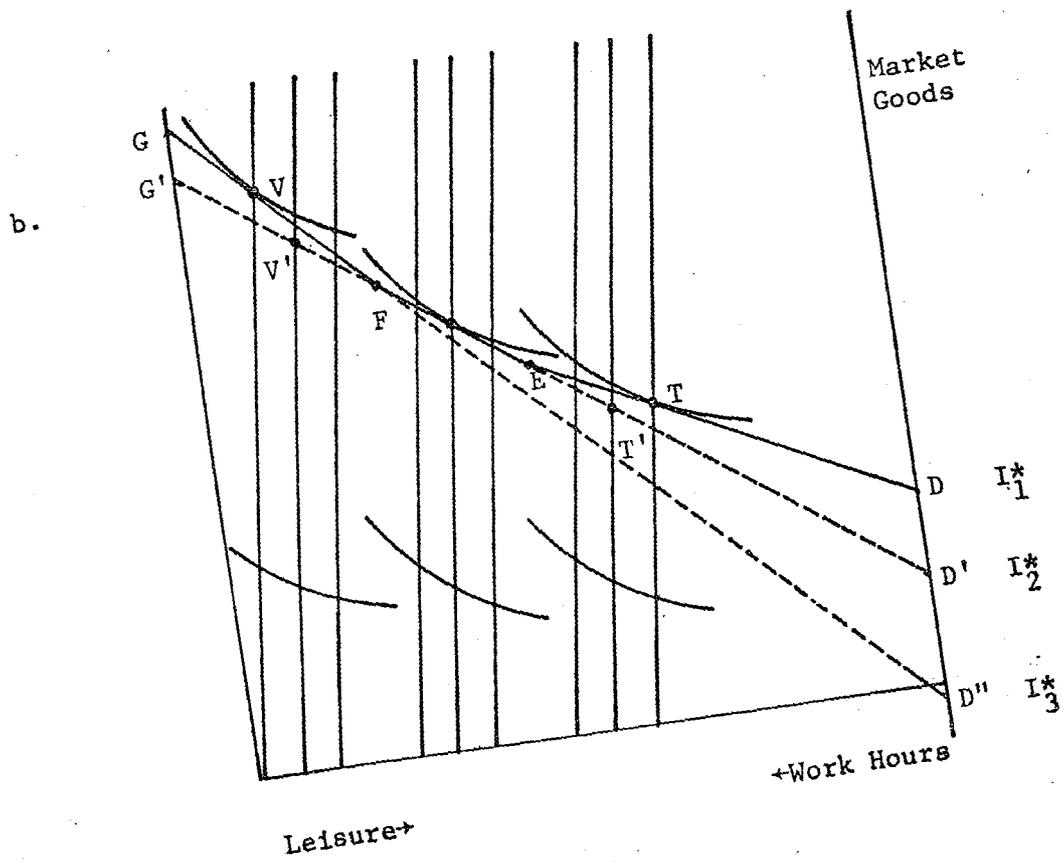
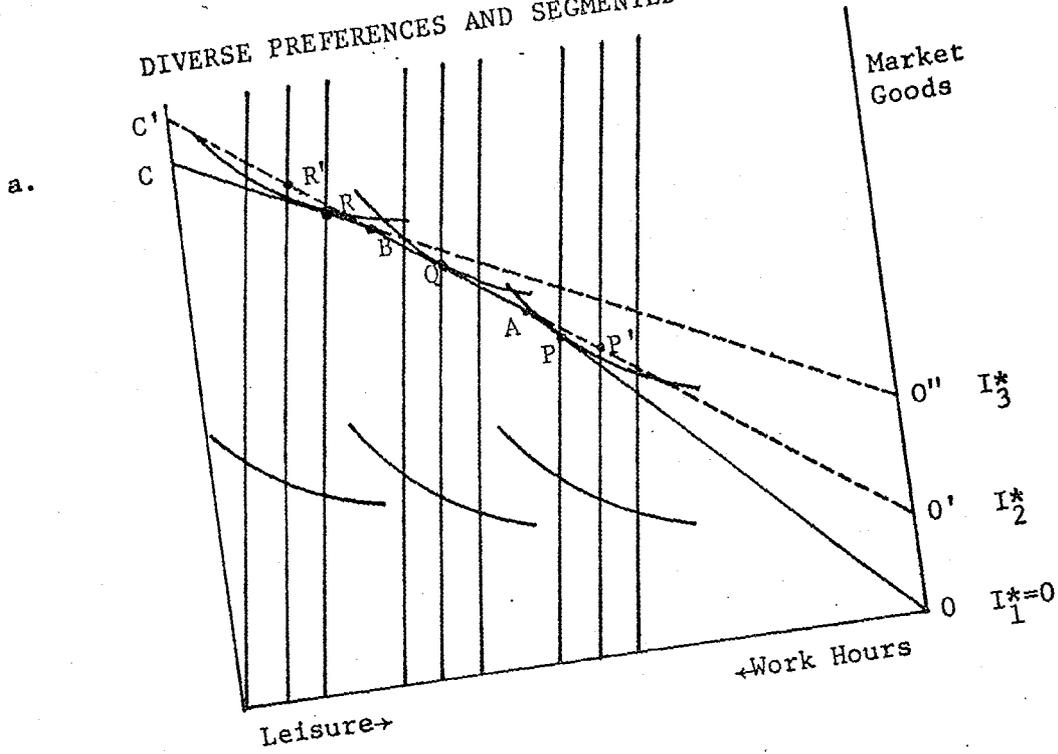
Simulation is essentially the obverse of estimation. In that case we are presumed to know or to have estimated the parameters of the utility function and the corresponding simple labor supply function, and we wish to describe, or simulate, the pattern of optimal labor supply behavior for a particular segmented budget structure. The results may be summarized as structure-specific labor supply functions that will generally involve intermittent corner solutions, dual values, and discontinuities.<sup>14</sup> The main complication in the procedure is in the determination of which segment the equilibrium will fall on for a given range of budget variables. Will a worker elect to work more in order to qualify for double overtime, or, at lower wage levels, will a worker reduce work effort in order to qualify for an income subsidy? Once these questions are resolved, the labor supply function for the particular segment follows immediately from the supply function for the equivalent simple budget line. These solutions are specific to the particular complex budget structures being considered and will not be further developed in this paper. The primary emphasis here will be on the problems of estimation.

Other authors concerned with the issue of segmented budget constraints and labor supply have approached the problem differently. Wales (1973) assumed a specific functional form for utility and focused on the budget structure associated with the income tax. He was then able to incorporate tax parameters in the form of his estimation model, so that the estimation equation was directly available for simulation. This method is not suitable for the current study, given our more general approach to both the form of the utility function and the nature of the budget structure. Brown, Ulph, and Levin (1974) specify the supply equation in terms of both the average wage and the marginal wage and then incorporate a second equation, based on the budget structure, that relates the two wage rates. While this is an innovative and potentially useful specification for the supply function itself, it appears to make inference about the utility function more difficult, and is thus poorly suited to our purposes.

#### Diverse Preferences and Segmented Budget Structures

The combination of diverse preferences and segmented budget constraints causes problems of estimation because different individuals, facing the same budget constraint, may choose optimal supply positions on different segments. The simple wage and income variables corresponding to the different segments thus become endogenous functions of unmeasured preference differences. The nature of resulting biases is illustrated in Figures 7a and 7b, using the well-behaved distribution of preferences introduced in Figure 3. In Figure 7a, the segmented budget

FIGURE 7  
DIVERSE PREFERENCES AND SEGMENTED BUDGET STRUCTURES



constraint OABC has tax rates that increase from 0 on segment OA to 33 percent on AB and 50 percent on BC. Individuals with the highest work-hours preference choose the equilibrium at point R with the lowest net marginal wage rate,  $0.5w$ , and the highest effective nonwage income,  $I_3^*$ . Individuals with lower preferences for work choose equilibria at Q and P, with successively higher marginal wage rates and lower levels of effective nonwage income. All individuals in the illustration face the same opportunity set, so the differences in values of effective simple budget variables are solely a result of differences in choices. If these differences in the levels of transformed budget variables from the same opportunity set were naively included in a cross-sectional regression analysis, they would cause a positive bias in the estimated response to exogenous income differentials and a negative bias in the estimated wage response.

Figure 7b shows an opportunity set with increasing premiums for overtime work. There is no premium in segment DE, "time and a half" in segment EF, and double time in segment FG. Again the different preference types are conveniently shown with equilibria on each segment. In this case the spurious correlations between work hours and the transformed budget variables are opposite from those in the case of increasing marginal taxation. Given that the workers from whom we have data face a combination of the two effects, the potential biases may be offsetting to some degree. The tax structure and the overtime structure are certainly not symmetrical, however, and in any case it is desirable to treat the potential bias problem more systematically.

The most feasible solution to the problem of bias due to self-selection of budget segments is to impute the same values of budget variables to all individuals who face the same opportunity set.<sup>15</sup> The imputed values of simple budget variables corresponding to a given segmented budget line should represent "average" values in the sense that the mean labor supply that would be observed if all individuals faced the imputed simple budget line closely approximates the mean value that actually is observed on the segmented line. An imputed budget line of this sort is shown in Figure 7a as the line O'ABC'. If all individuals faced that simple budget constraint, the equilibria would be at P', Q, and R'; the mean of those labor supply values closely approximates the mean of the observed points, P, Q, and R. In Figure 7b the imputed line is D'EF G'; the mean of the hypothetical equilibria T', U, and V' again closely approximates the mean of those actually observed. In Figure 7a the observed variance of unexplained differences between individuals is smaller than the variance that would be observed on the imputed simple line. The reverse is true for the case of Figure 7b. These variance differentials will have some consequences for the efficiency of estimates, but so long as the mean-value equivalence is maintained at all budget levels we have reasonable assurance of unbiased estimates.<sup>16</sup>

#### Net Budget Imputations Under the Income Tax

The particular methods of imputation and transformation are slightly different for the tax structure and the overtime premium structure. We will discuss the two cases separately and then combine the two procedures

in our empirical analysis. Under the income tax system a worker with a constant marginal wage rate has the gross market income as shown in equations (5a)-(5c). The variable X in the tax equation represents exemptions and deductions that affect tax liability but are not a function of gross income.

$$(5) \quad \begin{array}{ll} \text{a) } M_g = wH + I. & \text{(gross market income)} \\ \text{b) } \text{Tax} = T(M_g - X). & \text{(total tax liability)} \\ \text{c) } M_{\text{net}} = wH + I - T(M_g - X). & \text{(net market income)} \end{array}$$

In a given tax bracket, we denote the marginal tax rate by  $t_i$ , so that a worker's net marginal wage in the range of work hours that places him in that bracket is given by  $(1-t_i)w$ . Extending the budget line with this marginal wage rate back to zero work hours then yields the effective level of nonwage income shown in (6a)-(6c).

$$(6) \quad \begin{array}{ll} \text{a) } I^* = M_{\text{net}} - w_{\text{net}} H. \\ \text{b) } I^* = wH + I - T(M_g - X) - [1-t_i] \cdot wH. \\ \text{c) } I^* = I + [t_i wH - T(M_g - X)]. \end{array}$$

For a given value of I, the parenthesized expression in (5c) is a constant within a given tax bracket, since total tax liability varies directly as the marginal tax rate times earnings. As the wage rate and/or work hours increase so that earnings surpass the break point of a higher tax bracket, there is a discontinuous jump in the marginal tax

rate and in the effective level of nonwage income,  $I^*$ . However, if we consider a distribution of individuals facing successively higher gross wage rates, only a fraction of the distribution will pass over the break point for any given small change in the wage rate. The average budget line that we wish to impute to all individuals in the distribution at any given budget level thus evolves much more smoothly and continuously than the effective budget line for a single individual. A simple way to provide such a smooth evolution in the budget line is to approximate the stepwise increase in marginal tax rates by a continuous linear function of taxable income.<sup>17</sup> The corresponding function for total tax liability is then a simple quadratic function of taxable income. The points on each of these functions corresponding to the expected level of gross market income, given  $w_g$  and  $I$ , are taken as the appropriate tax values for the imputed average budget line. The elements of the transformation are shown in equations (7a)-(7d).

$$(7) \quad \begin{array}{ll} \text{a) } \hat{E} = \hat{E}(w, I). & \text{(estimated earnings (wH))} \\ \text{b) } \hat{M}_g = \hat{E} + I. & \text{(estimated gross market} \\ & \text{income)} \\ \text{c) } \hat{t} = \hat{t}(\hat{M}_g - X). & \text{(imputed marginal tax rate)} \\ \text{d) } \hat{T} = \hat{T}(\hat{M}_g - X). & \text{(imputed tax liability)} \end{array}$$

These imputed tax values are incorporated into the imputed net budget constraint as shown in equations (8a) and (8b).

$$(8) \quad a) \quad \tilde{M}_{\text{net}} = (1 - \hat{t})wH + I + (\hat{t} \cdot \hat{E} - T).$$

$$b) \quad \tilde{M}_{\text{net}} = \tilde{w}_{\text{net}}^* H + \tilde{I}^*.$$

The values of the imputed simple budget variables,  $w^*$  and  $I^*$ , are given explicitly in (9a) and (9b).

$$(9) \quad a) \quad \tilde{w}^* = (1 - \hat{t})w.$$

$$b) \quad \tilde{I}^* = I + (\hat{t} \cdot \hat{E} - \hat{T}).$$

The tilde notation is used in order to emphasize that only the tax parameters are estimated in the imputed budget constraint. Since these estimates involve a good approximation to a slowly varying known structure, the estimation error is virtually negligible by comparison with imputations in studies that involve estimates of the gross wage rate itself.<sup>18</sup>

#### Imputation of Simple Budget Lines For Structures With Overtime Premiums

The empirical analysis proposed in this study will be restricted to workers in jobs with hourly marginal wage rates that are known and are different from zero. The measured wage variables are derived from responses to direct questions about both the marginal wage rate and the regular time wage rate. These measures represent a substantial improvement over the wage data available for most previous studies, in which the typical wage measure has been average hourly earnings computed

as the ratio of reported total earnings to reported total work hours. That average hourly earnings measure not only neglects the structure of marginal wage rates<sup>19</sup> but also is the source of estimation problems due to potential errors of measurement that are correlated with errors in the dependent variable.<sup>20</sup>

The typical hourly wage worker receives a "straight time" wage for a standard work week, usually 40 hours, and a 50 percent premium, "time and a half," for overtime. Higher premiums are often paid for holiday work and for overtime in excess of a cutoff, which may depend on the particular labor contract. As shown in Table 1, more than 85 percent of hourly workers in the proposed analysis sample report that their rates for extra work are 1.5 times their regular wage rates. About 4 percent report double time or higher wage rates for extra work, while about 9 percent report no overtime premium. Of the latter group, it is apparent that fewer than 4 percent might receive overtime premiums but choose to work too little to qualify. Evidently the remaining 5 percent of workers are employed at jobs that do not offer overtime premiums, since they do not receive premium rates despite the fact that they regularly work more than 40 hours per week. If we ignore this small final category of workers for the moment, the work-hours choices of individuals who face the typical premium structure appear to be symmetrical with a large central majority choosing work hours on the budget segment with a 50 percent premium. The symmetry of the choices means that the modal budget segment also closely approximates the average budget line, which we wish to impute to all workers on the segmented structure. Given this close approximation and the simplicity of the procedure, we will impute overtime premiums of 50 percent to all workers who face a premium wage structure.

TABLE I  
 Marginal Wage Relative To Regular Wage  
 Analysis Sample of Employed Married Males  
 Who Receive Pay for Extra Work

	Incl. 2nd job
Marginal wage equal to regular wage and regularly works 40 hours of less*	3.8%
Marginal wage 1.5 times regular wage*	86.8%
Marginal wage twice * regular wage or more	3.8%
Works overtime but does not receive premium wage*	5.4%

Note: The analysis sample includes panel study respondents who reported both a regular wage rate and a wage rate for extra work on their main job in the years 1969 through 1972. The sample is also restricted to those who were free to vary their work hours in at least one direction and were not dissatisfied with the limits they faced. An individual could meet these criteria in any number of the four years. The unit of tabulation is an annual observation on a qualifying individual, so a given individual may be represented up to four times. There are 1288 such observations in the sample corresponding to 609 different individuals.

\* The tabulation is based on the following intervals of the ratio of overtime to regular wage; less than 1.25, 1.25-1.75, more than 1.75. The observations are highly concentrated at 1.0, 1.5, and 2.0, but there are a few intermediate values, perhaps corresponding to piece-rate employment or other unusual pay structures if not to response error.

In our proposed estimation we will also impute the 50 percent premium to the odd 5 percent of workers who work overtime but receive no premium. This decision, admittedly arrived at after some trial and error, is based on the inference that these workers might have chosen jobs with the conventional premium structure, but chose as they did because they faced fewer limits on the amount of available overtime. They are thus apparently a subgroup with higher-than-average work preferences, and if the wage and income imputations for them differ from those for the majority, the preference differential will bias the estimated wage and income effects. We thus choose to impose the same imputation. Separate estimates using the reported marginal wage rather than the imputed wage for this group will also be presented to show the sensitivity of the results to this particular imputation.

The mathematical formula used for the overtime transformation relies on the approximation that an individual qualifies for the "time and a half" overtime wage for all work hours in excess of 2000 per year. The imputed gross budget line thus has a slope 1.5 times the regular wage but passes through the market income point given by the regular wage budget line at 2000 hours. The relevant variables are shown in equations (10)-(12), using the notation  $w_r$  for the regular wage,  $w_{ot}$  for the marginal or overtime wage, and  $I^*_{ot}$  for the intercept of the simple transformed budget line.

$$(10) \quad M_{2000} = w_r \cdot 2000 + I.$$

$$(11) \quad w_{ot} = 1.5 w_r.$$

$$(12) \quad a) \quad I^*_{ot} = M_{2000} - 2000 \cdot w_{ot}.$$

$$b) \quad I^*_{ot} = I - 1000 w_r.$$

The budget constraint after the overtime imputation is then the simple expression in (13).

$$(13) \quad M_{ot} = w_{ot} \cdot H + I^*_{ot}.$$

The imputed budget constraint that combines the effects of overtime premiums and marginal tax rates is derived by using  $w_{ot}$  and  $I^*_{ot}$  as the input variables for the tax transformations of equations (8) and (9). It is possible to use this simple successful transformation method since the estimated tax rates used in the latter are based on the regular wage, which defines the basic budget level, and not on the reported marginal wage, which reflects endogenous choice. The results of the combined transformation are given in equations (14) and (15).

$$(14) \quad a) \quad \tilde{w}^*_{ot} = 1.5(1-t)\hat{w}_r.$$

$$b) \quad \tilde{I}^*_{ot} = I - 1000\hat{w}_r + (t \hat{E} - \hat{T}).$$

$$(15) \quad c) \quad \tilde{M}_{net \ ot} = \tilde{w}^*_{ot} H + \tilde{I}^*_{ot}.$$

In estimation of the labor supply model, we also ran comparable models without the overtime and tax transformation to test the sensitivity of the estimates to the budget specification decisions [Dickinson, 1976, chapter 5].

#### V. Short-Run Marginal Disequilibria, Potential Long-Run Equilibria, and Moonlighting

In Section II, we argued that small disequilibrium displacements caused by institutional constraints may reasonably be incorporated into the stochastic specifications of a labor supply model. We also alluded to the problem of more binding constraints on labor supply, which give rise to worker dissatisfaction with available work time. The existence of such constraints has been acknowledged in a number of previous studies, but it has not generally been possible to distinguish between workers in true internal equilibrium and those facing limits on work hours. In the absence of good data on the problem, it has generally been argued that no serious departure from the assumed conditions occurs, since workers may be presumed to adjust their work hours in the long run by changing jobs or taking second jobs.<sup>21</sup> In the following discussion we present the contrary case that numerous workers are in corner solutions at significant discontinuities in their opportunity sets and that serious biases may result if all workers are simply assumed to be in internal equilibrium at their given average or marginal wage rates.

For a perspective on the problem we first review the unique information about opportunities for varying work hours that is available from the Michigan Panel Study of Income Dynamics. Information was

collected on two major dimensions of work opportunities on a worker's present job or jobs. The first is whether the worker would be paid for marginal variations in work hours. The second dimension, discussed briefly in Section II, is whether the worker faced constraints on his work hours and whether he was dissatisfied with those constraints.<sup>22</sup>

A categorization that combines these two dimensions of equilibrium status is shown in Table 2 along with the percentage distribution of the sample of employed male family heads across these categories in the years 1969-1972. If we accept this classification at face value, we are immediately struck by the large proportion of the working population for which the assumptions of the classical labor supply model are not strictly met. Nearly half of the workers in the sample do not have a defined marginal wage rate, and among those who are paid for marginal variations in labor supply, more than 80 percent face some limits on work hours and 35 percent are dissatisfied with those limits. Overall, only about 11 percent of workers are fully free to vary their work hours at a defined marginal wage rate as assumed in the labor supply theory, and one-third of those gain that freedom by moonlighting.

The classification in Table 2 paints an extreme picture of the departures from the classical assumptions, so it is useful to consider the seriousness of the various problems individually. In some cases, such as in the minor disequilibrium effects discussed earlier, institutional constraints pose no grave problems for the classical equilibrium model. Other, more major, violations of basic assumptions of the

TABLE 2

## Equilibrium Status by Type of Employment

	Percentage of Full Sample	Percentage of Subsample
a. Workers employed at a single job that pays nonzero marginal wages		
1. Free to increase or decrease work hours	7.4	16.6
2. Free to increase only--don't want to work less	5.3	12.0
3. Free to decrease only--don't want to work more	6.7	15.2
4. Constrained in both directions but satisfied	9.4	21.2
5. Want more work	12.5	28.2
6. Want less work	3.1	6.9
	<u>44.2</u>	<u>100.0</u>
b. Workers employed at a single job with a zero short-run marginal wage rate		
7. Apparently satisfied with work hours	36.4	86.4
8. Want more work	3.3	7.7
9. Want less work	2.5	5.9
	<u>42.2</u>	<u>100.0</u>
c. Workers with extra jobs		
10. Free to increase or decrease work hours	3.5	26.2
11. Some constraints but satisfied	5.8	42.8
12. Want more work	4.2	31.0
	<u>13.5</u>	<u>100.0</u>

Note: Annual observations on a sample of employed married men for the period 1969-1972. Farmers and self-employed businessmen are excluded.

model may have very serious consequences for labor supply estimation and inference about income-leisure preferences. We treat the various difficulties in turn.

#### Marginal Wage Rate is Zero or Unknown

For the large groups of workers who do not receive defined marginal wage rates, the analysis difficulties are a mixture of discontinuous employment opportunities and measurement problems. Many of these workers, salesmen and self-employed businessmen for instance, do face reasonably smooth opportunity sets with positive effective marginal wage rates and are able to optimize their labor supply in the classical sense.<sup>23</sup> Unfortunately, no direct measures of their effective marginal wages are available. It is possible to measure average hourly earnings for such workers, but no data are available on the relationship between their marginal and average wage rates. It is possible that the ratio of marginal to average wage rates for such workers has an expected value quite different from unity. And, whatever the expected value of that ratio, the variance across individuals is likely to be large. As such, labor supply analysis that relies on average hourly earnings for workers in this group should be considered to be conditional on rather strong assumptions about the unknown structure of opportunity sets.

For salaried workers<sup>24</sup> opportunity sets are both discontinuous and unmeasured. A salaried person may realize some return to additional work hours in the form of possible future advancement, but it is unlikely

that that return is equivalent to his salary rate on an hourly basis. Thus, to a reasonable first approximation, such workers face a zero marginal wage rate on their current jobs. A large proportion of them report freedom to vary their work hours, but this is not meaningful in the absence of payment for extra work. The realistic options for varying work and earnings thus involve job changes and second jobs. These are discussed below in the context of workers facing limited work hours.

#### Binding Constraints on Work Hours

Among workers who are paid for marginal variations in work hours, a large proportion report limits on their work hours. In an earlier section, we argued that for those who report no dissatisfaction with their work hours it is reasonable to incorporate disequilibrium effects within the stochastic specification of the model. However, for workers who face limits on work hours and are dissatisfied with those limits, the discontinuities must be presumed to be more substantial. Such workers do have the option of changing jobs or taking a second job, but have implicitly refused those options. We know the marginal wage rate at which the worker would like to work more (or less). We do not know the effective marginal wage rate for the options he has refused, but we can assemble some general information about the nature of those options.

A worker who wishes to adjust work hours by changing jobs may be discouraged from doing so by two basic factors: the lump-sum costs of a job change and possible unfavorable wage rate differentials. Lump-sum costs, such as moving expenses or loss of pension benefits, may be

relatively small in some cases but are likely to be a significant barrier to mobility in others. Wage rate differentials, on the other hand, need not be large to present the worker with a substantial discontinuity, since any such differential will affect all hours of work, not just those at the margin. In effect, the worker faces the analog of a marginal revenue curve. As an example, an increase of 10 percent in work hours achieved by changing to a job with a 2 percent lower wage rate would result in an effective wage rate for those additional hours that was 22 percent below the original rate.<sup>25</sup> Workers who have found jobs with above average wage rates would thus be unlikely to change jobs despite substantial marginal disequilibrium at their stated wage rates.

A second job might offer somewhat more favorable opportunities for upward adjustments of labor supply than would a job change. Overhead costs such as travel time and expense are likely to be disproportionately large for moonlighters and may deter some persons from second jobs. They are unlikely to be as important as the costs of a job change, however. Wage differentials that a moonlighter may face will not, of course, affect his primary job earnings and thus will not have the compounded effect discussed above. There is some evidence that, on average, second-job holders do not fare badly on their second jobs relative to their regular wage rates on their primary jobs. The data reported by Schiffman (1963, pp. 520-522) suggest that approximate median hourly earnings for moonlighters on their second jobs are slightly higher than their hourly earnings on primary jobs.<sup>26</sup> The Michigan data permit us to take a more detailed look at comparative wage rates on primary

and second jobs. These data are tabulated in Table 3 in the form of ratios of moonlighting wage rates to regular and overtime wage rates on primary jobs. The sample mean of the ratio of moonlighting wage rates to primary regular wage rates, given in column 6 of Table 3A, is 0.92, which is reasonably comparable to the 1.13 figure based on the ratio of medians in the Schiffman study. However, the Michigan data also show substantial variability in relative opportunities, along with a strong systematic relationship to the level of the primary job wage. Even for those workers with regular primary wages below \$4 per hour for whom the mean ratio is unity, the distribution is skewed, with 36 percent having a ratio below 0.75, while a fortunate 22 percent have ratios of 1.24 or above. For workers with regular wage rates above \$4 per hour, the relative moonlighting opportunities are much less favorable, with 46 percent facing a wage ratio below 0.75 as compared with 17 percent with ratios above 1.25.

Table 3B presents the comparisons with overtime wage rates that are viewed as the relevant marginal wage rate for individual labor supply decisions. Not surprisingly, all of the effects noted in the regular wage comparison are evident and stronger here. Overall, the mean ratio of moonlighting to overtime wage rates is 0.66, and the ratio declines with increasing primary job wage. Nearly three out of four moonlighters have extra jobs with wage rates less than 75 percent of their primary overtime wage, and more than 40 percent take wage cuts of 50 percent or more.

TABLE 3

SECOND-JOB WAGE RATES RELATIVE TO REGULAR AND OVERTIME WAGE RATES. Prime-Age Married Male Moonlighters  
Who Reported Wage Rates on Both First and Second Jobs<sup>a</sup>

	under .5	.5-.74	.75-1.24	1.25-1.49	1.5+	Mean Ratio	Percentage of Sample
<u>A. Ratio of Moonlight Wage Rate to Regular Wage Rate</u>							
<u>Regular Wage Rate</u>							
under \$4/hr.	7.7	28.5	41.8	5.4	16.5	1.00	41.7
\$4/hr. +	23.3	23.2	36.3	6.7	10.4	0.87	58.3
ALL	16.8	25.4	38.6	6.2	12.9	0.92	100.0
<u>B. Ratio of Moonlight Wage Rate to Overtime Wage Rates</u>							
<u>Overtime Wage (gross)</u>							
under \$5/hr.	28.3	35.6	23.6	2.9	9.6	0.86	26.0
\$5/hr.+	45.8	32.6	15.8	2.3	3.6	0.59	74.0
ALL	41.2	33.4	17.8	2.4	5.1	0.66	100.0
<u>C. Ratio of Moonlight Wage Rate to Overtime Wage Rate for Subsample of Reporting Moonlighters Who Wanted Still More Work (28% of above sample; 115 observations)</u>							
<u>Overtime Wage (gross)</u>							
under \$5/hr.	36.1	43.9	9.1	3.1	7.8	0.78	31.3
\$5/hr.+	47.7	39.7	8.6	2.3	1.7	0.54	68.7
ALL	44.1	41.0	8.8	2.6	3.6	0.61	100.0

Note: <sup>a</sup>413 observations.

We may presume that all of the workers who wanted more work but did not take second jobs faced some effective wage penalty, else they would not rationally have remained in disequilibrium. The lower portion of the distribution of observed wage rates is thus indicative of the magnitude of the discontinuity in marginal wage rate faced by this group. If we assume that those who did not take second jobs faced a distribution of opportunities as good as those who did, then the magnitude of the average discontinuity may be placed in the neighborhood of a 50 percent deficit relative to marginal wages on the primary job. In fact, those who remained in disequilibrium may be expected to include disproportionate numbers of those who faced unusually poor second-job opportunities. The tabulation in Table 3C for moonlighters who wanted still more work than they were able to find may give a better sense of the opportunities faced by marginal second-job holders.<sup>27</sup> Of these, 85 percent took at least a 25 percent wage cut and 44 percent accepted a second-job wage rate of less than half their primary overtime wage rate.

It is clear from the above tabulations that many if not all underemployed workers face substantial unfavorable discontinuities in marginal wage rates. Overhead costs such as transportation, tools, and training add to the magnitude of the discontinuity. It is thus evident that these corner solutions cannot simply be assumed away and that the inclusion of such observations in the estimation sample at their nominal wage rates could result in serious biases.

In our empirical work, reported elsewhere [Dickinson, 1976 chapter 5], we estimate the parameters of the model for a select sample of workers whose responses to the set of work constraint questions indicate that they

are in labor supply equilibrium or suffer only minor displacement from such an equilibrium.<sup>28</sup> A comparative estimation is proposed for the sample of workers who are in determinate corner solutions, specifically those workers who reported that they were unable to find as much work as they wanted. The contrast between the two estimation samples abstracts from periods of full unemployment, which are separately controlled for, as discussed in the following section.

If workers in the constrained sample are significantly displaced from their desired labor supply positions, as we contend, then we expect the estimated parameters to reflect the demand conditions that underlie the constraints rather than the supply parameters corresponding to our utility optimization model. Coefficients of variables that are related only to supply effects are expected to be smaller in magnitude in the constrained sample than in the equilibrium sample. We expect this contrast in magnitude of effects to be observed both for economic variables that affect only the supply side and for those demographic variables that are related to systematic preference differentials for labor supply.

In the judgment of this author, the evidence presented above and the empirical results presented elsewhere (Dickinson, 1976, chapter V) provide persuasive support for our decision to exclude from our primary analysis those workers who face binding constraints. However, some question remains as to whether the remaining select equilibrium sample satisfies the equal sampling conditions necessary for unbiased cross-sectional estimates. Before addressing this question we need to consider

the estimation problem caused by workers who do take moonlighting jobs as a solution to constraints on their primary jobs.

### Moonlighting

Workers who do take second jobs represent the opposite end of the spectrum from those who report inadequate work on their primary job but implicitly refuse the moonlighting option. About 70 percent of moonlighters are in reasonable labor supply equilibrium, as required for our select sample, but the observed behavior of these workers creates a number of potential problems for labor supply estimation. If we were to enter moonlighting observations in the regression model at their observed marginal wage rates, we would encounter endogeneity problems similar to, but more serious than, those due to increasing marginal tax rates as illustrated in Figure 7a. Given the relatively unfavorable earnings opportunities available to moonlighters, only those individuals with above average preferences for market consumption relative to leisure would choose to take second jobs. The resulting endogenous correlations between high work hours, low marginal wage rates, and high effective levels of nonwage income would result in serious biases to cross-sectional estimates.

An alternative estimation procedure involves imputing the primary-job budget line to moonlighters. This specification is still problematical, but it will be one of those tried in the empirical analysis. The imputation does avoid endogeneity of the budget variables, but at the expense of major misspecifications. Some misspecifications were involved in the tax and overtime imputations discussed earlier, but the

imputations in those cases were chosen so that misspecification effects approximately averaged out. It is clear from Table 3 that the differential between first- and second-job wage rates depends on the wage level, so that the misspecification effects will not average out in the present case. Our rationale in experimenting with this specification is that it is similar to those from previous analyses that ignore the status of moonlighters,<sup>29</sup> and thus that it provides a point of comparison for the estimates obtained under our preferred specification.

A more fully satisfactory imputation procedure for moonlighters would require explicit modeling of differential preference effects and of the full distribution of second-job opportunities, both accepted and refused. Such a model is beyond the scope of the present study. Our simpler alternative involves the effective exclusion of moonlighters from the estimation. The exclusion is accomplished by means of a dummy variable and a complete set of wage and income interactions for moonlighters. The basic coefficients then represent parameter estimates for workers who are in equilibrium on their primary jobs, and the interaction coefficients give a direct reading on the nature of misspecification biases in models that ignore moonlighting.

Given all of the specification decisions discussed above, the estimates from our proposed empirical work will be conditional upon the highly selected sample. The individuals to whom the estimates apply are healthy, prime-age married males employed at a single job who are in approximate equilibrium at a known, nonzero marginal wage rate. We have argued that these restrictions are necessary if we are to make inferences about the underlying preference structures using the

classical utility maximization model. We do not contend that the estimates that we obtain for the highly select sample will represent all preferences nor that they necessarily will be close to the average preferences in the full population. The reader should recall that it is not necessary that the sample of workers be representative of all workers in the population. So long as it is a consistent subsample at different budget levels, we can obtain unbiased estimates of labor supply parameters for that subsample.

One small<sup>30</sup> but important group of married males who are not represented consists of those who choose not to participate in the labor force. The exclusion of these individuals follows from the sampling criterion that requires a directly reported marginal wage rate. Even if the marginal wage rates available to nonparticipants were known, however, it is not clear that it would be appropriate to include them in a single-stage estimation procedure. The observed level of labor supply for nonparticipants is at a corner solution more than four standard deviations below the mean supply position of participants. It is thus apparent that the labor supply behavior of nonparticipants is influenced by unmeasured factors, be they personal preferences or market opportunities, which are radically different from those influencing the behavior of their employed counterparts. It is the opinion of this author that the different behavior of nonparticipants requires explicit modeling of their special personal and economic circumstances. Such modeling is beyond the scope of the present study. Hall (1973) takes the contrary position that nonparticipants should be included in a single-stage estimation procedure. He provides no

discussion of the factors that might cause nonparticipants to behave so differently from employed workers, however. Garfinkel and Masters (1974) also include nonparticipants in a single-stage estimation procedure, primarily because of the resulting simplicity of their simulation procedure.

A final question that we are not able to answer definitively is whether the select sample satisfies the equal sampling condition. That is, within the restricted sample, do the individuals who are sampled at one budget level represent the same underlying distribution of preferences as those sampled at other budget levels, so that the equalities of expectations in equations (1a) and (1b) hold true. Full resolution of the question would require knowledge of the unobserved joint distribution of individual preferences and work-hours opportunities on current jobs. We can address the question only on the grounds of plausibility. Disproportionate sampling is unlikely to be a significant problem for those jobs that offer complete freedom of choice in work hours. The only potential sampling biases from this subsample of jobs would arise if there were differential selection into such jobs at different budget levels. We know of no evidence of such a problem. Potentially more serious problems arise with primary jobs that allow freedom of choice within limits. Workers whose preferences lead to approximate equilibria within the free interval thus qualify for our select equilibrium sample while those who are dissatisfied with the limits are excluded. It is entirely possible that this selection process could result in different slices from the preference distribution at different wage and income levels. At this writing, however, the likely direction of biases that would result is not apparent to the

author. It is our judgment that the selection of an equilibrium sample markedly reduces the biases from demand-related institutional constraints. Nonetheless, the problem may not have been fully eliminated and warrants further study.

#### VI. Empirical Treatment of Unemployment, Illness, and Commuting Time

The preceding discussion has focused on constraints on work time while employed, and our primary analysis sample has been selected so as to minimize the estimation problems caused by such constraints. However, a number of individuals in the select sample did experience periods of full unemployment, usually in the form of temporary layoffs or work stoppages. Presumably, those periods of time do not represent voluntary consumption of leisure and must be accounted for in our estimation of labor supply parameters. The treatment of illness is conceptually similar and will be discussed at the conclusion of this section. Commuting time is also similar in some respects but may require a more elaborate model.

One common approach to the treatment of unemployment has been to use hours of labor supplied as the dependent variable in the model, where labor supply is defined as the sum of work hours and time spent in the labor force while unemployed. Hours of labor supplied during a week of unemployment are usually assumed to be equal to weekly work hours while employed. Samuel Rea (1971, 1974) has proposed a more general treatment of unemployment that allows for a worker to compensate for lost wages by adjustment of his work hours subsequent to a period of unemployment. Rea's model allows possible substitution of unemployment time for leisure.

A graphical treatment of the Rea model is presented in Figure 8. In the absence of unemployment, hours of work definitionally comprise all of nonleisure time, and the equilibrium level of work hours at a given wage rate and level of nonwage income is shown as point P on budget line APE in the figure. In functional notation we denote this labor supply position by the following expression:

$$(16) \quad H = H_0(w, I).$$

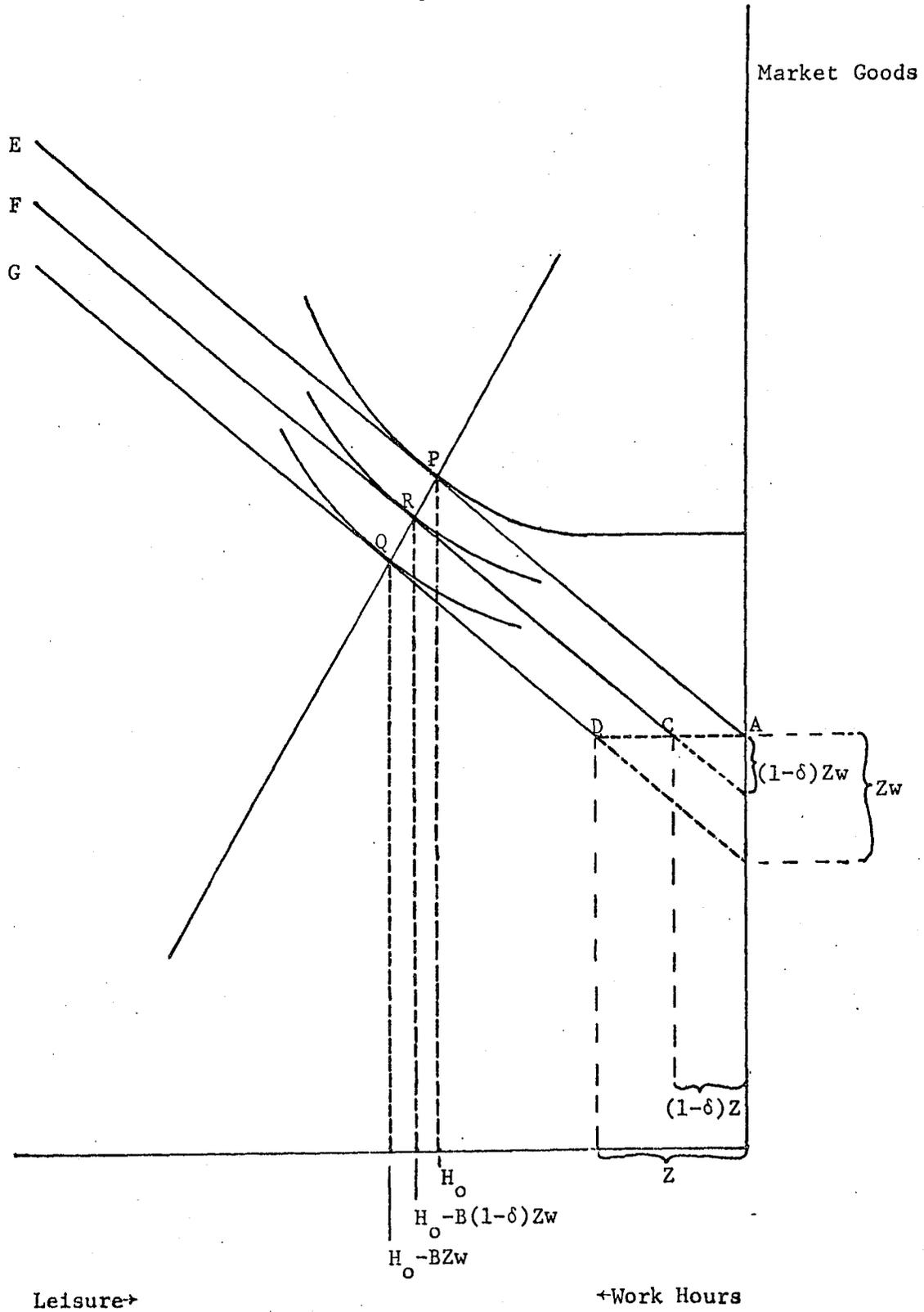
If a worker has Z hours of unemployment, none of which are considered to be leisure, then the equilibrium level of labor supply (work hours plus unemployment) will be equal to equilibrium work hours in the absence of unemployment plus an adjustment due to wages lost while unemployed. In the figure, the flat portion of the budget line from A to D represents unemployment time during which no wages are earned. The rising portion of the budget line after unemployment is then parallel to APE but lower by an amount Zw. The equilibrium at Q thus differs from that at P by the amount of the income effect, as shown in equation (17), with the income coefficient denoted by B.

$$(17) \quad (H + Z) = H_0(w, I) - BZw.$$

If some fraction of unemployment time,  $\delta Z$ , is counted as leisure, then the complement,  $(1-\delta)Z$ , will be nonleisure time expended without earnings.<sup>31</sup> The budget line will be flat over the interval AC and then rise parallel to APE but lower by the amount  $(1-\delta)Zw$ . The equilibrium supply of nonleisure at point R will differ from that at P, by the amount of income effect  $-B(1-\delta)Zw$ . Equation (18) expresses the function

FIGURE 8

THE REA MODEL OF RESPONSES TO UNEMPLOYMENT



for the supply of nonleisure in terms of the function for hours of work in the absence of unemployment.

$$(18) \quad H + (1-\delta)Z = H_0(w, I) - B(1-\delta)Zw.$$

Taking the incidence and duration of unemployment to be exogenous, we may solve (17) for hours worked as a function of wage rate, nonwage income, and hours of unemployment.

$$(19) \quad a) \quad H(w, I, Z) = H(w, I) - B(1-\delta)Zw - (1-\delta)Z.$$

$$b) \quad H(w, I, Z) = H(w, I) - (1-\delta)(1+Bw)Z.$$

For workers who are free to adjust their work hours subsequent to unemployment and who are observed over a sufficient period of time for the adjustment to take place, equation (19b) provides the form in which unemployment hours may be entered as an independent variable in the estimating function for hours of work. The coefficient may then be interpreted as the product of an income effect due to lost wages and an effect representing the substitution of part of unemployment time for voluntary leisure. Since  $B$  is negative if leisure is a normal good, both factors  $(1-\delta)$  and  $(1+Bw)$  are less than or equal to one. The overall coefficient is thus expected to have a magnitude less than unity. The coefficient will not in general be a constant with respect to wage rate and other economic variables, although it may in some cases be adequately approximated by a constant.

In many cases workers will receive compensation from unemployment insurance, so that their decline in income is not as large as the

loss in wages. Once the direct effect of unemployment time is controlled for in this model, the response to unemployment compensation is expected to be the same as the response to any other component of nonwage income. The supply of nonleisure, in the presence of unemployment compensation,  $\Delta I_z$ , is shown in equations (20a) and (20b).

$$(20) \quad a) \quad H + (1-\delta)Z = H_o(w, I) + B\Delta I_z - B(1-\delta)Zw.$$

$$b) \quad H = (1-\delta)Z = H_o(w, I + \Delta I_z) - B(1-\delta)Zw.$$

The solution for work hours is then given in (21).

$$(21) \quad H(w, I, Z, \Delta I_z) = H(w, I + \Delta I_z) - (1-\delta)(1+Bw)Z.$$

Once unemployment compensation is included in the overall nonwage income variable, the expected coefficient of unemployment time is exactly the same as in equation (19b).

The Rea unemployment model is also appropriate for the treatment of time lost due to illness. Illness time is presumed to be primarily nonleisure, but some portion may be substitutable for voluntary leisure. Sick time is frequently compensated by sick pay, which presents an empirical problem since no measure of this income is available in the current data set. Thus, to the extent that sick time is fully compensated at regular earnings levels, and that compensation is not included in nonwage income, the factor  $(1+Bw)$  will be biased toward unity in the coefficient of illness time.

If residential location were exogenous, commuting time would enter the conceptual model in the same way as unemployment and illness. Even under that assumption, however, the actual mechanism of a worker's

adjustment of work hours is quite different. Illness and unemployment replace usual work time with enforced idleness. The first-round effect, in the absence of any behavioral response, is a one-for-one reduction in work hours. Then, to the extent that there was an earnings loss and/or an excess of leisure because part of the lost work time was counted as leisure, the worker spends more time working subsequent to the unemployment or illness, so as to achieve leisure-income equilibrium for the whole period. Thus, after behavioral responses, we expect less than one-for-one reductions in work hours. Commuting time, on the other hand, has no direct effect on work time in the absence of a behavioral response. It is only after a worker has adjusted his work hours to reach leisure-income equilibrium that his response to commuting time is expected to be similar to the response to unemployment and illness. The responses will still differ, of course, if a different proportion of each time component is counted as leisure.

The simple model of labor supply response to commuting time is weakened when we recognize that workers may choose their places of residence and thus their time of commuting. Those for whom travel time is more highly substitutable for leisure would choose greater commuting distances without reduction of work effort, giving rise to biases in the estimates of the simple response coefficients.

#### VII. Summary

In this paper we have considered a variety of specific problems that are encountered in adapting the simple utility maximization model of labor supply for use as an empirical tool in the study of observable

labor market behavior. The first two sections were relatively general and dealt with the approximations inherent in the simple utility model and with the stochastic specification that bridges the gap between the exact theoretical model and the obviously nondeterministic nature of observed behavior.

The utility model, based on the market consumption and leisure time of the worker, was seen to be a powerful simplification that enables us to model those aspects of work-leisure choices that are systematically related to economic incentives. A potential weakness of the model lies in the highly composite nature of leisure time, which is defined to encompass all time not spent in labor market activities. Major changes in institutional factors, such as work scheduling, or in opportunities for leisure time activities might change the nature of the composite good, leisure, and in turn change the nature of the approximate preference structure based on that composite.

The stochastic structure of the empirical labor supply model was seen to have four major components: measurement error, minor disequilibrium effects, interpersonal diversity of labor supply preferences, and intertemporal variance in preferences. The first two components were shown to be compatible with the standard stochastic specification of an additive disturbance term. Reporting bias or asymmetry of utility losses from disequilibrium effects might result in a nonzero expected value for the disturbance, but that would have negligible effects on the properties of parameter estimates. Diversity of preferences, whether interpersonal or intertemporal, appeared to require the more elaborate stochastic specification of a random coefficients model. Under the

expanded model, the estimates of the income effect and the uncompensated wage effect are expected to have essentially the same basic properties and interpretations as in the simple model. However, the conventional estimate of the substitution effect was shown to be subject to a bias due to the neglect of the covariance between individual equilibrium levels of work hours and individual income effects.

The latter portion of this paper has been devoted to numerous specification questions that are addressed to our proposed empirical analysis using data from the Michigan Panel Study of Income Dynamics. These include the development of a budget specification to account for overtime wage premiums and progressive income tax rates, the determination of sample selection criteria to minimize biases from demand-related institutional constraints, and the consideration of a variety of complications that arise when budget complexities and demand constraints are observed in the context of a population of diverse individuals. In the course of the discussion, we have indicated our preferred solutions to the various empirical problems. The major features of the preferred specification are (1) a budget specification that imputes a 50 percent overtime premium to all workers and adjusts for marginal income taxes, (2) a set of control variables for diverse preferences that includes interaction variables for moonlighters, the Michigan measure of achievement motivation, and an analog of the Greenberg-Kosters preference variable, and (3) a selected sample for empirical analysis that is restricted to individuals who are in approximate equilibrium at a directly reported marginal hourly wage

rate. While we have justified these specification decisions at some length, we have also outlined alternative specifications in each of these dimensions.

In the final section of the paper we reviewed the Rea model for incorporating unemployment time as an independent variable in the empirical model of annual work hours. The model was also extended to cover illness time and time spent commuting to work.

## NOTES

<sup>1</sup>For family units with more than one worker, the opportunity set is a plane or hyperplane, but our attention is focused on the simplest case.

<sup>2</sup>The classification of such time components as unemployment, illness, and commuting time requires further discussion, which is provided later in this chapter.

<sup>3</sup>For a general discussion of models of this type, see the Symposium, "Time In Economic Life," Quarterly Journal of Economics, 87 (November 1973). The extension of the model envisioned here would entail the adaptation of the Lancaster-Becker model to an opportunity set in which possible tradeoffs among different allocations of time are discontinuous. Successful implementation of the model would then permit the study of institutional changes that would change the structure of the discontinuities.

<sup>4</sup>Figures reported by Bailar (1973) indicate that the proportion of male full-time workers reporting 41 or more hours per week as opposed to 35 to 40 hours drops from 50 percent in the first interview to 42 percent in the eighth interview, with most of the decline in the early months. If we assume that all of the difference represents a bias in the initial report and take reasonable mean values of 52 and 38 hours for the two groups, we obtain a limiting figure of a 3 percent upward bias.

<sup>5</sup>The specific questions are (i) Was there more work available on (your job) (any of your jobs) so that you could have worked more if you had wanted to? (ii) (IF NO TO i.) Would you have liked to have worked more if you could have found more work? (iii) Could you have worked less if you had wanted to? (iv) (IF NO TO iii.) Would you have preferred to work less even if you had earned less money?

Distributions of responses and further discussion of the interpretation of serious disequilibria are provided in Section IV of this paper.

<sup>6</sup>To obtain a meaningful distribution on the incidence of labor supply constraints we must also consider whether the worker would be paid for marginal work hours. The combination of these factors is discussed more fully in Section IV, and distributions are presented in Table 2. The quoted percentages refer to workers who have nonzero short-run marginal wage rates.

<sup>7</sup>A constant substitution effect and a zero income effect will result in exact symmetry since those properties imply indifference curves that are identical parabolas directly above one another. In Figure 1a, the lower portion of the curve has been flattened slightly, free hand, to compensate for the nonzero income effect.

<sup>8</sup>In an earlier analysis, (Dickinson, 1974, p. 220, Table 4.6, model 3) the mean deficit in annual hours for workers who wanted more work relative to those with constrained but satisfactory work hours was estimated to be 150 hours. The reference group does not necessarily have mean hours at the optimum, but those who want more work have hours below  $H_L$ , by definition. The figure of 150 hours per year is thus a rough upper bound for the magnitude of the lower half-interval

$H_L$ ,  $H_E$ . If the upper half interval were very much larger, the net asymmetry of the interval could be greater than the 150-hour figure, but more plausible expectations would place the mean of disequilibrium displacements within a range of  $\pm 100$  hours per year. Such values would result in biases of less than 5 percent in annual work hours and essentially negligible impact on the mean square error of the substitution effect.

<sup>9</sup>The assymetry of disequilibrium effect may be an important factor for married women. Estimates restricted to participants during a five-year period (Dickinson, 1974) indicated a steeply rising supply curve for wage rates up to \$2.50 per hour and an inelastic response for higher wage rates. While not definitive, these results suggest a strong substitution effect in the region of low marginal rates of substitution (low-work-hours region) and a much weaker substitution effect at higher marginal rates of substitution. Under these circumstances disequilibrium displacement toward fewer work hours would cause small utility losses relative to equal displacement on the high side. A related effect is that modest lump-sum costs of working may be sufficient to value labor force withdrawal even if marginal conditions favor participation.

<sup>10</sup>In the illustrated case the income and substitution effects are the same for the three preference types but the uncompensated wage effect is greater for those with lower work preferences as required by the Slutsky equation.

Each of the illustrated preference functions is a parallel preference function. For these functions the indifference curves at different utility levels are identical in shape, and expansion paths (loci of equilibrium points with a given marginal rate of substitution) are parallel. The analytical properties of these functions are discussed in more detail in Dickinson (1975).

<sup>11</sup> For a simple additive disturbance to be strictly appropriate, the uncompensated wage effect rather than the substitution effect should be the same across individuals.

<sup>12</sup> A large discrete compensated change in the wage rate is used for purposes of illustration, but the direction of the apparent substitution effect would be the same in the limit of small changes.

<sup>13</sup> Our discussion in this section draws on the work of Hall (1974), Rea (1971) and Wales (1973). Greenberg (1972) and Brown, Levin and Ulph (1974) address similar issues in the context of labor supply. The problem of segmented budget constraints also arises in the price structure of electrical power; see Taylor (1975).

<sup>14</sup> Corner solutions occur when a kink between budget segments is concave to the origin and the highest attainable utility is at the point of the kink. Dual values will occur for kinks in the opposite direction when a given indifference curve is simultaneously tangent to two segments. Discontinuities go hand-in-hand with dual values as equilibria shift from one segment to another.

<sup>15</sup> A more elaborate imputation system might involve identifying and controlling for differential individual preferences. Successful implementation of such an approach would preserve variances as well as

mean values, but it is not clear that this advantage warrants the additional complexity. It is clearly beyond the scope of the present study.

<sup>16</sup>Proportionate sampling is, of course, also necessary. The distribution of individuals facing a given segmented budget line must be the same as the distributions at other budget levels.

<sup>17</sup>The actual marginal rate approximation has two linear segments with a slope of 1 percent per \$1000 taxable income in the lower brackets and  $3/4$  percent per \$1000 in the upper brackets. The maximum difference between the stepped rate function and the linear approximation is 1.5 percent at the break points of the upper brackets, and the maximum difference in total tax liability is under \$40.

<sup>18</sup>See, for instance, Hall (1973) and Boskin (1973).

<sup>19</sup>One dimension of the structure is that many workers in salaried jobs report zero marginal wage rates. These workers are excluded from the current analysis but have typically been included in previous studies under the implicit assumption that they had marginal wage rates equal to their average hourly earnings. We comment briefly on this issue in the next section.

<sup>20</sup>Errors in measurement of work hours will result in negatively correlated errors in the wage variable, since work hours appear as the denominator of the latter. The result is a negative bias in the estimated wage coefficient in an otherwise correctly specified model.

<sup>21</sup>See, for instance, Kosters (1966, p. 26), and Rea (1971, pp. 48-49).

<sup>22</sup>The questions about constraints were described in footnote 5.

The questions pertaining to marginal wage rates are (i) If you were to work more than usual during some week, would you get paid for those extra hours of work? (ii) (If yes to i) What would be your hourly rate for that overtime? (iii) (If no to i) Do you have an hourly wage rate for your regular work? (iv) (all except no to iii) What is your hourly wage rate for your regular work time?

<sup>23</sup>Wales (1973) cites this freedom to vary work hours as a primary reason for choosing to analyze the labor supply of a sample of self-employed businessmen. However, he then assumes that the gross marginal wage is equal to average hourly earnings, with no consideration of the measurement problems.

<sup>24</sup>We use the term "salaried" to cover all employees whose earnings are not variable on an hourly basis. It covers a scattering of non-unionized blue-collar workers as well as more obvious salaried positions in professional, managerial, or clerical occupations. An analysis of the characteristics associated with such employment is presented in Dickinson (1974, pp. 191-194).

<sup>25</sup>Calculated as the change in earnings divided by the change in work hours  $(.98w_o * 1.1H_o - w_o H_o) / 0.1H_o = 0.78w_o$ .

<sup>26</sup>Schiffman's figures are based on the Current Population Survey for May 1962. Using those figures, we may calculate approximate hourly wage rates for men as the ratio of median weekly earnings to median hours on primary and secondary jobs. The resulting estimates are \$2.40 and \$2.70, respectively.

<sup>27</sup> This comparison is confounded, since the "want more" group presumably includes disproportionate numbers of high-work-preference individuals who have low reservation wages. The distribution of best offers that are refused by workers who choose to remain in disequilibrium depends on the unknown joint distribution of reservation wages and wage offers, with a wage offer being accepted only if it exceeds the reservation wage. Under reasonable assumptions, the unobserved distribution of refused best offers will include a disproportionate number of low wage rates relative to the observed distribution of accepted offers.

<sup>28</sup> Workers are included in the select sample if they reported freedom to increase and/or decrease their work hours or if they reported a constraint in one direction and no dissatisfaction with the constraint. Workers who were fully constrained but satisfied are excluded on the basis of a judgment that their observed work hours are too subject to a contamination by demand-related institutional factors.

<sup>29</sup> It is slightly different in that most previous studies have used average wage rates, which are affected by moonlighting, and thus involve a combination of endogeneity and misspecification problems.

<sup>30</sup> Greenberg (1972, p. 10) reported that among 9872 civilian married males under 62 who were observed in the Survey of Economic Opportunity, only 329 did not work and 250 of those were ill or disabled.

<sup>31</sup> Rea used somewhat more general notation with the leisure portion of unemployment time denoted as a function,  $g(Z)$ , so that  $\frac{dg}{dz}$  corresponds to our  $\delta$ . Our notation of  $II$  for work hours and  $L$  for leisure is also reversed from Rea's usage.

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