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THE STRUCTURE OF INTRAGENERATIONAL MOBILITY

Aage B. Sørensen

UNIVERSITY OF WISCONSIN ~ MADISON

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Abstract

This paper analyzes mobility rates for black and white men using life-history data on intragenerational mobility. Mobility is linked, in this paper, to the process of occupational achievement. It is argued that job mobility is generated by persons' attempts to maximize their status and income. Since opportunities for better jobs will be fewer the higher the occupational achievement already obtained, the rate of mobility will depend on time. The mathematical formulation of this time dependency is derived from a simple change model for the occupational achievement process. With this formulation, a redefinition of time is possible and job shifts in the redefined time scale can be described by a Poisson process. One component in the formulation of a realistic and theoretically meaningful stochastic model of mobility is then obtained. The empirical analysis indicates that the proposed model describes reasonably well the observed change in mobility rates over time.

THE STRUCTURE OF INTRAGENERATIONAL MOBILITY

Introduction

The distinction between intergenerational and intragenerational mobility is commonplace in research on social mobility. It is a distinction that apparently does not separate two equally important forms of mobility. Studies of father-son mobility clearly dominate the field. This emphasis seems easily justified. Intergenerational studies speak to a dominant concern of Western societies: the concern for equality of opportunity. The degree of "openness" of societies, as measured by father-son mobility rates, is generally held to be a major characteristic of societies. Svalastoga (1965) even constructs a typology of stratification systems based on the rate of intergenerational mobility.

Despite the prevalence of intergenerational studies, it can be argued that it is essential to study intragenerational mobility in order to achieve the objectives of mobility research. This is so even given the descriptive purposes that dominated early mobility research. The positions of fathers and sons in the social structure that are compared in measurement of the "openness" of societies are not fixed attributes of individuals over their lifetime. Rather all available evidence indicates a considerable mobility over age in industrial societies. Furthermore, a person's position seems to be systematically related to his age. The results of intergenerational mobility studies will therefore depend on the ages of fathers and sons used in the comparison. Some attention to patterns of intragenerational mobility is needed in order to obtain precise measurement and ensure comparability between different studies.

The need for intragenerational mobility data is even more apparent when the objective of research is causal analysis of mobility. A traditional distinction separates two sources of mobility: structural and individual. Research has rather sharply diverged according to which set of causal forces are analyzed. However, the causal analysis of both profits from the use of intragenerational data. This is to some extent already recognized in research on the effect of individual attributes on status that has come to be known as status attainment research. The causal models so successfully employed in this research use status obtained as the dependent variable. This status depends on a person's age, and already Blau and Duncan's (1967) pioneering study introduced status of first job in their analysis, and some analysis was done on age-variation in the process using synthetic cohorts. This work has subsequently been extended by Kelley (1973) and Featherman (1971), and in the research by Sewell and associates (for example, Sewell and Hauser, 1974) on a panel of Wisconsin youth. Although status attainment research shares the concern for equality of opportunity with traditional intergenerational mobility research, it is apparent that the research questions asked are intragenerational in nature, for they are questions about what determines a person's status (with special emphasis on the influence of the origin), and a person's status will vary over time.

The analysis of the occupational achievement process found in status attainment research does not satisfy those who insist that there are important structural features of society that influence the process by which a person obtains a certain position in society. The existence of "barriers" to mobility and the influence of supply and demand for positions in the social structure are largely ignored in status attainment research. Direct analysis of mobility patterns seems needed.[1] Again the proper data source for an analysis of these structural causes of mobility would seem to be intragenerational data. Intergenerational data does not present the actual mobility experiences of any real cohort, nor does it enable the location of mobility in time. Intragenerational data, in contrast, may permit the investigator to precisely identify single acts of mobility and to link those to structural characteristics of society. Although analysis of intragenerational mobility patterns are very scarce, the usefulness of these data for analyzing structural sources of mobility is apparent. The most successful attempt of analysis of this kind probably is White's (1970) analysis of vacancy chains that uses intragenerational data.

The analysis of intragenerational data presents problems. First of all, such data are difficult to obtain. They will either be prospective, in which case a panel is the most likely design (cf., the ongoing study of Wisconsin youth), or the data will be retrospective. Panel data does not provide complete information on mobility or status changes, as only discrete points in time will be observed -- points that for reasons of economy may be quite widely spaced. Retrospective life-history data in contrast may enable the observation of every act of mobility undertaken by respondents over the period of observation. The main drawback of these data are possible errors of recall. Retrospective life-history data for a cohort of 30-39 year old men are the data source for the empirical analysis to be presented in this paper.[2]

The major problems with intragenerational data are, however, problems of analysis. Since these data are change data, less well-established methods of analysis are available. Although the age variation in status is recognized in status attainment research, direct analysis of change in status typically is not attempted. Rather, the analysis focuses on the level of status at various ages, a procedure that cannot fully capture the dynamic nature of the process being investigated (Sørensen, 1974a). The use of causal analysis of mobility patterns based on intragenerational data presents the problem of how to summarize the wealth of information on mobility in a suitable mathematical form. With the exception of White's (1970) vacancy chain model, little work has been done that goes beyond the attempts to fit observed mobility patterns to a stochastic model. The typical choice of model has been the Markov model. However, this model in its simplest form never has been found to fit observed mobility patterns very well. A number of modifications have been made, in most instances using ad hoc procedures to improve the fit of the Markov model. These modifications have most often been statistical in nature and have not been derived from an explicit theory of the forces that produce mobility.

It is the purpose of this paper to analyze intragenerational mobility rates using a Markovian model of mobility as the framework. An attempt will be made to modify the simple Markov model in a way that both improves the fit of the model and can be justified by a theory of mobility. This means that the properties of the model must be derived from a reasonable set of assumptions about the process of mobility. These assumptions concerning how mobility is brought about will be derived from considerations of the occupational achievement process, that is, the process of attaining status and income. Although status attainment research that analyzes this process has become diverged from research on mobility patterns, this separation cannot be justified by the nature of the phenomena being investigated in status attainment and mobility research. Occupational achievement takes place through mobility and the mechanisms that produce change in status and income over time are mobility mechanisms. The two processes are intimately linked, and it will be demonstrated in this paper that the properties for a model of the rate of mobility can be derived from an analysis of change in occupational achievement.

It will be shown that these considerations necessitate a model that mirrors the interplay between structural and individual characteristics that traditionally has been held to generate mobility, an interplay that, however, never has been made very explicit. Comparison of mobility rates for blacks and whites will be used to demonstrate how this interplay works.

The next section will present the simple Markov model that is a point of departure for the analysis that follows.

The Markov Model of Mobility

In this section, I shall outline a stochastic model of mobility to be further specified in subsequent sections. The final product will technically be a nonstationary Markov model of a jump process.[3] It will be a nonstationary Markov model as the parameters of the model will be allowed to change over time. It will be a jump process, as mobility is conceived of as taking place in discrete steps: individuals spend some time in a job and then move (jump) to another job. Furthermore, the duration of stay in each job may depend on characteristics of that job, in particular the occupational group to which it belongs. The notion that moves take place in discrete steps does not imply a discrete time Markov Chain model. Rather, moves can take place at any point in time, so the process is conceived of as a continuous time process. However, as in a Markov Chain, individuals move in a discrete state space formed by job categories (in this application, they are occupational groups).

The elementary act of mobility is conceived of as a job shift. The analysis of such shifts can be broken down into two components: (1) the analysis of the occurrence of shifts, and (2) the analysis of the outcome of shifts given that they occur. This distinction is basic for the remainder of this paper. A model for mobility may be derived by making certain assumptions about the two components of the process. A simple model, useful as a point of departure, is obtained by making two sets of assumptions.

First, job shifts are assumed to occur according to a Poisson process, that is, they occur randomly in time with an intensity λ, where this parameter is assumed constant over time and across individuals. This means that the probability $p(t)$ that a person who entered a job at time 0, will remain there at time t, will change according to the differential equation:

$$\frac{dp(t)}{dt} = -\lambda p(t) \tag{1}$$

Equation (1) is the defining equation for a Poisson process. It implies that the number of job shifts in a period T will be Poisson distributed, and the waiting times between shifts will be exponentially distributed, a property to be used later.

The second assumption is that once a move occurs, its outcome will be given by a set of conditional probabilities m_{ij}'s that give the probabilities of moving from a job in some occupational category, or state i, to a job in state j, where i may equal j. These "direction" probabilities are assumed conditional on the state of departure only, not on any previous state the individual has occupied -- an assumption often referred to as the Markov Property. Again, these probabilities shall be assumed identical for all individuals and constant in time. These two assumptions and the parallel ones for the rate of job shifts shall be referred to in the sequel as the assumption of nonheterogeneity among individuals, and the assumption of the processes being stationary in time.[4]

With these assumptions, consider the probability $p_{ij}(t)$ that a person will be in state j at time t given that he was in state i at time 0, where 0 and t can be taken as arbitrary time points because of the stationarity assumption. The rate of change in this probability will be a function of the rate at which job shifts occur, λ, and the direction probabilities, the m_{ij}'s. This change can be described by a differential equation. Focus only on those that start out in state i, that is, the group of individuals for which $p_{ii}(0) = 1$ and $p_{ik}(0) = 0$, where $k \neq i$. The proportion of persons in state j at time t who originated in state i is $p_{ij}(t)$. The probability that a person will undertake a job shift in the small time interval dt is λdt. The probability that a person who originated in state i and at t is in j will leave state j in dt is $\lambda(1 - m_{jj})dt$, where m_{jj} is the probability of a shift to another job in state j. Similarly, the probability that a person in state k who originated in i will move to j in dt is $\lambda m_{kj}dt$. The change in $p_{ij}(t)$ in a small interval of time then will be given by the sum of the proportions of persons in states k $(k \neq j)$ at time t who moved to state j in dt, minus the proportion of persons in state j who left j in dt, or:

$$dp_{ij}(t) = \sum_{k \neq j} \lambda m_{kj} p_{ik}(t)dt - \lambda(1 - m_{jj})p_{ij}(t)dt \qquad (2)$$

which gives rise to the differential equation:

$$\frac{dp_{ij}(t)}{dt} = -\lambda(1 - m_{jj}) \cdot p_{ij}(t) + \sum_{k} \lambda m_{kj} p_{ik}(t) \qquad (3)$$

If the $p_{ij}(t)$ quantities are arranged in a matrix $P(t)$, and the quantities m_{ij} in a matrix M, then this differential equation can be written as:

$$\frac{dP(t)}{dt} = \lambda(M - I)P(t) \tag{4}$$

where I is the identity matrix. Equation (4) has the solution:

$$P(t) = e^{\lambda(M - I)t} \tag{5}$$

keeping the assumption $P(0) = I$.[5] If quantities q_{ij}'s are defined as:

$$q_{ij} \begin{cases} q_{ii} = \lambda(m_{ii} - 1) & i = j \\ q_{ij} = \lambda \cdot m_{ij} & i \neq j \end{cases} \tag{6}$$

then $\lambda(M - I)$ will be equal to a matrix Q with elements q_{ij} that represent transition rates in a continuous time discrete space Markov process.[6] The representation (5) is preferred here as it directly shows the dependency of the mobility process on the rate at which job shifts occur (as given by λ) and the direction of the move once they occur (m_{ij}).

The representation (5) is also useful for estimation purposes. On life-history data, all job shifts are known as well as the time intervals between them. Since the rate of moves is assumed to be governed by a Poisson process, it follows that waiting times between moves are exponentially distributed; that is, durations of jobs w, have the density:

$$f(w) = \lambda e^{-\lambda w} \tag{7}$$

with mean

$$E(w) = \int_0^\infty \lambda t e^{-\lambda t} dt = \frac{1}{\lambda} e \tag{8}$$

The expression for the mean suggests estimating λ as the inverse of the mean duration of jobs. The probability m_{ij}'s may be estimated by counting all moves originating in state (occupation) i according to their destination, irrespective of when they occur; that is, an estimator for m_{ij} is:

$$m_{ij} = \frac{n_{ij}}{n_{i.}} \tag{9}$$

where $n_{i.}$ is the total number of moves originating in i and n_{ij} is the number of moves to j (where j may equal i).

Even though (5) is a useful representation of a Markov model of mobility, it is no more valid than other formulations found to be a poor description of mobility data,[7] for these formulations can be derived from (5) without any change in assumptions. Furthermore the formulation (5) only represents a most rudimentary theory of mobility. The derivation of (5) did not rely on a causal analysis of the process, but only provided a description of the process.

The lack of fit calls for either some modification of (5), if the basic framework is to be kept, or for rejection of the Markovian framework altogether. The latter strategy has been advocated for conceptual reasons by White (1970) and has also been adopted by Boudon (1974). They are both concerned about implementing a basic axiom of mobility theory -- that mobility at least partly is determined by social structure. White (1970) argues that this implies that the analysis of mobility of men as a stochastic process is unreasonable, for men's mobility will be interdependent, as movement of one man will affect the movement of others. This violates a fundamental assumption needed to formulate a model for the process: the assumption of stochastic independence of the elements. White's solution is to formulate the model for vacancies, not for men. Boudon's strategy is to model the access to positions in social structure, not the mobility process over time.

White (1970) analyzes mobility in organizations where there are few vacancies and many men, and where the assumption about independence of the movement of men obviously is false. Models of mobility of vacancies in macro systems are not available, and a straightforward generalization of the vacancy chain models, even though conceivable, would be a purely formal exercise, for data to test such models are not available. Data are available on the movements of individuals. When these data (as the life-history data used here) only deal with the movement of one cohort in a much larger occupational structure, it does seem justified to analyze them as though they represent independent moves of individuals, and thus keep the framework of a stochastic process for mobility of men. It is not inconceivable that the structural forces that concern Boudon may be represented in a stochastic process model. As a matter of fact, the modifications of the model (5) to be presented represent such an attempt.

This paper then shall keep the basic framework suggested by (5) and attempt to modify it in order to obtain a more realistic model. From a statistical point of view the failure of the Markov model, if moves are assumed stochastically independent, may be attributed to failure of one or both of the basic assumptions of stationarity and nonheterogeneity of the parameters. These failures may be remedied in a number of ways, but if the objective of a theoretically meaningful model is to be achieved, one should attempt to introduce modifications that are derived from substantive considerations of the forces that govern mobility processes.

The remainder of the paper attempts such a modification of (5). The separation of the process into two components implies that the model may be modified both with respect to the assumptions concerning the rate of mobility, and the assumption regarding the direction probabilities. Only the first task shall be attempted here, that is, the assumption that job shifts occur according to a stationary Poisson process shall be modified. The result will be the formulation of a nonstationary Poisson process. Analysis of the direction probabilities is the task for another paper.[8]

The Rate of Job Shifts as a Time Dependent Poisson Process

The simple Markov model (5) was derived from the assumption that job shifts occur according to a Poisson process with a parameter λ, constant over time and identical for all individuals. This assumption is both simple and convenient as it leads to an estimation of λ as the inverse of the mean duration of jobs. However, the assumption does not represent a reasonable approximation of reality nor can it be derived from commonly held beliefs about what generates mobility.

The assumption does not represent reality for it is well known that the rate at which job shifts occur depends strongly on age with older persons being less likely to shift jobs than younger persons. On the life-history data, it was found that the correlation between the duration of job and a person's labor force experience is .43 for whites and .44 for blacks. This suggests that λ should be a function of time, $\lambda(t)$, and the process of job shift be a nonstationary Poisson process. Furthermore, most would probably find it reasonable to assert that the rate of job shifts differs among individuals, which would imply some distribution of λ and a failure of the Poisson process to mirror the occurrence of job shifts.

The observation that rates of job shift should vary with age and among individuals could be incorporated directly into the model on an ad hoc basis. Models where $\lambda(t)$ is specified as a declining function of time are discussed by McGinnis (1968), Ginsberg (1971) and Mayer (1972). Spilerman (1972) has shown how an assumption of λ being gamma distributed among individuals can be incorporated into the model (5) and a reasonable empirical fit obtained. Such devices then can be used to improve the fit of a model to observed data. However, when these devices are justified mainly by statistical concerns, they are of rather small help in understanding the forces that generate mobility.

A theoretical rationale for specifying $\lambda(t)$ and/or its variation among individuals cannot be obtained directly from a body of well-specified theory of mobility, for such a body of theory does not exist. The traditional conceptualization of mobility states that mobility is the outcome of an interplay between structural and individual characteristics. This is a very vague and general statement that must be made considerably more specific in order to be of any help in modeling the variation in rates of job shifts.

The point of departure for this specification is the observation that job shifts can represent either a voluntary act on the part of the job holder or represent an involuntary dismissal from a job. In both cases structural characteristics interact with individual ones in producing the shift. If the shift is voluntary there must be some vacant job available that the person can move to, and the individual must have the qualifications needed to obtain the available job. If the shift is involuntary, it often represents the elimination of a job, and it is likely that employers will first dismiss those employees that are most expendable to the firm. The two types of job shifts cannot be described by the same process. In the sequel the analysis will concentrate on voluntary shifts for which an explicit formulation of their dependence on time can be given. However, involuntary shifts are of importance in the empirical analysis to follow.

A person may be assumed to shift jobs voluntarily if he can obtain a better job. This shift may be seen as the outcome of a process where job opportunities become known to the individual and create impulses to leave a job. Denote the rate at which impulses to leave arrive in time by $z(t)$. Assume further that a person has a constant probability λ^* of acting on any one of these impulses. The relation between the probability that a person will occupy a job at time t entered earlier and the quantities λ^* and $z(t)$ can now be written, in analogy to the simple Poisson process (1), as:

$$\frac{dp(t)}{dt} = -\lambda^* z(t)p(t) \tag{10}$$

or, slightly rearranging:

$$\frac{dp(t)}{p(t)} = -\lambda^* z(t)dt \tag{11}$$

Define a quantity $v(t)$ by the relation $dv(t) = z(t)dt$, so that:

$$v(t) = \int_0^t z(s)ds \tag{12}$$

It is useful to conceive of $v(t)$ as a new time scale[9] defined by the rate at which impulses to leave arrive, which in turn reflects the rate at which opportunities for better jobs appear. In this new time scale the process will be Poisson, as can be seen by substituting dv for $z(t)dt$ in (7). The intensity λ^* of leaving jobs therefore may be estimated as the inverse of the duration of jobs in $v(t)$, a time scale that shall be called psychological time. Formally, the duration of a job in psychological time will be:

$$\hat{w} = v(2) - v(1) \tag{13}$$

where $v(2)$ and $v(1)$ denote a person's psychological age when he leaves and enters the job, at times t_2 and t_1.

The rate of impulses to leave a job $z(t)$ is considered a function of time. There is a finite number of jobs in any occupational structure and there will be an upper limit for how good a job a person can obtain given his qualifications. This means that the better job a person already has the less likely it is that an even better job will become available.

If $z(t)$ can be specified in a way that can be empirically tested, the formulation of the Poisson process in $v(t)$ is extremely convenient. The Markov model (5) will then be a stationary process in $v(t)$, that is,

$$p[v(t)] = e^{\lambda^*(M - I)v(t)} \tag{14}$$

will be a process where λ^* will be independent of time $v(t)$. Since non-stationarity in λ^* will imply heterogeneity in λ^*'s observed for a group of individuals observed at a point in time,[10] it is also possible that the re-definition of time will remove some of this heterogeneity.

The notion that $z(t)$ depends on the rate at which job-opportunities appear suggests that the specification of $z(t)$ should be based on a model of the career process, i.e., a model of the occupational achievement process.

Rates of Job-Shifts and the Occupational Achievement Process

Although persons may consider a number of different aspects of jobs desirable, it is reasonable to assume that the status and income a job provides will be especially important. Denote by $y(t)$ the occupational achievement of a person at time t, as measured by his status and income. The occupational achievement will change over time as a person obtains better jobs. A simple model[11] for this process will be:

$$\frac{dy(t)}{dt} = c_1 + by(t) + c_2x_2 \tag{15}$$

where x_2 is a measure of a person's occupational resources (family background, education, etc.), and c_1 stands for unmeasured resource variables. Other things being equal, the higher a person's occupational resources the greater the rate of change in occupational achievement. However, the rate of change is also determined by $y(t)$ the level of achievement already obtained. This influence can be assumed to be negative $(b < 0)$ so that the higher the achievement already obtained the less gain will take place. Eventually the rate of change will be zero and at that time the level of achievement will be:

$$y(e) = -\frac{c_1}{b} - \frac{c_2}{b} x_2 \tag{16}$$

The quantity $y(e)$ shall be referred to as the equilibrium level of occupational achievement.[12] It is the highest level of achievement a person can hope to obtain and keep, for if his achievement exceeds this level the change in achievement will be negative.

The larger the absolute value of b the sooner the equilibrium value will be reached, other things equal. The quantity b then determines how much growth is possible in occupational achievement, and therefore is a measure of the opportunity structure of society. The closer b is to zero the more opportunities for better jobs will be available to a person, in the extreme case where $b = 0$, better jobs will always be available regardless of how much status and income a person obtains.

The career curve determined by (15) will be concave to the time axis, that is, achievement will increase rapidly in the beginning and then gradually taper off until the status and income reach the equilibrium level given by (16). This is indeed the pattern of growth observed, as can be seen in Figure 1 which gives the mean occupational prestige for blacks and whites by years of labor force experience. Both graphs show the expected pattern. It can be noted that the curve for blacks not only overall is at a lower level than for whites, it is also flatter. This may be due to the lower levels of occupational resources

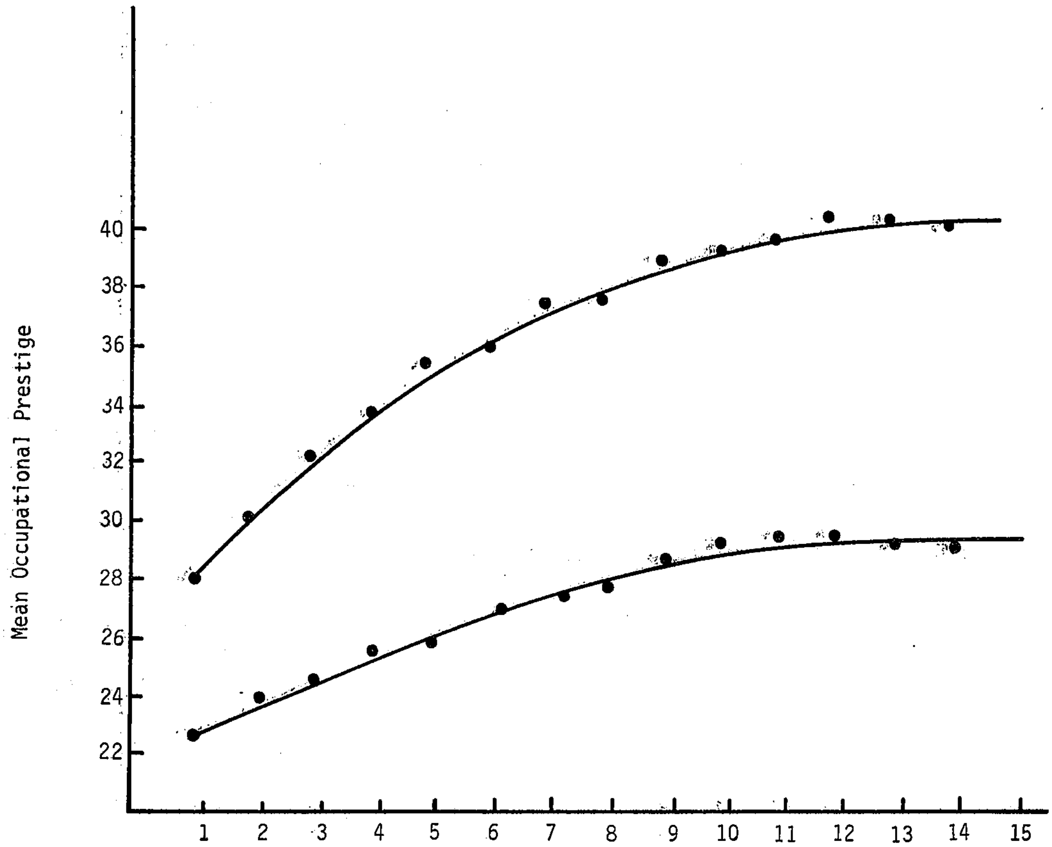
Mean Occupational Prestige
40
38
36
34
32
30
28
26
24
22
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

Figure 1

of blacks, but could also reflect a difference in b due to different opportunity structures for blacks and whites.

The curves represent the solutions to the differential equation (15). Integrating (15) from time 0 to time t_n, where time 0 represents a person's entry into the labor force, gives:

$$y(t) = \frac{c_1}{b}(e^{bt} - 1) + e^{bt}y(0) + \frac{c_2}{b}(e^{bt} - 1)x_2 \qquad 0 \leq t \leq t_n \qquad (17)$$

where $y(0)$ represents the occupational achievement at entry into the labor force.

The model of the process of occupational achievement given by (15) shall now be linked to the rate of voluntary job shifts. Gains in achievement may be assumed to entail job shifts. This is trivially true for occupational prestige, because prestige is defined as an attribute of occupational groups. It is a reasonable approximation for income, as major shifts in earnings usually entail job shift, while within-job gains represent real and inflationary increases common to all jobs. If gains in achievement then necessitate job shifts, it follows that the rate of shifts can be derived from (15). The pattern of rapid change in the beginning of the career and less change later on must entail high rates of shifts in the beginning of the career and low rates later on. It is both instructive and necessary to derive the explicit formulation for the relationship between the rate of job shifts and the achievement process.

A slight reformulation of (17) is useful. Define a quantity $v(t)$ as:

$$v(t) = \frac{1}{b}(e^{bt} - 1) \qquad (18)$$

Equation (17) can now be written as:

$$y(t) = e^{bt}y(0) + v(t)(c_1 + c_2x_2) \tag{19}$$

Subtracting $y(0)$ from both sides of (19) and rearranging gives:

$$y(t) - y(0) = y(0)(e^{bt} - 1) + v(t)(c_1 + c_2x_2) \tag{20}$$

Since,

$$y(0)(e^{bt} - 1) = \frac{1}{b} \cdot by(0)(e^{bt} - 1) = v(t)by(0), \tag{21}$$

equation (20) may be written as:

$$y(t) = y(0) + v(t)[c_1 + c_2x_2 + by(0)]. \tag{21}$$

Consider $v(t)$ a redefinition of time. In this time scale the career curve will be a straight line with a slope equal to $[c_1 + c_2x_2 + by(0)]$ and an intercept equal to $y(0)$. The slope will be zero when

$$y(0) = -\frac{c_1}{b} - \frac{c_2}{b}x_2 \tag{22}$$

that is, when the first job provides a person with the equilibrium level of achievement. In general, the slope will be determined by a person's level of resources in relation to the achievement at entry into the labor force: the lower this achievement in relation to the occupational achievement that can be obtained [i.e., y(e)], for given occupational resources, the higher the slope.

The fact that the career is linear in $v(t)$ must mean that there will be a constant gain per unit time in $v(t)$. Since gains are assumed obtained through job shifts, it follows that the rate at which shifts occur must be constant in $v(t)$. Using the notion of time suggested before, the underlying mechanisms may be seen as one where job opportunities (that generate impulses to leave jobs) occur at a constant rate in $v(t)$.

The redefinition of time $v(t)$ is then the desired transformation of time so that the rate of job shifts will follow a Poisson process. The function $z(t)$ that gives the rate of impulses to leave in real time, can be obtained from $v(t)$ defined in (18) by differentiation, or

$$z(t) = \frac{dv(t)}{dt} = e^{bt} \tag{23}$$

Further, it is easily shown that the basic process at the individual level, that produces the decrease in the rate of impulses to leave, is one in which the rate of impulses decreases proportional to $z(t)$ by the constant b, or, by differentiating (23)

$$\frac{dz(t)}{dt} = bz(t) \qquad b < 0 \tag{24}$$

This completes the derivation of the desired redefinition of time that expresses the dependency of the rate of job shifts on time. However, the model for the occupational achievement process can also give some insights into what determines the overall rate of job shifts. Assume that a person only will undertake a job shift if a gain of a certain size in occupational achievement can be realized. If the growth in achievement follows the model (15), the inverse of this differential equation will give the increase in time needed to realize a gain of a certain size. In $v(t)$ the basic differential equation is

$$\frac{dy(t)}{dv(t)} = c_1 + c_2x_2 + by(0) \tag{25}$$

with the inverse

$$\frac{dv(t)}{dy(t)} = \frac{1}{c_1 + c_2x_2 + by(0)} = \frac{-b}{y(e) - y(0)} \quad . \tag{26}$$

For a given change in $y(t)$, $dv(t)$ will be determined by b and by the inverse of the distance between the equilibrium level of achievement and the achievement of the first job. The second quantity reflects the level of a person's occupational resources relative to the achievement of the first job. It follows that for given b, the better the occupational resources relative to the achievement of the first job, the smaller $dv(t)$ and the higher the rate of shift. Should the first job provide the equilibrium level of achievement $dv(t)$ will be infinite and the rate of shift will be zero.

The theory of the occupational achievement process formulated here then can provide a specification of the dependency of the rate or intensity of job shifts on time, and also provide a framework for interpreting the size of the rate of job shift. The theory specifies the traditional notion of mobility being an interplay between structural and individual characteristics, by conceiving job shifts as brought about by individuals attempting to utilize opportunities for better jobs to maximize their occupational achievement. Because opportunities are finite, the higher the achievement already obtained, the fewer opportunities for better jobs there will be, and the lower the rate of shift.

The formulation here only concerns voluntary job shifts. The rate of involuntary shifts clearly should reflect the opportunity level in society. Furthermore, the rate should depend on personal resources in relation to the achievement obtained, but be different in value from the rate of voluntary shift. Further specification will not be attempted in this paper.

The derivation of the dependency $\lambda(t)$ on time can be used to redefine time in such a way that the Markov model is stationary in the new time scale. One of the means needed to construct a realistic stochastic model of mobility has been obtained, but only if the formulation here is a reasonable approximation to reality. While the observed career curve does show the pattern to be expected from the proposed model of the achievement process, this is a too impressionistic validation. A direct verification of the expression for the dependency of rates of shifts on time must be attempted, and procedures developed to estimate the rate of job shifts in the redefined time scale, $v(t)$.

Tests and Applications

The model for the dependency of the rate of job shift on time formulated above relied on the redefinition of time into the new time scale $v(t)$ in which job shifts are assumed to occur according to a Poisson process. The rate of shift λ^* in psychological time can be estimated as the inverse of the mean durations measured in $v(t)$. Inserting the expression (18) for $v(t)$ in the definition of the duration in psychological time (13) gives, for the duration $\hat{w}$:

$$\begin{aligned}\hat{w} &= v(2) - v(1)\\ &= \frac{1}{b}(e^{bt_2} - 1) - \frac{1}{b}(e^{bt_1} - 1)\\ &= \frac{1}{b} e^{bt_2}(e^{bw} - 1)\,, \end{aligned} \qquad (27)$$

where t_2 and t_1 denote the time of leaving and entering the job, respectively, and $w = t_2 - t_1$ is the duration of the job in real time.

In order to use (27) it is necessary to know b. In addition, the results of the time transformation is only meaningful if the model (18) is a reasonable approximation to reality. The model can be tested and estimates of b obtained in several ways.

One procedure is to study whether the observed mean durations measured in $v(t)$ can indeed be made time independent, using an iterative procedure to obtain estimates of b. This is a cumbersome procedure especially since the interruption of the process at the time of interview causes estimation problems -- problems to be described below. An alternative procedure is to rely directly on some property of the model both to test it and to obtain estimates of b -- the parameter that determines the redefinition of time.

The latter procedure shall be attempted first. Assume that the process indeed is Poisson in $v(t)$. The solution to the defining differential equation (1) is

$$p[v(s)] = e^{-\lambda^{*}[v(s) - v(0)]} \tag{28}$$

or

$$\log p[v(s)] = -\lambda^{*}[v(s) - v(0)] , \tag{29}$$

where $p[v(s)]$ is the proportion remaining in a job at time $v(s)$ of those that have entered at time $v(0)$.

Denote by t_2 the time (in real time) at which a person enters a job and by $s = t_s - t_1$ a time interval of given length s. Inserting for s and t_1 in (29), gives:

$$\log p[v(s)] = -\lambda^{*}[\frac{1}{b} e^{bt_1}(e^{bs} - 1)] . \tag{30}$$

Multiplying through by -1 and taking logarithms again gives:

$$\log \log p[v(s)] = \log \lambda^{*} - \log b + bt_1 + \log (e^{bs} - 1) \tag{31}$$

If a set of t_1 values is chosen and the proportion remaining after s time units is observed, a linear relation between $\log \log p[v(s)]$ and t_1 is indicated. For given s, all terms on the right hand side will be constant except bt_1 or,

$$\log \log p[v(s)] = k + bt_1 . \tag{32}$$

This expression may be used both to test the model and obtain an estimate of b.

The life-history data provides information on all jobs held by a sample of 30-39 year old black and white men from the time of entry into the labor force until the time of interview. A special file was created with jobs as the unit of observation, that is, the total number of observations equals the number of respondents times the number of jobs held by them. For blacks this gives 3678 jobs, for whites 4992. The starting and ending times of jobs were registered in months. For purposes of this analysis all time points were measured from time of entry into the labor force. Only jobs left voluntarily are used below. The separation of shifts into involuntary and voluntary shifts was based on respondents' reports on whether a job was left voluntarily or not. For blacks 2651 jobs, and for whites 3877 jobs were left voluntarily.

The opportunity structure for blacks and whites is generally believed to be different, and possibly this may be reflected in different values of b. The test and the following analysis is therefore carried out separately for blacks and whites.

The test of the model suggested by (32) is displayed in the graphs given in Figure 2. Unfortunately, much of the information contained in the life-history data was not usable for the test. When a group of persons that enter a new job, say four years after entering the labor force, are followed over time, some will be registered as having terminated their jobs, say two years after, simply because they at that time were interviewed and no further information was obtained. All respondents were between 30 and 39 when they were interviewed, but since there is a great deal of variation with respect to age at entry into the labor force, the proportion $p[v(s)]$ will be influenced by respondents leaving the sample, the more so the larger the value of s. To minimize the influence of sample drop-out and still obtain a reasonable range of values of s and t_1, only persons who entered the labor force before age 23 are used in Figure 2.

Figure 2 Here

Overall the expected linear relationship between $\log \log p[v(s)]$ and t_1 (given in years of labor force experience) comes through reasonably well in the graphs. Also the slope of the line relating the two quantities appears very similar. However, there are exceptions to the expected relationship. For blacks, the graph for $s = 1$ does not show the expected pattern, and also for whites the fit is less impressive for this value of s. Furthermore, it can be observed that for blacks the points corresponding to $t_1 = 1$ in all graphs systematically deviate from the straight line relationship. Both phenomena may be ascribed to lower reliability of the information pertaining to these early years in the labor force. Unfortunately, the deviations, although explainable, invite caution in using estimates of b obtained from equation (32) to calculate $\hat{w}$.

The graphs given in Figure 2 do seem to indicate that the model proposed here is a reasonable approximation to reality. Estimates of b can be obtained by pooling least squares estimates of b for

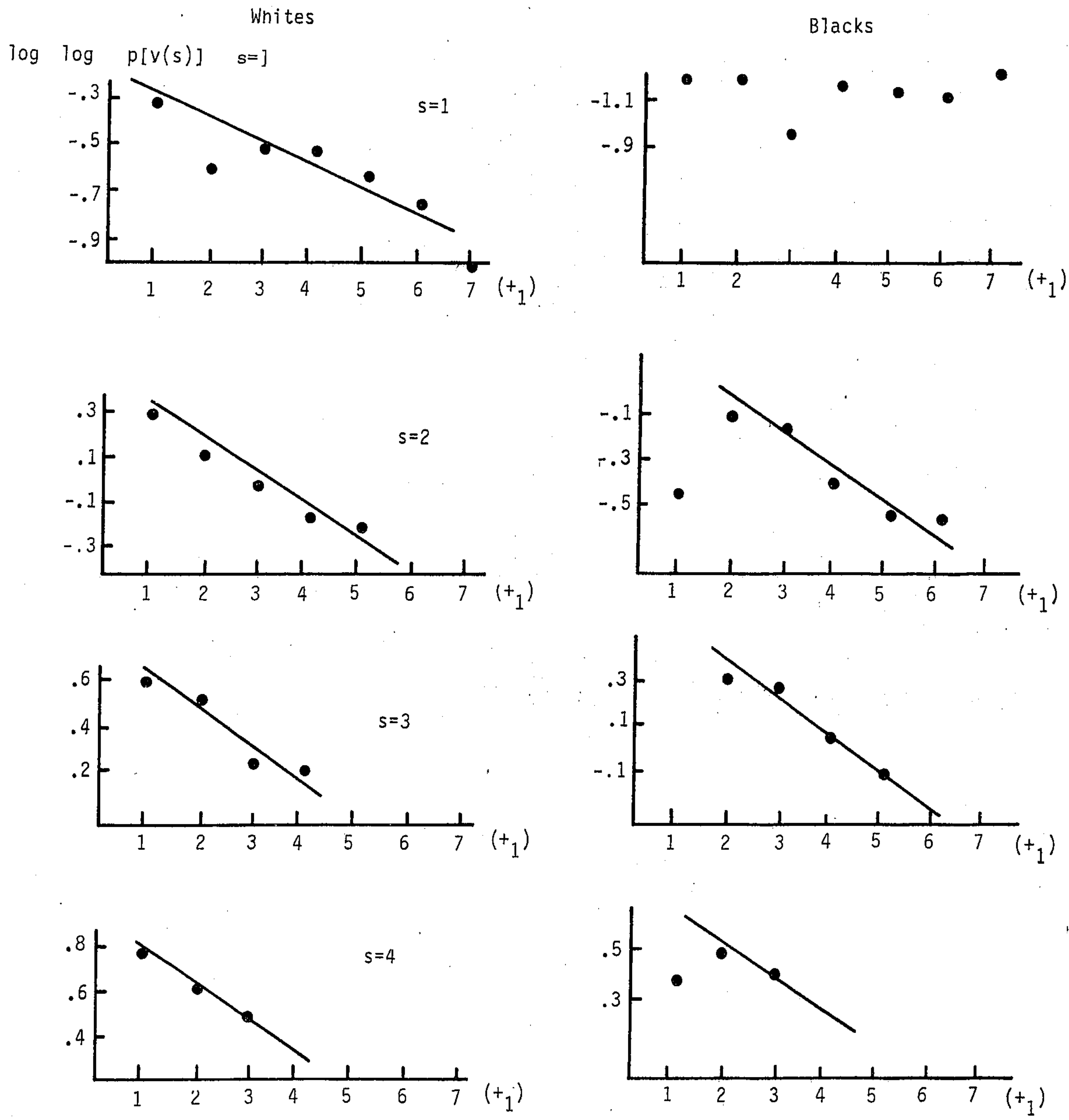
Whites
Blacks
log log p[v(s)] s=]
s=1
s=2
s=3
s=4
-.3
-.5
-.7
-.9
-1.1
.3
.1
-.1
-.3
-.5
.6
.4
.2
.8
.5
1 2 3 4 5 6 7
(+1)

Figure 2

the various values of s. For whites, the resulting b is $-.0065$, with an R^2 of $.79$, where b is given with months as units. For blacks, inclusion of the observations for $s = 1$ and $t_1 = 1$ gives a bad fit of the linear equation: $R^2 = 21$. Deleting these observations gives a much stronger fit. The obtained b is $-.0086$ with an R^2 of $.94$. This appears to indicate that b is indeed higher in absolute value for blacks than it is for whites, indicating a more unfavorable opportunity structure for blacks. However, since these estimates are based on only part of the sample, and since they by necessity will use the more unreliable information from the early job history, they are not necessarily the best estimates. It is necessary to complement the test by an analysis that directly shows whether mean durations are time independent after the transformation of time.

Mean durations in $v(t)$ can be calculated from equation (27). However most respondents held a job at the time of interview. The duration of this job in real time is not known. One might suggest deleting these jobs from the sample, but it turns out that this will produce a biased estimate of λ^* based on the remaining jobs. The problem has to do with the distribution of the truncated duration of the job held at interview. The nature of the problem shall be briefly indicated. A more detailed treatment is given in the appendix.

One might argue that for reasons of symmetry, the observed portion of the duration of the job held at interview should be half the expected duration. If job durations are exponentially distributed (as they will be when job shifts follow a Poisson process) they will have the mean $1/\lambda^*$. As a consequence, it might be argued that the truncated durations should have an expected value of $1/2\ (1/\lambda^*)$. While both arguments that lead to this conclusion are correct, the consequence does not follow; rather the expected value of the truncated durations is $1/\lambda^*$. This paradox is due to the fact that the distribution of the duration of the jobs that are interrupted by the interview is not exponential, even if the assumptions for the stationary Poisson process are upheld. Rather, the expected duration of the interrupted jobs is $2/\lambda^*$, and the symmetry argument then gives an expectation of the observed, truncated portion of the job of $1/\lambda^*$. The paradox is demonstrated by Feller (1971) who also derives the distribution of the truncated waiting times in a Poisson process that is interrupted. The proofs and some modifications to the above statements are given in the appendix.

Intuitively the reasons for the paradox is that the interruption of the process (i.e., the interview) will tend to select longer durations among the set of randomly distributed durations, that is, a long job duration has a greater likelihood of covering the interview than a short one. Since the expected duration can be shown to be $2/\lambda^*$ there is no reason to exclude the job durations for those jobs held at the time of interview; they may apparently be treated just as all other job durations are. But, if the interruption selects longer jobs among those held by an individual from the time of entering the labor force until the time of interivew, it will leave the remainder job durations to be shorter. Since the time of interview in this sample is age dependent (between 30 and 39) it is necessary to adjust for this effect of the interruption, otherwise the observed mean duration of jobs held at a certain time point will depend on the proportion of these jobs that are truncated. The proper adjustment is shown in the appendix to be:

$$\hat{w}_i = \frac{\hat{w}_{xi} \cdot \hat{T}}{\hat{T} - \hat{w}_y} \tag{33}$$

where $\hat{w}_{xi}$ is the observed duration on $v(t)$ of a nontruncated job, $\hat{T}$ is the time from entry into the labor force until interview [also measured in $v(t)$], and $\hat{w}_y$ is the mean truncated duration of jobs held at the time of interview.

Using the estimates of b given above and the adjustment procedure just described, durations were computed in psychological time. The mean duration for jobs was then obtained and their dependency on the time of leaving the job [measured in $v(t)$] studied.[13] It was apparent that the estimate for b used did not completely remove the time dependency, especially not for whites. This apparent failure of the model could simply be due to poor estimates of b. Using essentially a trial and error method, a number of different values of b in the neighborhood of the first estimates were used. This procedure, quite cumbersome because of the truncation problem, gave $\hat{w}$'s that behaved fairly reasonably for $b = .008$ for both blacks and whites. The correlation between $\hat{w}$ and time of leaving job is .03 for blacks and .08 for whites, as opposed to .44 and .43 before the transformation of time. The mean values of $\hat{w}$ by time of leaving the job, denoted $\hat{t}$, are presented in Table 1. The first value of $\hat{t}$ shown is 30 ("months of psychological time"), as truly time independent durations cannot be obtained for the smallest values of $\hat{t}$, not because of a failure of the model, but because of the necessary dependency between $\hat{w}$ and $\hat{t}$ for small values of $\hat{t}$.

Although the mean $\hat{w}$'s do appear to be fairly stable over time, there is a slight curvilinearity indicated for both blacks and whites. The curvilinearity means that b is not completely constant over time, contrary to our assumption. The absolute values of the true b apparently is underestimated by a $|b| = .008$ in the early years of labor force participation, and somewhat over-estimated in later years. This is a result that most reasonably can be interpreted to reflect changes in the overall opportunity structure of society in the period covered by the life-histories used here. This period roughly covers the two decades from the late 1940s to the late 1960s, and the general expansion of the economy, especially in the 1960s, likely provided for more opportunities for better jobs toward the end of the period. This would be reflected in a decline in the absolute value of b, as argued earlier: the more opportunities for better jobs there are, the closer to zero should be the value of b.

A direct test of this interpretation of the slight curvilinearity in $\hat{w}$ can be given by analyzing the jobs held in 1968 separately. Using the iterative procedure, it turned out that the best fitting values for b were $b = -.005$ for whites and $b = -.007$ for blacks. Blacks seem somewhat less sensitive to the presumed changes in opportunity structure than whites. The durations in $v(t)$ using these values of b for jobs held at the time of interview are presented in Table 2.

Since $\hat{w}$ for these jobs are independent of $\hat{t}$ with b's lower in absolute value than the best fitting b's for all jobs, the interpretation given to Table 1 seems correct. This means that the model for the dependency of rates of job shifts on time should be revised to take into account overall changes in opportunity structures in the period in which careers take place. Alternatively, separate estimates should be obtained for various epochs. The modifications shall not be attempted here. The overall estimates of b used do seem to behave reasonably well for the purposes of this paper. The development of more refined method of estimation shall be attempted in future research.

The overall mean $\hat{w}$ using $b = -.008$ for whites is 14.0 and for blacks 17.5, indicating that the rate of voluntary job shifts is lower for blacks than for whites. If durations in $v(t)$ are exponentially distributed their standard deviations should be $1/\lambda^*$. The overall estimates of the standard deviation for blacks is 18.7 and for whites 16.2. However, the truncated jobs are known not to be exponentially distributed (cf., the Appendix). The standard deviation for the nontruncated jobs for blacks is 17.9. This is close to the value of the mean (17.3) indicating that the durations conform reasonably well to the exponential distribution for blacks. By this test, job shifts do seem to be adequately described by a Poisson process for blacks. For whites, the results are somewhat less satisfactory; the standard deviation is 15.4, higher than the mean of 13.3 for the nontruncated jobs. This indicates some heterogeneity in λ^* for whites that might be described by a gamma distribution as demonstrated by Spilerman (1972). However, the heterogeneity is slight, and for purposes of this paper, the methods employed here seem adequate.[14]

Next we discuss substantive implications of the results obtained by the model for the dependency of rates of job shifts on time.

Black-White Comparisons

The preceding section would mainly be a quite elaborate statistical procedure for obtaining a stationary Markov model of mobility if it were not possible to give an interpretation to the results. To demonstrate that the framework developed does lend itself to substantive analysis, some results comparing rates of job shifts for blacks and whites are discussed in this section.

The sample consisted of 851 whites and 738 blacks. The black respondents held a total of 3678 jobs, while whites held 4992 jobs. This gives a mean number of jobs equal to 4.98 for blacks and 5.87 for whites. Blacks have fewer jobs and thus appear to hold their jobs longer (in real time) than whites. This difference could reflect either a difference in the opportunity structure blacks and whites are exposed to, or a difference in the likelihood of changing jobs. The latter would reflect that the occupational achievements of the early jobs blacks obtain are closer to the maximum blacks can hope to obtain -- the equilibrium value of the occupational achievement given the occupational resources.

A difference in opportunity structure between blacks and whites would show up in the duration of jobs because the higher the absolute value of b the fewer impulses to leave the job a person will receive, and hence -- even if λ^* is identical for the two groups -- the longer the durations will be in real time. The trial and error method used to establish b did, however, indicate that a b of $-.008$ for both groups gave the best estimates. Presumably, the reason for the overall longer duration of jobs for blacks in real time then is that blacks early in their career obtain an occupational achievement closer to the best they can expect to obtain than do whites. Technically, the difference in average duration in real time could also be explained by a difference in age of entry into the labor force: if blacks enter earlier they have a greater chance to hold long jobs. However, the difference in mean age of entry is only a little more than half a year; and more important, our calculation of the mean duration in $v(t)$ (using $b = -.008$) indicated that blacks do indeed have a lower rate of shifts ($\lambda^* = .057$) than do whites ($\lambda^* = .071$).

This conclusion should however be modified in several ways. The calculation of b used only voluntary shifts. If the average black is exposed to a more unfavorable opportunity structure than are whites, this should produce more involuntary shifts for blacks than for whites. This is indeed the case: of all jobs held by blacks, 28.7 percent were left involuntarily, while 20.7 percent of the jobs held by whites were terminated this way. Only focusing on voluntary shifts captures a more select group of job shifts undertaken by blacks than by whites.

Including involuntary shifts in the estimation of b for blacks and whites would not lead to better estimates of b. In fact, worse estimates would result. This is because involuntary shifts obviously occur when a person would not otherwise undertake the shift. A person would not voluntarily undertake a shift if he already has obtained the equilibrium value of occupational achievement. If he nevertheless is fired, a job shift would appear at a point in time where a shift should only occur voluntarily if the opportunity structure is favorable. Including the involuntary shifts among the voluntary shifts when estimating b would therefore underestimate b indicating a more favorable opportunity structure than actually exists.

The somewhat perverse influence of involuntary shifts on the estimation of b can be used to support an argument that the absolute value of b is underestimated by our trial and error method especially for blacks. Even though only voluntary shifts are used in the estimation, it might be argued that the voluntary shifts of blacks were "less voluntary" than those of whites. The indicator of whether a job shift was voluntary or not is a self-report, and obviously fallible. If it is considered an indicator of an underlying variable that expresses the degree of control over the employment situation, the higher observed percentage of involuntary shifts for blacks should indicate an overall lower level of control. Hence also for stated voluntary shift the level of control could be lower, and the absolute value of b correspondingly underestimated. For this reason, a conclusive statement regarding the difference, or lack of it, between b's estimated for blacks and whites should await more refined methods of estimation than the ones used here.

It is interesting to note in this connection that the mean duration of jobs left involuntarily, measured in $v(t)$, is 12.8 for blacks and 8.9 for whites. Blacks are on the average fired when they have held jobs longer than are whites. No one should stay in a job if a better one becomes available before he is fired, and this result therefore again reflects that blacks have less to gain by job shifts and may have fewer opportunities for shifts.

Involuntary job shifts are important for the occupational achievement process because they are likely to produce a loss in status and/or income. Any gain a person may obtain should be obtained through a voluntary shift, before the firing takes place. The combined impact of less frequent voluntary shifts (and thus fewer gains in achievement) and a greater likelihood of loss, not only makes the career line of blacks flatter but also more likely to undergo a decline. This clearly illustrates that for blacks the lower observed levels of occupational achievement not only is a question of lower levels of occupational resources but also of more tenuous employment situations.

The apparent decline in b with the expansion of the economy is a result that, even though it points to a limitation of the model, is of interest in its own right. It demonstrates that structural changes in society indeed are reflected in the mobility of men, and that the impact can be given a precise expression in the model for the rate of mobility. Furthermore, it appears that blacks have profited somewhat less from this expansion in terms of opportunities for better jobs than have whites. Another apparent failure of the model also points to an interesting substantive phenomenon. While for blacks it is the case that the standard deviation in durations in $v(t)$ is very close to the mean, a difference was observed for whites indicating that there is heterogeneity in λ^* for whites. The rate of shifts in psychological time has been shown earlier to reflect the slope of the career line in $v(t)$. The result then indicates that blacks have very similar careers insofar as they can undertake voluntary job shifts, while there is some variation in slopes for whites. A greater homogeniety in starting points of the career as well as in occupational resources is indicated for blacks by this result, a result that is in accordance with the known concentration of blacks at lower levels of occupational achievement and their lower levels of occupational resources, and hence smaller dispersion in these attributes.

Conclusion

The occupational achievement process is a change process. Few person's status and income remain constant over their lifetime. The change in status and income is brought about by mobility. In this paper, it has been attempted to show how a stochastic model of mobility may be derived from a simple model for the occupational achievement process.

The intimate relation between occupational achievement and intragenerational mobility seems obvious. Research in the two areas has nevertheless diverged rather sharply. Status attainment research has focused on the level of status and income obtained with little concern for analyzing change in occupational achievement. Much mobility research has been directed toward the formulation of stochastic models of mobility, with little concern for the forces that generate both mobility and occupational achievement. However, research that synthesizes the two traditions seems needed if basic problems in both areas are to be solved.

From the perspective of status attainment research, a synthesis is necessary to provide a framework for interpreting and measuring the effect of individual attributes on occupational achievement, and ascertaining how various structural characteristics modify these effects. Status attainment research sometimes, and to some, leaves the impression that the measured effects of individual attributes on status, somehow indicates how important these attributes are for the degree of inequality in society. In particular, the effect of education has been interpreted this way (Jencks et al., 1972). This inference runs counter to most theories of inequality, which explain inequality by power, class, functional importance, etc., but not by the occupational resources of persons. If the structure of inequality is seen as fairly independent of the distribution of individual attributes such as education, then the occupational achievement process should be conceived of as an allocation process. Individual attributes will be important for this process, but so will the availability of jobs at various status and income levels. This means that the achievement process should be seen as the outcome of an interplay between structural and individual characteristics, a conception that can be implemented by focusing on change, that is mobility, as the model proposed here does.

From the perspective of mobility research the need for a synthesis derives from the need to obtain a mathematical model that not only fits the data, but also gives some useful information about the process. A model is needed to summarize the wealth of information that is contained in intragenerational mobility data, but this can hardly be a goal in itself. The objective should be to establish a theory of mobility in mathematical form. The simple Markov model is an unreasonable theory and also fits the data poorly. The needed modifications of the model should, however, be guided by considerations of why mobility takes place, not solely by statistical considerations, and a theory of the occupational achievement process is essential for such an effort.

This paper represents the beginning of such a synthesis. Only one part of the process has been dealt with: the rate of mobility. However, it was possible to derive a formulation of the dependency of this rate on time that is implied by a theory of the achievement process, and this formulation seems a reasonable approximation to reality. The model conceived of mobility as generated by an interplay between individual and structural characteristics; more specifically mobility was seen as the outcome of individual's attempts to improve their occupational achievement in an occupational structure, where opportunities for better jobs are finite.

The proposed model is a simplification. Differences in rates of job shifts not caused by the opportunity structure were ignored. The influence of change in the opportunity structure on the rate of shifts likewise was not incorporated explicitly in the model. The model only described shifts that are voluntary. Finally, the estimation techniques used might be improved. These are problems for further research.

Table 1

Mean $\hat{w}$ as a Function of $\hat{t}$.

Voluntary Shifts using $b = -.008$

$\hat{t}$	Whites	Blacks
	Mean $\hat{w}$	
30	14.3	18.2
40	14.3	18.4
50	13.0	18.6
60	14.2	20.1
70	14.4	19.4
80	16.7	17.3
90	15.3	18.4
100	15.0	16.5
110	12.4	17.0
Overall	14.0	17.5

Table 2

Duration in $v(t)$ of Jobs Held at Time of Interview by Total Labor Force Experience ($\hat{T}$)

Whites ($b = -.005$)		Blacks ($b = -.007$)	
$\hat{T}$	$\hat{w}_x$	$\hat{T}$	$\hat{w}_x$
100	25.2	80	21.4
110	25.7	90	17.8
120	24.1	100	20.0
130	26.2	110	20.6
140	23.2	120	21.8
150	26.0	130	21.6

Note: The scales for $\hat{T}$ differ for blacks and whites because of different values of b used.

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Appendix

Estimating Waiting Times in an Interrupted Poisson Process

In the redefined time scale $v(t)$, durations of jobs are assumed to be exponentially distributed. However, completed durations for some jobs -- those held at the time of interview -- are not known. This creates special estimation problems that briefly shall be outlined in this appendix. The problems stem from a paradox about the distribution of waiting times in an interrupted Poisson process described by Feller (1971).

Consider a Poisson process with parameter λ that starts at time 0 and is interrupted at time t. Let s_n be the time at which the n'th event occurs. Let s_{n-1} be the time at which the last event before the interruption occurs and s_n be the time at which the next event would have occurred had the interruption not taken place. The distribution function for the interval $L_t = s_n - s_{n-1}$ is derived by Feller (1971:12-13).

The desired probability is $P(L_t \leq x)$, where x is an arbitrary number so that $0 < x < \infty$. Consider first the situation where $0 < x \leq t$. The inequality $L_t \leq x$ holds if the n'th event before t occurs at time y and the waiting time w until the $n + 1$ event is greater than $t - y$ and smaller than or equal to x. The probability $g_n(y)$ that the n'th event will occur at y has a gamma density (Feller, 1971:11). The probability that $t - y < w \leq x$ is given by the exponential distribution function as:

$$P(t - y < w \leq x) = (1 - e^{-\lambda x}) - [1 - e^{-\lambda(t - y)}]$$

$$= e^{-\lambda(t - y)} - e^{-\lambda x} \qquad \text{(A.1)}$$

The desired probability $P(L_t \leq x)$ then is:

$$P(L_t \leq x) = \sum_{n-1}^{\infty} \int_{t-y}^{t} g_n(y) \; [e^{-\lambda(t - y)} - e^{-\lambda x}] \qquad \text{(A.2)}$$

It can be shown that $g_1(y) + g_2(y) \ldots = \lambda$. Hence carrying out the integration

$$P(L_t \leq x) = 1 - e^{-\lambda x} - \lambda x e^{-\lambda x} \tag{A.3}$$

with density

$$f(x) = \lambda^2 x e^{-\lambda x} \qquad 0 < x \leq t \tag{A.4}$$

Using a similar argument, the density for $x > t$ is found to be

$$f(x) = \lambda(1 + \lambda t)e^{-\lambda x} \qquad x > t \tag{A.5}$$

Feller goes on to show that the mean of the waiting time from t to the first event after the interruption is $1/\lambda$. However, the interest here is in the mean of L_t, since in this application the mean of L_t is the expected duration of jobs held at interview. This expectation can be found by integrating the two parts of the density function, or

$$\begin{aligned} E(x) &= \int_o^t \lambda^2 x^2 e^{-\lambda x} dx + \int_t^\infty \lambda x(1 + \lambda t)e^{-\lambda x} dx \\ &= \frac{2}{\lambda}\,[1 - e^{-\lambda t}(1 + \lambda t + 1/2\ \lambda^2 t^2)] + e^{-\lambda t}(1 + \lambda t)(t + \frac{1}{\lambda}) \\ &= \frac{2}{\lambda} - \frac{1}{\lambda}\, e^{-\lambda t} \end{aligned} \tag{A.6}$$

The second term in this expression will vanish as $t \to \infty$, leaving the mean to be $2/\lambda$. For reasons of symmetry, the observed portion of the truncated jobs then will have a mean of $1/\lambda$. The symmetry argument does not, however, hold when t is close to zero. The expected waiting time from the interruption to the first event after it, will be $1/\lambda$ regardless of when the interruption occurs. The exact expectation for the period w_x from the last event before the interruption until the interruption, that is the observed truncated job duration, will therefore be:

$$E(w_x) = E(x) - 1/\lambda$$

$$= \frac{1}{\lambda}\,(1 - e^{-\lambda t}) \qquad (A.7)$$

If the interruption takes place close to the origin of the process, the observed truncated durations will underestimate $1/\lambda$, more so the closer t is to zero. In this paper it has been assumed that this problem did not occur.

Intuitively, the reason for these results is that the interruption is more likely to take place between events for which the time interval is large. This implies that the waiting times that precede the interruption will be correspondingly small.[15] If it is desired that any observed duration can be used to estimate the true value of the expectation $(1/\lambda)$, it is necessary to adjust the nontruncated durations.

The expected number of events in period 0 to t is λt. Each event is preceded by a waiting time with expected value w_y. The following identity then holds:

$$w_x + \lambda t \cdot w_y = t \qquad (A.8)$$

Rearranging gives:

$$\frac{1}{\lambda} = \frac{w_y \cdot t}{t - w_x} \qquad (A.9)$$

which is the formula used in the paper to adjust the nontruncated durations.

Further detail and simulations testing the above results are given in a forthcoming paper (Sørensen, 1974b).

Notes

1. Boudon (1974) thus returns to the analysis of mobility to establish the importance of education for inequality of opportunity, because he wants to incorporate social structural variables in the analysis. Such analysis was rejected by Blau and Duncan (1967), because of the serious methodological problems present in causal analysis using mobility as a dependent variable. However, the strategy of using status obtained as the dependent variable, adopted in status attainment research, does not give answers to the kind of questions Boudon poses.

2. The Life History Study dealt with the occupational, educational, familial and residential experiences from age 14 to time of interview. The universe is the total population of males 30-39 years of age, in 1968, residing in households in the United States. Two samples were drawn: (a) a national sample, and (b) a supplementary sample of blacks. The total number of interviews obtained was 1589: 738 blacks and 851 whites. The completion rates were 76.1 percent for sample (a) and 78.2 percent for sample (b). The Life History Study was initiated by James S. Coleman and Peter H. Rossi of the Department of Social Relations, The Johns Hopkins University.

3. The term "jump process" is used by Feller (1971) to denote continuous time, discrete space processes, where moves take place in discrete steps. The imaginary conveyed by this representation of the Markov model seems especially useful in applications to mobility. I am indebted to Seymour Spilerman for introducing me to this formulation. For an excellent survey of various formulations of this model and a number of estimation problems, see Singer and Spilerman (1974).

4. The use of the term stationary is adopted here, although "homogeneous" probably is the more commonly used term to denote time independence. The

latter term is however apt to get confused with nonheterogeneity used to denote identical parameters for all individuals.

5. This formulation is sometimes referred to as a Generalized Poisson process. It is the simplest version of a jump process. The more general formulation assumes that λ depends on the state, so that the scalar λ is replaced by a diagonal matrix Λ with elements $\lambda_1, \lambda_2, \ldots \lambda_i, \ldots \lambda_n$. Further generalization is obtained by letting the m_{ij}'s depend on time. Introduction of state specific λ_i's would be a reasonable modification of (5) for use on mobility data. However, if differences among occupational groups in the λ_i's are caused by a variation in the desirability of occupations, the formulation of the dependency of λ on time to be introduced should remove this variation.

6. The definition of the continuous time Markov process using the quantities q_{ij} is the well-known formulation introduced into sociology by Coleman (1964).

7. The best known test of the Markov model on intragenerational mobility data is probably the one given by Blumen, McCarthy, and Kogan (1955). They tested a Markov Chain, but the failure of this model also implies that the representation (5) will provide a poor fit to the data.

8. The basic idea is to conceive the m_{ij}'s as determined by the availability of jobs, denoted c_j, in an occupational group, and the distance or affinity between occupational groups, denoted d_{ij}, so that $m_{ij} = c_j \cdot d_{ij}$. The parameters c_j and d_{ij} can be estimated using the techniques developed by Goodman (1970). See Sørensen and Rosenfeld (1974).

9. A similar formulation, using age as the time scale, is developed in an earlier paper (Sørensen, 1972).

10. A discussion of the relation between heterogeneity and nonstationarity is presented by McFarland (1970). His argument easily generalizes to the

formulation brought here.

11. This model is a special case of the linear model for change discussed by Coleman (1968). The assumptions of the model are that parameters are constant over time and identical for all individuals. These assumptions are similar to the ones made in the simple Markov model. Interestingly, they imply a nonstationary model for the mobility process, as will be shown below.

12. In status attainment research, where the level of status is the dependent variable, there is an implicit (but false) assumption that $y(e)$ is achieved for all respondents, as (16) corresponds to the linear models used in this research. Some consequence of making this assumption for causal inferences in status attainment research are discussed in Sørensen (1974a).

13. Because of the truncation, there is a necessary dependency between $\hat{w}$, and $\hat{t}$ measured at the start of a job, also after adjustment of $\hat{w}$. A simulation established that the use of $\hat{t}$ at the end of the job was the most reasonable for demonstrating independence of $\hat{t}$ and $\hat{w}$.

14. James (1974) has attempted to incorporate the model for change in $\lambda(t)$ into a gamma distribution of λ.

15. I am indebted to James Coleman for this observation.

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