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ABSTRACT

The phenomenon of economic discrimination, which is conventionally defined as different pay for workers of the same ability, is explored with the aid of a simple stochastic framework, based on the idea that employers must predict workers' abilities from imperfect information about them. The available information consists of both group information (black, white; male, female) and information about individual performance on some indicator of ability (i.e., a test). Several types of economic discrimination within the context of neoclassical, competitive market assumptions are revealed and the question of the empirical plausibility and implications of these several models of discrimination are discussed.

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I. INTRODUCTION

Economic discrimination, which is clearly an issue of enormous social importance, has proven to be difficult to explain by means of standard neoclassical economic models that assume pervasive competition. If two groups of workers have the same productivity, why should they receive different remuneration? The existence of marked differentials in wages and earnings between blacks and whites and between men and women--differentials that remain substantial despite diligent efforts to control for supply-side productivity traits--signals the phenomenon of economic discrimination, which has so far resisted our full understanding.

This paper examines this issue from a perspective suggested by Edmund Phelps (1972) in a communication that purported to introduce "The Statistical Theory of Racism and Sexism." Despite the title, the Phelps paper is not only quite limited in scope but even misleading regarding the meaning of discrimination. One objective of our paper is to clarify the meaning of "economic discrimination" and to indicate how such discrimination might arise in labor markets. Our main purpose, however, is to offer alternative "statistical" models that, compared with Phelps's models, demonstrate economic discrimination in a more theoretically satisfactory way, yield more plausible empirical implications, and are simpler.[1] The alternative models retain the central idea that employers must estimate the productivity of workers. This idea has been applied to the discrimination issue by Arrow (1972, 1973) and McCall (1972), as well as by Phelps, but we treat it differently. We do not claim, however, that our alternative models provide anything close to a complete or fully satisfactory economic theory of discrimination, and their shortcomings will be noted. Let us begin with a critique of the Phelps model.

II. THE ESSENTIALS OF THE PHELPS MODEL

The essential features of the Phelps model are as follows. In the hiring and placement of workers, employers will base their decisions on some indicator of skill, y, such as a performance test, that measures the true skill level, q. In practice, y would undoubtedly involve a number of trait measures, but we will simply assume throughout this paper that a single test score is all that is measured by y. The structural relationship may be expressed as

$$(1) \qquad y = q + u,$$

where u is an error term, independent of q. Further it is assumed that u and q are normally distributed with means equal to zero and α, respectively, and with constant variances.

Let us accept Phelps's definition of q as the employers' subjective assessment of qualifications, or skill, or productivity. We would argue that this subjective assessment of q, given y, should equal the expectation of the actual and/or realized q, conditional on y. This assumption is, after all, in keeping with profit maximizing behavior of employers, whose function is, among others, to assess (or predict) factor productivity, given the costs of available information, and to pay the factors of production accordingly. Indeed, competitive forces would tend to weed out employers who failed in this function.[2] <u>Thus, it is reasonable to assume employers will pay workers according to their expected or predicted q.</u> These points are not explicit in Phelps' paper, but we believe they are implicit or at least do not conflict with his model.

To proceed with the essentials of Phelps's model, we write equation (1) for the two groups of workers of special interest, whites (Ws) and blacks (Bs) as

(2a) $$y^W = q^W + u^W \text{ , and}$$

(2b) $$y^B = q^B + u^B \text{ .}$$

In either population, the "least squares predictor," as Phelps refers to it, is given (from normal distribution theory) by the conditional mean of q given y,

(3) $$E(q|y) = (1-\gamma)\alpha + \gamma y,$$

which is a linear function of y,[3] involving a "group" effect $[(1-\gamma)\alpha]$ and an "individual" effect $(\gamma\, y)$. In (3), $\gamma = \mathrm{Var}\,(q)/[\mathrm{Var}\,(q) + \mathrm{Var}\,(u)]$, the population regression coefficient. (Note also that in this model $\gamma = r^2$, the squared correlation coefficient between q and y.)

At this point, Phelps makes three critical assumptions. First, (for the bulk of his paper) he assumes that u^W and u^B have the same (zero) means and (constant) variances. Second, an employer's knowledge that a worker is black is assumed to convey the additional information that the variance in abilities is greater for blacks than whites, i.e., $\mathrm{Var}\,(q^B) > \mathrm{Var}\,(q^W)$. Finally, he assumes that the black worker's ability, q, is on average lower than the white worker's; that is, $E(q^B) < E(q^W)$ or $\alpha^B < \alpha^W$.[4]

III. CRITERIA FOR DEFINING ECONOMIC DISCRIMINATION

It is a bit disconcerting that Phelps assumed a difference in average abilities at the outset, because discrimination is so often defined as differences in pay for workers of the _same_ ability. Economic discrimination can be analyzed more clearly when "equal abilities" for the two groups is assumed. Nevertheless, a special kind of discrimination is revealed in the case of unequal average abilities, and this case will be discussed later in this paper.

One type of labor market discrimination is suggested by focusing on discriminatory _practices_ by employers: Discrimination exists when employers do not pay (or hire) workers in accordance with their expected contributions to output, i.e., their expected marginal productivity.[5] Thus, if we have no reason to believe that different wages are being paid to black and white workers with the same expected productivity, $\hat{q}[=E(q|y)]$, then there is no discrimination by this criterion.

A second type of discrimination, one that concentrates more on labor market _outcomes_, is revealed when the total compensation (wage bill) for each of the two groups is not proportional to their respective total contributions to output. In the simple case of equal-sized groups with equal abilities, for example, total compensation (and average wages) should be the same for the two groups if no discrimination is present.

One model we present below exhibits both of these kinds of discrimination. We contend, incidentally, that the cases presented by Phelps fail to show discrimination by either criterion. A second model, in which we relax an assumption typically made for competitive labor markets, reveals discrimination by the second criterion concerning _outcomes_, but not by the first criterion concerning _employer practices_.

Finally, we refer to another definition of economic discrimination involving outcomes, wherein workers of the same true ability receive different pay on an individual basis. Symbolically, the test is whether $E(\hat{q}^B|q) = E(\hat{q}^W|q)$. The relevant expressions for these expectations are given by equation (3) with $\hat{q}$ and q substituted for q and y, respectively. In general, this equality will not hold in the models we discuss. Thus, there will generally be within-group "discrimination" (or individual "discrimination"). However, there need not be any overall difference in compensation between groups, as the individual inequalities of the above expectation over the range of q may be offsetting as between whites and blacks.

All of the points just raised will become clearer as we proceed to analyze particular models.

IV. ECONOMIC DISCRIMINATION WHEN THE AVERAGE ABILITIES OF BLACKS AND WHITES ARE EQUAL: $\alpha^B = \alpha^W = \alpha$.

A. A Phelps Model

We first illustrate the analysis of the model outlined in section II under the assumption that the variance of black abilities exceeds that for white abilities (Phelps's "Case 2").

In conjunction with the assumption of equal variances in u, Phelps correctly deduces that the slope, γ, of the q on y regression is steeper for Bs than Ws, which means that the test score, y, is a more "reliable" predictor of the all-important q-value for blacks than whites.[6] Accepting this unusual result for the time being, let us examine its implications.

As Phelps remarks, "at some high test score and higher ones the black applicant is predicted by the employer to excel over any white applicant with the same or lower test scores" (p. 661). This is evident in Figure 1, as is the corollary proposition that at low test scores the white worker is predicted to excel over a black worker with the same test score. But in what sense does this picture depict economic discrimination? Each worker is paid in accordance with his expected productivity, based on an unbiased predictor. Moreover, the two groups, which have (by assumption) the same average ability, receive the same mean (and total) wages.[7]

It is true that the wage rates ($\hat{q}$'s) of whites are distributed toward the mean, α, relative to blacks. But what blacks "win" at the highest q-values (relative to whites) they "lose" at the lowest q-values.[8]

The apparent definition of economic discrimination revealed by Figure 1, and which we must ascribe to Phelps, is "different pay for different y-scores." But since y-scores are intended only to indicate expected productivity, it is discrimination with respect to $\hat{q}$ and not y that is economically relevant. Even a legal requirement that payments be equal for equal y-scores would contribute nothing to the overall improvement of the status of blacks.

Actually, the assumption that $\gamma^B > \gamma^W$, i.e., that the y-score is a more reliable indicator of q for blacks than whites, is intuitively unappealing. Indicators such as SAT and GRE scores or years-of-schooling-completed are usually viewed as less reliable indicators of "real ability" for blacks than for whites. The empirical evidence that we are aware of supports this impression.[9] At the same time, we see no reason to assume the variance in real ability differs for the two races, although arguments can be made for a difference in either direction.[10] Moreover, the implication of the

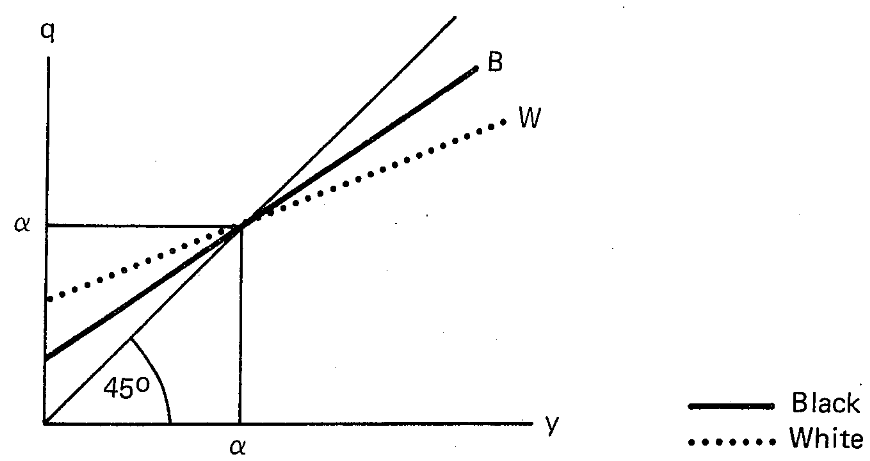

Figure 1. Predictions of Productivity (q) by Race and Test Score (y), Assuming a Steeper Slope for Blacks

hypothesis that $\gamma^B > \gamma^W$ is that we should observe the white-black differential in pay--reflecting a differential in expected q--to narrow (and eventually become negative) as the y-indicator increases. The bulk of the empirical evidence points to the opposite result. If y were measured by years of school completed or by years of experience--two of the most important and commonly used indicators of productivity--the empirical relation between y and earnings (or wages) would show blacks faring worse relative to whites as y increases.[11]

We examine in the next section a model that reflects this evidence and assumes that the testing process is less reliable for blacks. Thus, we assume $\mathrm{Var}(u^B) > \mathrm{Var}(u^W)$ in conjunction with the assumption that $\mathrm{Var}(q^B) = \mathrm{Var}(q^W)$. It follows that $\mathrm{Var}(y^B) > \mathrm{Var}(y^W)$ and $\gamma^B < \gamma^W$.

B. An Alternative Model

Up to this point we have assumed explicitly that the employer knows $E(q|y)$, and we have assumed implicitly that the dispersion of $q|y$ is costless. This is equivalent to assuming that q enters the profit function linearly, or that the employer is risk-neutral with respect to q. Clearly it is more realistic to permit q to enter the profit function (or the "utility of profit" function) nonlinearly. The correct decision rule for hiring labor may then involve higher moments of q. In the simple model adopted below, only the second moment (variance) of q is required to reflect risk aversion and to yield a theoretical explanation for economic discrimination.

To simplify the problem, assume that labor is the only factor of production, that output is fixed, and that output-price and wage rates are exogenously determined. Thus, profits, Π, are solely a function of labor services. To maximize the utility-of-profits function, $U(\Pi)$, the employer need only choose the type of labor, here B or W, to maximize $U(q)$.

There exist in the literature several well-known utility functions that will result in a decision rule that depends on the variance of the argument. One is the quadratic function used by Tobin (1958). Another, which we adopt, is the function used by Parkin (1970) and suggested by McCall (1971), which for our purposes may be written

(4) $$U(q|y) = a - be^{-cq} \qquad b,c > 0,$$

whence[12]

(5) $$E[U(q|y)] = a - be^{-c\, E(q|y) + \frac{c^2}{2}\ \mathrm{Var}\ (q|y)},$$

where

(6) $$\mathrm{Var}\ (q|y) = \mathrm{Var}\ (q)(1 - \gamma).$$[13]

It is easily seen that

(7) $$\max E[U(q|y)] \equiv \max\ [E(q|y) - k\ \mathrm{Var}\ (q|y)],$$

where $k = c/2$.

It follows that an employer with this utility function will attempt to choose labor services from the group of workers that maximizes expected productivity-ability, q, discounted for "risk." This risk can arise from differing variances in the distribution of q, of the indicator, y, or both. Substantively, the risk-costs of variance in worker abilities may stem from variance in output within homogeneous jobs or, perhaps, from the costs of mistakes in assigning workers to particular job slots within a job pool. Note that the conditional variance, (6), does not depend on the level of y, so that the risk factor is constant over the range of indicator scores.[14]

The empirical question of whether the conditional variance in q, given y, is larger or smaller for black or white workers is, therefore, crucial in determining the direction of discrimination. Given racial equality in

Var (q), this question hinges on the reliability of y as a predictor of q, namely γ. From (6) and (7) we see that the group of workers with a lower γ will be discriminated against. If we accept as fact the existence of discrimination against blacks, then assuming that black y-scores are less reliable is clearly the more plausible. (Recall that $\gamma^B < \gamma^W$ will follow from the assumption Var $(u^B) >$ Var (u^W) and Var $(q^B) =$ Var (q^W).[15])

Figure 2 shows the new y, q relationships incorporating these assumptions plus $\alpha^B = \alpha^W = \alpha$. We define q - R as the "risk-discounted q," where $R^W = k \text{ Var } (q^W)(1 - \gamma^W)$, and correspondingly for R^B. The lines W and B are from equation (4) in conjunction with the assumption that $\alpha^W = \alpha^B$ and $\gamma^B < \gamma^W$, whereas their risk-discounted counterparts are based on $E(q|y) - k \text{ Var } (q|y)$. To see clearly that the figure reveals economic discrimination against blacks in the conventional sense--that they receive lower pay on average for the same expected ability--look at the lower value of (q-R) given $y=\alpha$ for blacks. The source of the discrimination can be shown in Figure 3 where we graph the conditional distribution of q for precisely this convenient point, E(y), E(q)--both of which equal α for both races. (If three dimensional diagrams were available, the information in Figure 3 could be shown in Figure 2.) The observed smaller conditional variance of q^W in Figure 3 is, of course, precisely the source of the large magnitude of γ^W when Var $(q^B) =$ Var (q^W).

Note that the risk-discount borne by black workers in the form of a lower relative wage could be defined in terms of the extra search costs employers would have to bear to reduce the conditional variance of q^B to equal Var $(q^W|y)$. However, the model does not require any ad hoc assumptions about the direct hiring costs being larger for blacks compared to whites.

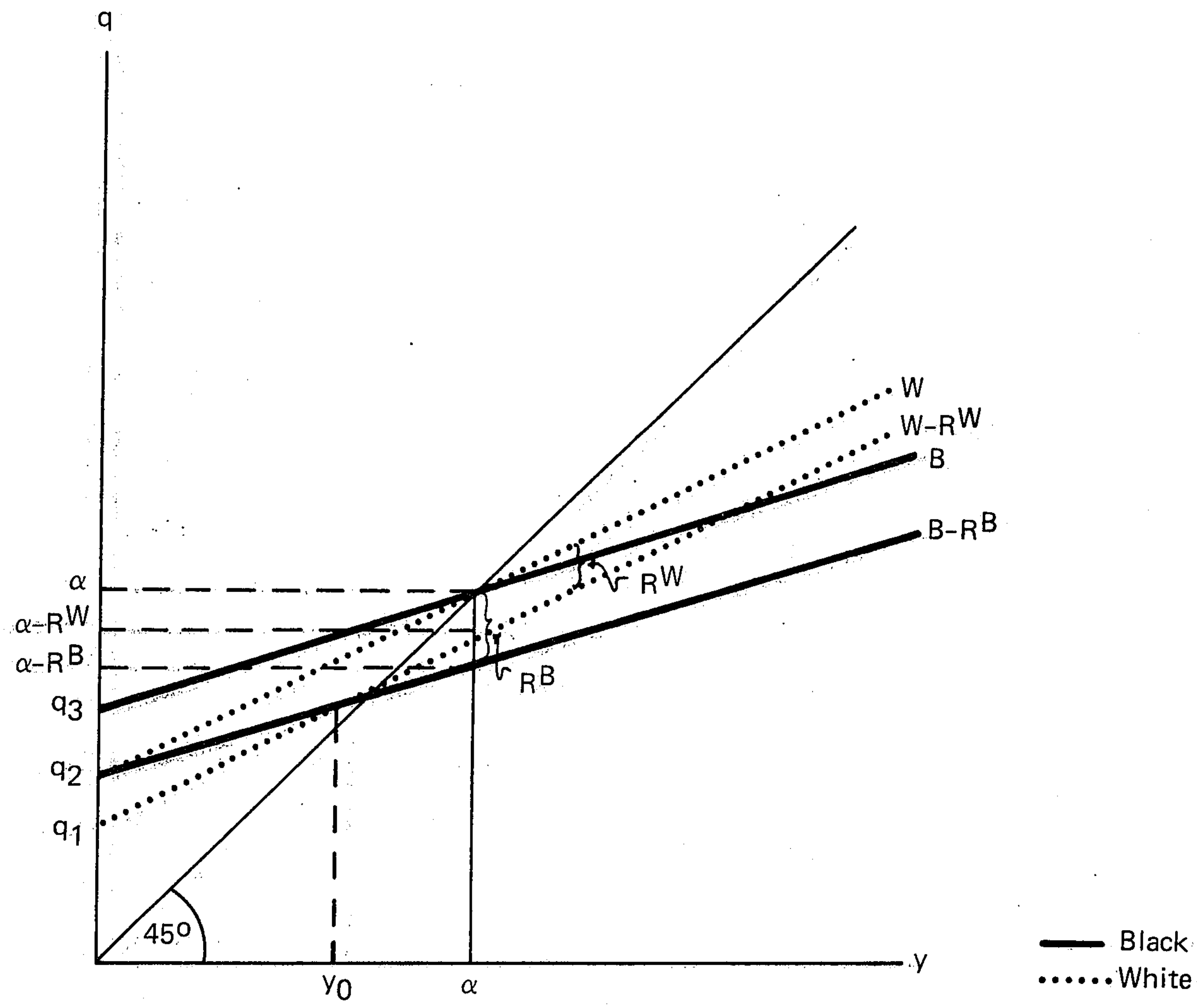

Figure 2. Predictions of Productivity (q) by Race and Test Score (y), with "Risk-Discounts" and Flatter Slope for Blacks

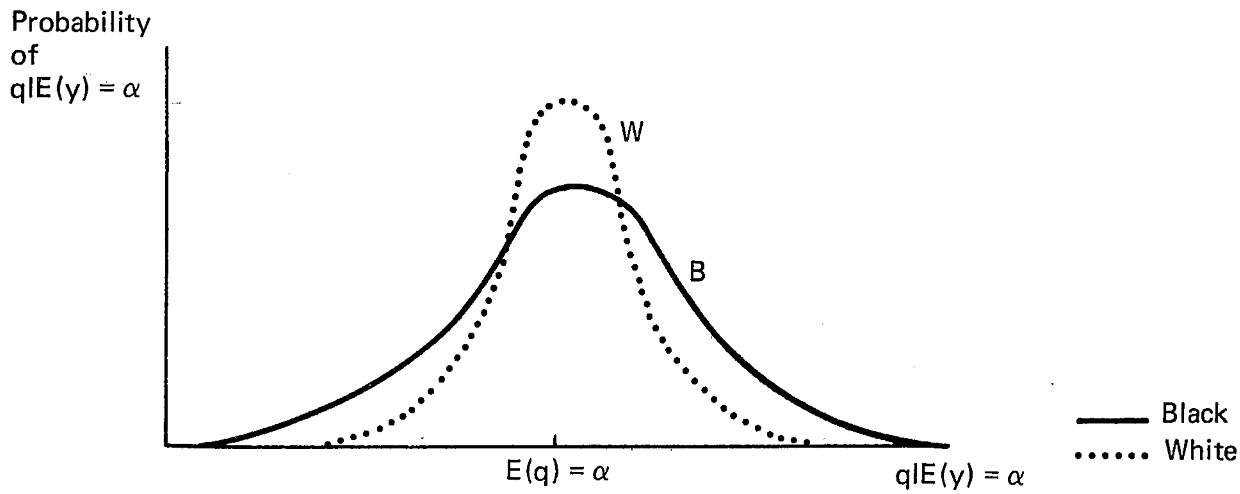

Figure 3. The Distribution of Productivity (q), Given the Test Score $y = E(y)$, by Race

Note: $E\,(q|E(y)) = E(q) = E\,(q^B) = E(q^W) = E(y^B) = E(y^W)$.

This is not to deny that these costs, or cost differentials, exist. For example, the geographic segregation of black workers away from white employers (firms) may well impose extra search and transportation costs upon black workers.[16]

Figure 2 represents, of course, a hypothetical model, but it is consistent with our view of reality in two important respects. Economic discrimination against blacks, women, and other groups exists, and the differential wage or income advantage of white male workers increases as the indicator variable increases. However, only if the uncertainty penalty (as drawn) were as large as $q_3 - q_1$ would the graph show all W workers earning higher wages (on average) than B workers over the whole range of y. As the graph is now drawn, the wages of B workers exceed those of W workers with the same y-scores for $y < y_0$.

We are not aware of data revealing a smaller wage for Ws compared to Bs for low scores of productivity indicators. Furthermore, although we have not expressed dollar equivalencies to the q scale, the empirical magnitudes of the negative differential borne by black and women workers--perhaps 10 to 30 percent for workers with the same number of years of schooling completed--seems too large to be rationalized by risk aversion. Indeed, large firms have some capacity to self-insure against risks of output variability or mistaken job assignments. In perfect capital markets, even small firms could "purchase" such insurance through various pooling devices.

C. A Second Model Depicting Economic Discrimination

The model incorporating a lower reliability of an indicator score for blacks may illuminate another case of economic discrimination, this time without recourse to risk aversion. We adopt a definition of discrimination whereby the black workers have equal average ability but do not receive equal total compensation. The application is suggested by the work of the psychologist, R. L. Thorndike, which was reported in Linn (1973).

When the hiring or selection decision is <u>confined</u> to the upper end of the y distribution, whites will be preferred on the basis of expected values of q, given y, even when the q distribution is identical for blacks and whites. If the hirings involve only "high" y-values, then the preference for blacks at the lower end of the y distribution would be inoperative. Consider a rule that did not permit workers with low y-scores to be hired at all. Clearly, a higher average value of q (or wage rate) for whites would emerge--evidence of economic discrimination in total compensation--despite the fact that employers are not race-biased in their hiring: that is, they hire workers solely on the basis of $E(q|y)$.

Figure 4 shows this result in an extreme form. We assume perfect reliability for Ws and zero reliability for Bs. The distribution of q is identical for Ws and Bs (as indicated on the vertical axis). Only values of $q > \alpha$ are eligible for hire. (Assume that α now represents a comprehensive, legal minium wage, here unrealistically set equal to the wage corresponding to the overall average value of productivity.) Given the costs of hiring and associated costs of making a mistake, all blacks, but only half the whites, would be unemployed or not in the labor force. The model has, of course, greater relevance to, say, college admissions than to the labor market, but it may be at least suggestive of some economic situations.

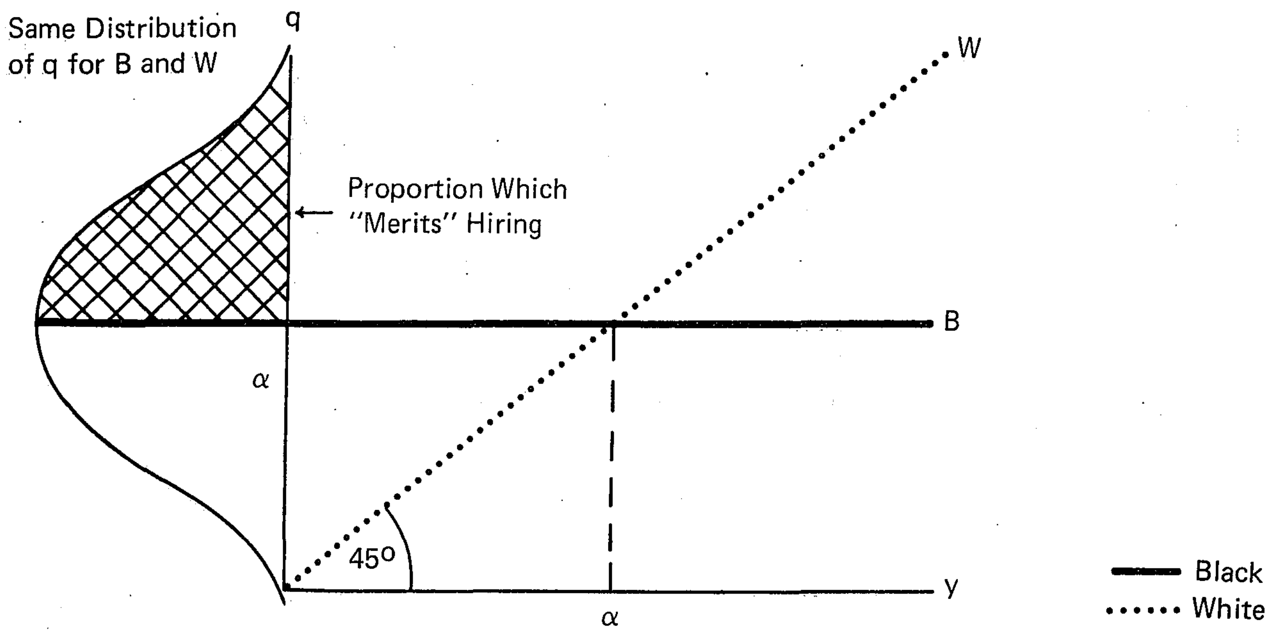

Figure 4. Predictions of Productivity (q) by Race and Test Score (y), Assuming Perfect and Zero Predictive Relations for Whites and Blacks, Respectively.

V. ECONOMIC DISCRIMINATION WHEN THE AVERAGE ABILITY OF BLACKS IS LESS THAN WHITES

We return now to the initial example in Phelps's paper (his "Case 1"), in which the variances of q and u are equal for the two groups, but it is assumed that the mean ability of blacks is less than the mean ability of whites. Then the systematic effect of blackness, $\alpha^W - \alpha^B$, would by itself lead to a lower predicted value of q for blacks than whites, given equal y scores. Phelps remarks that the $\alpha^W - \alpha^B$ effect might reflect "disadvantageous social factors."[17]

The graph relating the y, q relation for Ws and Bs is shown in Figure 5 (and in Phelps's paper).[18] The B line is below and parallel to the W line; the same slope is a consequence of the assumption (in this case) of equal variances of q, u, and, therefore, y. Clearly, our natural assumption that employers are interested in q and are willing to pay workers accordingly, compels us to conclude that white workers will be preferred to (and get higher wages than) black workers with the same y-score. As before, we dispute that different pay for the same y-score demonstrates economic discrimination. Indeed, were Bs to get paid the same as Ws when both had the same y-scores, there would manifestly be discrimination against Ws, since the latter are more productive (i.e., higher average q).

One could argue, of course, that the very existence of different average ability, $\alpha^W - \alpha^B$, demonstrates a type of discrimination, and we would agree. Within the confines of the foregoing model we would generally refer to it as premarket discrimination--discrimination in the acquisition of various forms and amounts of human capital that workers possess when they enter the labor market. But, given these handicaps, the differential pay (or

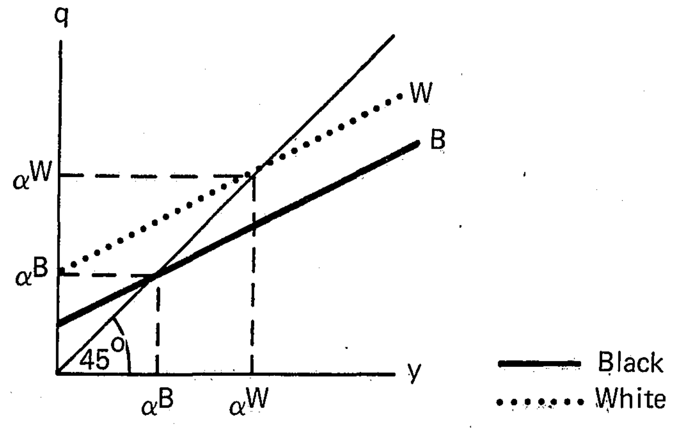

Figure 5. Prediction of Productivity (q), by Race and Test Score (y), Assuming the Slopes Are Equal

differences in employer demand) appears to be no more discriminatory--in the pejorative sense of the word--than are the lower wages that would be paid to workers with less experience, other factors (like the y-score) equal.

It is interesting to recognize that the assumed linear relation between q and y (hence $\hat{q}$ and y) demonstrates nondiscrimination by the "outcome" criterion of a proportional relation between total compensation for the groups and their respective productivities. Despite this, it is the case that for every ability level, a black with that ability will always get paid less than his white counterpart.[19] This apparent paradox is resolved in one sense by recalling that we are assuming here less ability on average for blacks <u>and</u> imperfect information. Thus, blackness is assumed to provide information that $E(\hat{q}^B|y)$, and therefore $E(\hat{q}^B|q)$, is lower than $E(\hat{q}^W|y)$, and therefore $E(\hat{q}^W|q)$, for all y(or q). The smaller wage bill for blacks is the result of every black getting paid less than a comparable white. In the presence of perfect information (γ=1), the systematic difference in $E(\hat{q}|y)$ or $E(\hat{q}|q)$ disappears (and the lines for both color groups coincide with the 45° line).

In light of known premarket discrimination against blacks and women, the assumption by employers of unequal average abilities is not at all unrealistic, nor is the assumption of imperfect information. The systematic inequality in $E(\hat{q}|q)$ that results is, therefore, profoundly disturbing. One consolation is that this inequality should decline as employers assimilate more knowledge over time, thereby reducing Var(u) and raising γ.

We end this discussion with the cautionary remark that the combination of the assumptions $\alpha^B < \alpha^W$ with $\gamma^B = \gamma^W$ may not be very realistic. We hold that the "raw-labor" abilities of blacks and whites are equal. Different average abilities in the labor market would, therefore, reflect variations in human capital acquisitions. Under a condition in which some whites and blacks have identical "raw-labor" values of q, but more whites have higher capital-augmented abilities, then whites may well have a larger variance in q upon entering the labor market. Depending on concomitant assumptions made about Var(u), the regression slopes for the two groups would not generally be the same.

VI. CONCLUSIONS

Although the Phelps model of statistical discrimination does not, in our opinion, explain or describe racial or sex discrimination, it provides a useful point of departure for several models that do. On empirical grounds we have argued that one feature of his model--a race differential in reliability of test scores (representing productivity indicators)--is more plausibly introduced when blacks (or women) are assumed to have less reliable scores. When we combine this reliability differential with risk aversion by employers, we produce a model showing economic discrimination that is broadly consistent with empirical evidence. Finally, the combination of lesser reliability for blacks (or women) on tests with truncation of lower scoring applicants also reveals a kind of economic discrimination, and reinforces the potential inequities that may stem from lower test reliabilities for minority groups.

We are reluctant, however, to claim too much for these models. In the model that uses risk aversion, there are questions of the size of the risk premium and/or the doubtful empirical support for a "crossover" point at the lowest end of the indicator scale, where blacks earn more than whites for comparable indicator scores. Obviously, we have made no thorough attempt to test the model, or even to give more satisfactory empirical definitions of the y-variable. The $\hat{q}$-variable itself has been assumed to represent a wage rate throughout, implicitly relying on the proposition that wages measure productivity and that competition will, on average, equate equal productive abilities with equal wages. Thus, one may argue that there is no discrimination when "productivity" is defined in terms of contributions to a utility function that allows for risk aversion.

Many real world influences that affect economic discrimination have been ignored. One is the presence of monopoly and/or monopsony power.[20] Two other influences are tastes for discrimination by employers and/or their systematic subjective underevaluations of the abilities (q-values) of the discriminated groups. While neither of these latter two hypotheses, by itself, is consistent with long-run economic discrimination in a competitive model, introducing additional factors may do so. It would take a more extensive discussion to deal with, say, Arrow's list of additional considerations (1973, pp. 26-32). They include capital market imperfections, wage rate rigidities, discontinuities in hiring decisions, and self-fulfilling prophecies (or self-perpetuating syndromes). But it is fair to say that they were offered very tentatively and leave a number of unanswered questions.

The models we have presented in this paper offer an explanation for economic discrimination pertaining to a plausible but limited aspect of labor market behavior, but the problem is far more extensive and more complicated.

FOOTNOTES

[1]Throughout, many details of our comments on Phelps's paper will appear in footnotes so as not to obscure the main theme.

[2]A contrary assumption--that the employers' subjective assessments of q differ from the expected value of actual q--could lead to undervaluing, and therefore underpaying, an identifiable groups of workers who are thereby discriminated against. However, without some special ad hoc assumptions--not introduced by Phelps, but mentioned by Arrow (1972)--this behavior by employers would not be viable unless all current and potential employers made the same error. Otherwise, the forces of competition would lead to an expansion of output by employers who erred the least (or not all) at the expense of those who erred the most. Thus, imposing a wedge between the subjective expectation of q and the actual expected value of q is analytically equivalent to imposing employers' "tastes for discrimination" as a wedge between the employers' subjective evaluation of the worth (or productivity) of a worker and his actual worth. As both Becker (1971) and Arrow (1972, 1973) have made clear, variance in tastes for discrimination among employers will tend to drive out of business those employers with such tastes in the long-run competitive equilibrium. We return to several of these points below.

[3]Alternatively, (3) may be written as the population regression

$$q = \phi + \gamma y + u',$$

where u' is the usual well-behaved error term and the inequality $0 < \gamma < 1$ reflects the error in y as a measure of true ability, q. Phelps writes this as his equation (2) with slightly different notation ($\gamma = a_1$) and with the variables expressed as deviations from means.

[4]The difference in expected values of q is represented in Phelps's paper by a dummy variable for race (1 if black) with a negative coefficient, but the presentation is not entirely clear. His equation (5') would appear to represent a single regression model for workers of both races, in which z is an additive term that is equivalent to a dummy variable (0 if white). However, the additivity of z gives the wrong impression, because the all-worker regression requires a zy interaction to capture Phelps's assumption that the slope of y on q is different for the two races. (We might also register at this point our confession that the reference in footnote 4, which Phelps suggests is an aid in understanding his equation (5'), was a complete mystery to us.)

[5]We adopt the prevailing convention in analyses of discrimination of defining "equal productivity" in terms of physical output or actual work performance. As others have pointed out, discrimination can always be explained away by attaching a cost to some characteristic of blacks or women that is not directly related to their work abilities. This is not to deny the real or semantic complexities that are involved--what defines "actual work performance"; what is meant by "directly related" and "work abilities"? However, it is expedient to suppress these questions temporarily.

[6]Reliability is customarily defined in terms of the squared correlation coefficent (r^2) between the true score and the predictor variable. The r^2 equals γ in the model discussed in the text--equation (3).

[7]We note that the definition of economic discrimination as wage differences among workers with the same productivity implies pervasive within group discrimination, given the conditional variance in q. Thus, some Ws with a given y score, y_0, who will be hired for a wage commensurate with $E(q|y_0)$, will have an actual q that is greater than $E(q|y_0)$; others will turn out to have an actual q that is below the expected value. We could fairly say that the former (positive residuals) receive preferential treatment. Presumably, there is more of this sort of discrimination at the time of initial hirings than after the elapse of time, when the experience of workers and employers will narrow the conditional variance of q given what would now be an augmented y. But this mere presence of conditional variance within a group does not imply the socially pathological case of discrimination between groups.

[8]If the slope of the regression is flatter for Bs than for Ws, the consequences for black-white wage differentials over the range of q-values are opposite to those described. Phelps mentions this ["A Further Case" (1972, p. 661)], but it conveys no more information about discrimination than the model we show in Figure 1.

[9]Some 22 studies that examine the ability of the Scholastic Aptitude Test (SAT) to predict college grades are reviewed in Linn (1973). A flatter slope of the regression line for black students relative to whites is strongly supported. (On the other hand, the slope is flatter for men than for women, although, as discussed below, standard wage-productivity indicators for women are probably less reliable than for men.)

[10]Thus, blacks confront environmental restrictions on fulfilling their capacities, and this may lead to a lower variance of q. On the other hand, maybe whites face a more homogeneous set of environmental determinants of q. Any number of possibilities suggest themselves.

[11]For example, see Welch (1973), who remarks: "It is well known that, on balance, the ability of schooling to boost Negro earnings has been less than for whites, at least for males" (p. 43). Weiss (1970) supports this finding and also finds that "scholastic achievment" (as measured by test scores) was a better predictor of earnings for white males compared with black males. Finally, a flatter age (experience)/wage profile is shown for black males and women generally relative to white males in Hall (1970, p. 394).

[12]Since q is normally distributed, e^{-cq} is lognormal, and its expected value is $e^{-cE(q) + (c^2/2)\mathrm{Var}\,(q)}$.

[13]This relation is derived from normal distribution theory. It is an analogue of the expression for the residual variance in a simple linear regression as: $(1-r^2)$ Var (dependent variable). Thus, in the population regression equation associated with equation (3), specified in footnote 3, we have

$$q = \phi + \gamma y + u'$$

where Var $(u'|y)$ is a constant and:

$$\mathrm{Var}\,(q|y) = \mathrm{Var}\,(u') = \mathrm{Var}\,(q) - \gamma^2\,\mathrm{Var}\,(y)$$

$$= \mathrm{Var}\,(q) - \left[\frac{\mathrm{Var}\,(q)}{\mathrm{Var}\,(y)}\right]^2 \mathrm{Var}\,(y)$$

$$= \mathrm{Var}\,(q)\,(1 - \gamma).$$

[14]Using the Tobin quadratic utility function in q, one finds a different result, where the risk term does vary over y. Another possible risk-oriented framework that yields a constant risk factor is the "safety-first" model [see Day et al. (1971)].

[15]A lower slope for blacks would also result from the assumption that Var (u^B) = Var (u^W) and Var (q^B) < Var (q^W). Indeed, the risk-discount, Var $(q|y)$, is symmetric with respect to Var (u) and Var (q) since, by manipulations of relations in footnote 13, we see

$$\mathrm{Var}\,(q|y) = \frac{\mathrm{Var}\,(q)\,\mathrm{Var}\,(u)}{\mathrm{Var}\,(q) + \mathrm{Var}\,(u)}$$

In our comparisons between blacks and whites we cannot, however, interchange the terms "reliability" (= $\gamma = r^2$) with "risk-discount" [= Var $(q|y)$] unless we hold equal either Var (q) or Var (u) for the two groups.

[16]See Kain (1974) and the cited references and McCall (1972) for attention to this disadvantage to blacks.

[17]The three words in quotes are used by Phelps but are not written in a single phrase, although the expression fairly conveys his meaning. Without more information, however, the interpretation of this expression--and of $\alpha^W - \alpha^B$--could be ambiguous. Does $\alpha^W - \alpha^B$ reflect a real deficiency in skills, as would be the case if the social factors referred to factors such as less schooling, less training, and poorer health? Or does $\alpha^W - \alpha^B$ reflect merely a misconception or a false sterotype held by employers? In accordance with our earlier expressed preference for believing that employers can accurately assess q on average, we assume the first interpretation. (See footnote 2 above.)

[18]This corresponds to Phelps's Case 1: "black if Var $\varepsilon_i = 0$" in his Figure 1. Note that his curve is not drawn correctly; its height should not be $\alpha - \beta$ at $y = \alpha$. His $\alpha - \beta$ corresponds to our α^B and at $y = \alpha$ (our α^W), $E(q^B|y) = \alpha^B + \gamma(\alpha^W - \alpha^B)$. In his terms, at $y = \alpha$, $E(q^B|y) = \alpha - \beta + a_1\beta$.

[19]Recall, $E(\hat{q}|q) = (1 - \gamma)\alpha + \gamma q$. For the same ability (q-value) and regression slope (γ) but different mean ability, we have $(1 - \gamma)\alpha^B < (1 - \gamma)\alpha^W$.

[20]As the reader may know, however, the evidence for these anti-competitive sources for sustained economic discrimination is meager. Ashenfelter (1972) produces evidence against a net antiblack discrimination effect of unions. Becker (1971, pp. 7-8) and others have disputed Thurow's claim (1969) that monopsony power by employers is an important explanation. And Alchian and Kessel (1972) have argued that monopoly power in the product market is consistent with long-run economic discrimination only (or mainly) when there are constraints on the employers' ability to maximize money profits, as in regulated monopoly industries.

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