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ABSTRACT

This paper describes a dynamic mathematical model of the labor market which can simulate its equilibrium under a variety of circumstances. The model is neoclassical in origin, but frictions have been built into it in a variety of ways in an attempt to replicate the effect of information costs, uncertainty, and capital market imperfections. The study is designed to explore the general behavior of a non-homogeneous labor market, but the simulations can shed light on many specific questions in that context.

One thousand laborers are specified to differ from each other by a normally distributed characteristic--called talent--which affects their productivity. The market is divided into ten skill groupings, which differ from each other in their talent requirements, and laborers attempt to get into the highest skill class in which they can find work. Ten firms offer employment in each of these skill classes to those workers whose talent is sufficient to make them productive in that class.

With hiring, firing, quit, and production decisions being made endogenously, the model determines a level of frictional unemployment that depends on the various frictional parameters and the nature of the shocks to which the model is exposed.

This paper describes the model itself. Subsequent papers will describe the simulation experiments which are run with the model.

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Solow says "The art of successful theorising is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive." Unfortunately, for a variety of labor market problems, the results seem to be exceptionally sensitive to the kinds of assumptions economists usually make. Solow defines a crucial assumption as "one on which the conclusions do depend sensitively," and he notes that crucial assumptions should be realistic. Several of the assumptions most frequently made by economists in other contexts--presumably where they are not crucial--seem to be crucial in labor market analysis. Thus, at present, labor market economists are finding great difficulty in choosing a set of assumptions which will reduce their problems to a mathematically tractable level without affecting significantly the conclusions that can be drawn about several important phenomena. Specifications that are simple enough to yield results do not seem to be able to lend insight into many questions of great importance.

Part of this is due to the historical development of economics as we know it. Our greatest accomplishments have been our concise characterizations of competitive markets along with our imaginative manipulations of those characterizations. With monopoly, however, we have done much worse. While it is true that we have a precise theory of how a single monopolist should behave when he finds himself in a world otherwise characterized by perfect competition, or how a single laborer should carry out a policy of optimal search for employment in a variety of given environments, we have no good way of aggregating those monopolies or workers into an economy which simultaneously determines the environments within which each of those agents behaves. This is true both of the most abstract, mathematical models of general equilibrium and of practical, applied models of the labor market. Our theory is most powerful when it can be applied to problems that can be conveniently represented by competitive markets for homogeneous goods and factors.

For many problems, this shortcoming of conventional theory is no more than an annoyance; it comprises but one of the many awkward but realistic factors that a streamlined theory does well to ignore. But in the labor market, many of the things we wish most to explain exist precisely because of the behavior of these awkward factors. In these cases, conventional models of general equilibrium, or even of supply and demand, lend little insight into the behavior in question.

Part of the awkwardness may be circumvented in the future by theoretical breakthroughs--by what Solow calls the art of successful theorizing. It is idle to speculate here about the nature of these breakthroughs, but it should be clear that a great contribution will have been made by the person who finds a way to represent with great economy the many complex search, informational, and subjective factors that exist in markets for heterogeneous commodities like land, labor, structures, or used equipment.

Another part of the awkwardness is simply computational. Even an economical theory of how a heterogeneous market functions may require a great amount of computational work if it is ever to be applied. In this vein, it is interesting to ask how much of our present inability to grapple with some important labor market problems is a result of the weakness of our theory, and how much is simply due to the computational problems that arise when our theory is applied to markets of unique commodities.

The simulation model I report on in this paper is intended to shed light on this question. This model uses a very simple specification of hiring, firing, quit, and wage decisions that are applied to 1,000 workers of differing abilities. These decisions constitute a labor market that has certain realistic characteristics not seen in previous models. The computational complexity that necessitates the use of simulation as opposed to analytical methods results simply from the fact that the productive power of each worker is different. While this makes the numerical detail of the problem enormous, the economic behavior can be kept to a minimum so that the effect on the equilibrium of a change in the level of one parameter or another can be easily determined.

The following list of questions can be answered by a simulation of this kind, but are intractable using conventional analytical methods. The list includes a great many of the issues that are most troublesome to labor economists today.

(1) How do aggregate demand policies affect the distribution among unemployed workers of such characteristics as work experience or ability?

(2) Is there a nonzero unemployment rate which maximizes steady state GNP, and how does this rate vary with certain behavioral or technological labor market parameters?

(3) What are the effects of a wage subsidy or a minimum wage rate on aggregate output and on unemployment among the low-income labor force?

(4) How does the equilibrium rate of unemployment depend on the characteristics of the labor market and on aggregate demand?

It is true, of course, that the answers to these questions that are generated by any simulation model will be of direct policy applicability only if the model is a careful representation of the economy in question. The model I will describe is a pilot, or laboratory, model whose purpose is to explore in the abstract the relations between certain economic concepts which can be related only with great difficulty using analytic techniques. Therefore, the quantitative results I derive are of little interest; it is the qualitative dependencies that I wish to isolate.

The questions, it should be noted, cover rather broad areas of labor economics and macroeconomics, some of which are not usually thought of in conjunction with each other. Thus the antecedents of this model can be found in a diverse literature that I can only briefly mention here.

The macroeconomic issues were described in the volume by Phelps et al. (1970) which explored the link between inflation and unemployment. Various factors must be considered when describing that link, and a large literature now exists on each. Holt and David (1966), in a seminal paper, had first described the links between turnover and unemployment, thereby giving an empirical foundation to the concept of frictional unemployment. Recently, estimates of many aspects of these complex hypotheses have appeared in several issues of The Brookings Papers, most notably the work of Hall (1970, 1972), Gordon (1971, 1973), and Perry (1970, 1972). The emphasis in this literature has been macroeconomic in the sense that its purpose has been to improve our understanding of the effect of macroeconomic policy on inflation and unemployment.

Various institutional theories have also grown up to explain other labor market phenomena not easily described by neoclassical theory. These include the work of Thurow and Lucas (1972), Piore and Doeringer (1971), and the more radical market segmentation theories of Reich, Gordon and Edwards (1973). Feldstein (1973) also examined the effect of a group of institutional forces on the equilibrium unemployment rate.

The reader of this literature cannot fail to be impressed with the difficulty of the problems being considered, and the inadequacy of existing theory to give concise, satisfactory answers to the important questions being asked.

II. An Overview of the Model

In section III below, I describe the basic functional forms and parameter values which are to be used in the subsequent simulations. Here I describe broadly the model's structure.

There are 1,000 workers in the simulation who differ from each other by a single normally distributed characteristic called talent. There are ten firms, each of which produces output according to a production function that uses ten different labor skills as inputs. The amount of a particular skill that an individual worker can contribute depends on his talent. The functional dependency is nonlinear so that high talent individuals have a comparative advantage at high skill jobs.

Thus there are 100 different jobs (firm-skill combinations) that a worker might acquire, each with its own wage rate. Workers attempt to get the best jobs they can--those with the highest wage rates--while firms attempt to hire the best workers they can--those with the most talent. The heart of the simulation is the set of rules which govern the search processes that are carried out in each time period in order to match workers and jobs. An outline of that process follows.

Taken as given is some allocation of workers to jobs (or to unemployment), and a wage rate for each job. For the first time period, the wage rates are determined exogenously, while the allocation of workers to jobs is done randomly. For subsequent periods, those data are carried over from the preceding period.

Each worker, if employed, decides whether or not to quit. He makes this decision after considering the unemployment rate, the wages available on his present job and elsewhere, and his own talent relative to that of his co-workers. If he quits, he determines an asking wage and becomes unemployed. If unemployed at the outset of the period, he lowers his asking wage by five percent, and decides whether or not to lower his skill classification and search for less desirable jobs.

Firms examine their employees and fire those workers whose production is less than eighty percent of the wage being paid at that level. I will use the term marginal product to denote that production, though the heterogeneity of the labor force implies that each worker will have a different marginal product at each job. It is marginal in the sense that the production is calculated taking as given the allocation of the other workers to their jobs within that firm.

Firms then search the unemployed for workers who have a level of talent that is high enough to make them productive at the job in question. The search is carried out for each job in order according to the wage rate being offered. The unemployed are classified by skill, and only those classed one above, at, or one below the skill listing of the jobs are searched. An offer is made to any unemployed worker discovered through this process whose talent is sufficiently large that his marginal product will exceed the wage at the job in question. A worker accepts the first job offer which has a wage in excess of his asking wage. These hire, fire, and quit decisions determine a new allocation of workers to jobs which is maintained until the next period. The only behavior of importance that is not contained in this sequence is that which determines wage rates. Firms determine wage offers in a rather complex manner that is described more fully below. Here, we need only note that wages are increased when they are less than the marginal product of the worst worker on the job, and decreased when they are greater.

Aggregate demand can be simulated by changing the demand for the output of all firms. This demand is an important element of the demand for labor. These output demand functions can also be subjected to random shocks in order to create the need for labor turnover. At present, this is the only stochastic force which I intend to use in this model.

III. Specific Functions and Parameter Values

The 1,000 workers in the model are numbered consecutively from one and are indexed by the letter K. The single characteristic, TALENT, which distinguishes workers from each other is normally distributed with mean of 1.0 and standard deviation of .15. Specifically, each worker is assigned a level of talent according to the following implicit function.

$$\text{(1)} \qquad \frac{K-1/2}{1000} = \int_{-\infty}^{\text{TALENT (K)}} \frac{1}{.15\sqrt{2\Pi}}\, e^{-1/2\left(\frac{x-1}{.15}\right)^2} dx$$

Thus worker #100 has that talent level which is greater than 9.95 percent of all talent levels while worker #500 has the level which is greater than 49.95 percent of all other levels. For these workers, talent levels of .803 and 1.000 respectively are assigned.

TALENT is transformed into the various labor SKILLS by a set of nonlinear functions. SKILLS are indexed by the letter I. The quantity of the I-th SKILL input that the K-th worker can produce is determined by 2).

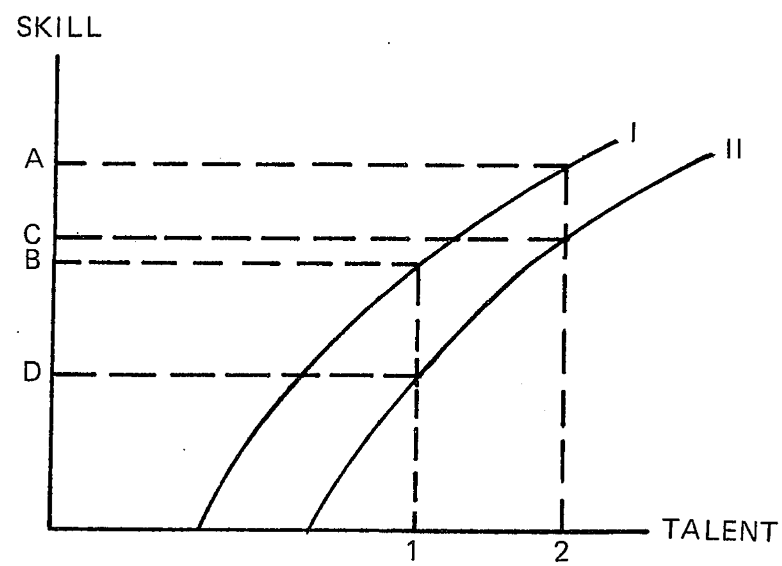
$$\text{(2)} \qquad \text{SKILL(I,K)} = \text{LOG(TALENT(K)} + .68 - .08{*}\text{I)}$$

Thus in the most demanding skill class, (I = 10), it takes a level of talent greater than 1.12 for a worker to be productive while in the least demanding skill class, a level of talent greater than .4 is required. Even worker #1 has a talent level of .5, however, so the possibility for productive employment of that worker exists. The numbers .68 and .08 are arbitrary, of course, and are chosen relative to the distribution of talent so as to exert strong pressure for certain men to gravitate to certain jobs without completely dominating that allocation. The sensitivity of the results to this arbitrary choice will be examined.

Equation (2) guarantees diminishing returns to talent in any skill classification, and it guarantees high talent individuals a comparative advantage at high talent jobs. Since the functions for each skill classification differ from each other by a constant, it will be true that the ratio of the outputs of a specific skill of a high talent individual to a low talent individual will be higher the higher the skill classification. In the diagram, this means that A/B will always be less than C/D. Since C and D represent the levels of skill II of individuals with talents 1 and 2, while A and B represent their skill levels at less difficult job I, it is easily seen why the logarithmic form of these equations guarantees that high talent individuals will have a comparative advantage at high skill jobs. This should guarantee the existence of a unique optimal allocation of men to jobs in the absence of stochastic disturbances and market frictions.

Each of the ten different firms in the economy faces a separate demand curve for its output and must produce that output using a Cobb-Douglas production function defined over the ten labor skill classes. Firms are indexed by the letter J. The skill of the I-th class that is

FIGURE 1



used as an input by the J-th firm is simply the sum of the effective levels of skill of all workers employed by that firm at that skill level.

(3) $$\text{SKILLS}(I,J) = \sum_{K} \text{SKILL}(I,J,K)$$

These skill aggregates are used to produce the firm's output.

(4) $$\text{OUTPUT}(J) = \prod_{I=1}^{10} \text{SKILLS}(I,J)^{.1}$$

Thus there are 100 different jobs that a worker might acquire. He can also be unemployed and seek work in any of the ten different skill classifications. Because the production functions are Cobb-Douglas, each firm must hire some labor at each skill classification.

Demand curves are assumed to be rectangular hyperbolae.

(5) $$\text{OUTPUT}(J) = \text{B}(J)/\text{PRICE}(J)$$

When random shocks are used, they enter in the form of changes in the constants B(J). Changes in aggregate demand are simulated by increasing all the B(J) simultaneously. For the experiments reported below, B(J) = 10.0 unless otherwise noted.

These five equations complete the environment within which decisions are to be made. The environment is technically very simple, yet it leads to difficult decisions because of the problems introduced by heterogeneous labor. Next, we examine the decisions that must be made in order to allocate the workers to the correct jobs.

While the number of decisions to be made in this model are small, the environment within which these decisions are to be made is complex. Since each worker is different and since each firm has at any point in time a work force of differing composition, the marginal product of a particular worker may vary dramatically from firm to firm even at the same skill classification. A very lengthy search procedure for both workers and firms would be necessary if a state of perfect knowledge were to be characterized and which guaranteed each worker that job at which his productivity was highest. The procedure followed here does not replicate a state of perfect knowledge. Instead, a few simple rules of behavior are followed which it is felt are generally consistent with profit and utility maximization in the long run.

Each of the 100 firm-skill job classifications has an individually determined wage rate. Workers examine these rates and determine whether they feel they can improve their income by quitting their present job and looking for a different one. This calculation is made by comparing two numbers, one to represent the costs of search and possible unemployment, the other to represent the expected income gain to be attained once the job switch has been completed. Since all hiring is done from the pool of unemployed workers, it is necessary for a worker first to quit before he can attain a better job. However, it is possible for a worker to accept employment in the same time period in which he had quit. Thus he need not be unemployed for any finite time since all production takes place at the end of the period. There are no internal promotions in the model.

The costs of unemployment are assumed by the worker to be his present wage rate, WAGE(I,J), multiplied by the present unemployment rate for workers in the skill class in question with a constant added to the unemployment rate and another constant multiplying the whole expression. These constants are to be varied to determine their effect on the labor market's adjustment to equilibrium. The expression denotes the cost of being unemployed for one time period, (the present wage rate) multiplied by terms which represent the probability of being unemployed and the expected duration of that unemployment. The numbers exhibited in equation (6) imply that when unemployment rates are ten percent, the worker estimates the cost of quitting as exactly equal to his present wage. This is the same cost as would result from knowing with certainty that he would be unemployed one time period.

The benefits to be gained from switching jobs are estimated to be equal to the difference in wage rates between the present job and that paid on average at the next higher classification multiplied times the difference in talent between the worker in question and the average of his co-workers multiplied by a constant. The worker is assumed to feel underpaid only if he feels he's better than his co-workers. The constant in this expression serves two purposes. It converts talent into man-time periods, and it multiplies the resulting expression by the number of time periods the new job is expected to be held. The effect of the constant is to determine the talent differential necessary to make quitting profitable.

(6) WORKER(K) QUITS IF

60.0*[TALENT(K) - AVERAGE TALENT(I,J)]*[AVERAGE WAGE(I+1)
- WAGE(I,J)] ≥ WAGE(I,J)*[.1
+ UNEMP(I)/LABOR FORCE(I)]*5.0.

Once the worker quits, he seeks work in the job classification immediately above the one he just left. His asking wage is set equal to a weighted average of his old wage and the average wage paid at the new classification.

(7) ASK = .8*WAGE(I,J) + .2*WAGE(I+1)

Fired workers must also determine an asking wage. It is a fixed percentage of the wage on the job they just left. Fired workers seek work in the job classification below the one they just left.

(8) ASK = .9*WAGE(I,J)

Each period, all those unemployed who do not find work lower their asking wages by five percent. When the asking wage falls to be equal to the average wage paid in the next lower classification, the worker drops to that classification. There is no other worker behavior.

The behavior of firms is a bit less simple. The firms must determine employment and the wage rates at each skill classification. Each worker at a given firm in a given skill class earns the same wage. Thus the firm must determine how much to pay a diverse group of employees, and it must take account of several factors when making this decision.

- The firm realizes that workers' talents differ and that it can generally hire better workers by paying higher wage rates.

- The firm realizes that its best present workers will quit if wages are too low.

- The firm knows that it is easier to hire and retain workers when unemployment rates are high.

- For a given labor force, the firm obviously makes higher profits the lower are the wages it pays.

- The firm wishes to hire anyone whose marginal product exceeds the real wage.

These factors make the problem sufficiently difficult that I confess to have little prior idea of how wages must be set if profits are to be maximized in the long run. I do constrain the problem somewhat by requiring that the firm behave competitively; that is, it moves in the general direction of having real wages equal to marginal products. I attempt to have the firm act as a price taker, but there are no natural functions to use to generate marginal revenues or prices for workers in particular classes. Each worker has his own level of talent and his own wage demand. I can arbitrarily align these workers in order according to talent and/or wage demand, but the resulting alignment is not a labor supply curve to the firm; it is still an ordered shopping list of individual workers. Economists have not yet, to my knowledge, derived any general results concerning the optimal behavior of firms in such an environment, whether or not they assume the firms to be price takers.

At present, I determine wages in the following fashion. The worst worker employed in a particular skill class is the marginal worker. His marginal product, MPL(I,J), is attributed to the skill class.

(9) MPL(I,J) = .1*PRICE(J)*OUTPUT(J)*SKILL(I,J,K)/SKILLS(I,J)

Note that this marginal product is defined for the worker while output is a function of skill units. Thus an extra term appears in the formula for the marginal product in order to convert skill units into workers. This term is the number of units of skill possessed by the worst worker in the relevant skill class currently employed by the firm. Note also that the use of the Cobb-Douglas production function and the unitary elastic output demand curve simplify this formula a great deal. If the marginal product exceeds the wage being paid, the firm attempts to expand employment in that skill class while it attempts to contract in the opposite case. When the firm is expanding, wages are determined according to (10a).

(10a) WAGE(I,J) = (.2 + .6*U(I,J))*WAGE(I,J)
+ (.8 - .6*U(I,J))*MPL(I,J)

where U(I,J) is a measure of unemployment or the availability of labor relative to the size of the firm.

$$U(I,J) = UNEMP(I)/(UNEMP(I) + LABOR(I,J))$$

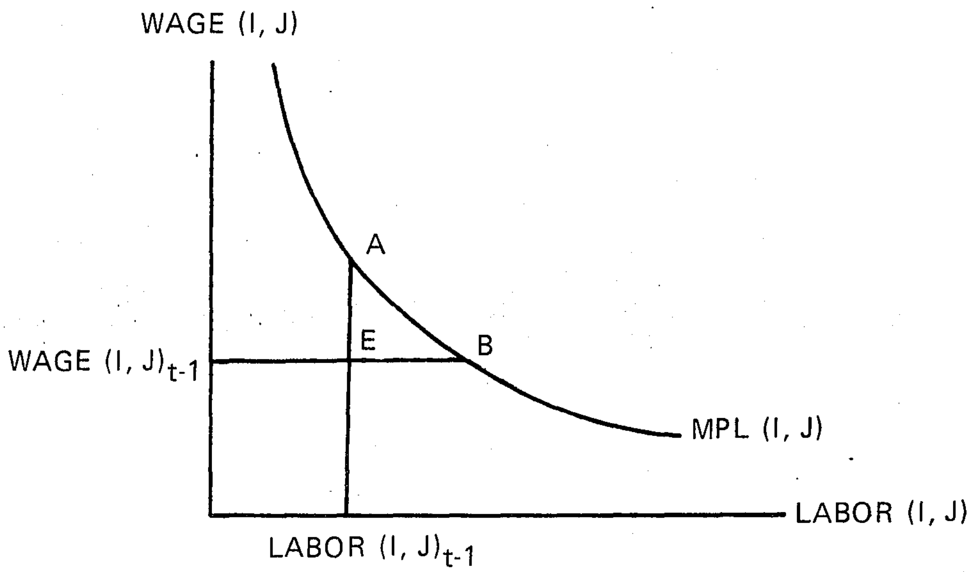
There are some further constraints on the rate at which wages can go up which are merely designed to prevent awkward results during unusual periods of turmoil (such as the period of adjustment to the initial random allocation of workers). These constraints prevent real wage rates from going up more than 25 percent per period unless the firm's wage would still be below the average asking wage of the unemployed in that class.

For contraction, the wage equation is (10b) which merely reverses the weights used in (10a).

$$\text{(10b)} \qquad WAGE(I,J) = (.2 + .6*U(I,J))*MPL(I,J) + (.8 - .6*U(I,J))*WAGE(I,J)$$

The sense of these functions can be understood with reference to the accompanying figures. In Figure 2, the downward sloping marginal product function acts as a demand curve for labor of this class. If E is taken to be a point of historical, long-run, stable equilibrium, then the demand curve drawn indicates an expansion in demand has occurred. The firm must select some point between A and B as its target for the coming time period. If it raises wages to be exactly equal to the present marginal product, the solution will be at point A and the firm need do no hiring or firing in the coming time period. If it keeps wages fixed, the solution will be at point B and the firm must hire EB laborers in the coming time period. Its choice depends on the unemployment rate of the relevant workers. If unemployment is high, it will choose a point near B, while if unemployment is low, it will choose a point near A.

FIGURE 2



I have described why there is no supply curve to the firm. However, it is possible to trace out loci of the points just described by varying demand with unemployment held constant. Figure 3 shows these equilibria for high rates of unemployment while Figure 4 shows them for low unemployment. Note that both functions are discontinuous at the prevailing equilibrium.

Once the firm has chosen a set of wage rates, the rest of its behavior is simple. Firms hire those unemployed workers whose marginal products exceed the real wage of the relevant labor classification, who are looking for work at that classification, and whose asking wage is less than the firm's offer. They search for these workers in the pool of unemployed, and, generally, offer work to the most talented workers first. As each worker is hired, he reduces the marginal product of a unit of labor at that classification.

A worker is fired if his marginal product is less than 80 percent of the real wage he is to be paid. This requirement is checked immediately before and after the firm searches for new workers in the given skill class.

Behavior is simulated in the following manner. At the beginning, workers are assigned job classifications according to a pseudo-random process.* The initial wage offers are supplied exogenously and various behavioral parameters are assigned.

The program then enters the basic loop which determines a complete time period of behavior. First, some basic housekeeping calculations are performed to generate various aggregates which are needed as inputs to the

*Pseudo-random numbers are numbers that appear to be random for statistical purposes, but are in fact generated by a deterministic process. Since the process can be replicated, it is possible to use the same set of random numbers for successive experiments.

FIGURE 3

LOCUS OF EQUILIBRIA WHEN
UNEMPLOYMENT IS HIGH

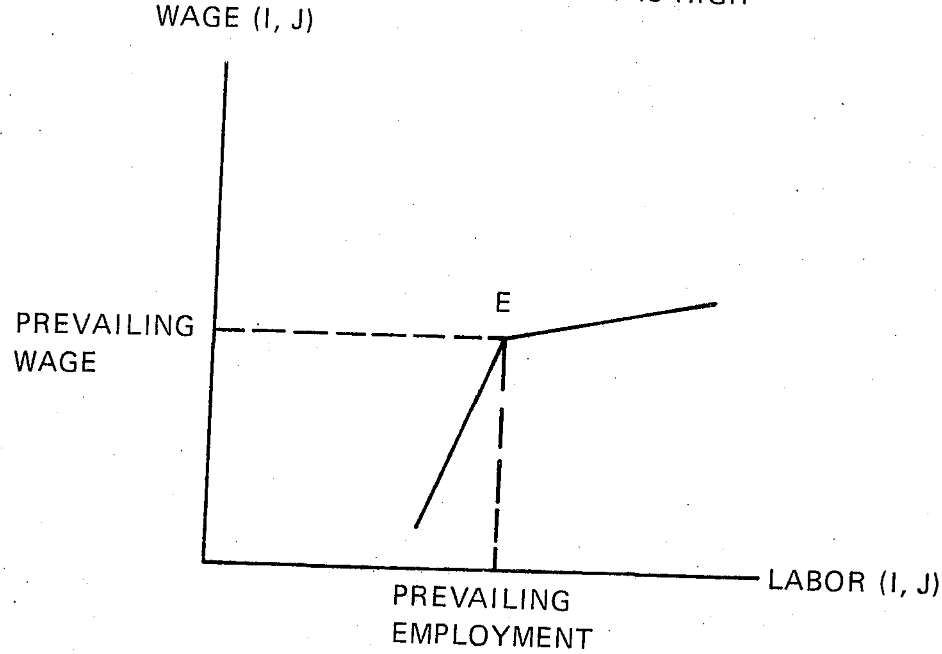
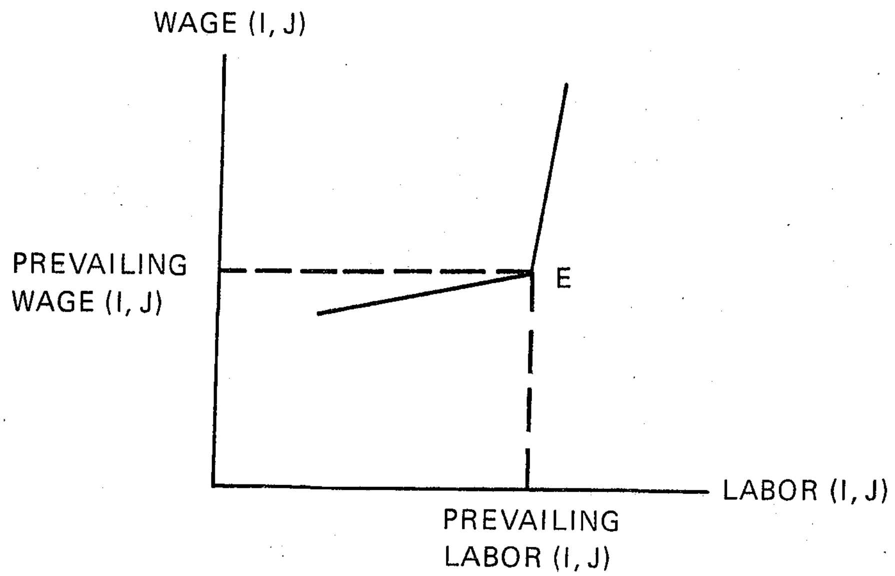


FIGURE 4

LOCUS OF EQUILIBRIA WHEN
UNEMPLOYMENT IS LOW



behavioral decisions. These include the calculations of average skill levels by job and wage rates by skill class. Various rankings are performed which affect the order in which certain behavior occurs later.

Then come the behavioral decisions which form the heart of the model. First, workers decide whether or not to quit, and once they become unemployed, they determine an asking wage. In subsequent time periods, those already unemployed determine whether or not to drop to a lower skill class. There now exists a pool of unemployed with fixed wage demands in each skill class, and a set of firms with a stock of employees and fixed wage offers. At this point, market clearing behavior occurs.

The 100 jobs, each denoting a firm and skill classification, are considered in order according to wage offers, highest first. The firm fires workers whose marginal products are less than 80 percent of the wage rate. These workers immediately join the unemployed of the next lowest skill class. The unemployed in the relevant skill classes are then searched to see if job offers should be made. These classes include the ones immediately above and below that of the job in question, as well as its own class. When an unemployed worker is found to have a marginal product in excess of the real wage, he is offered a job. He accepts if the wage offer equals or exceeds his asking wage. If any hiring is done, it reduces the marginal product of labor in the class in question, and it is again necessary to determine that the workers' marginal products are at least 80 percent of the real wage.

After the market clearing behavior is completed, the program proceeds to calculate output and prices for each firm. Real GNP is calculated, aggregate unemployment rates, wage rates and prices are determined and recorded. In the final time period, the program exits at this point.

For all other time periods, the housekeeping calculations are performed again, beginning this time with a calculation of new wage offers for each job. This done, the program repeats the calculations within the main loop until the required number of time periods has been reached.

IV. Some Conceptual Considerations

The weaknesses of simulation are well known. Arbitrary choices concerning specific functional forms or parameter values can have important effects on the results. While one hopes to derive results that can be generalized to many different environments, one can never be sure that this desire is realized. As a particular problem is modeled, the larger the model becomes, the more one is tempted to simulate because of the increasing difficulty of deriving analytic results. Yet the increased size of the model makes it more difficult to isolate with confidence the role of any single parameter in the simulation model. Thus as the expected benefits from simulation increase, so do its weaknesses.

If the pitfalls of simulation are to be avoided, it is important that a careful account be taken of the usual hazards to minimize the likelihood of their appearance in the present case. The discussion of these hazards and the steps taken to avoid them can also serve as a useful way to describe the model being simulated.

It was noted above that the very size of the model that leads one to simulate in the first place often obscures the nature of the results that are attained. There are several ways in which this can happen. First, there may be very many structural equations and parameters. The complex interaction of these factors may obscure the role of any single parameter, or may yield results that are specific to the assumed values for the other parameters.

The model described here does not suffer from this problem. There are only a few equations and parameters in this model. The computational difficulties arise not because of complexity in the structure of the model, but because the agents are not identical. With talent distributed unequally, different men make different decisions when faced with an identical environment. The description of these many decisions is tedious, and while the summary of it by a few statistics may be useful for descriptive purposes, a model built around those statistics would lose many of the characteristics felt to be most important in this study. In this model, then, a few simple behavioral rules are used by a heterogeneous group of laborers. It is the aggregation of their behavior which must be performed numerically, not the determination of that behavior for any single person. Thus, there are only a small number of parameters of interest, and it should be easy to determine the effect of any one of them on the important summary statistics.

The complexity of a simulation model may also obscure the manner in which irregularities in the equilibrium can occur. These irregularities can take the form of non-existence, non-uniqueness or instability in the equilibrium of the model, and these characteristics are thought to be sufficiently interesting properties of an equilibrium that theoretical economists spend a good deal of their time deriving conditions under which one or another of these phenomena will or will not occur. It would be nice to know if those irregularities occur in new ways in the heterogeneous labor context, and under which conditions. Unfortunately, with a large model, it is often difficult to determine the source of peculiar behavior of this kind, and it is possible that its appearance denotes nothing more than the hidden existence of one of the well-known causes for such behavior. To prevent this from happening, care should be taken when specifying the structure of the model to guarantee that the usual requirements for existence, uniqueness, and stability of the equilibrium are satisfied.

Unfortunately, I know of no way to prove rigorously that the necessary conditions are satisfied by this model. While there are a small number of basic equations, they apply to a thousand unique individuals, each of whose behavior affects the environment within which the others must decide. Thus it remains possible that the interactions between the distributional, structural, and dynamic equations of the model can lead to peculiar solutions for some parameter values and not for others thereby giving misleading impressions that the peculiarities are due to parameter values rather than faulty model structure.

It is difficult to prevent this from happening. However, several issues are known to affect these phenomena in all economic models, and they should be considered in any attempt to minimize the probability of this happening.

Convexity

In the absence of convex preferences and technologies, demand curves need not slope downward or supply curves upward, equilibria need not be unique and marginal changes in controlled parameters can lead to large discontinuous changes in solution values. I have accordingly attempted to build the essential kinds of convexity into the model, though I confess to no knowledge of how to prove that I have accomplished my intentions.

First, demand curves are assumed to be rectangular hyperbolae while production functions are Cobb-Douglas. Both of these assumptions are much stricter in terms of the regularity imposed on the equilibrium than they need be for that equilibrium to be unique and stable.

Second, and probably most important for the problem being modeled here, is the fact that the skill functions [equation (2) above] are non-linear and guarantee a unique optimal allocation of men to jobs. Only stochastic shocks and market frictions will prevent that unique allocation from being obtained.

Money Illusion

If any conclusions about long-run adjustments to inflation are to be drawn, it is important to be aware of any sources of money illusion built into the model. I have attempted to purge them completely from the long-run equilibrium relationships, though they have not been purged from the short-run adjustment equations. Equations such as the quit decisions equation, which consider wage rates and other variables, are all specified so that the wage rate (or some variable generally proportional to it) is multiplied by all other elements in the equation rather than added to them. Thus all terms in these equations have the same dimension and their relative size is thought to be independent of the absolute level of the wage rate.

While there is a total absence of money illusion in the long run, this is not true of all short-run relationships. Money wage demands by the unemployed are lowered by 5 percent per time period. This may put a floor on the rate at which wages can fall. Firms may lower wage offers at any rate without having present employees quit, however, so this rigidity on the part of the unemployed does not preclude wage reductions at a rate greater than 5 percent per time period.

Lags

The time paths of the dynamic adjustments of variables to long-run equilibria depend importantly on the lag structure of the model. The lag structure has been purposely kept simple here in order to observe the effect of other forces on this behavior.

The system can best be described as first order recursive. No simultaneous equations are solved. The program is a large loop or sequence of operations in which any new value that is generated for any variable is immediately used for the generation of the next variable. Thus causation is of the form illustrated in Figure 5.1 and not as in 5.2.

5.1

$$\begin{tikzcd} X_1 \arrow[r] \arrow[rr, bend left] & Y_1 \arrow[r] \arrow[rr, bend right] & X_2 \arrow[r] \arrow[rr, bend left] & Y_2 \arrow[r] & X_3 \end{tikzcd}$$

5.2

$$\begin{tikzcd} X_1 \arrow[r, shift left] \arrow[rr, bend left] & Y_1 \arrow[l, shift left] & X_2 \arrow[r, shift left] & Y_2 \arrow[l, shift left] \end{tikzcd}$$

Stochastic Disturbances

The only way in which disturbances enter this model is through the demand functions for output. The constant term in the demand function is multiplied by a term with a random component, and this generates all the labor market action once the initial random allocation of workers has been adjusted to.

Market Frictions

To simulate behavior under uncertainty explicitly or to calculate optimal decisions where search costs exist would be tedious. Instead, certain functions and various parameter values have been specified so as to exhibit behavior of the kind that could be expected in a world of uncertainty or where decision and search costs exist. These have the effect of slowing down any adjustment to long-run equilibrium and to make the equilibrium that is finally attained be just an approximation to the one that would occur in the absence of the friction imposed by these parameters.

Workers compare the expected costs of being employed to the expected benefits of attaining a higher wage when they decide whether or not to quit. Expected unemployment costs are estimated as 5*(.1 + UNEMPLOYMENT RATE)*PRESENT WAGE, while the gains at the new job are estimated as the wage differential times the amount by which the worker's skill exceeds that of his co-workers (a general magnitude of .1) times 60. These numbers may be a bit conservative and cause the worker to retain his job in some cases where a quit would be profitable.

Firms hire new workers only if their productivity is substantially above that of the worst workers already on the job. Workers in nonshrinking firms are fired only if their productivity is 80 percent of that of the best available unemployed. This differential represents the difficulty of ascertaining the true abilities of workers as well as the costs of hiring and firing.

These parameters plus the structure of the search procedure (wherein workers are considered only for skill classifications adjacent to the one for which they have applied) exert forces of inertia which tend to keep workers on the same jobs longer than it is profitable to do so. It will be interesting to see how variations in these parameter values affect the equilibrium.

V. Summary

In summary, the empirical inputs to the simulation are very "regular," and are not expected to be the source of any peculiar output. The model is best suited to answering questions about how labor market structure affects the distribution of work experience over the workers, rather than to answering questions about macroeconomic equilibria. The macro questions are interesting, however, and I intend to explore them first while gaining an understanding of the sensitivity of the results to various parameter changes. Only then will I add minimum wage rates, unions, wage subsidies, or perhaps a second discrete dimension of differences among workers to represent skills or race-sex differences.

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