

**On-the-Job Search, Minimum Wages, and Labor Market  
Outcomes in an Equilibrium Bargaining Framework**

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## **Abstract**

We look at the impact of a binding minimum wage on labor market outcomes and welfare distributions in a partial equilibrium model of matching and bargaining in the presence of on-the-job search. We use two different specifications of the Nash bargaining problem. In one, firms engage in a Bertrand competition for the services of an individual, as in Postel-Vinay and Robin (2002). In the other, firms do not engage in such competitions, and the outside option used in bargaining is always the value of unemployed search. We estimate both bargaining specifications using a Method of Simulated Moments estimator applied to data from a recent wave of the Survey of Income and Program Participation. Even though individuals will be paid the minimum wage for a small proportion of their labor market careers, we find significant effects of the minimum wage on the ex ante value of labor market careers, particularly in the case of Bertrand competition between firms. An important future goal of this research agenda is to develop tests capable of determining which bargaining framework is more consistent with observed patterns of turnover and wage change at the individual level.

# 1 Introduction

The impact of minimum wages on the welfare of agents on the supply and demand sides of the labor market has been at the center of an age-old policy debate on the proper role of the government in the economy. The standard elementary treatment of minimum wage policy views its impact as unambiguously negative. In a competitive market in which unrestricted supply and demand forces combine to determine a unique equilibrium employment and wage level, the imposition of a minimum wage greater than the market clearing wage creates true unemployment, defined as the situation in which individuals who are willing to supply labor at the going wage rate are unable to find jobs. It creates *ex post* inequality as well, in that those individuals fortunate enough to find jobs have a higher welfare level than they would have had in the competitive equilibrium, while those who do not will have lower welfare levels. If individuals are risk-averse, this increased “uncertainty” may be welfare decreasing in an *ex ante* sense, as well.

It has long been appreciated that, for minimum wages to have beneficial effects (at least for the supply side of the market), there must exist labor market frictions and/or multiple equilibria.<sup>1</sup> The multiple equilibria case (see van den Berg, 2003) is perhaps the strongest one for the beneficial effects of government-imposed wage policies. There are a large number of labor market models that can produce multiple equilibria, which occurs when the “primitives” of a labor market environment can produce a number of different labor market equilibria.<sup>2</sup> If, for example, two equilibrium outcomes are possible, with the supports of the wage distributions associated with the two equilibria non-overlapping, then a minimum wage placed below the lower bound of the support of the higher wage distribution and above the upper bound of the support of the lower wage distribution can serve as an equilibrium selection device that ensures selection of the preferred equilibrium.<sup>3</sup>

The equilibrium search models of Albrecht and Axell (1984) and Burdett and Mortensen (1998)

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<sup>1</sup>There are a number of papers in the literature that consider the possibility of welfare-enhancing minimum wage rates (e.g., Drazen (1986), Lang (1987), Rebitzer and Taylor (1996), Swinnerton (1996)), though the frameworks in which the models are set tend to be relatively abstract and the models themselves are typically not estimable. These are important contributions, but in this paper we focus more on models that have been or are capable of being taken to data.

<sup>2</sup>Some examples of labor market models that can easily produce multiple equilibria involve statistical discrimination (see, e.g., Moro (2003)) and the general equilibrium search model with contact rates determined through a non-constant returns to scale matching function (see, e.g., Diamond (1982)).

<sup>3</sup>This presumes that the higher wage distribution is associated with the Pareto optimal equilibrium. If this is not the case, implementing a maximum wage policy would select the preferred equilibrium.

offer another venue for positive minimum wage effects on welfare, once again, at least for the supply side of the market. In the Albrecht and Axell model, potential firm entrants into the market are heterogeneous in terms of quality. A high minimum wage prevents low-quality firms from making nonnegative profits, and hence improves the labor market through a selection effect on the demand side. Having higher quality firms competing for their services improves the wage distributions individuals face while searching. As the model is written, there is no adverse impact on employment rates.

The Burdett and Mortensen (1998) equilibrium framework allows for on-the-job (OTJ) search in addition to unemployed search. As do Albrecht and Axell (1984), they assume a wage-posting equilibrium in which firms offer a fixed wage to all potential and actual employees. The Burdett and Mortensen framework does not require heterogeneity in the populations of (potential) firms and workers, in fact it assumes no heterogeneity. They prove the existence of an equilibrium in which the probability that any two firms offer the same wage is zero. Adding a minimum wage to their model simply shifts the equilibrium wage offer distribution to the right and has no adverse employment effects. As a result, binding minimum wages are beneficial to the supply side of the market.

The framework we use posits search frictions, as do the two we have just discussed. Differently from those two models, an important component of our model is heterogeneous productivity. In particular, when a potential employment opportunity is found after some period of search, the productivity of the match,  $\theta$ , is determined by taking a draw from some fixed distribution  $G(\theta)$ . It is typically assumed that the value of the match for a given possible worker-firm pairing is observed immediately by both sides, an admittedly strong assumption.<sup>4</sup> Once the draw is made, the pair can determine if there exists positive surplus to the potential match. “Positive surplus” is said to exist if there exists any wage rate at which both sides would prefer creating the match to their next best options of continued search. If there exists positive surplus to the match, the worker and firm bargain over its division. In general, both sides have some degree of bargaining power, because the other side cannot find a perfect replacement for them without spending additional effort and/or resources. In this case, the ultimate source of bargaining power to both is search frictions.

As is common in this literature, we use axiomatic Nash bargaining to determine the division

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<sup>4</sup>Jovanovic (1979) emphasizes the role of learning about match quality in explaining separation decisions and wage progression at a firm. He expands his framework to include unemployment in Jovanovic (1984). The bargaining process is not emphasized in his approach.

of the surplus between the worker and firm. The minimum wage acts as a side constraint on the Nash bargaining problem. Depending on the form of the equilibrium wage function, the minimum wage may act to preclude the formation of otherwise acceptable matches. This corresponds to the standard negative employment effect associated with a competitive markets framework. Secondly, for all acceptable matches after the imposition of the minimum wage, the minimum wage affects the bargained wage both directly and indirectly. The direct effect is obvious: for bargained wages less than  $m$  without the constraint, if the match is viable, the wage offer must be increased to  $m$  to comply with the law. The indirect effect is associated with the change in the outside option (in the Nash bargaining problem) associated with the minimum wage change, even when the constraint itself is not directly binding. In the minimum wage empirical literature, this is often referred to as “spillover.” Our framework gives a behavioral motivation for the existence of such an effect.

This research extends the model of Flinn (2002, 2006) to the case of on-the-job search. Using Current Population Survey (CPS) data, he estimated a continuous time model of matching, search, and bargaining (Flinn, 2006) in both partial and general equilibrium settings. In the partial equilibrium case, it was assumed that the rate of contact between unemployed searchers (there were no employed searchers) and firms was fixed; in particular, it was invariant with respect to changes in the minimum wage. In the general equilibrium case, the contact rate was modeled using the matching function setup (see, e.g., Pissarides, 2000), in which the contact rate is a constant returns to scale (CRS) function, with its arguments being the measure of unemployed searchers and the measure of posted vacancies. Given a social welfare function that weighted the average welfare of individuals and firms in the population equally, Flinn used estimates of primitive parameters to determine optimal minimum wage values under the partial and general equilibrium assumptions. He found that the partial equilibrium model implied optimal minimum wages of approximately \$8.50 an hour (when the mandated federal minimum wage was \$4.25), while in the general equilibrium framework the optimal minimum wage was only \$3.35. The results of testing exercises led him to conclude that the empirical evidence supported the partial equilibrium specification of the model.

The addition of on-the-job search to the bargaining model is a critical extension. Most obviously, from descriptive evidence we know that in the U.S. labor market there are a large number of job-to-job transitions that don’t involve an intervening spell of unemployment. By ignoring this fact, there exists the potential for a significant degree of model misspecification, leading to inconsistent estimates of model parameters and misleading policy implications drawn from those estimates.

The addition of on-the-job search is likely to be particularly relevant for purposes of investigating minimum wage effects on labor market outcomes. The work of Leighton and Mincer (1981), and, more recently, by Acemoglu and Pischke (2002), investigates the potential impacts of minimum wage laws on life-cycle wage profiles through reductions in general human capital investment by recent labor market entrants. Under either of the two bargaining specifications investigated here, minimum wages, by truncating the lower tail of the accepted wage distribution, tend to produce less wage growth over employment spells and the life cycle. In the bargaining specification that allows bidding between two competing employers, minimum wage effects on wage growth are expected to be especially pronounced, since the equilibrium wage function displays a “compensating differential” property, i.e., those jobs offering the highest growth prospects offer commensurately lower wages.<sup>5</sup> Without a minimum wage, low wages are an (imperfect) indicator of high wage growth prospects. The minimum wage limits the extent to which firms can “charge” an employee for this future wage growth potential, thus reducing average wage growth in the market.

In introducing on-the-job search, the econometric framework employed in Flinn (2006) has to be considerably revised. The point sample CPS data he used are no longer sufficient for determining the parameters characterizing the more complicated employment processes modeled in this paper. We utilize event history data taken from the Survey of Income and Program Participation (SIPP), with the data coming from the period 1997-2000, and estimate the primitive parameters of the model using a Method of Simulated Moments (MSM) estimator. We draw on some of the arguments in Flinn (2006) in discussing sources of identification in the model we use, and manage to obtain reasonable model estimates under both bargaining specifications. We then consider the selection of an optimal minimum wage under our two bargaining assumptions. We find that the answers we get differ markedly across the two specifications, and explore the reasons why.

The remainder of this paper proceeds as follows. In Section 2 we derive the model and present some analysis of the effects on labor market outcomes of minimum wages with OTJ search. Section 3 contains a discussion of the data used to estimate the equilibrium model, while Section 4 develops the MSM estimator we use. Section 5 presents the empirical results, and Section 6 presents the results of our policy experiments. In Section 7 we conclude.

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<sup>5</sup>The reason for this is that future firms will have to bid against a high-valued  $\theta$  to attract the worker. This effectively increases the outside option of the worker, and the firm demands compensation for this future bargaining advantage by reducing the current wage offer.

## 2 Model

In this section we describe the behavioral model of labor market search with matching and bargaining in which the interactions between applicants and firms are constrained by the presence of a minimum wage. The minimum wage,  $m$ , is set by the government and is assumed to apply to all potential matches. We assume that the only compensation provided by the firm is the wage. As a result, there are no other forms of compensation the firm can adjust so as to “undo” the minimum wage payment requirement.

We restrict our attention to the case of exogenous contact rates in this paper. In Flinn (2006), the model was estimated in both partial and general equilibrium settings, where the general equilibrium model made use of the matching function formulation typically used in macroeconomics. The policy experiments produced vastly different “optimal” minimum wages in the two cases, but formal testing found no support for the general equilibrium framework. While it is obviously preferable to conduct the policy analysis within a general equilibrium framework, the problem is that it is only possible to estimate a highly restricted model of the demand side of the market. Given this limitation, we have chosen to use a partial equilibrium framework throughout.<sup>6</sup>

As is common in the search literature, we use a Nash bargaining framework in the wage determination process. We estimate and evaluate the model under two alternative assumptions regarding outside options. The differences and similarities of the two approaches are simple to describe. Say that a firm currently employs a worker who has a certain productivity  $\theta$  and is paid a wage  $w$ . The employee meets another potential employer, and the (potential) productivity at that job is immediately revealed to be  $\theta'$ . In both of the bargaining settings we consider, efficient mobility decisions are made. That is, the employee will change firms if and only if  $\theta' > \theta$ . The models differ in the wage determination process.

We devote most of our attention to the most theoretically interesting case, which allows direct wage competition between firms. This setup has been used by Postel-Vinay and Robin (2002), Dey and Flinn (2005), and Cahuc et al. (2006). Firms, competing for the same individual’s services at a moment in time, engage in a Bertrand competition for the employee, with the firm associated with the worst productivity level dropping out of the auction at the point at which its profit level is zero. If, for example, the individual’s productivity at the new potential employer exceeds that

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<sup>6</sup>This is the route often taken in the estimation of equilibrium search models. Some other recent and noteworthy examples are Postel-Vinay and Robin (2002) and Cahuc et al. (2006).

at his current firm, so that  $\theta' > \theta$ , it is assumed that the current firm makes a (doomed) effort to retain her services by making higher and higher wage offers until it drops out of the bidding at a wage offer  $\tilde{w} = \theta$ . In this case, the employee moves to the new firm, and the outside option used to set her wage is the value of being employed at the original firm at a wage equal to  $\tilde{w} = \theta$  (i.e., when she receives all of the surplus of employment match).

The other situation that can arise under renegotiation is when the employee's productivity at the new firm is less than or equal to her current productivity, but greater than her current wage, or  $w < \theta' \leq \theta$ . Though the employee will not leave the current firm, given efficient mobility, she can use the threat of leaving to increase her current wage  $w$ . In this case, the potential employer bids for the individual's services until it reaches the point  $\tilde{w} = \theta'$ , at which point it drops out of the auction. The renegotiated wage at the current employer uses the value of being employed at the match value  $\theta'$  with a wage equal to  $\tilde{w} = \theta'$  as the outside option.

Our second bargaining scenario considers the case in which each labor market participant's outside option value is equal to the value of unemployed search independent of their current labor market state. Of course, this is the option value for those searching in the unemployment state at the time they encounter a potential employer. The second bargaining scenario posits that it also serves as the outside option for employed searchers. This may be due to the fact that employed individuals are not able to credibly convey their current employment conditions (including wage or wage offer) to a new potential employer, while at the same time not being able to credibly reveal current wage offers from potential employers to their current employers.

An alternative justification is one of lack of commitment. If offers must be rejected or accepted at the instant when they are tendered, then a worker loses his or her outside option the moment after it is received. When the option is lost, the only relevant one becomes quitting into the unemployment state, the value of which is always  $V_n$ . Thus wage payments will always be determined from this outside option. Knowing this, a worker may insist that the firm transfer a lump sum amount to them to obtain their services at the moment when they have two employment options. To the extent that this is not recorded as a wage payment, this will have no effect on the wage process. Since all mobility is efficient in any case, such payments will have no impact on the mobility process. Thus the empirical wage-mobility process should be consistent with model assumptions even in the presence of unobservable (to us) one-time payments associated with the receipt of an offer by an employed individual.

## 2.1 The Model with Renegotiation

The model assumes a stationary labor market environment and is formulated in continuous time. We assume that there exists an invariant, technologically-determined distribution of worker-firm productivity levels which is given by  $G(\theta)$ . To facilitate the numerical solution of the model, we assume that the random variable  $\theta$  is discrete, with the set of values  $\theta$  can assume being given by  $\Omega_\theta = \{\theta_j\}_{j=1}^L$ , where  $0 < \theta_1 < \dots < \theta_L$ . When a potential employee and a firm meet, the productive value of the match  $\theta$  is immediately observed by both the applicant and the firm. At this point a division of the match value is proposed using a Nash bargaining framework that is described below. The searcher's instantaneous discount rate is given by  $\rho > 0$ . The rate of (exogenous) termination of employment contracts is  $\eta > 0$ .

While unemployed individuals search, their instantaneous utility is given by  $b$ , which can assume positive or negative values. Unemployed workers meet firms at the exogenously-determined rate  $\lambda_n$ . If both the firm and the worker accept the match, then they split it using a Nash bargaining framework and determine a wage  $w(\theta, U)$ . The acceptance set of matches from the unemployment state is given by  $A(m)$ , with  $\theta^A(m)$  being the minimal  $\theta$  value in  $A(m)$  (further discussion of the decision rule is provided below). It is assumed that labor is the only factor of production and if an individual and a firm meet, but the firm "passes" on the applicant, then the firm receives a value of 0. This is the firm's disagreement value in the Nash bargaining framework.<sup>7</sup> Analogously, the disagreement value for an unemployed searcher is the value of continued search, which is denoted  $V_n(m)$ .

While employed, workers meet firms at the exogenous rate  $\lambda_e$  which is independent of the employed worker's current match value. For simplicity, we assume that OTJ search is costless. Letting  $w$  represent the worker's wage, we denote the current labor market state of an employed individual by  $(\theta, w)$  and any potential new state by  $(\theta', w')$ . We now consider the rent division problem facing a currently employed agent who encounters a new potential employer.

Let there be a currently employed individual with wage  $w$  and match value  $\theta \in A(m)$ , who meets a new potential employer at which the match value is  $\theta'$ . We assume that the potential match value will only be reported to the current employer if the employee has an incentive to do so. One situation in which this will be the case is when  $\theta' > \theta$ . When this occurs, we assume that a

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<sup>7</sup>In the general equilibrium version of the model, a free entry assumption implies that the expected value of a vacancy is zero, which is why we impose this condition even in the partial equilibrium case. To see such a condition imposed in an estimable general equilibrium version of this model without OTJ search, see Flinn (2006).

bargaining process for the individual's services begins between the current and potential employers and stops when one of the firms' surplus reaches zero, as in Bertrand competition. The winner of the competition will be the current employer when  $\theta' > \theta$ . Let the maximal value of the match  $\theta$  to the worker be given by  $Q(\theta)$ . Then the objective function for the Nash bargaining problem when  $\theta' > \theta$  is:

$$S(\theta', w', \theta) = \{V_e(\theta', w', \theta) - Q(\theta)\}^\alpha \times \{V_f(\theta', w', \theta) - 0\}^{1-\alpha}$$

where  $V_f(\theta', w', \theta)$  denotes the new firm's value of the match, assuming that each firm's threat point is zero, and  $\alpha \in (0, 1)$  represents the bargaining power of the worker.

The firm's value of the current employment contract is defined as follows. Over an infinitesimally small period of time  $\varepsilon$ , the firm earns a profit of  $(\theta - w)\varepsilon$ , which is discounted back to the present with the "infinitesimal" discount factor  $(1 + \rho\varepsilon)^{-1}$ . With "probability"  $\eta\varepsilon$ , the match is exogenously terminated and the firm earns no profit. With "probability"  $\lambda_e\varepsilon$ , the worker receives a job offer from an alternative firm. If he reports this offer to his current firm, his wage will be renegotiated.<sup>8</sup> With approximate probability  $(1 - \lambda_e\varepsilon - \eta\varepsilon)$ , the worker does not receive another job offer and is not exogenously dismissed over the period  $\varepsilon$ . In this case the status quo is maintained. The term  $o(\varepsilon)$  represents the probability that two or more events occur over the period  $\varepsilon$  and has the property that  $\lim_{\varepsilon \rightarrow 0} \left(\frac{o(\varepsilon)}{\varepsilon}\right) = 0$ . We denote the value to the firm as:

$$\begin{aligned} V_f(\theta', w', \theta) = & (1 + \rho\varepsilon)^{-1}\{(\theta' - w')\varepsilon + (\eta\varepsilon \times 0) + \left(\lambda_e\varepsilon \sum_{\tilde{\theta} \in B(\theta', \theta)} V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta})p(\tilde{\theta})\right) \\ & + \left(\lambda_e\varepsilon P(\tilde{\theta} \leq \theta) \times V_f(\theta', w', \theta)\right) + (\lambda_e\varepsilon P(\tilde{\theta} \geq \theta') \times 0) \\ & + \left((1 - \lambda_e\varepsilon - \eta\varepsilon) \times V_f(\theta', w', \theta)\right) + o(\varepsilon)\}, \end{aligned}$$

where  $V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta})$  represents the equilibrium value to a firm of the productive match  $\theta$  when the worker's next best option has a match  $\tilde{\theta}$ , and a value  $\theta_j \in B(\theta', \theta)$  if and only if  $\theta_j \leq \theta'$  and  $\theta_j > \theta$ .

The interpretation of the arguments involving  $\lambda_e$  is as follows. Given a dominant and dominated match value pair  $(\theta', \theta)$ , we can partition  $\Omega_\theta$  into three sets. Since efficient mobility is an implication of our model structure, any new draw of a match value at a prospective employer that is greater

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<sup>8</sup>Since the searcher has the option to not report any current match value, he will only do so when his employment value at the current firm will increase, which only occurs when the wage increases.

than the match value at the current employer,  $\theta'$ , results in an immediate departure. This event implies a value to the firm of 0 under our assumption regarding its outside option. Moreover, if the employee meets a prospective employer at which her productivity is equal to  $\theta'$ , then she will be indifferent regarding which offer to accept since both offer a contract giving her all of the rents.<sup>9</sup> Independent of whether she stays, the firm will receive a value of 0 in this case as well. By our definition of the set  $B$ , we have assumed that she stays in such a case. However, the value to the incumbent firm is 0, so it is indifferent as well.

The set  $B(\theta', \theta)$  contains all of those values that result in a renegotiation of the contract ( $w$ ) at the current firm and that don't result in a departure. Since the total surplus associated with a match value is strictly increasing in the match value, the amount of surplus the individual can appropriate from  $\theta'$  is strictly increasing in the value of the outside option. If the current outside option is  $\theta$ , then any outside option greater than  $\theta$  and less than  $\theta'$  will result in a renegotiation. The value of the job to the firm at that point in time will be  $V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta})$ , where  $w(\theta', \tilde{\theta})$  will be the new wage paid. Finally, all potential matches less than or equal to the current outside option will not be reported to the firm, since the value of the outside option associated with these values is no greater than the current one. When such an offer arrives, the value of the firm's problem does not change. After rearranging terms and taking limits as  $\varepsilon \rightarrow 0$ , we have

$$V_f(\theta', w', \theta) = \left( \rho + \eta + \lambda_e P(\tilde{\theta} > \theta) \right)^{-1} \times \left\{ \theta - w + \lambda_e \sum_{\tilde{\theta} \in B(\theta, \theta')} V_f(\theta', w(\theta', \tilde{\theta}), \tilde{\theta}) p(\tilde{\theta}) \right\}.$$

The worker's value of being employed is defined similarly. For the employee, the value of employment at a current match value  $\theta$  and wage  $w$  is given by

$$\begin{aligned} V_e(\theta', w', \theta) &= (1 + \rho\varepsilon)^{-1} \{ w\varepsilon + \eta\varepsilon V_n(m) + \lambda_e\varepsilon [ \sum_{\tilde{\theta} \in B(\theta, \theta')} V_e(\theta', w(\theta', \tilde{\theta}), \tilde{\theta}) p(\tilde{\theta}) \\ &\quad + \sum_{\tilde{\theta} \in C(\theta')} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta'), \theta') P(\tilde{\theta}) + P(\tilde{\theta} \leq \theta) \times V_e(\theta', w', \theta) ] \\ &\quad + (1 - \lambda_e\varepsilon - \eta\varepsilon) \times V_e(\theta', w', \theta) + o(\varepsilon) \}, \end{aligned}$$

where  $V_e(\theta', w(\theta', \theta), \theta)$  is the equilibrium value of employment to a worker with match value  $\theta'$  when his next best option has a match value of  $\theta$  and the set  $C(\theta')$  is the set of all  $\theta_j \in \Omega_\theta$  such

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<sup>9</sup>This event has positive probability given our assumption that  $\theta$  is a discrete random variable.

that  $\theta_j > \theta'$ . As we saw in the case of the firm, when an employee encounters a firm with a new match value  $\tilde{\theta}$  which is lower than his current match  $\theta$  but belongs to the set  $B(\theta', \theta)$ , his new value of employment at the current firm becomes  $V_e(\theta', w(\theta', \tilde{\theta}), \tilde{\theta})$ . Instead, when the match value at the newly-contacted firm is greater than the current match value  $\theta'$ , the employee will change employers, and the new value of employment is given by  $V_e(\tilde{\theta}, w(\tilde{\theta}, \theta'), \theta')$ . Thus, the match value at the current firm becomes the determinant of the threat point faced by the new firm and plays a role in the determination of the new wage. Finally, when the match value at the prospective new employer is less than the current dominated match value  $\theta$ , the new contact is not reported to the current firm since it would not improve the current contract. Because of this selective reporting, the value of employment contracts must be monotonically increasing within a job with a particular firm. As originally noted by Postel-Vinay and Robin (2002), wage declines can be observed when moving directly between firms, though, of course, the value of the employment match must always be increasing. After rearranging terms and taking limits, we have

$$V_e(\theta', w', \theta) = \left( \rho + \eta + \lambda_e P(\tilde{\theta} > \theta) \right)^{-1} \times \{w + \eta V_n(m) + \lambda_e [ \sum_{\tilde{\theta} \in B(\theta, \theta')} V_e(\theta', w(\theta', \tilde{\theta}), \tilde{\theta}) p(\tilde{\theta}) + \sum_{\tilde{\theta} \in C(\theta')} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta'), \theta') P(\tilde{\theta}) ] \}.$$

With a new match value of  $\theta' > \theta$ , the surplus attained by the individual at the new match value with respect to the value she could attain at the old match value after extracting all the surplus associated with it is

$$V_e(\theta', w(\theta', \theta), \theta) - Q(\theta),$$

where  $Q(\theta) \equiv V_e(\theta, w(\theta, \theta), \theta)$  is the value of employment to the employee if he receives the total surplus of the match  $\theta$ . In this case, the equilibrium wage function has the property that  $\theta = w(\theta, \theta)$ . Then,

$$Q(\theta) = \left( \rho + \eta + \lambda_e P(\tilde{\theta} > \theta) \right)^{-1} \times \{ \theta + \eta V_n(m) + \lambda_e \sum_{\tilde{\theta} \in C(\theta)} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta), \theta) P(\tilde{\theta}) \}.$$

The model is closed after specifying the value of nonemployment,  $V_n(m)$ . We will discuss the manner in which minimum wages impact job acceptance, the unemployment rate, and the equilibrium wage offer function in detail below. As we mentioned above, there is a minimal acceptable match value from the unemployment state denoted by  $\theta^A(m)$ , such that for all  $\theta_j \geq \theta^A(m)$  the match

is accepted. Define the set of acceptable match values out of the unemployment state by  $D(\theta^A(m))$ . We note that for the firm to earn nonnegative flow profits, it is necessary that  $\theta^A(m) \geq m$ .

The searcher's value of being unemployed is defined as follows. The  $\varepsilon$ -look ahead formulation of the Bellman equation for the unemployed searcher takes the form

$$V_n(m) = (1 + \rho\varepsilon)^{-1} \{ b\varepsilon + \lambda_n \varepsilon [ \sum_{\tilde{\theta} \in D(\theta^A(m))} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta^*(m)), \theta^*(m)) P(\tilde{\theta}) ] + P(\tilde{\theta} < \theta^A(m)) V_n(m) \} + (1 - \lambda_n \varepsilon) \times V_n(m) + o(\varepsilon).$$

The value  $b$  is the flow utility in the unemployment state. The ‘‘implicit’’ match value  $\theta^*(m)$  is that used as the outside option when determining the wage rate at the first acceptable job offer received out of the state of unemployment. The value  $\theta^*(m)$ , is not, in general, an element of  $\Omega_\theta$ . The second term is the probability-weighted sum of the values of employment in all of the acceptable match states. The third term corresponds to receiving an unacceptable offer, and the fourth represents the value of receiving no offer. Taking the limit as  $\varepsilon \rightarrow 0$ , we have

$$V_n(m) = \left( \rho + \lambda_n P(\tilde{\theta} \geq \theta^A(m)) \right)^{-1} \times \{ b + \lambda_n \sum_{\tilde{\theta} \in D(\theta^A(m))} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta^*(m)), \theta^*(m)) P(\tilde{\theta}) \}.$$

The equilibrium wage function,  $w(\theta', \theta)$ , is defined as follows. When an employed agent with an (acceptable) outside option  $\theta$  meets a dominating match  $\theta'$ , Nash-bargained wage is given by

$$w(\theta', \theta) = \arg \max_{w \geq m} S(\theta', w, \theta).$$

When an unemployed agent meets a new potential employer, the solution to the Nash bargaining problem is given by

$$w(\theta, \theta^*(m)) = \arg \max_{w \geq m} S_n(\theta, w, \theta^*(m)),$$

where  $S_n(\theta, w, \theta^*(m)) = \{V_e(\theta, w) - V_n\}^\alpha \times \{V_f(\theta, w) - 0\}^{1-\alpha}$ . The minimum wage acts as a side-constraint in the Nash bargaining problem in both cases.

## 2.2 Analysis of the Model

In Flinn's (2006) model without OTJ search, a binding minimum wage always served as the minimal acceptable match value from the unemployment state. In our notation, with no OTJ search, it was

always the case that

$$\theta^*(m) \leq m. \tag{1}$$

A binding minimum wage created a (positive-valued) difference between the minimal accepted match value and what was referred to as the “implicit” reservation match value,  $\theta^*(m)$ .

The situation can be markedly different in the case in which there is a “significant” amount of OTJ search and when we allow for renegotiation. In this subsection, we illustrate why this is so. We first consider characteristics of the equilibrium wage function  $w(\theta', \theta)$  when there is no minimum wage constraint. We then look at the introduction of a minimum wage and pay particular attention to the various ways in which it may be “binding.” These distinctions will have important ramifications for the determination of “optimal” minimum wage policies.

### 2.2.1 The Wage Function with $m = 0$

In this section we describe the method of solution of the model and the properties of the equilibrium wage function. The discrete  $\theta$  assumption greatly facilitates solving and analyzing the model. Recall that the set of values that  $\theta$  can take is contained in the finite set  $\Omega_\theta$ , where the  $L$  elements of the set are ordered

$$0 < \theta_1 < \dots < \theta_L < \infty. \tag{2}$$

The equilibrium wage function is described via the matrix:

**Table 1.1**  
**Equilibrium Wage Matrix**

	<i>Dominant Value</i>				
<i>Dominated Value</i>	$\theta_j$	$\theta_{j+1}$	$\dots$	$\theta_{L-1}$	$\theta_L$
$\theta_L$					$\theta_L$
$\theta_{L-1}$				$\theta_{L-1}$	$w(\theta_L, \theta_{L-1})$
$\vdots$					$\vdots$
$\theta_{j+1}$	$\theta_{j+1}$		$\dots$	$w(\theta_{L-1}, \theta_{j+1})$	$w(\theta_L, \theta_{j+1})$
$\theta_j$	$\theta_j$	$w(\theta_{j+1}, \theta_j)$	$\dots$	$w(\theta_{L-1}, \theta_j)$	$w(\theta_L, \theta_j)$
$U$	$w(\theta_j, U)$	$w(\theta_{j+1}, U)$	$\dots$	$w(\theta_{L-1}, U)$	$w(\theta_L, U)$

$\cdot \tag{3}$

The value  $\theta_j = \theta^A(0)$  in this case is the minimum acceptable match value for an unemployed searcher. The wage function is not defined for values of  $\theta_k < \theta^A(0)$ . Moreover, the bargaining

mechanism always produces efficient mobility, meaning that the current match value (i.e., the “dominant” one) is always at least as large as the “dominated” match value, which generates the outside option value in the Bertrand competition between firms.

An important feature of the Bertrand competition for workers and the discreteness of  $\theta$  is that when the dominated value is equal to the dominant value, a positive-probability event, the worker captures all of the rents from the match. This means that the wage rate in this case is simply equal to the match value, simplifying the computation of the equilibrium wage function.

The equilibrium wage function computation is conducted in the following recursive manner. We begin by assuming that the only acceptable match value to an unemployed searcher is  $\theta_L$ , which is the largest match in the set  $\Omega_\theta$ , so  $\theta^A(0) = \theta_L$ . We begin with a guess of the value of unemployment,  $\tilde{V}_n(\theta^A)$ . In terms of an employment spell, the state  $(\theta_L, \theta_L)$  is an absorbing state, since no further job mobility can take place from that state during the current employment spell. The only way such a spell can end is through exogenous termination, which occurs at the constant rate  $\eta$ . The individual’s value of being in such a spell (given the value of unemployment  $\tilde{V}_n$ ) is given by

$$\tilde{V}_e(\theta_L, \theta_L, \theta_L) = \frac{\theta_L + \eta\tilde{V}_n(\theta_L)}{\rho + \eta}, \quad (4)$$

where the second argument in  $\tilde{V}_e$  is the wage rate associated with the state  $(\theta_L, \theta_L)$ , which happens to be  $\theta_L$ . The firm’s value is 0.

Now an unemployed searcher only accepts a match of  $\theta_L$ , the probability of which is  $p(\theta_L)$ . When the unemployed searcher accepts the one employment contract available to her, it has the associated value

$$\tilde{V}_e(\theta_L, w, U) = \frac{w + \lambda_e p(\theta_L)\tilde{V}_e(\theta_L, \theta_L, \theta_L) + \eta\tilde{V}_n(\theta_L)}{\rho + \lambda_e p(\theta_L) + \eta},$$

while the value to the firm is

$$\tilde{V}_f(\theta_L, w, U) = \frac{\theta_L - w}{\rho + \lambda_e p(\theta_L) + \eta}.$$

The wage associated with this state is then given by

$$\tilde{w}(\theta_L, U) = \arg \max_w (\tilde{V}_e(\theta_L, w, U) - \tilde{V}_n(\theta_L))^\alpha \tilde{V}_f(\theta_L, w, U)^{1-\alpha}.$$

Then the (new) implied value of unemployed search is given by

$$\tilde{V}_n'(\theta_L) = \frac{b + \lambda_n p(\theta_L)\tilde{V}_e(\theta_L, w, U)}{\rho + \lambda_n p(\theta_L)}.$$

If  $\tilde{V}'_n(\theta_L)$  is sufficiently “close to the initial guess  $\tilde{V}_n(\theta_L)$ , then we say that the value of search when only  $\theta_L$  is acceptable is given by  $V_n^*(\theta_L) = \tilde{V}'_n(\theta_L)$ . If not, replace  $\tilde{V}_n(\theta_L)$  with  $\tilde{V}'_n(\theta_L)$ , and repeat the process. We then continue the iterations until convergence.<sup>10</sup>

A similar technique is used for the cases in which we set  $\theta^A = \theta_j$ ,  $j = 1, \dots, L - 1$ . Each different “potential” critical value implies a unique wage distribution associated with it and a value of unemployed search given by  $V_n^*(\theta_j)$ . The optimal acceptance match chosen by the individual is the one that produces that highest value of searching in the unemployment state, i.e.,

$$\theta^A = \theta_j \Leftrightarrow V_n^*(\theta_j) = \max\{V_n^*(\theta_k)\}_{k=1}^L.$$

The equilibrium wage matrix is the one associated with that value of  $\theta^A$ . If, for example,  $L = 10$  and the  $\theta^A = \theta_4$ , then the (lower triangular) wage matrix is  $8 \times 7$ .

This is the algorithmic approach used to compute the wage matrix in Table 1.1. Changes in primitive parameters will of course sometimes change the critical acceptance match  $\theta^A$ , but not always due to the discreteness of the distribution. This will be observed in some of the examples that we turn to now. While the discrete distribution assumption does have some negative aspects, computation of the equilibrium wages/values is simplified and some of the impacts of minimum wages on labor market outcomes and welfare are somewhat more transparent.

We now present an example of the wage function computation. We set the parameters of the search environment at  $\alpha = .25$ ,  $\lambda_n = .2$ ,  $\lambda_e = .05$ ,  $\eta = .01$ ,  $\rho = .01$ , and  $b = -5$ . We assume a six point match distribuiton, with  $\Omega_\theta = \{5, 8, 11, 14, 17, 20\}$ , with an associated vector of probabilities given by (.1 .2 .25 .2 .15 .1). The equilibrium wage distribution is given below:

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<sup>10</sup>The mapping  $V_n = TV_n$ , while typically not a contraction, is monotone increasing. Subject to existence conditions, there exists a unique fixed point solution.

**Table 1.2**  
**Wage Matrix**  
 **$m = 0$**

	<i>Dominant <math>\theta</math> Value</i>					
<i>Dominated <math>\theta</math> Value</i>	5	8	11	14	17	20
20						20
17					17.00	17.35
14				14.00	13.84	14.19
11			11.00	10.27	10.11	10.46
8		8.00	6.70	5.96	5.80	6.15
5	5.00	3.32	2.02	1.80	1.12	1.47
<i>U</i>	4.78	3.10	1.79	1.06	0.90	1.25

(5)

At this set of labor market parameters, we note that all elements of  $\Omega_\theta$  are acceptable from the unemployment state. The most striking feature of the matrix is probably the degree of non-monotonicity in the wages given and outside option  $U$  or some  $\theta_j$ . For individuals coming out of unemployment, the highest wage offer attainable is the one associated with the lowest acceptable match value,  $\theta = 5$ . Although the value of the employment contract is strictly increasing in the match value found by the unemployed searcher, wages are not. In fact, were it not for the wage associated with the highest match value, exactly the opposite would be true. A similar pattern is observed in every row of the matrix (those with more than three entries).

The low wages associated with the high match values, holding constant the dominated value, are the employee's payment for the future bargaining advantages the match conveys during the employment spell. Under this set of parameters, the rate of meeting other employers, 0.05, is quite high relative to the rate of exogenous termination of the job (and employment) spell, 0.01. Combined with the relatively low discount rate of 0.01, the wage "compensation" for the future bargaining advantage is high.

Obviously, when there is no OTJ search, a high match value delivers no future bargaining advantage, and there are no compensating differentials observed in the wage function. As an illustration, we determine the equilibrium wage rates for the same parameter values used to generate Table 1.2 except that we set  $\lambda_e = 0$ . The wage function in this case is

**Table 1.3**  
**Wage Matrix**

$$\lambda_c = 0$$

	<i>Dominant <math>\theta</math> Value</i>						
<i>Dominated <math>\theta</math> Value</i>	5	8	11	14	17	20	.
$U$	7.60	8.35	9.10	9.85	10.60		(6)

In this case, the match value of 5 that was previously acceptable is no longer so. The wages are much higher coming out of the unemployment state, since there is no bargaining advantage component of remuneration. Most importantly, for our purposes, the wage function is monotonically increasing in  $\theta$  due to the fact that wages are the only compensation mechanism and the outside option is the same,  $V_u$ , at all jobs.

Before concluding this subsection, we return to the example wage function in Table 1.2 to discuss implications of the model regarding patterns of wage changes over an employment spell. Given the arrival of a “reportable” competing match value, that is, which is larger than the current dominated match value, two things can occur. If the new match arrival is larger than the current dominated match value and less than or equal to the current match value associated with the job, there is no consequent mobility but there is renegotiation of the wage contract. Since the only “negotiable” element of the employment contract is the wage, this is increased. Thus, while the worker remains at the same firm, all wage changes are positive.

In the second case, where the new match value exceeds the value of the current match, there is job mobility and contract negotiation. The old dominant match value becomes the new dominated match value and serves as the outside option. Though both the dominant and dominated match value increase with a job-to-job move, the wage rate need not. Once again, this is due to the fact that part of the employee’s share of the surplus is generated by the OTJ bargaining option, and this option value may increase to such an extent in the job-to-job move that a wage reduction is required to satisfy the surplus division rule.

### 2.2.2 The Wage Function with $m$ Binding

In the Flinn (2006) analysis without OTJ search, minimum wages could only be binding in one particular manner, which was by constraining the choice set of the worker and the firm. In that analysis, a binding minimum wage always produced an acceptance match value,  $\theta^A$  in our notation,

that was greater than what was called the “implicit” acceptable match value. In other words, workers would have accepted lower matches than  $m$ , but were constrained not to by minimum wage law. In that setting, the minimum wage essentially served as a coordination device that enabled workers with little bargaining power to achieve more of the surplus produced by the match. The cost of this gain was a lower probability of finding an acceptable match.

The minimum wage potentially plays an altogether different role in the presence of OTJ search, at least when the wage function displays nonmonotonicities of the type discussed in the previous section. We illustrate the role of the minimum wage with two separate examples. All of the parameters of the labor market environment are the same as they were in the example discussed in the previous subsection. The only state variable that differs from the example above and among the two presented here is the minimum wage rate,  $m$ .

In the first example, the minimum wage is set at \$1.50. Since all match values are acceptable when  $m = 0$ , and since the the minimum match value is 5, clearly all matches are still in the choice set of the bargaining worker-firm pair, by which we mean that the firm can earn nonzero flow profits when paying  $m$  for all  $\theta \in \Omega_\theta$ . In this case, the equilibrium wage matrix is given by

**Table 1.4**  
**Wage Matrix**  
 $m = 1.5$

	<i>Dominant <math>\theta</math> Value</i>					
<i>Dominated <math>\theta</math> Value</i>	5	8	11	14	17	20
20						20
17					17.00	17.35
14				14.00	13.84	14.19
11			11.00	10.27	10.11	10.46
8		8.00	6.70	5.96	5.80	6.15
5	5.00	3.40	2.10	1.50	1.50	1.55
$U$	4.82	3.32	2.02	1.50	1.50	1.50

(7)

The first important thing to note about this example is that the acceptance match value has remained the same at  $\theta^A = 5$ . All matches that were previously accepted in equilibrium still are. However, the minimum wage constraint on the bargaining process has changed the bargained

outcomes for most of the wages associated with dominated values of  $\theta = 5$  and  $U$ . The impact has come through the improvement in the wage distribution that resulted from not allowing for the firm to be fully compensated for its contribution to the future bargaining power of the individual during the current employment spell. Where the minimum wage is binding, the implication is that individual is receiving more than their share of the surplus, which in our example is set at 0.25. This is a plus for the individual's side of the market, particularly those in unemployment and who have found employment at the minimal acceptable match value. Firms still earn positive profits whenever they employ an individual who has a dominated match value less than the dominant one, and minimum wages cannot affect the wage payment when  $\theta' = \theta$ .

We conclude this section with an illustration of the wage function when the minimum wage is set at such a high level that an otherwise acceptable match value (to a searching individual) is smaller than the minimum wage, and thus cannot lead to an employment match. Table 1.5 contains the wage function when  $m = 13$ .

**Table 1.5**  
**Wage Matrix**  
 **$m = 13$**

	<i>Dominant <math>\theta</math> Value</i>		
<i>Dominated <math>\theta</math> Value</i>	14	17	20
20	20		
17	17.00    17.35		
14	14.00	13.84	14.19
$U$	13.00	13.00	13.00

(8)

In comparison with the relevant rows and columns of Table 1.4, the new, extremely high, value of the minimum wage has no discernible impact on the wages negotiated during OTJ search. The new minimum wage has a large impact on the wage offers to currently unemployed workers, though the probability of getting an acceptable offer has substantially decreased.

### 2.3 Model with No Renegotiation

The model with renegotiation is considerably simpler to describe. As was discussed in the introduction to this section, the outside option in this case is always equal to the value of unemployed

search,  $V_n(m)$ . There is efficient mobility, which means that the match value at a new firm in a job-to-job transition is strictly greater than the match value at the firm the individual is leaving. This implies that all job-to-job transitions are associated with an increase in an individual's wage. Since there is no renegotiation, the wage at a given firm is constant over the duration of employment at that firm.

### 2.3.1 The Case of $m = 0$

We now characterize the mobility-wage process. With the value of unemployed search given by  $V_n$ , the value of employment at an acceptable match value  $\theta_i$  is given by

$$V_e(\theta_i) = \frac{w(\theta_i) + \eta V_n + \lambda_e \sum_{j>i} p_j V_e(\theta_j)}{\rho + \eta + \lambda_e p_i^+},$$

where  $p_i^+ \equiv \sum_{j>i} p_j$ . Given our assumption regarding the outside option in the bargaining problem, the value of the wage associated with acceptable match value  $\theta_i$  is determined as

$$\begin{aligned} w(\theta_i) &= \arg \max_w \left( \frac{w + \eta V_n + \lambda_e \sum_{j>i} p_j V_e(\theta_j)}{\rho + \eta + \lambda_e p_i^+} - V_n \right)^\alpha \\ &\quad \times \left( \frac{\theta_i - w}{\rho + \eta + \lambda_e p_i^+} \right)^{1-\alpha} \\ &= \arg \max_w (w - (\rho + \lambda_e p_i^+) V_n + \lambda_e \sum_{j>i} p_j V_e(\theta_j))^\alpha \\ &\quad \times (\theta_i - w)^{1-\alpha} \\ &= \alpha \theta_i + (1 - \alpha) ((\rho + \lambda_e p_i^+) V_n - \lambda_e \sum_{j>i} p_j V_e(\theta_j)). \end{aligned} \tag{9}$$

The wage-setting rule is a simple extension of that associated with the bargaining problem in the absence of on-the-job search (see, e.g., Flinn, 2006), when  $\lambda_e = 0$ . This is to be expected given our restriction on the threat point.

**Proposition 1** *Let  $\theta^A = \theta_i$  denote the minimal acceptable match values.*

$$w(\theta_i) < w(\theta_{i+1}) < \dots < w(\theta_L).$$

Proof: Since

$$\lambda_e p_i^+ V_n - \lambda_e \sum_{j>i} p_j V_e(\theta_j) = \lambda_e \sum_{j>i} p_j (V_n - V_e(\theta_j))$$

is an increasing function of  $i$ , the result is obvious. ■

The monotonicity in the wage function in this case will have important implications in determining the optimal minimum wage rate. In the strategic renegotiation case, monotonicity of the

wage in current match value depended on the values of the primitive parameters. In this case, it does not.

The model, without minimum wages, is completed by determining the minimal acceptable match value from the unemployment state. This is accomplished by computing the value of search for each of the  $K$  possible acceptance sets, differentiated by the lowest match value included in the set. Let the value of search be given by  $\tilde{V}_n(\theta_i)$ , when  $\theta_i$  is the minimal acceptable value. Then  $V_n = \max\{\tilde{V}_n(\theta_i)\}_{i=1}^L$ , and  $\theta^A = \theta_j$  is the associated minimal acceptable match value when the argmax of the right hand side is equal to  $j$ .

### 2.3.2 Minimum Wages

Since the wage function is monotonically increasing in the match value, binding minimum wage rates will have qualitatively similar effects to those described in Flinn (2006), where there was no on-the-job search. In particular, we will want to define the value of search in the unemployment state as a function of the minimum wage rate,  $V_n(m)$ . As was true in the model with renegotiation, the set of feasible acceptable match values is truncated from below by the minimum wage, so that the lowest acceptable match value must be at least equal to  $m$ .

Given the value of search, the wage associated with an acceptable match value  $\theta_i$  is given by

$$w_i(m) = \max\{m, \alpha\theta_i + (1 - \alpha)((\rho + \lambda_e p_i^+)V_n(m) - \lambda_e \sum_{j>i} p_j V_e(\theta_j; m))\}.$$

Since the second term in the max function is an increasing in  $\theta_i$ , we see that the match values that yield minimum wages are the lowest ones in the acceptance set, which is not true under renegotiation when the wage functions are not monotone-increasing in the current match value. We saw that, under our model estimates, minimum wage workers were likely to have high match values coming out of the unemployment state due to the pronounced nonmonotonicity in the wage functions.

The value of search in the unemployment state is determined in the same way as described for the case of no binding minimum wage, except that  $\theta^A$  is restricted to the subset of  $\Omega_\theta$  that includes only those values of  $\theta_i \geq m$ .

### 3 Data

The data used to estimate the model contain information on individuals from the 1996 panel of the Survey of Income and Program Participation (SIPP). A main objective of the SIPP is to provide accurate and comprehensive information about the principal determinants of the income of individual households in the United States. The SIPP collects monthly information regarding individual's labor market activity including earnings, average hours worked, and whether the individual changed jobs during the month, making it an attractive data set with which to study employment dynamics of job seekers and workers.

Although the size of the SIPP's target sample is quite large, our sample size has been greatly reduced by several restrictions. We only consider individuals ages 16 to 30 who do not participate in the armed services or in any welfare program (e.g., TANF, Food Stamps, WIC) during the sample period. We focus on this age group since minimum wage earners are typically young. The parameter estimates, particularly the bargaining power parameter, as well as the results from policy simulations presented in subsequent sections, should be interpreted with this younger sample in mind.

In addition to these general selection criteria, we impose a restriction that is particular to estimating a stationary on-the-job search model with minimum wages. The minimum wage changed from \$4.75 to \$5.15 on September 1, 1997, and remained at \$5.15 for the remainder of the SIPP survey period. Although the SIPP interviews individuals every four months for up to twelve times from 1996 to 2000, we use data only from February 1998 to February 2000 in order to allow adequate time for the labor market to adjust to the policy change and to avoid minimum wage changes within the survey period. A drawback of defining a sample window close to the end of the panel, however, is that discontinuities in respondents' employment histories become increasingly present as individuals approach the end of the survey period. Because our econometric specification relies heavily on identifying transitions between labor market states, it is essential that individuals have complete labor market histories. After excluding individuals with incomplete histories, our final sample consists of 3,048 individuals.

As we discuss in the next section, we use a moment-based estimation procedure to estimate the model. The set of sample moments is estimated using cross-sectional data from February 1998 and February 1999, as well as data that describes individuals' labor market dynamics between these two points in time. The cross-sectional moments include the proportion of the sample that is

unemployed, the mean and standard deviation of wages, and the proportion of workers who earn the minimum wage. Other moments describe employment transitions and wage changes between the two points in time. These moments include the proportion of individuals employed in February 1998 who lose their job before February 1999, and the mean wage change among workers who have a job-to-job transition during the year.

Table 2 contains descriptive statistics generated from the data. In February 1998, 4.3 percent of the sample was unemployed. Among employed workers, the mean and standard deviation of wages were \$9.47 and \$4.67, with 3.3 percent of workers earning the minimum wage. Our moment-based estimation strategy allows us to include workers in the sample with wages below the minimum wage, although they compose a very small proportion of the sample.

Turning to measures of employment and wage dynamics, we find that 29 percent of workers in the sample who are employed in February 1998 transition directly to another job before February 1999 (i.e., there is no intervening spell of unemployment). The mean wage in February 1998 for these workers is \$8.43, about one dollar less than the mean wage of the full sample. While the mean change in wages across jobs for these workers of \$0.90 is fairly small, the distribution of changes is considerably dispersed (the standard deviation of the change is \$4.32). The size of the mean wage change is due partially to the existence of wage decreases across consecutive jobs, as 32.2 percent of employed workers accept new jobs with lower wages (see Figure 1). However, the nature of the wage-bargaining process between individuals and firms in our model does not predict that wages have to be at least as large at the destination job as at the current job in order for the individual to leave the current job. That is, workers may leave their current jobs to accept new, lower wage jobs if the option value of doing so is large enough.

The transition rate from employment to unemployment is also included in the set of moments that measure employment dynamics. The percentage of individuals in the sample who are employed in February 1998 who exit into unemployment before February 1999 is 6.4 percent. The group of individuals who make this transition consists of both those who voluntarily leave their job and those who are involuntarily dismissed. Among those individuals who make the opposite transition, from unemployment to employment, the mean wage at the first job is \$8.48. This is about one dollar less than the mean of the cross-sectional wage distribution in February 1998 for the full sample. The distribution of initial wages for the individuals making this transition is also slightly less dispersed than the cross-sectional distribution.

## 4 Estimation Method

In this section we discuss the simulation-based method used to estimate the primitive parameters of the model developed above. Given the rather rich patterns of wage mobility and turnover that the model generates, and the fact that the wage function is not (in general) monotonic in the value of the current match under the bargaining model that allows renegotiation, the use of a maximum likelihood estimator is problematic. We have opted to use a Method of Simulated Moments (MSM) estimator instead.

The procedure used is similar to that employed by Dey and Flinn (2008) in their estimation of a model of household (husband and wife) labor market search, which was also used implemented using SIPP data. The panel data we have access to is used to construct an event-history data set, in which the labor market status of each sample member is considered known at each point in time during their sample participation period. Let  $m_{98}$  denote a set of sample characteristics from the point sample constructed in February 1998, and let  $m_{99}$  denote the analagous sample characteristics computed from the February 1999 point sample (recall that these are the same set of individuals). Finally, let  $P_{99|98}$  denote a set of sample characteristics computed from February 1998 to February 1999 transitions of the sample members. For example, one such characteristic could be the probability of observing the individual in the unemployment state in 1999 given that they were in the employment state in 1998. Another might be the average wage of employed individuals at the 1999 survey date who were unemployed at the time of the 1998 survey.

In using these moments in the estimation procedure we must make some strong assumptions regarding initial conditions. For a number of reasons to be discussed below, we assume that the population from which the sample is drawn is in the stationary portion of their labor market careers. In the model, each individual begins his labor market career in the unemployment state, so this assumption essentially implies that this initial condition has “worn off” by the time a sample member’s career is point-sampled in February 1998. This assumption is likely to be violated for extremely young sample members whose labor market careers are relatively short. Depending on the primitive parameters, it may be reasonably appropriate for the older members of our sample. One indication of whether this assumption is approximately satisfied involves a comparison of the point-sampled sample characteristics from February 1998 and February 1999. If the steady state assumption is reasonable, then

$$\text{plim}_{N \rightarrow \infty} m_{98} = \text{plim}_{N \rightarrow \infty} m_{99} = m_{SS},$$

where  $m_{SS}$  are the true steady state moments of the selected labor market characteristics. Given the relatively large sample size of  $N = 3,048$ , it is reasonable to assume that the asymptotic arguments apply.

We performed a small, but formal, testing exercise to check the stationarity assumption. Because our sample size is relatively large, we decided to divide the sample into two equal-size subsamples to induce independence in the point samples across the two years; i.e., for one half of the (total) sample (subsample  $S_{98}$ ), we used moment information from 1998, and for the other half (subsample  $S_{99}$ ), we used moment information from 1999. We examined the equality of five sample characteristics: the proportion unemployed, the average wage of the employed, the standard deviation in wages of the unemployed, the proportion of the employed paid the minimum wage, and the proportion of the employed paid more than 20 dollars an hour. Let the value of these sample characteristics be denoted by  $\hat{m}_i$ ,  $i = 98, 99$ . For each subsample, we then estimated a covariance matrix of these sample characteristics by resampling  $S_i$  500 times. Let the estimated covariance matrices be given by  $Z_i$ ,  $i = 98, 99$ . Invoking Central Limit Theorem arguments, we claim that under the null of  $H_0 : m_{98} = m_{99}$ ,

$$\chi_{(5)}^2 \stackrel{a}{\sim} (\hat{m}_{99} - \hat{m}_{98})'(Z_{98} + Z_{99})^{-1}(\hat{m}_{99} - \hat{m}_{98}).$$

The value of the test statistic was a large 90.804, leading us to decisively neglect the null hypothesis of stationarity. There are two primary reasons for such a large value of the test statistic, in our opinion. The first is nonstationarity in the labor market environment. This is the most serious type of misspecification we could face, since the entire modeling framework is premised on the constancy of the “primitive” parameters characterizing the labor market environment. While the actual labor market environment cannot be assumed constant over time, the question is whether the actual fluctuations are so large that our approximation of constancy is poor. Unfortunately, there seems to be no way to access this without developing a full-blown search model in which the primitive parameters are actually stochastic processes.

The second source of misspecification is generated by the assumption that individuals are in the steady state portion of their labor market careers. Since we study minimum wage workers, and since these workers are largely young, we are forced to include recent labor market entrants in the sample. Under our modeling assumptions, all participants begin their careers in the unemployment state. This initial condition has probably not “worn off” for many of the youngest workers in our sample, thus rendering inconsistent our estimates based on matching (approximately) steady state moments from the model simulations with sample moments. This kind of concern is consistent

with the empirical evidence. For example, the path to the steady state unemployment rate should be monotonically decreasing as the cohort ages, while the average wage of the employed should be monotonically increasing. We observe the proportion of the unemployed to decrease from 0.043 to 0.029 across the two years, while the average wage of the employed increases from \$9.46 to \$10.39.

Practically speaking, if we can accurately determine the date of entry into the labor market environment of each sample member, there is no reason to use the steady state assumption in forming the estimator. If we know that sample member  $i$  has been in the labor market for  $\tau_i$  periods as of the point sample date, then each simulated path can be point-sampled at time  $\tau_i$  to determine that sample member's state. Such a procedure is the focus of our current estimation efforts, although it must be mentioned that the determination of the entry date into the labor market is not straightforward for many sample members, particularly those who are currently enrolled in school. For the present, we use the steady state assumption when computing the sample characteristics from the simulated paths.<sup>11</sup>

Under the steady state assumption, it is straightforward to define the estimator. Let the value of the sample characteristics used be given by  $X = (m'_{98} P'_{99|98})'$ , which is a column vector containing  $K$  sample characteristics. Let the corresponding model counterparts, conditional on the parameter vector  $\varphi$ , be given by  $\tilde{X}(\varphi) = (\mu^{SS}(\varphi)' \pi(\varphi)')'$ . Given a method for computing  $\tilde{X}(\varphi)$ , discussed immediately below, the minimum distance estimator of  $\varphi$  is given by

$$\hat{\varphi} = \arg \min_{\varphi} (X - \tilde{X}(\varphi))' A (X - \tilde{X}(\varphi)),$$

where  $A$  is an  $K \times K$  symmetric, positive definite weighting matrix. We compute the weighting matrix by resampling the SIPP data matrix from which the sample characteristics,  $X$ , are computed. The matrix  $A$  is the inverse of this estimated covariance matrix.

We assume that the sample is drawn from a homogeneous population, i.e., there exist no observable or unobservable persistent differences across population members. To compute  $\tilde{X}(\varphi)$ , we first solve the model at the parameter vector  $\varphi$ , which gives us the equilibrium wage functions and job acceptance rule. We then simulate  $R$  sample paths, which all begin in the unemployment state. After a sufficiently long period of time (about 20 years here), we point-sample each path, and then point-sample the same path 12 months later. We then use the distributions of the states at the

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<sup>11</sup>We are currently recomputing model estimates using the cross-sectional sample characteristics from 1999, instead of those from 1998, upon which our current estimates are based. We hope to get some idea of the sensitivity of parameter estimates to the lack of stationarity observed in the data.

first point sample of the  $R$  paths to compute the steady state characteristics,  $\mu^{SS}(\varphi)$ . We use the transition information between the two point samples to compute  $\pi(\varphi)$ .

We have computed estimates of the standard errors by bootstrapping; this involved resampling the original individual data to compute new values of  $X$ . For each bargaining structure, we generated over 100 resamples.

## 4.1 Identification

We estimate two specifications of the equilibrium model that vary in terms of the possibility of renegotiation. The same parameters characterize both specifications, and there are no identification issues that are particular to either. Therefore we consider just the simpler (in terms of characterizing the equilibrium of the model) case of no-renegotiation.

From the Flinn and Heckman (1982) identification analysis of the partial-partial equilibrium search model, we know that the c.d.f.  $G$  is not identified except under a class of parametric assumptions. In our analysis, we utilize a discrete distribution of  $\theta$ , so that the results of Flinn and Heckman are not strictly applicable. The distribution of  $\theta$  contains  $L$  points in its support, and it is assumed that this set is known. The distribution of  $\theta$  is then characterized in terms of  $K - 1$  unknown probabilities, which can be a large number of parameters with the value  $K = 30$  that we utilize in the estimation. To eliminate this overparameterization, we define cut points at midpoints between the successive  $\theta$  values. We then assign the probability of  $\theta_i$  as

$$p(\theta = \theta_i) = \Phi\left(\frac{\ln c_i - \mu_\theta}{\sigma_\theta}\right) - \Phi\left(\frac{\ln c_{i-1} - \mu_\theta}{\sigma_\theta}\right),$$

where  $c_0 = 0$ ,  $c_K = +\infty$ , and  $c_i = \frac{\theta_i + \theta_{i+1}}{2}$  for  $i = 1, \dots, K - 1$ . In other words, we use a lognormal assumption to assign probability mass across the support points. This makes the Flinn and Heckman results applicable to the extent that we can think of our discrete  $\theta$  distribution as an approximation to an underlying continuous lognormal distribution.

They also showed that the parameters  $b$  and  $\rho$  were not individually identified. As is commonly done, we fix the parameter  $\rho$  at a value reasonably commensurate with the interest rate. The value we use (denominated in months) is  $\rho = 0.05/12$ . With  $\rho$  fixed, it is in principle straightforward to point identify  $b$  if the model is parameterized in terms of the critical match value given when the match value is continuously distributed. With a discrete match distribution, the critical match value will lie between points of support of the distribution, and hence is not uniquely determined. It is possible to identify  $b$  by explicitly solving the value function at each new trial vector of primitive

parameters  $\varphi$ , where  $\varphi$  contains the unknown value of  $b$ . This is what we do in our estimation procedure, though at present we have not attempted to estimate  $b$ , but merely fix it at the value  $-2$ .<sup>12</sup>

While we do not use event history data in the estimator, the steady state distributions and transition probabilities have been found sufficient to yield reasonable and precise estimates of the transition rate parameters  $(\lambda_n, \lambda_e, \eta)$  using the SIPP data. For example, Dey and Flinn (2008) estimate a joint husband-wife labor market search model using a similar estimation strategy, and obtain precise estimates of the transition rate parameters for *both* husbands and wives. While their model is not set in an equilibrium framework, the transition dynamics are more complex than those generated by ours. We are confident that a MSM approach based on reasonable sample characteristics can yield “good” estimates of these parameters, which is borne out in our empirical results.

The most vexing problem we face is the estimation of the bargaining power parameter  $\alpha$  using only information from the supply side of the market. Flinn (2006) showed that, for a continuously distributed  $\theta$ , a sufficient condition for identification of  $\alpha$  was the  $G$  not belong to a location-scale family. He assumed that  $G$  was lognormal, which belongs to a log location-scale family, but not a location-scale family. Monte Carlo experiments showed that  $\alpha$  could be recovered when extremely large samples were available under this functional form assumption.

In our application, the situation is worse on one hand and better on the other. We do not have a continuously-distributed  $\theta$ , so his results are not directly applicable. As we argued above, thinking of our discrete distribution as an approximation to an underlying continuous lognormal ameliorates this problem. On the positive side, our model with on-the-job search provides a richer mapping from a fixed-population wage-offer distribution, a set of outside options (determined by primitive parameters), and the bargaining power parameter than did the one he faced in the no OTJ case. Wage changes across firms during the same employment spell provide a rich potential source of identifying information about primitive parameters, including  $\alpha$ , that is not present when all employment spells consist of one job spell.

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<sup>12</sup>It is notoriously difficult to precisely estimate  $b$ , the evidence being the large standard errors associated with point estimates of this parameter, usually “backed out” of the model in a final estimation step (see Flinn and Heckman, 1982). Given the length of time it took to estimate the model that allows renegotiation of employment contracts, we decided to merely fix this value in the estimation and policy experiments.

## 5 Results

Table 3 contains estimates from the model in which workers are allowed to renegotiate their wage with their current firm and the model in which renegotiation is not allowed. We refer to these as the "renegotiation" and "no renegotiation" models when describing the estimates below.

We first report the estimates from the model with renegotiation and then compare these estimates with those obtained without renegotiation. The estimates in the first column of Table 3 indicate that the average time between contacts for unemployed searchers is  $(0.421)^{-1}$ , slightly over 2.4 months. When an unemployed searcher receives a job offer, the probability he accepts it is  $\tilde{G}(m) = 0.691$ . Thus, the estimated length of an unemployment spell is  $(\lambda_n \tilde{G}(m))^{-1} = 3.4$  months. Once employed, an individual receives an alternative job offer from a competing firm approximately every  $(0.063)^{-1} = 15.9$  months. Given an alternative job offer, the probability that the worker accepts it depends on his or her current wage and current match value. The estimated exogenous dissolution rate of jobs is 0.007, implying that workers are exogenously terminated from their jobs every 11.4 years on average.

The estimates of the rate parameters differ depending on whether renegotiation is allowed. The estimates of the model without renegotiation indicate that the average time between contacts for unemployed searchers is  $(0.306)^{-1}$ , or 3.3 months, slightly less than one month greater than the time between contacts in the model with renegotiation. When a job offer is received by an unemployed searcher, the probability he accepts it is  $\tilde{G}(m) = 0.455$ . Thus, the estimated length of an unemployment spell is  $(\lambda_n \tilde{G}(m))^{-1} = 7.2$  months. Once employed, an individual receives an alternative job offer from a competing firm approximately every  $(0.152)^{-1} = 6.6$  months. This is approximately twice as frequent as in the model with renegotiation. This is expected, since in order to generate the same amount of wage dispersion in a model without renegotiation, offers must arrive more frequently. The estimated exogenous dissolution rate of jobs is 0.008, which is slightly greater than in the model with renegotiation. This implies that workers are exogenously terminated from their jobs every 9.5 years on average.

In the model with renegotiation, the average ln match draw in the population is 2.309 with a standard deviation of 0.339 (the implied mean and standard deviation of the match draw  $\theta$  in levels are 10.66 and 3.72). Without renegotiation, these values are 1.634 and 0.625 (with the mean and standard deviation of the match draw  $\theta$  in levels being 6.234 and 4.313). The differences in these parameters across models stem from differences in the estimates of the bargaining power parameter.

When renegotiation is allowed, the estimate of the bargaining power parameter for workers is 0.20, while it is 0.45 when workers are not allowed to renegotiate. Thus, low bargaining power in the first model causes observed wages to be mapped to a match distribution that is centered to the right of the match distribution corresponding to a higher bargaining power.<sup>13</sup>

Figures 2.a and 2.b plot the wage function against workers' current match value. Figure 2.a depicts the case in which workers are not allowed to renegotiate their wage at their current job. The wage function is increasing in the current match value, as expected. Figure 2.b depicts the case in which workers are allowed to renegotiate their wage at their current job. Each wage function plotted in this figure corresponds to a different outside option. The wage functions corresponding to the outside options of unemployment,  $U$ , and the minimal acceptable match value,  $\theta_9$ , are nonmonotone.<sup>14</sup> Workers coming out of unemployment, for example, accept lower initial wages at jobs with higher match values as payment for future bargaining advantages. The wage function loses this feature as the outside option increases. Outside options of at least  $\theta_{14}$ , for example, produce a wage function that is increasing in the dominant match value.

## 6 Policy Experiments and Comparative Statics Results

In this section we look at the impact of the minimum wage on labor market outcomes and welfare, using the estimates of primitive parameters we obtained under our two bargaining specifications. As we will see, the implications regarding the optimal minimum wage level depends critically on which specification of the bargaining environment we assume. We begin our analysis by looking at the impact of the minimum wage on wage growth over the life cycle and over the course of an employment spell (i.e., a sequence of job spells with no intervening unemployment spell).

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<sup>13</sup>The estimate of the bargaining power parameter is of particular importance in determining the effect of an increase in the minimum wage on labor market outcomes. In Flinn (2006), in which workers were not allowed to search on the job, the estimate of the bargaining power parameter was 0.424 with a standard deviation of 0.007. In the model in which renegotiation was allowed, the value of the bargaining power parameter was significantly lower than 0.424. While the data sets and econometric specifications of the models are different, we attribute this result to allowing workers to receive alternative job offers from competing firms with which they can renegotiate their wage at their current job. We will return to a discussion of the implications of this result on labor market welfare in the following section.

<sup>14</sup>The model is estimated using a 30-point discrete match distribution. Given our parameter estimates, the first eight match values are not feasible in equilibrium.

## 6.1 Minimum Wage Effects on Wage Profiles

The work of Leighton and Mincer (1981), and, more recently, by Acemoglu and Pischke (2002), investigates the potential impacts of minimum wage laws on life-cycle wage profiles through reductions in general human capital investment by recent labor market entrants. While we do not consider human capital investment in our model, the potential for minimum wage impacts on the shape of lifetime wage profiles exists due to effects on the bargaining environment. Under either of the two bargaining specifications investigated here, minimum wages, by truncating the lower tail of the accepted wage distribution, tend to produce less wage growth over employment spells and the life cycle.

We perform two simulations. The first examines how the minimum wage affects workers' wage profiles in their first employment spell, while the second examines how the minimum wage affects workers' age-earnings profiles (i.e., over a labor market career). In both cases, we simulate profiles using models with and without renegotiation at the worker's current job using minimum wages of \$5.15, \$7.15, \$9.15, and \$11.15. For each value of the minimum wage, we re-solve the model holding constant all parameter estimates.

Figures 3.a and 3.b plot the average wage as a function of the time elapsed in individuals' first employment spells for various minimum wage values for the model without and with renegotiation. All individuals begin the simulation in unemployment and the average wage is obtained at each month of the first employment spell. This spell can consist of one job or many jobs and ends when the worker is exogenously dismissed from employment. For both models and for all four minimum wages, the wage profiles increase with time. In the model without renegotiation at the current job, the average wage increases solely due to job-to-job transitions. While this effect is also present in the model in which renegotiation is allowed, intrafirm competition generates increasing wages at workers' current job as well. Comparing the wage profiles across models given a minimum wage of \$5.15, we observe that average wages increase more quickly in the early months of the workers' first employment spells when renegotiation is not allowed. This may be attributed to workers experiencing wage decreases when renegotiating their wage at their current firm in the model with renegotiation.

The effect of increasing the minimum wage is the same across models, with higher minimum wages flattening the wage profiles in each figure. In both models, higher minimum wages increase the minimally acceptable match value for unemployed workers. Thus, workers that find jobs earn

higher wages initially, but the smaller set of viable (larger) match values decreases the chances of a worker finding an alternative firm that will offer him a higher wage. There is less wage growth as a result. In the bargaining specification that allows bidding between two competing employers, the equilibrium wage function displays a “compensating differential” property, i.e., those jobs offering the highest growth prospects offer commensurately lower wages. The reason for this is that future firms will have to bid against a high-valued match value to attract the worker. This effectively increases the outside option of the worker, and the firm demands compensation for this future bargaining advantage by reducing the current wage offer. In this case, the minimum wage limits the extent to which firms can “charge” an employee for this future wage growth potential, thus reducing average wage growth in the market.

Figures 4.a and 4.b depict the effect of the minimum wage on workers’ age-earnings profiles. The labor market career of any individual can consist of many labor market cycles, defined as sequences of labor market states beginning with an unemployment spell and ending with the last job prior to the following unemployment spell for a given individual. Thus, throughout their labor market career, individuals can become employed at a job, renegotiate their wage at their current job (in one of our specifications), change jobs to receive a higher wage, and become exogenously terminated and return to unemployment.

Figures 4.a and 4.b show that individuals’ age-earnings profiles are increasing in age. Most wage growth occurs early in their labor market careers. There are two effects of increasing the minimum wage on these age-earnings profiles. The first is that average wages are higher at any point in time in the labor market career. By shrinking the set of viable matches available to the searcher while unemployed (the standard (negative) employment effect), the minimum wage impacts workers’ wage profiles by delaying the start of the firm competition process during which significant wage gains occur. Thus, higher minimum wages delay entry into employment, but guarantee better job offers once employed. The second effect is that wage profiles flatten out earlier in individuals’ labor market careers when minimum wages are higher. Workers receive better offers once employed, but these offers arrive less frequently when the minimum wage is higher. This is similar to the effects observed in Figures 3.a and 3.b.

## 6.2 Optimal Minimum Wages

In this subsection we determine the optimal minimum wage under a few different welfare measures for both of our bargaining environment assumptions. Since our analysis is set in a partial equilibrium

framework, we don't look at efficiency issues in determining an optimal minimum wage, as in Flinn (2006); our minimum wages are optimal solely in a distributional sense.

Since we condition on labor market participation, a dubious assumption depending on the level of the minimum wage, we confine our attention to four distinct sets of agents at any moment in time. The first are the unemployed. Everyone in this state has the same value,  $V_n(m)$ . The proportion of individuals who are unemployed in the steady state when the minimum wage is  $m$  is given by

$$\pi_U(m) = \frac{\eta}{\eta + \lambda_n p_U(m)},$$

where  $p_U(m)$  is the probability of drawing a match value in the acceptable set under the minimum wage of  $m$ .

The proportion of employed individuals in the labor market is simple  $1 - \pi_U(m)$ , obviously. The average welfare of employed individuals is a complicated expression under OTJ search, since the wage distribution is a mapping from the steady state distribution of dominant and dominated match values,  $(\theta', \theta)$ . We can represent this expected value as

$$EV_e(m) = \sum_{\theta'} \sum_{\theta} p_{SS}(\theta', \theta|m) V_e(\theta', w(\theta', \theta|m), \theta).$$

We can obtain an arbitrarily good approximation to  $EV_e(m)$  through simulation, without explicitly having to determine  $p_{SS}(\theta', \theta|m)$ , fortunately.

Turning to the firms' side of the market, a similar situation prevails. For firms with vacancies, the value of the vacancy is zero. Outside of general equilibrium, where the number of vacancies is determined, the measure of firms with vacancies is indeterminate. However, for each worker, there is a firm with a filled vacancy. Therefore the measure of firms with employees is  $1 - p_U(m)$ . The expected value of firms with an employee under minimum wage  $m$  is

$$EV_f(m) = \sum_{\theta'} \sum_{\theta} p_{SS}(\theta', \theta|m) V_f(\theta', w(\theta', \theta|m), \theta).$$

Putting all of these terms together, an egalitarian social welfare function can be written

$$W(m) = p_U(m)V_n(m) + (1 - p_U(m))EV_e(m) + (1 - p_U(m))EV_f(m).$$

This function is egalitarian in the sense that each individual and firm is given the same weight in determining aggregate welfare.

We first examine the impact of the minimum wage on the equilibrium steady state probability of unemployment. The situation for the two bargaining environments is presented in Figures 5.a and

5.b. Due to the discreteness of the match distribution, we see that the unemployment probability function is a step function in  $m$ . We can also note that, at each value of  $m$ , the unemployment probability is greater under the no-renegotiation case than under renegotiation. This ordering is not pre-ordained by the theory, since we use different sets of parameter estimates to compute these functions for the two bargaining situations. What should be true is that, at the same set of parameter values, the equilibrium unemployment rate would be no less under no renegotiation, since the value of employment across all states can be no greater than under renegotiation. The main thing to note, for future reference, is that high minimum wages can induce a substantial increase in the steady state unemployment rate under either bargaining situation.

In Figures 6.a and 6.b we graph the values or expected values for the sets of agents just described as a function of the minimum wage, and we present the aggregate measure  $W(m)$  as well, which is weighted by the class proportions. Once again, we see discontinuities in these curves as increases in the minimum wages result in a reduction in the feasible set of match values. These eliminations can be “voluntary” or “involuntary,” loosely speaking. In the case of no renegotiation, all of these discontinuities occur when the minimum wage is increased above an otherwise acceptable match value, which we label “involuntary,” since it arises directly from the increase in an external constraint. In the case of renegotiation, there also exist “voluntary” eliminations, when the equilibrium wage function is non-monotone. In this case, an increase in the minimum wage can improve the value of unemployed search to such a degree that agents eliminate a match value from their acceptance set even if it still feasible (i.e., yields non-negative flow profits to the firm).

If we just focus attention on the aggregate welfare measure in these figures, we see that they have a roughly concave shape, once the minimum wage starts to bind and ignoring the discontinuities. This was found in Flinn (2006), as well. We see that the minimum wage has a much less beneficial effect in the case without renegotiation. While a binding minimum wage does improve aggregate welfare, the maximizing level is at \$8.00 an hour. When we allow for renegotiation between workers and firms, the situation changes dramatically. In this case, the optimal minimum wage in terms of maximizing  $W$  is \$15.00. This is mainly due to the fact that adverse employment effects are essentially absent in the model with renegotiation until we get to very high levels of the minimum wage.

Turning to Table 4, we see a sizable amount of variation in the minimum wage that maximizes the welfare of each set of agents, the percent change in average welfare for each set of agents from moving from the baseline minimum wage of \$5.15 to the optimal minimum wage, and the unem-

ployment rate at the optimal minimum wage. There are gains in the average welfare of unemployed and employed workers of 1.7 percent and 2.4 percent, respectively, in the model with no renegotiation which are significantly less than the gains of 19.3 percent and 21.3 percent, respectively, in the model with renegotiation. While a different set of parameters governs the search environment in each model, setting a high minimum wage impacts worker welfare less in the model with renegotiation. While this is also the case in the aggregate labor market, consisting of all workers and firms, the difference in the percent change in welfare is mitigated slightly by the effect of the minimum wage on the welfare of firms with filled vacancies. In the model without renegotiation, firms that employ workers experience a 42.5 percent increase in average welfare at the optimal minimum wage, whereas the increase is 28.9 percent in the model with renegotiation. While increases in the minimum wage render match values less than the minimum wage unacceptable in both models, they prevent the firm from being fully compensated for its contribution to the future bargaining power of the individual in the model in which renegotiation is allowed. Though the parameter estimates differ across simulations of each model, we attribute the lower welfare gain for firms with filled vacancies to this second effect.

## 7 Conclusion

In a matching model with search frictions and on-the-job search, minimum wages may “bind” in different ways depending on the nature of the worker-firm bargaining problem. We found that in a model that allowed for “bidding wars” between firms, minimum wages may bind at relatively high match values, somewhat counterintuitively. The reason is that high match values, particularly when the individual is coming from the unemployment state, may have a high value in terms of the future bargaining advantages they convey during the current employment spell. To pay for this advantage, workers obtain a lower flow wage rate. Hence, over a certain range, minimum wages may merely impact the degree to which firms can charge employees for the bargaining advantage associated with the match value. Only at high values of  $m$  does the minimum wage eliminate otherwise advantageous match values.

The situation is different in the no-renegotiation case, where the value of unemployed search, given  $m$ , always served as the outside option in the Nash bargaining problem. In that case, as was true in the no OTJ framework investigated in Flinn (2006), binding minimum wages always eliminated otherwise acceptable match values. This leads to “nonadvantageous” increases in the

equilibrium unemployment rate, and increases the costs of imposing high values of  $m$ . As a result, the optimal minimum wages found under this regime were about one-half as high as they were under Bertrand competition for workers. There seems to be some anecdotal evidence suggesting that firms often follow the policy of not responding to outside offers (except, notably, in the U.S. academic sector), which is more consistent with our no renegotiation case. The optimal minimum wage implications of that case also seem more reasonable, though we have not, as yet, developed a formal apparatus to test between the two bargaining specifications. This is an objective of our current research.

In their search, matching, and Nash bargaining frameworks, Dey and Flinn (2005) and Cahuc et al. (2006) found that allowing for OTJ search substantially reduced the estimate of the worker's bargaining power parameter in comparison with the case in which OTJ search was not introduced (e.g., Flinn, 2006). Our results here show that this is an artifact of allowing for Bertrand competition. When competition between firms is introduced, substantial wage gains over an employment spell can be generated simply from this phenomenon, even when the individual possesses little or no bargaining power in terms of  $\alpha$ .<sup>15</sup> When we allow for Bertrand competition, the estimated value of  $\alpha$  is 0.20. When we assume no wage competition between firms, the estimate increases to 0.45. This latter estimate is very similar to Flinn's (2006) estimate of  $\alpha$  using a sample of younger workers drawn from the Current Population Survey. Thus, the outside option appearing in the Nash bargaining problem is a key determinant of the estimates obtained of primitive parameters and the optimal minimum wage.

Aside from the important goal of testing between the alternative bargaining scenarios, we are pursuing two extensions of the research reported here. The first, described in the section on estimation, is to relax the steady state assumption when generating simulated sample paths from which simulated sample characteristics are computed. Given such young individuals, the stationarity assumption is questionable, as we noted in that section, can formally be rejected. At the present time, we don't know how sensitive our model estimates are to a misspecification along these lines. The second extension is to add a labor market participation decision to the model, as was done in Flinn (2006). Under the Bertrand competition assumption, the optimal minimum wage was found to be \$15.00. It is hard to believe that such a high value of  $m$  would not have potentially profound effects on the number of individuals in the labor market. We think it is important to try to capture

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<sup>15</sup>The (approximately) limiting case of this is that considered by Postel-Vinay and Robin (2002), in which workers possessed no bargaining power whatsoever.

such changes in conducting a more robust welfare analysis.

## References

- [1] Acemoglu, D., and J.-S. Pischke. 2002. "Minimum Wages and On-the-Job Training." CEP Discussion Paper no. 527, Centre for Economic Performance, LSE, London.
- [2] Albrecht, J., and B. Axell. 1984. "An Equilibrium Model of Search Employment." *Journal of Political Economy* 92: 824-840.
- [3] Burdett, K., and D. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review* 39: 257-273.
- [4] Cahuc, P., F. Postel-Vinay, and J.-M. Robin. 2006. "Wage Bargaining with On-the-Job Search: Theory and Evidence." *Econometrica* 74(2), 323-64.
- [5] Dey, M., and C. Flinn. 2005. "An Equilibrium Model of Health Insurance Provision and Wage Determination." *Econometrica* 73: 571-627.
- [6] Dey, M., and C. Flinn. 2008. "Household Search and Health Insurance Coverage." Forthcoming, *Journal of Econometrics*.
- [7] Diamond, P. 1982. "Wage Determination and Efficiency in Search Equilibrium." *Review of Economic Studies* XLIX: 217-227.
- [8] Drazen, A. 1986. "Optimum Minimum Wage Legislation." *Economic Journal* 96: 774-784.
- [9] Flinn, C. 2006. "Minimum Wage Effects on Labor Market Outcomes under Search, Bargaining, and Endogenous Contact Rates." *Econometrica* 74: 1013-1062.
- [10] Flinn, C. 2002. "Interpreting Minimum Wage Effects on Wage Distributions: A Cautionary Tale." *Annales d'Économie et de Statistique* 67-68: 309-355.
- [11] Flinn, C., and J. Heckman. 1982. "New Methods for Analyzing Structural Models of Labor Force Dynamics." *Journal of Econometrics* 18: 115-68.
- [12] Jovanovic, B. 1979. "Job Matching and the Theory of Turnover." *Journal of Political Economy* 87: 972-990.
- [13] Jovanovic, B. 1984. "Matching, Turnover, and Unemployment." *Journal of Political Economy* 92: 108-122.

- [14] Leighton, L., and J. Mincer. 1981. "The Effects of the Minimum Wage on Human Capital Formation." In *The Economics of Legal Minimum Wages*, edited by S. Rottenberg.. Washington D.C.: American Enterprise Institute for Public Policy Research, pp. 155-173.
- [15] Lang, K. 1987. "Pareto Improving Minimum Wage Laws." *Economic Inquiry* 25: 145-158.
- [16] Moro, A. 2003. "The Effect of Statistical Discrimination on Black-White Wage Inequality: Estimating a Model with Multiple Equilibria." *International Economic Review* 44: 467-500.
- [17] Pissarides, C. 2000. *Equilibrium Unemployment Theory*, 2nd edition. Cambridge, MA: MIT Press.
- [18] Postel-Vinay, F., and J-M Robin. 2002. "Equilibrium Wage Dispersion with Heterogeneous Workers and Firms." *Econometrica* 70: 2295-1350.
- [19] Rebitzer, J., and L. Taylor. 1996. "The Consequences of Minimum Wage Laws: Some New Theoretical Ideas." *Journal of Public Economics* 56: 245-256.
- [20] Swinnerton, K. 1996. "Minimum Wages in an Equilibrium Search Model with Diminishing Returns to Labor in Production." *Journal of Labor Economics* 14: 340-355.
- [21] Van den Berg, G. 2003. "Multiple Equilibria and Minimum Wages in Labor Markets with Informational Frictions and Heterogeneous Production Technologies." *International Economic Review* 44: 1337-1357.

**Table 2**  
**Descriptive Statistics**  
 Individuals Aged 16-30

	Feb 1998	Feb 1999
Proportion unemployed	0.043	0.030
Mean wage	\$9.47	\$10.39
Standard deviation of wages	\$4.67	\$5.09
Proportion of minimum wage earners	0.033	0.018
Proportion earning wage greater than \$20	0.043	0.053
Proportion employed in Feb 1998 that exit into unemployment within 12 months	0.064	
Proportion employed in Feb 1998 with at least one job change within 12 months	0.291	
Mean wage at initial job among individuals employed in Feb 1998 who have job-to-job transition before Feb 1999	\$8.43	
Mean wage change among individuals employed in Feb 1998 who have job-to-job transition before Feb 1999	\$0.90	
Standard deviation of wage change among individuals employed in Feb 1998 who have job-to-job transition before Feb 1999	\$4.33	
Mean wage at first job for those unemployed in Feb 1998 who become employed within 12 months	\$8.48	
Standard deviation of wages at first job for those unemployed in Feb 1998 who become employed within 12 months	\$4.43	

**Table 3**  
**Model Estimates**

	With Renegotiation	Without Renegotiation
$m$	5.15	5.15
$\lambda_n$	0.42126054 (0.055409)	0.30595648 (0.034104)
$\lambda_e$	0.0630453 (0.005197)	0.15210585 (0.022009)
$\eta$	0.007332899 (0.001113)	0.008807459 (0.000477)
$\mu_\theta$	2.3088722 (0.057633)	1.6344473 (0.074587)
$\sigma_\theta$	0.33904086 (0.058637)	0.62545427 (0.029256)
$\alpha$	0.20 (0.011731)	0.45 (0.014865)

Note: Instantaneous value of search,  $b$ , set equal to -2 in both estimations

**Table 4**  
**Policy Experiments**

	Unemployed Workers	Employed Workers	Firms with Filled Vacancies	Employed and Unemployed Workers	Aggregate Labor Market
<b><u>Without Renegotiation</u></b>					
Optimal $m$	\$7.00	\$9.00	\$14.05	\$8.00	\$8.00
Percent Change With Respect to Baseline ( $m = \$5.15$ )	0.017	0.024	0.425	0.017	0.016
Unemployment Rate (Baseline = 0.058)	0.075	0.122	0.411	0.096	0.096
<b><u>With Renegotiation</u></b>					
Optimal $m$	\$14.00	\$16.00	\$18.05	\$16.00	\$15.00
Percent Change With Respect to Baseline ( $m = \$5.15$ )	0.193	0.213	0.289	0.195	0.161
Unemployment Rate (Baseline = 0.058)	0.083	0.147	0.369	0.147	0.110

Figure 1  
Wage Changes in Sample  
for Individuals Making Job-to-Job Transition

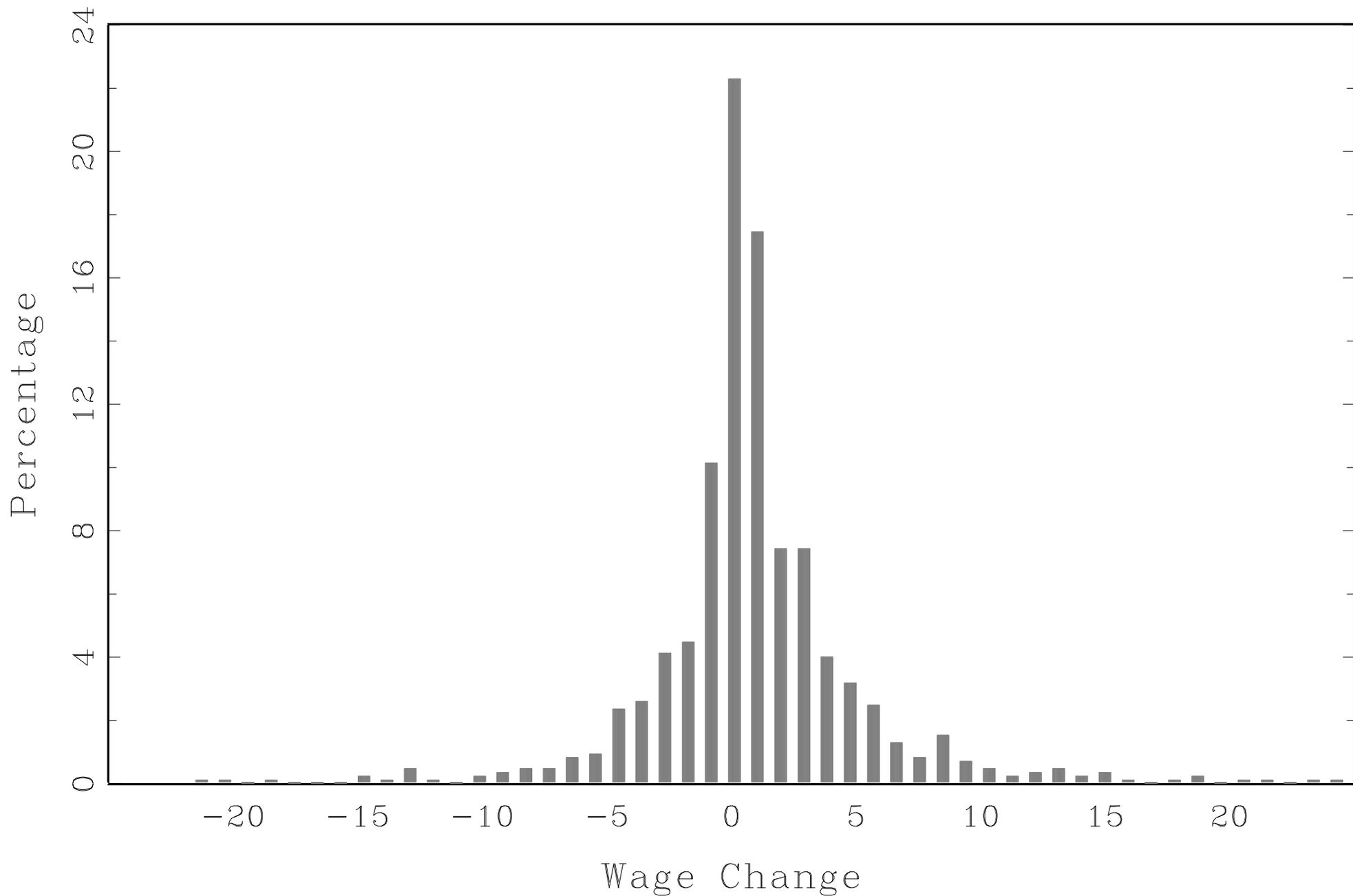


Figure 2.a  
Estimated Wage Function  
(Without Renegotiation)

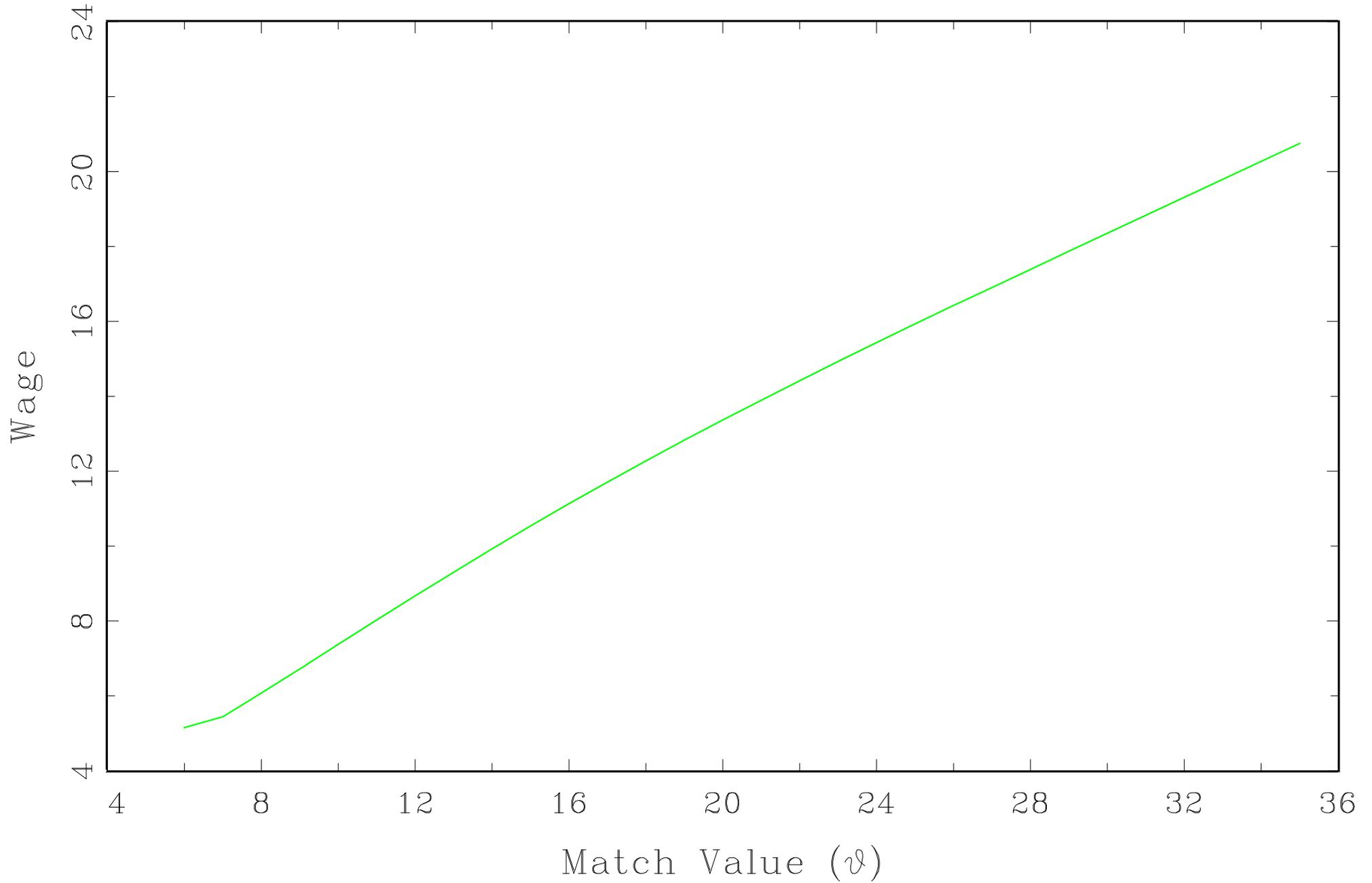


Figure 2.b  
Estimated Wage Function  
(With Renegotiation)

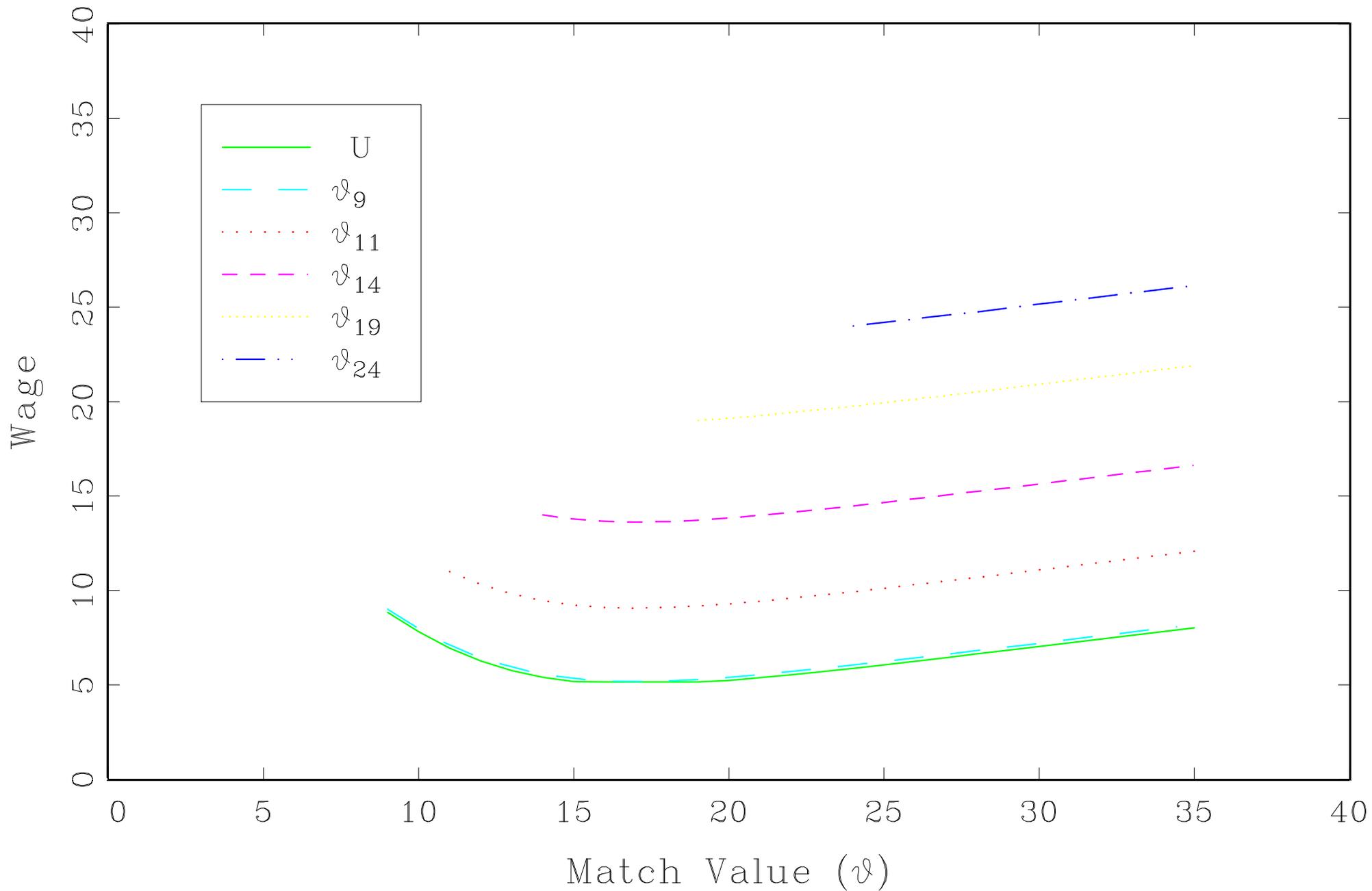


Figure 3.a  
Average Wage over First Employment Spell  
(No Wage Renegotiation Allowed)

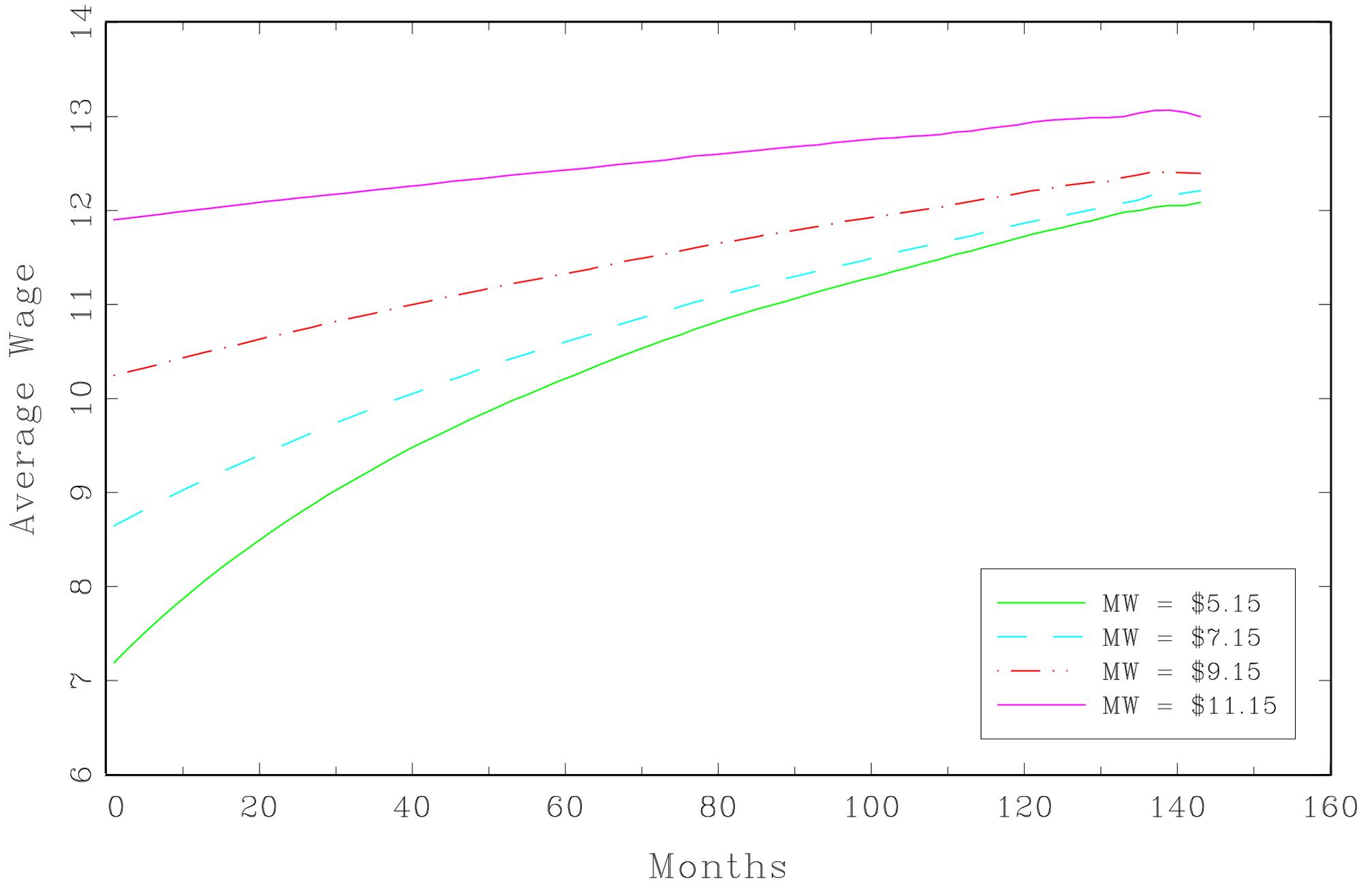


Figure 3.b  
Average Wage over First Employment Spell  
(Wage Renegotiation Allowed)

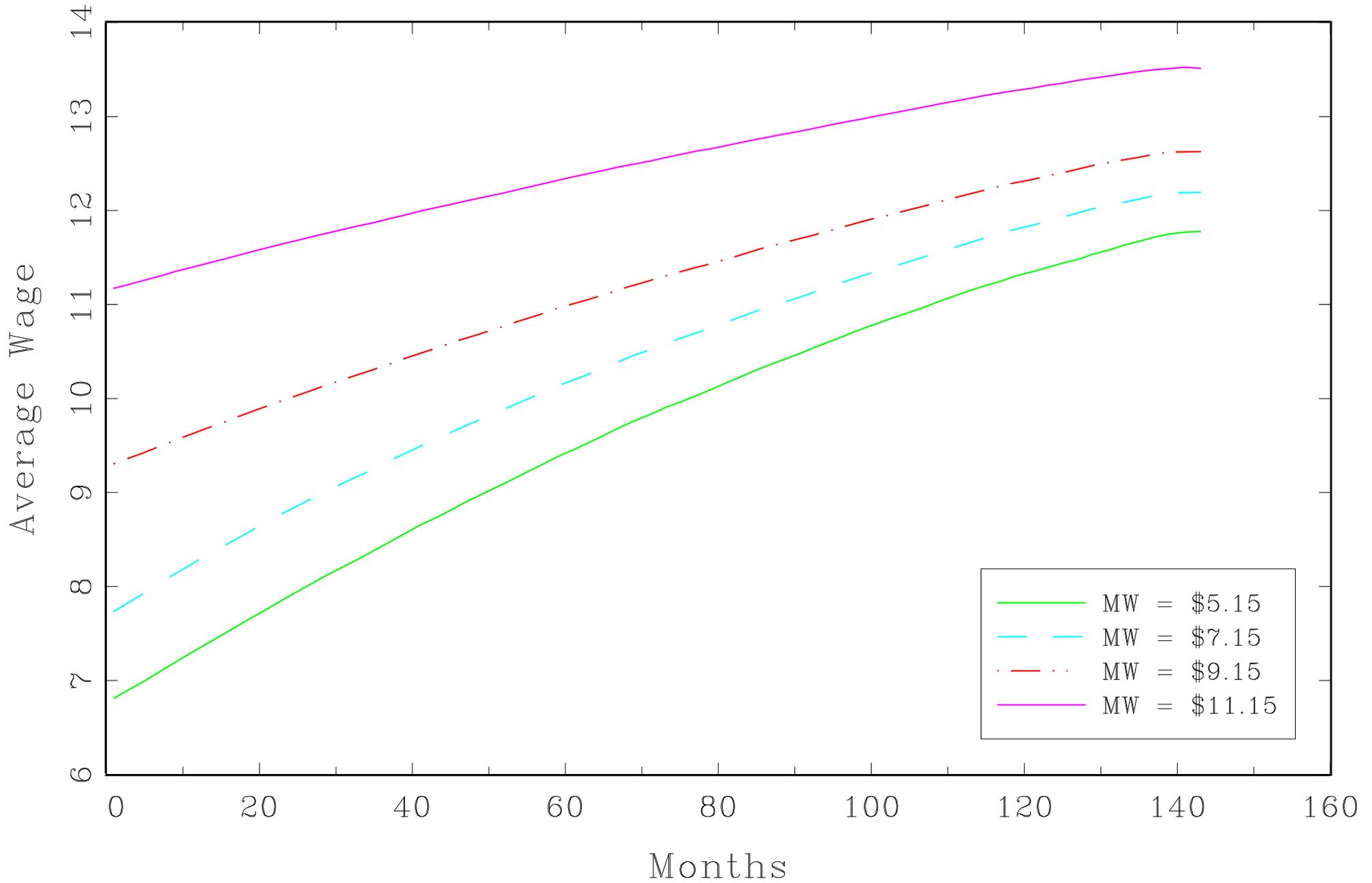


Figure 4.a  
Average Wage over Labor Market Career  
(No Wage Renegotiation Allowed)

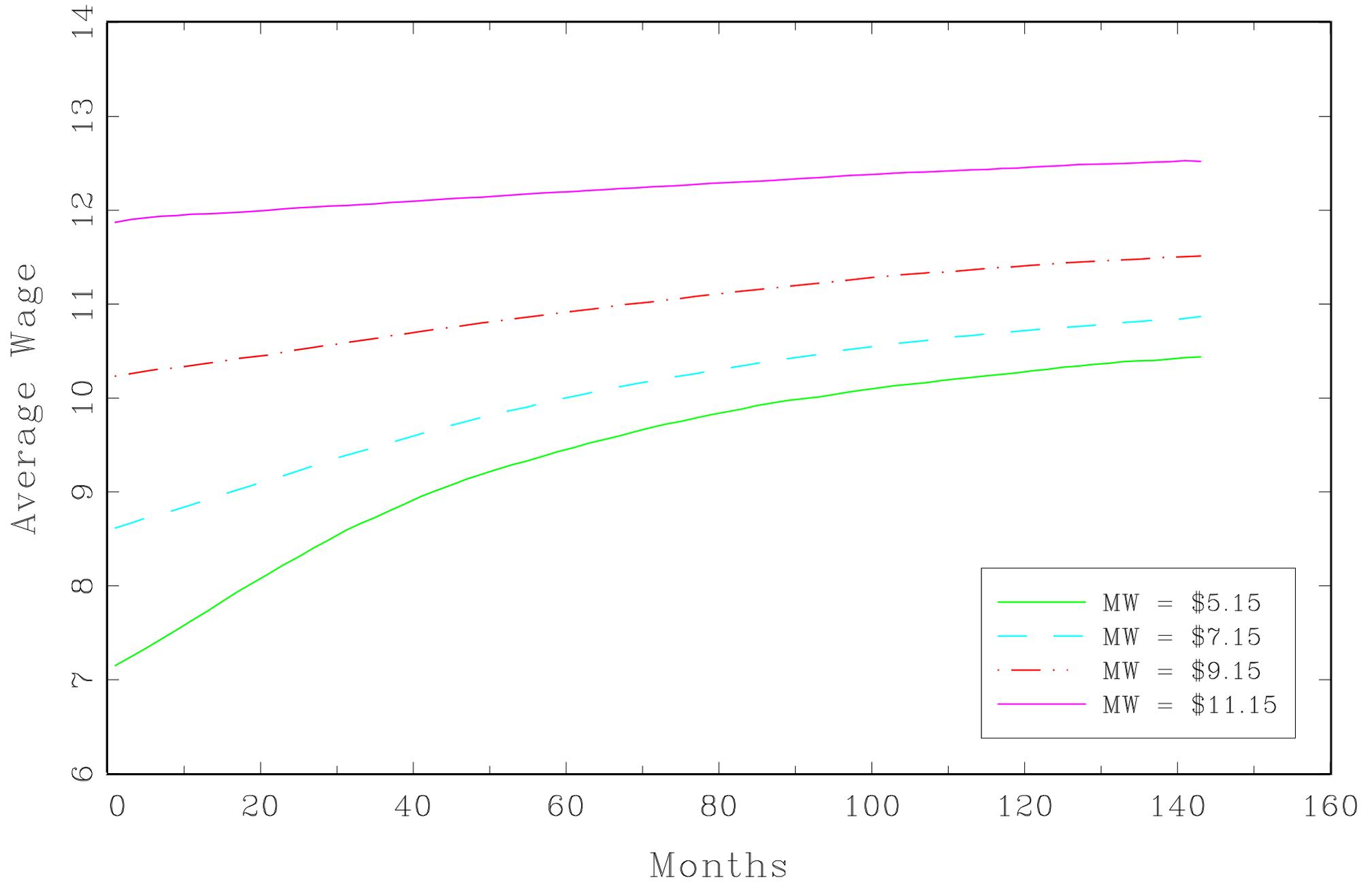


Figure 4.b  
Average Wage over Labor Market Career  
(Wage Renegotiation Allowed)

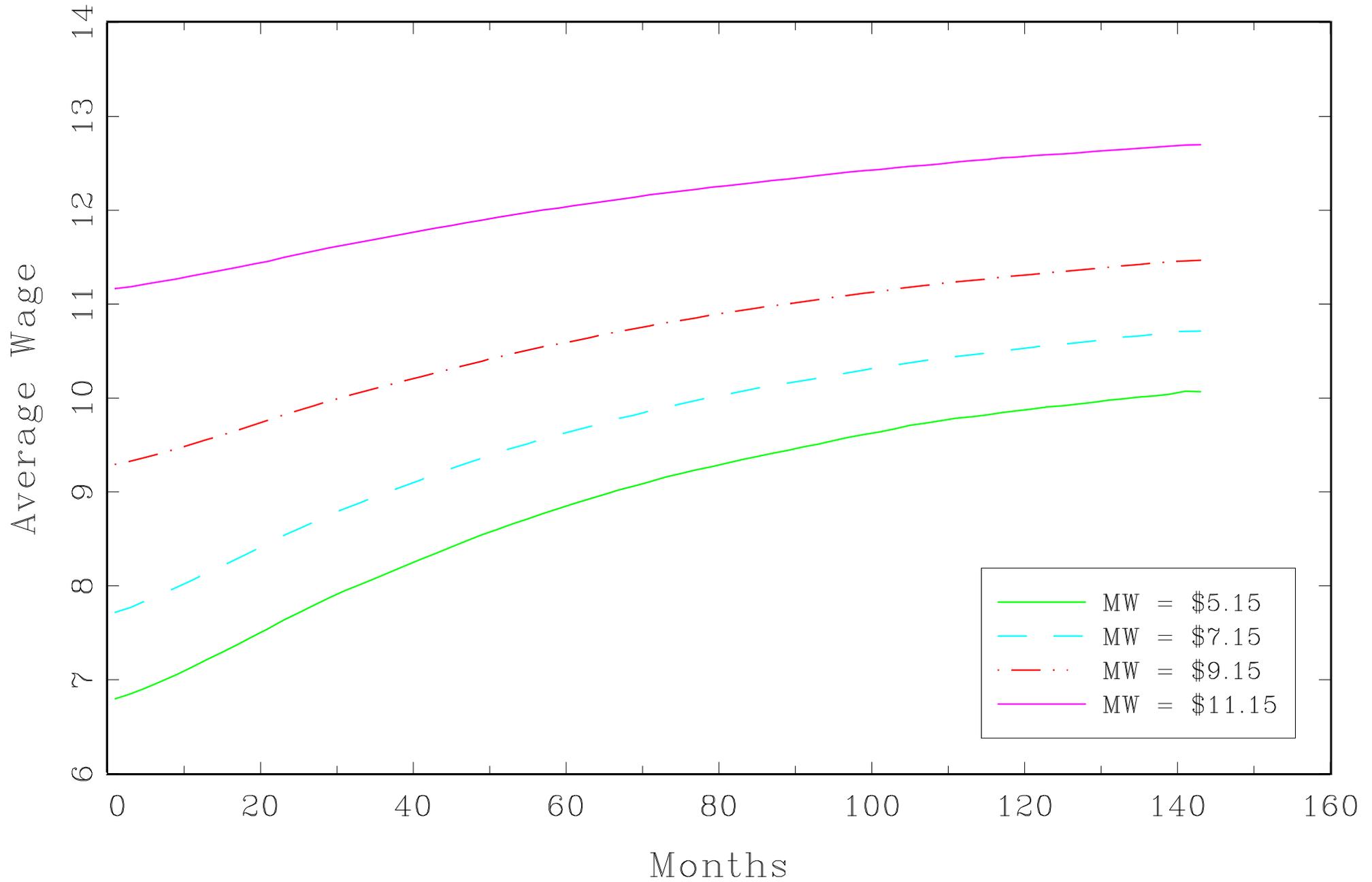


Figure 5.a  
Steady State Proportion of Unemployed  
(No Renegotiation Allowed)

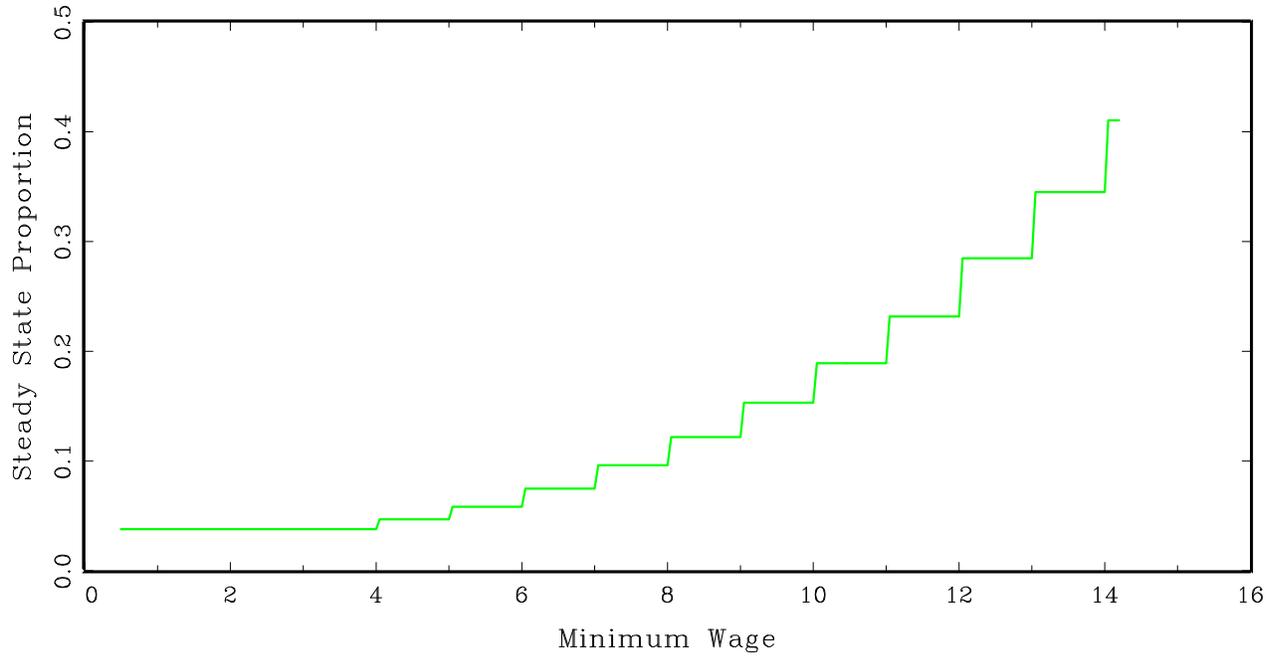


Figure 5.b  
Steady State Proportion of Unemployed  
(Renegotiation Allowed)

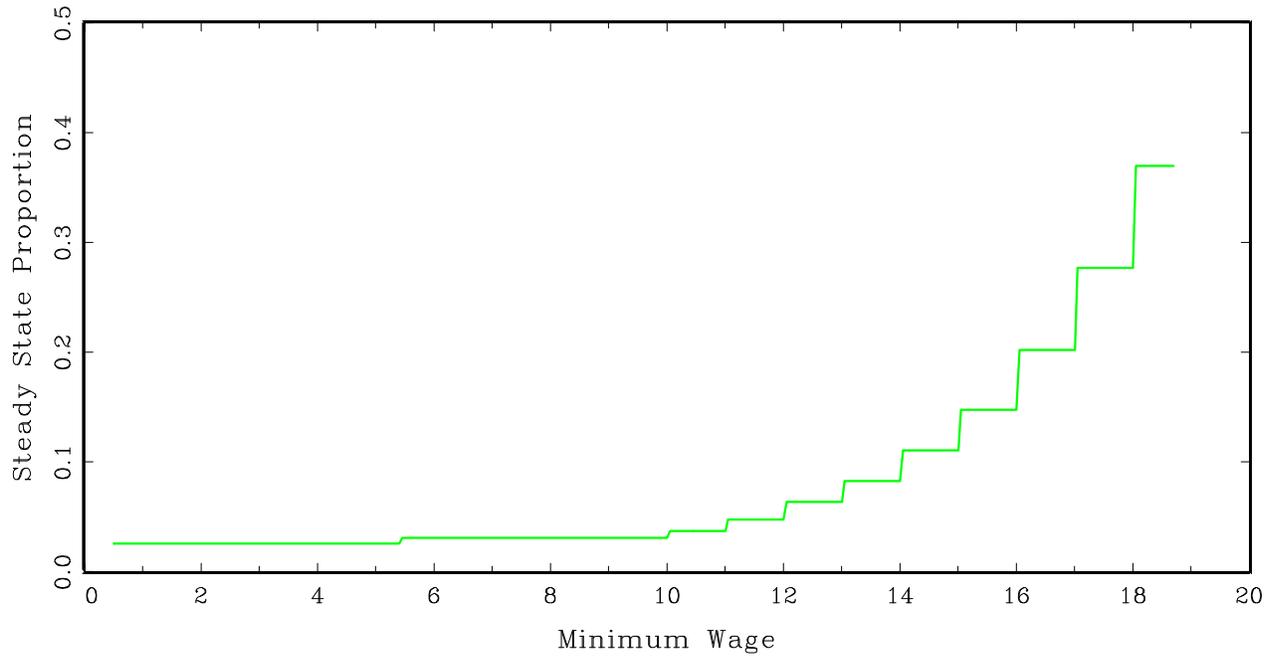


Figure 6.a  
Average Welfare  
(No Renegotiation Allowed)

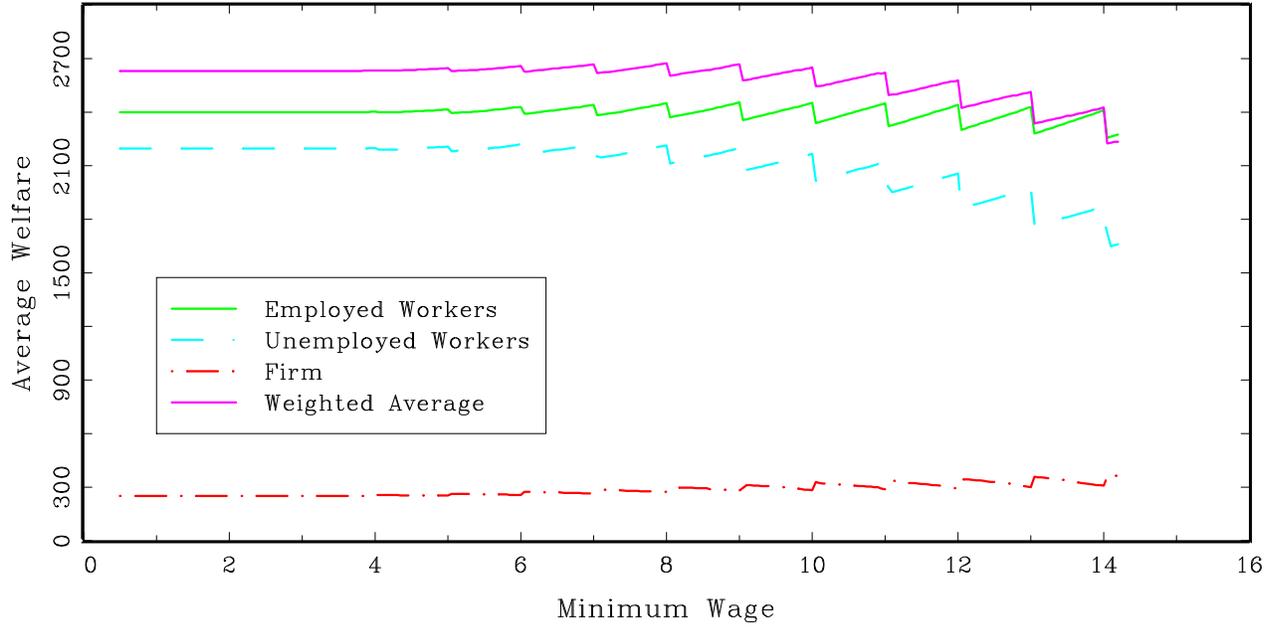


Figure 6.b  
Average Welfare  
(Renegotiation Allowed)

